A NOTE ON THE DICE MODEL

by

Sebastien Buttet

Major paper submitted to the Department of Economics

in partial fulfilment of the requirements for the

M.A. degree in Economics

Supervisor: Pr. Nguyen Van Quyen

Department of Economics, University of Ottawa

April, 2000

ACKNOWLEDGEMENT

The professor Nguyen Van Quyen is special to me. He has a truly remarkable and kind personality, is doted with exceptional communication skills and has a very broad knowledge of both mathematics and economics. During the MA program, he has always been present to help me, both when I was taking his courses and then when he supervised my major paper. I have learned a lot from him and I consider his education to be a very solid preparation to continue for a professional career in economics. For all of that, I would like to thank him.

I also would like to thank Pr. Philippe J. Crabbe for reviewing my work and providing insightful and constructive comments.

1. INTRODUCTION

Awareness of and concern over the environment has grown dramatically over the past two decades to the point where there is now a substantial amount of effort devoted to understanding its science and economics. Environmental concerns include increasing evidence of widespread damage from acid rain; the appearance of the Antarctic "ozone hole", interpreted as the signal of global ozone depletion that threatens to remove the shield that protects organisms from harmful ultraviolet radiation; deforestation, especially in the tropical rain forest, which may upset global and local ecological balance.

On top of these environmental stresses, global warming, which results from increased atmospheric concentration of the greenhouse gases could lead to dramatic climate change and have significant socioeconomic impacts. The greenhouse effect refers to the process by which trace gases, such as carbon dioxide CO_2, trap reflected long-wave solar radiation and warm the atmosphere. If the atmosphere contained no greenhouse gases, the mean global surface temperature would be 33°C cooler. The greenhouse effect also helps to explain the hot temperatures on Venus along with the frigid conditions of Mars. The concern about the greenhouse effect arises because human activities are currently raising atmospheric concentrations of greenhouses and causing a significant and undesirable climate change.

The major greenhouses (GHG) are carbon dioxide (CO_2), methane (CH_4), nitrous oxides (N_2O) and chlorofluorocarbons (CFC). The GHG emissions derive from a number of human activities, including production and use of fossil fuels, non-energy industrial processes, deforestation and agricultural practices. Estimations of the relative contribution to global warming between 1980 and 2030 arising from these activities and GHG are shown in Table 1.

	CO_2	CFCs	CH_4	N_2O	% warming by sector
Energy	35	-	4	10	49
Deforestation	10	-	4	-	14
Agriculture	3	-	8	2	13
Industry	2	20	-	2	24
% warming by sector	50	20	16	14	100

Source: World Resource Institute (1990)

Table 1: Estimated contribution to global warming by greenhouse gases and human activities between 1980-2030

It can be seen that CO_2 emissions alone represent about half of the global warming over the past 40 years period, with most of the CO_2 emissions coming from the energy sector and the burning of fossil fuels. The second most important source of GHG is CFC, coming from the industrial sector. When the contributions by the various sources of GHG are considered, energy production and use are expected to contribute about half of the increased greenhouse effect, while deforestation and agriculture together contribute 25%, and industry being responsible for the remaining 25% (World Resources Institute, 1990).

To project climate changes due to the increase in the GHG concentration in the atmosphere, large climate models (or general circulation models) have been used along with carbon emission scenarios. These models all predict that a doubling of atmospheric CO_2 will lead to global surface warming ranging from 0.9°C to 3.5°C (IPCC, 1996). The range of temperature change reflects some uncertainties that remain in climate modeling, such as the ocean-atmosphere interaction, sea ice, transfer of heat and moisture from the land surface[1].

Global warming will have a variety of effects, which can be classified as either market related - variations of output in various sectors of the economy – and non-market related – e.g., damages on ecosystems or human activity. Figure 1 provides a summary of these impacts.

[1] Confidence in the models has increased due to measures such as incorporating the effects of aerosols. Aerosols are small particles suspended in the atmosphere, which reflect sunlight and tend to cool the earth. The main sources of aerosols include fossil fuel combustion and biomass burning. Aerosols have a short lifetime and vary regionally, so their overall impact on the global climate is hard to assess. However, it has been concluded that the increase in sulfate aerosols since 1850 has had a cooling effect (IPCC, 1996).

Agriculture, forestry and fishing are examples of the sectors most sensitive to climate change. In the agricultural sector, it is expected that yields would decline due to less soil moisture and greater heat stress; on the other hand, higher levels of CO_2 will induce higher fertilization rate that will compensate for the lower yields. Cline (1992) estimated that the net effect resulting from a doubling of the GHG concentration would be a loss of 7% of world agricultural production. In another study, Kane, Reilly, and Tobey (1992) used a general equilibrium model to estimate the impacts of climate change in the agricultural sector worldwide. Their estimate range from an optimistic finding that world real income would increase by 0.1 percent to a pessimistic scenario in which world output would decrease by 0.47 percent.

