Total Factor Productivity Measurement

and Returns to Scale

by

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1. Introduction

This study deals with returns to scale and total factor productivity growth (TFPG), two concepts that are intimately related. Returns to scale measures the contribution of inputs to the growth of outputs, while TFPG measures the residual contribution which is not accounted for by the growth of inputs. Thus TFPG reflects the combined effects of technical progress and other influences not explicitly taken into account.

The measurement of TFPG and returns to scale depend heavily on the methodology used and the definition of variables. This paper describes the different models used and compares the different methods of measuring variables, and then chooses the most appropriate methodology to estimate TFPG and returns to scale for the U.S. economy over the period 1950-1988.

Returns to scale and TFPG are highly relevant topics in economic theory and practice. The degree of returns to scale is fundamental to some basic economic theories. Decreasing returns to scale, which results in a concave production possibility frontier, is a sufficient condition for the existence of a perfectly competitive general equilibrium. In turn, the existence of a competitive general equilibrium, at least theoretically, provides the foundation for a variety of economic policies. For example, some anti-monopoly policies are based on the theory that perfect competition is a sufficient condition for Pareto optimality and thus provides superior outcomes to monopoly.

Similarly, the assumption of constant returns to scale (CRS) has a long history in economic theory. In a survey article, Hicks (1989,12) pointed out that: " Walras is pure CRS... from Walras to Cassel, from Cassel to Leontief and Wald and von Newmann, ... and much of Samuelson, to Linear Programming and Activity Analysis... All I need to do is to emphasize that it is all CRS." Constant returns to scale is also a key assumption in international trade theory. The mathematical proofs of such fundamental theorems of international trade as Samuelson's Factor Price Equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and Johnson's theorem all require the assumption of CRS (Bhagwati 1983, Appendix A).

However, some modern growth theorists (Romer 1986; Lucas 1988) have argued that CRS or DRS provide no compensation and incentive to innovate, and thus no explanation as to why a free-market economy generates technical change. Therefore, they have emphasized increasing returns to scale. It is possible, since many investments involve fixed costs, that successive increments of investment in an economy, an industry, or a firm may yield a range of increasing returns as costs that were incurred earlier provide publicly available knowledge and as customers become more receptive to new products.

Although total factor productivity (TFP) has perhaps played a less important role in economic theory than returns to scale, it is an important measure of the efficiency of an economy. It measures the amount by which output will increase as a result of improvements in methods of production, with the quantities of all inputs unchanged. For a given quantity of inputs, higher TFP yields more output. TFP at the national level provides an objective criterion for comparing economic efficiency and technological improvement over time or across countries, while TFP at the industry or firm level measures the competitiveness of various domestic industries vis-a-vis similar industries in the rest of the global economy. Thus information pertaining to TFP can help policy makers design industrial policy and international industrial strategy.

The outline of this study is as follows. After the introduction, section 2 describes three popular models -- the Solow model, the translog model and the productivity index model.

Section 3 then gives a brief survey of the empirical literature relating to these three models. Next, section 4 discusses the problems involved in objectively measuring returns to scale and TFP; more precisely, it focuses on the functional forms used, and the definition and measurement of outputs and inputs, particularly capital input. Based on this evaluation and comparison of the different models and the different methods of measuring inputs and output, the most appropriate regression model and measurement methods are selected, and applied to the U.S. economy in section 5. The stability of the regression model over time is also tested. The last section discusses and compares the results with those of other researchers and draws some conclusions.

2. Methodologies for Measuring Returns to Scale and Total Factor Productivity Growth

All methods of measuring returns to scale and productivity are based on mathematical models. Three models are particularly popular: the Solow model, the translog model and the productivity index model. Here, the term "Solow model" refers to the model in his paper of 1957, not Solow's theory of growth. The Solow model is widely used at the national level and the industry or firm level. The translog and productivity index models are used primarily at the industry or firm level.