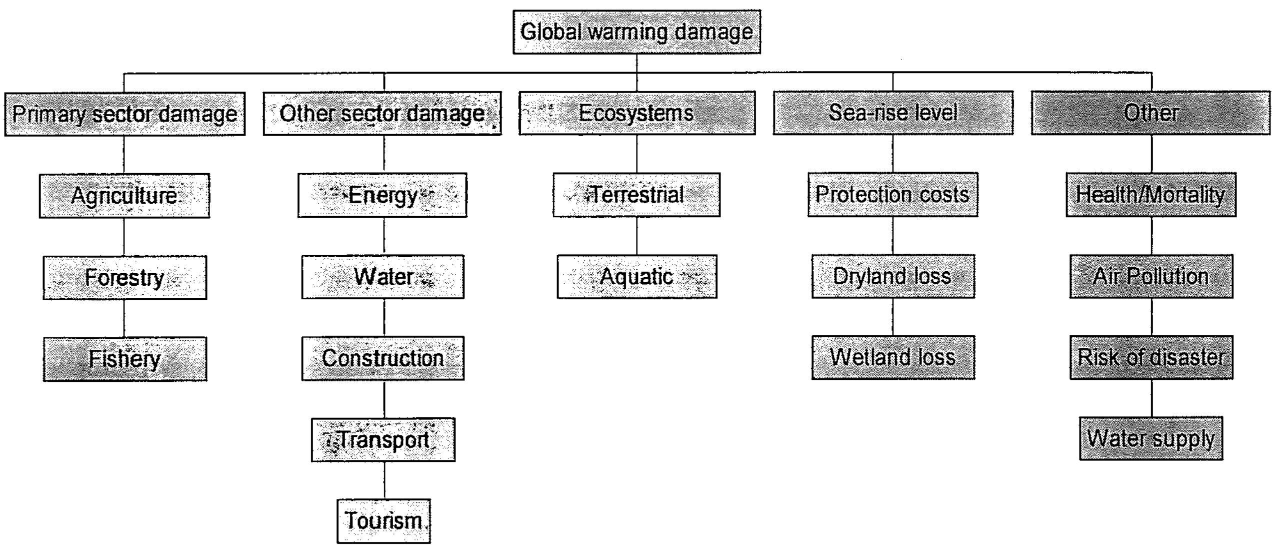

Figure 1: Overview of global warming impacts

Sea level rise is another important consequence of climate change. Global sea level has risen by about 10-25cm over the last 100 years (IPCC, 1996). Increased temperatures will cause the sea to expand and glaciers as well as ice caps[2] to retreat. The physical impacts of the sea level rise include inundation of low-lying areas and island nations; penetration of salt into drinking and agricultural water supplies; damage of coastal infrastructures such as harbors and roads. The associated socioeconomic impacts could be significant in terms of the cost of protecting and defending coastlines. For instance, it is expected that the rise of one meter of the sea level would inundate 12-15% of Egypt's arable land, 14% of Bangladesh's net cropped area, and would eliminate 3% of the earth's land area (IPCC, 1996).

Climate change will also alter the composition and geographic distribution of many ecosystems (e.g., forests, deserts, mountain systems, lakes, wetlands, and oceans). There will be reductions in biological diversity and in the goods and services that ecosystems provide. Some ecological systems may not reach a new equilibrium for several centuries after the climate reaches a new balance. For example, in the case of forestry, climate change is projected to occur at a rapid rate relative to the speed at which forest species grow, reproduce, and reestablish themselves. For mid-latitude regions, an average warming of 1-3.5°C over the next 100 years would be equivalent to a northward shift approximately 250-550 km, or an altitude shift of about 150-550 m. Assuming that forests could not migrate faster than 100 km per century, this implies that there would be substantial forest loss and extinction of tree species. The societies that rely the most on natural ecosystems for their welfare are expected to suffer significant losses.

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[2] The IPCC (1996) estimate that if no actions were taken (business-as-usual), the global warming induced by GHG will accelerate the sea level rise, leading to an increase of 18 cm by 2030, 44 cm by 2070, 70 cm at the end of next century.

Other impacts of global warming include increased mortality during the summer (with some offsets in the winter), increased incidence of vector-borne and infectious diseases, increased air pollution, increased destruction from hurricanes, increased stress on water supply and its related problem of energy supply from hydropower generation. IPCC (1997) provides a complete review of the potential damages of global warming.

Nordhaus (1991a, 1991b, 1991c) was one of the first economists who attempted to estimate the social costs that would result from a doubling of atmospheric CO_2-equivalent concentration. His studies culminated in a model called the Dynamic Integrated model of Climate and the Economy; see Nordhaus (1993, 1994). DICE (the Dynamic Integrated model of Climate and the Economy) follows the Ramsey approach to optimal economic growth and adds a climate sector as well as a closed-loop interaction between the climate and the economy. One of the interesting features of the model is to allow for state variables in the transition path to be different from those in the ultimate steady state. which better accounts for the long time lags involved in the reaction of the climate and the economy to greenhouse gas emissions. Using DICE, Nordhaus estimated that the marginal social costs (shadow value) would start at \$5.3/tC in 1995 and gradually rise to \$10/tC in 2025.

Cline (1992) replicated the DICE model and concluded that Nordhaus' choice of parameter values (of particular importance is the discount rate) may have led to an underestimation of the "true" social costs. Using lower values for the discount rate, Cline (1992) and Peck and Teisberg (1992) found that social cost of GHG emission is at \$12.4/tC in 1995 and will rise to \$23/tC in 2025. Other criticisms were also voiced by Grubb (1993), Ayres and Walter (1991), and Fankhauser (1995).

These authors criticized $2xCO_2$ damage estimates as being based on largely subjective valuation of non-market impacts. Being able to assess the quantitative impacts of $2xCO_2$ on non-market activities, such as ecosystems, air pollution and health and morbidity, would significantly improve the accuracy of future studies. Also, they questioned the promise that the estimates based on a developed country (the US) can be extended to the world as a whole. They suggested that the damages due to global climate change may be larger in amounts of dollars and that Nordhaus failed to account for the regional impact of climate change. For example, Ayres and Walter (1991) estimated the world impacts of sea level rise and formulated more precise assumptions about the area of land lost and land price; the environmental refugees from the lost land and resettlement costs; the cost of protecting the coast line. Altogether, the total cost was estimated at $18.5-21 trillion for the world or 2.1%-2.4% of the world gross income in 2040. In comparison, this cost is 10 times the total cost projected in Nordhaus's study.

Another weakness of the DICE model is the exogenous treatment of technology. Our modest contribution to the global warming debate is to propose an extended version of the DICE model by incorporating the theory of endogenous growth such as proposed in Lucas (1988). Investment in human capital presents several advantages. better understanding and reduction of uncertainties associated with climate change, discovery of non-polluting technologies... The paper is organized as follows. Section 2 presents the new model. Section 3 gives the solution of the model and presents the steady-state equilibrium. Section 4 provides a discussion of the results. Finally, Section 5 concludes and outlines directions for future research.

2. THE MODEL

The aim of the model is to study the optimal path of reductions of GHG gases in a dynamic framework, along the lines of the Ramsey model of optimal growth, with certain adjustments. The economy modeled is the global economy. Each country has identical preferences and seeks to maximize a social welfare function that is the discounted sum of utility of per capita consumption[3], $(c_t)_{t\geq 0}$. Preferences over consumption are given by

$$(1) \qquad \int_0^{\infty} e^{-\rho t} u(c_t, L_t) dt,$$

where u is the utility function. The population of the economy at time t, say L_t, grows at rate $\lambda>0$, and ρ (the only parameter[4]) is the pure social rate of time preference. In the rest of the paper, the utility function is assumed to have the following specific functional form $u(c_t, L_t) = L_t \dfrac{c_t^{1-\alpha} - 1}{1-\alpha}$,

where α is a measure of the social valuation of different levels of consumption. A value of $\alpha = 0$ indicates no aversion to inequality, and as α gets larger, society is willing to reduce the welfare of

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[3] By consumption, we mean a broad concept that includes not only purchases of goods and services but also nonmarket activities such as leisure and enjoyment of the environment.

[4] The pure rate of social preference ρ is closely linked with the market interest and the saving rate. According to Nordhaus (1994), a value of ρ of .03 per year is consistent with the historical value of saving and interest rates. Cline (1992) advocated a value of ρ of .01 per year but this implies both too high a saving rate and too low a rate of return on capital.

high-income generations to improve the welfare of low-income generations. In our model, $\alpha = 1$, which yields the logarithmic or Bernoulli utility function $u(c_t, L_t) = Log(c_t)L_t$.

The economy has two sectors of production: the manufacturing sector where the consumption goods are produced and the R&D sector where knowledge is accumulated. In each period t, a worker with skill level h_t devotes a fraction u_t of his non-leisure time to current production and the remaining 1 - u_t to human capital accumulation. For the economy as a whole, the production function is given by $Y_t = \Omega_t A K_t^{\beta} [u_t h_t L_t]^{1-\beta}$, where Ω_t is the term which relates to the impact of climate change, K_t is the stock of physical capital. A_t is the technology level, and β represents the elasticity of output with respect to capital $(0 \prec \beta \prec 1)$. Production per capita of the consumption good is divided into consumption and capital accumulation. Let $\dot{K}_t$ denotes rate of change of physical capital and δ_K the depreciation rate, then

(2) $$L_t c_t + \dot{K}_t + \delta_K K_t = \Omega_t A K_t^{\beta} [u_t h_t L_t]^{1-\beta}.$$

As in Lucas (1988), human capital accumulates according to

(3) $$\dot{h}_t = \delta_h [1 - u_t] h_t,$$

which means that, if no effort is devoted to human capital accumulation (u_t = 1), then none accumulates. On the contrary, if all effort is devoted to capital accumulation (u_t = 0), h_t grows at its maximal rate δ_h. This will end the description of the economic model.

Let us now turn to the description of the climate model. The emission of carbon dioxide is represented by

$$(4) \quad E_t = [1 - \mu_t]\sigma_t Y_t,$$

where μ_t represents the proportion of emissions which is controlled by the central planner and σ_t represents the trend in CO_2 emissions per unit of gross output in the absence of emissions control. In our model, μ_t is a control variable determined by optimization and σ_t is a decreasing function of human capital, reflecting the idea that a higher state of knowledge will induce energy efficiency improvements (e.g., transition away from coal). In the model, we choose

$$(5) \quad \sigma_t = \frac{\sigma_0}{h_t^{\alpha}}, \qquad \text{with } 1 - \alpha - \beta > 0.$$

However, since the current model does not explicitly model the energy sector, one must rely on existing studies and historical data to estimate the growth rate of σ_t, g_{σ_t}. Relying on historical data, Nordhaus (1994) estimated g_{σ_t} to be constant between -1% and -1.5% per annum[5].

[5] Dean and Hoeller (1992) have discussed the importance of this factor in sensitivity testing, a low value of g_{σ_t} inducing higher GHG emissions path. Unfortunately, there is little empirical backing in the econometric literature for specific values of g_{σ_t} and its value must be determined by a subjective assessment. For example, Manne and Richels (1994) assumed different initial values for different regions, with the growth rate of σ_t lowest in the former USSR (-0.5% per annum) and highest in China (-1.0% per annum). These values are based on the assumptions that the countries with the lowest level of industrialization have the greatest potential for energy efficiency improvements.

The accumulation of greenhouse gases (GHG) in the atmosphere is given by:

$$(6) \qquad \dot{M}_t = \phi E_t - \delta_M \left(M_t - 590 \right),$$

where M_t is the atmospheric concentration of CO_2 or its equivalent. Next, accumulation of GHG leads to global warming through increasing warming at the surface by increased radiation. The relationship between GHG accumulation and increased radiative forcing, F_t, is derived from empirical measurements and climate models and takes the following form:

$$(7) \qquad F_t = 4.1 Log_2 \left(\frac{M_t}{590} \right),$$

where F_t is the increase of surface warming in watts per square meter (W/m^2).

The next equation provides the link between radiative forcing and climate change. Higher radiative forcing warms the atmospheric layer, which then warms the upper ocean and gradually warms the deep oceans

$$(8) \qquad \dot{T} = \left(1/R_1 \right) \left\{ F_t - \Lambda T_t - \left(R_2/\tau_{12} \right) \left[T_t - T_t^* \right] \right\},$$

$$(9) \qquad \dot{T}_t^* = \frac{1}{\tau_{12}} \left[T_t - T_t^* \right],$$

where T_t is the increase in the average temperature in the atmosphere and the upper level of the oceans, T_t^* the increase in the average temperature in deep oceans, R_l the thermal capacity of the upper stratum, R_2 the thermal capacity of the deep oceans, τ_{12} the transfer rate from the upper layer to the lower layer, and Λ a feedback parameter.

Following Nordhaus (1994), we assume that there is a relationship between global temperature increase and the income loss, $\frac{D_t}{Y_t} = \theta_1 T^{\theta_2}$, where D_t is the loss of global output, θ_l a parameter representing the scale of the damage, θ_2 a parameter that represents the nonlinearity in the damage function. The cost of reducing GHG emissions satisfies the following relationship: $\frac{TC_t}{Y_t} = b_1 \mu_t^{b_2}$, where TC_t is the total cost of GHG reduction, b_l is a parameter that represents the scale of the damage, and b_2 is a parameter that represents the nonlinearity in the damage function. Combining the cost and damage function, we can define Ω_t, which appears in the production function and relates to the impact of emissions reduction and climate change, as follows:

$$(10) \quad \Omega_t = \frac{1 - b_1 \mu_t^{b_2}}{1 + \theta_1 T_t^{\theta_2}}.$$

The last equation completes the model, the analytical solution of which is given in the following section. Given the public goods nature of global warming, the perfectly competitive model will be inadequate to capture the essence of the problem. That is, it is doubtful to assume that, through the price mechanism, countries have internalized the *global* costs of their emissions decisions. Therefore, the model will be solved within an optimizing framework (i.e., the problem solved by a central planner). Although the solution may differ from the perfectly competitive equilibrium, the current approach has the virtue of calculating the equilibrium that would emerge were each country to behave in a farsighted and altruistic manner. The central planner has to choose the control variables u_t, c_t, μ_t to maximize the objective (utility) function (1) subject to constraints (2), (3), (6), (8), and (9).

3. SOLUTION OF THE MODEL: BALANCED GROWTH PATH ANALYSIS

The model will be solved according to the standard optimization techniques. Let H be the Hamiltonian defined as follows:

$$
\begin{aligned}
& H\left(K,h,M,T,T^*,c,u,\mu,\varepsilon^K,\varepsilon^h,\varepsilon^M,\varepsilon^T,\varepsilon^{T^*},t\right)= \\
& e^{-\rho t}Log(c)L+\varepsilon^K\left\{\frac{1-b_1\mu^{b_2}}{1+\theta_1 T^{\theta_2}}AK^\beta[uhL]^{1-\beta}-\delta_K K-Lc\right\}+\varepsilon^h\delta_h[1-u]h \\
& +\varepsilon^M\left\{\phi[1-\mu]\frac{\sigma_0}{h^\alpha}\frac{1-b_1\mu^{b_2}}{1+\theta_1 T^{\theta_2}}AK^\beta[uhL]^{1-\beta}-\delta_M(M-590)\right\} \\
& +\varepsilon^T\left(\frac{1}{R_1}\right)\left\{4.1Log_2\left(\frac{M}{590}\right)-\Lambda T-\left({R_2}/{\tau_{12}}\right)[T-T^*]\right\} \\
& +\varepsilon^{T^*}\frac{1}{\tau_{12}}[T-T^*]
\end{aligned}
$$

Maximizing the Hamiltonian, we obtain the following first-order conditions. First, for the optimal per-capita consumption at each instant t, we have

(11) $$\frac{e^{-\rho t}}{c} = \varepsilon^{K}.$$

Second, for the optimal division of non-leisure time between production and human capital accumulation at each instant t, we have

(12) $$(1-\beta)\frac{1-b_1\mu^{b_2}}{1+\theta_1 T^{\theta_2}} AK^{\beta}u^{-\beta}[hL]^{1-\beta}\left[\varepsilon^{K} + \varepsilon^{M}\phi[1-\mu]\frac{\sigma_0}{h^{\alpha}}\right] = \varepsilon^{h}\delta_h h.$$

Third, for the optimal control of GHG emissions at each instant t, we have

(13) $$\varepsilon^{K} b_1 b_2 \mu^{b_2-1} = -\varepsilon^{M}\phi\frac{\sigma_0}{h^{\alpha}}\left[1-b_1\mu^{b_2} + (1-\mu)b_1 b_2\mu^{b_2-1}\right].$$

Along the optimal trajectory, the behaviour of the costate variables is governed by the following adjoint equations

(14) $$\dot{\varepsilon}^{K} = \varepsilon^{K}\left\{\delta_K - \beta\frac{1-b_1\mu^{b_2}}{1+\theta_1 T^{\theta_2}} AK^{\beta-1}[uhL]^{1-\beta}\left[1+\frac{\varepsilon^{M}}{\varepsilon^{K}}\phi(1-\mu)\frac{\sigma_0}{h^{\alpha}}\right]\right\},$$

(15)

$$\dot{\varepsilon}^{h} = -\varepsilon^{h}\delta_h(1-u) - (1-\beta)\frac{1-b_1\mu^{b_2}}{1+\theta_1 T^{\theta_2}} AK^{\beta}h^{-\beta}[uL]^{1-\beta}\varepsilon^{K}\left[1+\frac{\varepsilon^{M}}{\varepsilon^{K}}\phi(1-\mu)\frac{1-\alpha-\beta}{1-\beta}\frac{\sigma_0}{h^{\alpha}}\right],$$

(16) $\dot{\varepsilon}^M = \varepsilon^M \delta_M - \dfrac{4.1}{R_1 \ln 2} \dfrac{\varepsilon^T}{M},$

(17) $\dot{\varepsilon}^T = \varepsilon^T \dfrac{1}{R_1}\left[\Lambda + \dfrac{R_2}{\tau_{12}}\right] - \dfrac{\varepsilon^{T^*}}{\tau_{12}},$

(18) $\dot{\varepsilon}^{T^*} = \varepsilon^{T^*} \dfrac{1}{\tau_{12}} - \dfrac{R_2}{R_1 \tau_{12}} \varepsilon^T .$

Along the optimal trajectory, the behaviour of the state variables $K_t, h_t, M_t, T_t, T_t^*$ are governed, respectively, by the differential equations (2), (3), (6), (8) and (9). The system is completely determined by five initial conditions $K_0, h_0, M_0, T_0, T_0^*$ and the following transversality conditions:

(19) $\lim_{t \to \infty} \varepsilon_t^K K_t = 0,$

(20) $\lim_{t \to \infty} \varepsilon_t^h h_t = 0.$

(21) $\lim_{t \to \infty} \varepsilon_t^M M_t = 0,$

(22) $\lim_{t \to \infty} \varepsilon_t^T T_t = 0.$

(23) $\lim_{t \to \infty} \varepsilon_t^{T^*} T_t^* = 0.$

Our interest is to study the behaviour of the system in its steady-state (balanced growth path), i.e. to find a particular solution $\left(K_t, \varepsilon_t^K, h_t, \varepsilon_t^h, M_t, \varepsilon_t^M, T_t, \varepsilon_t^T, T_t^*, \varepsilon_t^{T^*}, c_t, u_t, \mu_t\right)$ so that the growth rate of each of these variables is constant.

Let κ denote the rate of growth of per-capita consumption on a balanced growth path. From (11), we know that $\frac{\dot{\varepsilon}^K}{\varepsilon^K} = -\kappa - \rho$. Then, from (14), we have

$$\text{(24)} \quad \rho + \delta_K + \kappa = \beta \Omega_t A K_t^{\beta-1}\left[u_t h_t L_t\right]^{1-\beta}\left\{1 + \frac{\varepsilon^M}{\varepsilon^K}\phi\left[1-\mu\right]\frac{\sigma_0}{h^\alpha}\right\}.$$

Next dividing (2) by K_t and applying (24), we have

$$\text{(25)} \quad \frac{\dot{K}_t}{K_t} + \delta_K + \frac{L_t c_t}{K_t} = \frac{\rho + \delta_K + \kappa}{\beta}\frac{1}{1 + \frac{\varepsilon^M}{\varepsilon^K}\phi\left[1-\mu\right]\frac{\sigma_0}{h^\alpha}}.$$

From (13), we see that a necessary condition for a balanced growth path is that $\frac{\varepsilon^M}{\varepsilon^K h^\alpha}$ is constant and satisfies the following relation

$$\text{(26)} \quad \phi\sigma_0 \frac{\varepsilon^M}{\varepsilon^K h^\alpha} = -\frac{b_1 b_2 \bar{\mu}^{b_2 - 1}}{1 - b_1 \bar{\mu}^{b_2} + \left(1 - \bar{\mu}\right) b_1 b_2 \bar{\mu}^{b_2 - 1}}.$$

By definition, on a balanced growth path $\frac{\dot{K}_t}{K_t}$ is constant. Differentiating (25) and using (26), we see that $\frac{L_t c_t}{K_t}$ is also constant or equivalently, K_t grows at the constant rate $\kappa+\lambda$.

Next let ν denote the growth rate of human capital on a balanced growth path. It is clear from (3) that

(27) $\nu = \delta_h(1-\bar{u})$,

and from differentiating (24) that κ , the common growth rate of per-capita consumption and capital, is

(28) $\kappa = \nu$.

Then, differentiating (12) and using (28), we obtain

(29) $$\frac{\dot{\varepsilon}^h}{\varepsilon^h} = -\kappa + \lambda - \rho .$$

Together, (15) and (27) yield

(30) $$\frac{\dot{\varepsilon}^h}{\varepsilon^h} = -\delta_h - \frac{\alpha\left(\nu - \delta_h\right)}{1-\beta} .$$

Eliminating $\frac{\dot{\varepsilon}^h}{\varepsilon^h}$ between (29) and (30) yields

(31) $$\kappa = \delta_h + \frac{(\lambda - \rho)(1-\beta)}{1-\beta-\alpha}.$$

It follows from (27), (28), and (31) that

(32) $$\bar{u} = \frac{(\rho - \lambda)(1-\beta)}{\delta_h(1-\beta-\alpha)}.$$

Equation (31) gives the expression of the growth rate of per-capita capital κ (which is also the growth rate of per-capita consumption and human capital) as a function of the parameters of the model. On a balanced growth path, the growth rate of the other variables is zero:

(33) $$\frac{\dot{M}}{M} = \frac{\dot{T}}{T} = \frac{\dot{T}^*}{T^*} = 0.$$

and

(34) $$\frac{\dot{u}}{u} = \frac{\dot{\mu}}{\mu} = 0.$$

Equation (33) state that the atmospheric concentration of GHG cannot increase forever and should be constant at the equilibrium (the same argument applies to the increase of the average temperature). Equation (34) comes from the fact that both u and μ vary between 0 and 1 and should be constant on a balanced growth path. Equations (31) to (34) describe the asymptotic rates of change of the state and control variables of the system.

We will now study the levels of these variables. To this endeavour, it is useful to make the system stationary. Let

(35) $\widetilde{K}_t = e^{-(\kappa+\lambda)t} K_t$,

(36) $\widetilde{h}_t = e^{-\nu t} h_t$,

(37) $\widetilde{c}_t = e^{-\kappa t} c_t$,

(38) $\widetilde{L}_t = e^{-\lambda t} L_t$.

The dynamics of the system is now represented by the behaviour of the normalized variables $\widetilde{K}_t, \widetilde{h}_t, \widetilde{c}_t$, and $\widetilde{L}_t$ as well as the original differential equations (6), (7), (8), and (9) that describe the climate model. The dynamics of the normalized capital stock can be shown to be governed by the following differential equation:

(39) $$\frac{d\widetilde{K}_t}{dt} = \frac{1-b_1\mu^{b_2}}{1+\theta_1 T^{\theta_2}} A\widetilde{K}_t^{\beta}\left[\bar{u}\widetilde{h}_t\widetilde{L}_t\right]^{1-\beta} - \left(\delta_K + \kappa + \lambda\right)\widetilde{K}_t - L_0\widetilde{c}_t .$$

Now from (4), one can see that the growth rate of emissions along a balanced growth path if $\mu_t = \bar{\mu} \prec 1$ is

(40) $$\frac{\dot{E}_t}{E_t} = \frac{\dot{\sigma}_t}{\sigma_t} + \frac{\dot{Y}_t}{Y_t} = \lambda + (1-\alpha)\nu .$$

In the model, $\alpha < 1$ and both v and λ are positive, which means that the emissions growth rate $\frac{\dot{E}}{E}$ is also strictly positive. Therefore, the only acceptable value of μ for $\dot{M} = 0$ is $\mu = 1$, which means that, in the long run, all GHG emissions need to be controlled. With complete control of emissions in the long run, i.e., $\lim_{t \to \infty} E_t = 0$, it follows directly from (6) that

(41) $$\lim_{t \to \infty} M_t = \overline{M} = 590 .$$

Using (41) in (7), we obtain

(42) $$\lim_{t \to \infty} F_t = 0 .$$

Now the dynamics of the upper and deep oceans are governed by (8) and (9), respectively. Using (42) in (8), we obtain the following relation in the long run

(43) $$-\Lambda \overline{T} - \frac{R_2}{\tau_{12}}\left(\overline{T} - \overline{T}^*\right) = 0,$$

where $\overline{T} = \lim_{t \to \infty} T_t$, $\overline{T}^* = \lim_{t \to \infty} T_t^*$.

Using (9), we can assert that in the long run

$$(44)\quad \frac{1}{\tau_{12}}\left(\overline{T}-\overline{T}^{*}\right)=0.$$

Next, recall from (32) that the fraction of non leisure time devoted to the production of the consumption good in balanced growth is given by

$$(45)\quad \overline{u}=\frac{(\rho-\lambda)(1-\beta)}{\delta_{h}(1-\beta-\alpha)}.$$

In the model, $\rho \succ \lambda$, which ensures that $\bar{u} \succ 0$. On the other hand, $\alpha \in [0,\bar{\alpha}]$ so that $0 \prec \bar{u} \prec 1$.

Also, in balanced growth, normalized human capital is constant, i.e.,

$$(46)\quad \lim_{t\to\infty}\widetilde{h}_{t}=h_{0}=\text{Constant}.$$

As for the behaviour of the capital stock, note that on a balanced growth path, the normalized capital stock is constant, i.e., $\frac{d\widetilde{K}_{t}}{dt}=0$. This fact and (39) imply

$$(47)\quad (1-b_{1})A\widetilde{K}^{*\beta}\left[\overline{u}\widetilde{h}^{*}L_{0}\right]^{1-\beta}-\left(\delta_{K}+\lambda+\kappa\right)\widetilde{K}^{*}-L\widetilde{c}^{*}=0.$$

Finally, on a balanced growth path (24) assumes the following form

$$\beta(1-b_1)A\widetilde{K}^{*\beta-1}\left[\overline{u}\widetilde{h}^{*}L_0\right]^{1-\beta} = \rho+\delta_K+\kappa . \tag{48}$$

Therefore, the solution of the system along a balanced growth path is given by:

$$\overline{\mu} = 1, \tag{49}$$

$$\overline{M} = 590, \tag{50}$$

$$\overline{T} = \overline{T}^{*} = 0. \tag{51}$$

$$\overline{u} = \frac{(\rho-\lambda)(1-\beta)}{\delta_h(1-\beta-\alpha)}. \tag{52}$$

$$\widetilde{K}^{*} = \frac{(1-\beta)(\rho-\lambda)}{\delta_h}(1-\beta-\alpha)\left[\frac{\beta(1-b_1)A}{(1-\beta)(\delta_K+\delta_h+\lambda)-\alpha(\delta_K+\delta_h+\rho)}\right]^{1/1-\beta} h_0 L_0 . \tag{53}$$

(54)

$$\widetilde{c}^{*} = \frac{(1-\beta)(\rho-\lambda)}{\beta\delta_h h_0}(1-\beta-\alpha)\left[\frac{\beta(1-b_1)A}{(1-\beta)(\delta_K+\delta_h+\lambda)-\alpha(\delta_K+\delta_h+\rho)}\right]^{1/1-\beta} \times$$
$$\left[(1-\beta)(1-\beta-\alpha)(\delta_K+\delta_h)+(\lambda-\rho)(1-\beta)^2+(\rho-\beta\lambda)(1-\beta-\alpha)\right].$$

Using the parameter value given in Lucas (1988) and Nordhaus (1994): β=.25, ρ=.03, λ=.02, δ_h=.05 along with α=1/3, we obtain the following results:

The per-capita consumption and capital (physical and human) grow at the rate κ=3.3 percent per annum. The population ought to devote 66 percent of his time to human capital accumulation $(\overline{u} = 0.33)$. In the steady state equilibrium, the stock of CO_2 returns to the estimated pre-industrialized level, i.e. 590, and the deviation of the temperature at the surface of the earth as well as the deviation temperature of the ocean are both zero. In the next section, the dynamic properties of the model will be discussed.

4. DISCUSSION OF THE RESULTS

In the previous section, we have characterized the balanced-growth equilibrium of the economy. From this point on, there are two distinct paths that we can pursue. We can conduct a comparative static analysis to analyse the impact on the long run equilibrium of a change in ρ or α. Such analysis can be found in standard microeconomics textbooks at the graduate level, such as Varian (1996). The other path is to continue to explore some of its dynamic properties, namely how fast the economy will converge to its steady state. and this is our purpose in this section.

In the steady state equilibrium, physical capital grows exponentially at rate $\kappa + \lambda$; human capital grows exponentially at rate κ : and per-capita consumption grows exponentially at rate κ . Equations (53) and (54) give the expressions of $\widetilde{K}^*$ and $\widetilde{c}^*$, the normalized capital stock and the normalized per-capita consumption in the steady state, as functions of the parameters of the system.

However, one question that remains open is how fast the economy will enter the steady state.

The linear approximations, around their equilibrium values $\tilde{c}^*$ and $\tilde{K}^*$, of the rate of change of the normalized rate of consumption $\dot{\tilde{c}}$ and the rate of change of the normalized capital stock $\dot{\tilde{K}}$ are given, respectively, by [6]:

$$(55)\quad \dot{\tilde{c}} \approx \frac{\partial \dot{\tilde{c}}}{\partial \tilde{K}}\left[\tilde{K} - \tilde{K}^*\right] + \frac{\partial \dot{\tilde{c}}}{\partial \tilde{c}}\left[\tilde{c} - \tilde{c}^*\right].$$

$$(56)\quad \dot{\tilde{K}} \approx \frac{\partial \dot{\tilde{K}}}{\partial \tilde{K}}\left[\tilde{K} - \tilde{K}^*\right] + \frac{\partial \dot{\tilde{K}}}{\partial \tilde{c}}\left[\tilde{c} - \tilde{c}^*\right].$$

with the first order partial derivatives evaluated at $\tilde{K} = \tilde{K}^*$ and $\tilde{c} = \tilde{c}^*$.

Using the definitions (35) and (37) of $\tilde{K}_t$ and $\tilde{c}_t$, one sees that

$$(57)\quad \dot{\tilde{c}}_t = \frac{d}{dt}\left(c_t e^{-\kappa t}\right) = e^{-\kappa t}\left[\dot{c}_t - \kappa c_t\right] = e^{-\kappa t}\dot{c}_t - \kappa \tilde{c}_t,$$

$$(58)\quad \dot{\tilde{K}}_t = \frac{d}{dt}\left(K_t e^{-(\kappa+\lambda)t}\right) = e^{-(\kappa+\lambda)t}\left[\dot{K}_t - (\kappa+\lambda)K_t\right] = e^{-(\kappa+\lambda)t}\dot{K}_t - (\kappa+\lambda)\tilde{K}_t.$$

[6] The analysis is adapted from Romer (1997)

Using the property that $\frac{\partial}{\partial \widetilde{K}} = e^{(\kappa+\lambda)t}\frac{\partial}{\partial K}$ and $\frac{\partial}{\partial \widetilde{c}} = e^{\kappa t}\frac{\partial}{\partial c}$, we can rewrite (55) and (56) as:

$$(59)\quad \dot{\widetilde{c}} \approx \left\{e^{\lambda t}\frac{\partial \dot{c}}{\partial K} - \kappa\frac{\partial \widetilde{c}}{\partial \widetilde{K}}\right\}\left[\widetilde{K} - \widetilde{K}^*\right] + \left\{\frac{\partial \dot{c}}{\partial c} - \kappa\right\}\left[\widetilde{c} - \widetilde{c}^*\right],$$

$$(60)\quad \dot{\widetilde{K}} \approx \left\{\frac{\partial \dot{K}}{\partial K} - (\kappa+\lambda)\right\}\left[\widetilde{K} - \widetilde{K}^*\right] + \left\{e^{-\lambda t}\frac{\partial \dot{K}}{\partial c} - (\kappa+\lambda)\frac{\partial \widetilde{K}}{\partial \widetilde{c}}\right\}\left[\widetilde{c} - \widetilde{c}^*\right],$$

with the first order partial derivatives evaluated at $\widetilde{K} = \widetilde{K}^*$ and $\widetilde{c} = \widetilde{c}^*$.

From (25), (53), and (54) we have

$$(61)\quad e^{\lambda t}\left(\frac{\partial \dot{c}}{\partial K}\right)_{\widetilde{K}=\widetilde{K}^*} = \frac{1}{\beta L_0}\left[(1-\beta)(\delta_K+\kappa)+\rho-\beta\lambda\right]\left[(\beta-1)(\rho+\delta_K+\kappa)\right].$$

From (53) and (54),

$$(62)\quad \left(\frac{\partial \widetilde{c}}{\partial \widetilde{K}}\right)_{\widetilde{K}=\widetilde{K}^*} = \frac{(1-\beta)(\delta_K+\kappa)+\rho-\beta\lambda}{\beta L_0}.$$

From (25), it is easy to show that

$$(63)\quad \left(\frac{\partial \dot{c}}{\partial c}\right)_{\widetilde{c}=\widetilde{c}^*} - \kappa = 0.$$

Therefore, (57) can be rewritten as

$$(64)\quad \frac{d}{dt}\left(\tilde{c}-\tilde{c}^{*}\right)=A\left(\tilde{K}-\tilde{K}^{*}\right),$$

where

$$(65)\quad A=\frac{(1-\beta)(\delta_K+\kappa)+\rho-\beta\lambda}{\beta L_0}\left[(\beta-1)(\rho+\delta_K+\kappa)-\kappa\right].$$

On the other hand, using (24) and (48), we have

$$(66)\quad \left(\frac{\partial\dot{K}}{\partial K}\right)_{\tilde{K}=\tilde{K}^{*}}=\rho+\kappa\,.$$

From (2), (53) and (54), one sees that

$$(67)\quad e^{-\lambda t}\left(\frac{\partial\dot{K}}{\partial c}\right)_{\tilde{c}=\tilde{c}^{*}}-(\kappa+\lambda)\left(\frac{\partial\tilde{K}}{\partial\tilde{c}}\right)_{\tilde{c}=\tilde{c}^{*}}=-L_0\frac{\rho+\kappa+(1-\beta)\delta_K}{(1-\beta)(\delta_K+\kappa)+\rho-\beta\lambda}\,.$$

Then equation (60) can be restated as:

$$(68)\quad \frac{d}{dt}\left(\tilde{K}-\tilde{K}^{*}\right)=(\rho-\lambda)\left[\tilde{K}-\tilde{K}^{*}\right]+B\left[\tilde{c}-\tilde{c}^{*}\right],$$

where

(69) $$B = -L_0 \frac{\rho + \kappa + (1-\beta)\delta_K}{(1-\beta)(\delta_K + \kappa) + \rho - \beta\lambda}.$$

Next divide (64) by $\tilde{c} - \tilde{c}^*$ and (68) by $\tilde{K} - \tilde{K}^*$, we obtain

(70) $$\frac{d/dt(\tilde{c} - \tilde{c}^*)}{(\tilde{c} - \tilde{c}^*)} = A \frac{(\tilde{K} - \tilde{K}^*)}{(\tilde{c} - \tilde{c}^*)}.$$

(71) $$\frac{d/dt(\tilde{K} - \tilde{K}^*)}{(\tilde{K} - \tilde{K}^*)} = (\rho - \lambda) + B \frac{(\tilde{c} - \tilde{c}^*)}{(\tilde{K} - \tilde{K}^*)}$$

Equations (70) and (71) show that the growth rate of $\tilde{K} - \tilde{K}^*$ and $\tilde{c} - \tilde{c}^*$ depends only on the ratio $\frac{(\tilde{c} - \tilde{c}^*)}{(\tilde{K} - \tilde{K}^*)}$. Let assume that $\tilde{K} - \tilde{K}^*$ and $\tilde{c} - \tilde{c}^*$ decrease at the same rate, ζ, we have from (70) and (71):

(72) $$\zeta^2 - (\rho - \lambda)\zeta + AB = 0.$$

Since $AB<0$, the equation (72) has two real roots, ζ_1 and ζ_2. However, only the negative root will lead the economy towards its long run equilibrium $(\tilde{K}^*, \tilde{c}^*)$. Let $\xi = \rho - \lambda$, then the two roots of (72) are given by

$$\zeta = \frac{\xi \pm \sqrt{\left(\xi^2 - 4AB\right)}}{2}. \tag{73}$$

For example, using the same values as in section 3 along with δ_K =0.05, we have that the economy will converge towards its equilibrium state at around 10 percent.

5. CONCLUSION

The research paper proposes an extension of the DICE model, by incorporating the theory of endogenous growth. The extended model was then solved under the optimization framework (the central planner approach) and the steady state equilibrium solution was analysed in some detail. Finally, the question of how fast the economy will converge towards this steady state was partially resolved. What can be concluded from this study? From a policy perspective, very little because the relevant question for policy design is not "What is the best course for the next 200 years?" but rather "What is the best course for the next few years?". A more suitable approach for policy design/implementation could be a sequential strategy ("act-and-learn" approach), where new information can be gathered as time passes and knowledge about the climate change process accumulates. Alternative policy choices would, for example, include (1) immediate investment in new plants and equipments, (2) aggressive research and development on greenhouse abatement technology, or (3) deferring large investments for 10 years, when the nature and size of the threat are better understood and when costs will have dropped due to the availability of new technology.

From a theoretical perceptive, however, this model could be part of a new generation of global warming models, in which human capital (and/or technology) are no longer treated as exogenous or based on historical data, but follows a dynamic and endogenous path. This type of model has already proven useful in macroeconomic theory and could contribute to the climate change debate. One aspect that clearly could be improved in the model is the treatment of uncertainty, by using a stochastic version of the DICE model. In the case of global warming, the question of uncertainty is directly related to irreversibility and the high cost of being wrong in either direction taken. In turn, the irreversibilities involved create an option value, the value of preserving choice for the future. In act-then-learn decision making, uncertainty is not fully resolved before a decision is taken so that the resolution of uncertainty must be viewed as either unacceptably costly or not helpful to the decision. However, reducing uncertainty in a way that can create future options should be highly valued. Accordingly, uncertainty should prompt decision makers to focus on the timing of crucial investment decisions and to accelerate those near-term activities that create options (Dixit and Pindyck, 1995).

Global warming is a complex multi-disciplinary problem (economics, meteorology, sustainable development, political aspects) and the way human beings will deal with this problem also depends on numerous factors, such as economic efficiency of proposed alternatives, social considerations, international cooperation... Given the large uncertainties that surround climate change, it may be worthwhile to invest in human capital now to better understand its process and then take the appropriate decisions. As a final message in my conclusion, I would like to quote the words of (Bolin, 1995), the former chairman of the IPCC: "The issue at stake is not to agree on policies for decades into the next century but rather to adopt a strategy whereby needed actions could be formulated as more knowledge becomes available. The climate change issue will in any case be with us for decades to come and the adequacy of the commitments under the Convention should be judged in that perspective."

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