All three models are based on a general production function:

$$Q^*(t)=F(A^*(t),H^*(t),L^*(t)), \tag{2.1}$$

where $Q^*(t)$, $H^*(t)$ and $L^*(t)$ are the aggregates of output, capital input and labour input respectively; and $A^*(t)$ is a set of state variables that represent the natural and nonnatural environment of production. The natural environment includes weather, pollution and the endowment of natural resources, while the nonnatural environment includes political and institutional variables, etc. All variables are functions of time t, and are subject to changes in both quantity and quality from one period to the next. In other words, the qualities and quantities of outputs depend on the qualities and quantities of capital and labour inputs and their environment. A change in the quality of capital inputs is reflected in increases in the productive capacity of capital goods, e.g., automatic assembly lines, stronger materials, etc. Changes in the quality of labour input take the form of advances in knowledge, improved management, more efficient operating skills, better educated and healthier workers, etc.

If we further assume the separability of inputs and the environment, that is that the state variable is independent of the input variables, then (2.1) can be re-written as:

$$Q^*(t)=A^*(t)f^*(H^*(t),L^*(t)). \tag{2.2}$$

This assumption helps to simplify empirical analysis. However, one problem that cannot be easily rectified is that the quality of inputs and outputs is difficult to measure. The qualities of inputs represent the state of technology and knowledge of the physical inputs. If we cannot measure the quality of inputs, then the production function we use for empirical work must be expressed as the function:

$$Q(t)=A(t)f(H(t),L(t)). \tag{2.3}$$

In (2.3) all variables and the function are different from (2.2). Q(t), H(t) and L(t) represent only the physical quantities of capital and labour inputs respectively. In this function, the effects of changes in the qualities of inputs are included in the state variable A(t). A(t) differs from $A^*(t)$ in that it incorporates the effects of changes in the quality of inputs as well as the environment variables, so A(t) represents two characteristics of the state -- environment and technology.

In the real world, the measurement of inputs and output is complicated. For example, the quantity of capital input is hard to measure precisely, although the quality of some inputs may be measured by certain indexes. Similarly, some components of the environment variables may be partially measured. Not surprisingly, different investigators often measure variables in different ways. Therefore, the theoretical TFP or TFPG defined in the introduction is usually measured by residuals which reflect the combined effects of the quality of inputs and environmental variables which are not explicitly accounted for by investigators.

Although there are some differences across investigators, their productivity measurements are all based on the same basic framework, equation (2.3). The three most widely used models are discussed below. A basic assumption made in this section is that the production function is continuous and differentiable. All variables are measured in real terms, unless otherwise noted.

2.1. The Solow Model

In 1957, Solow published a paper that provided a theoretical foundation for productivity measurement. Most subsequent work at the national level and much of the work on productivity measurement at the industry and firm levels is based on the Solow model. Solow used a production function to link the output of the economy to the input of factors of production and to the state of technical knowledge. To study how the growth rate of output changes as the growth rates of inputs and technology change, he differentiates both sides of (2.3) with respect to time, then divides it by Q, yielding:

$$\frac{dQ/dt}{Q}=\frac{dA/dt}{A}+w_H\frac{dH/dt}{H}+w_L\frac{dL/dt}{L} \tag{2.4.a}$$

or alternatively:

$$\frac{d\ln Q}{dt}=\frac{d\ln A}{dt}+w_H\frac{d\ln H}{dt}+w_L\frac{d\ln L}{dt}, \tag{2.4.b}$$

where $w_H=\partial Q/\partial H \bullet H/Q$ and $w_L=\partial Q/\partial L \bullet L/Q$ are the elasticities of output with respect to inputs. Since (2.4) is for continuous data, while in practice data are discrete, in applied work (2.4) needs to be replaced by an approximation. For discrete year-to-year data, replace d by Δ and set $\Delta t=1$; then (2.4.a) becomes:

$$\frac{\Delta Q}{Q}=\frac{\Delta A}{A}+w_H\frac{\Delta H}{H}+w_L\frac{\Delta L}{L}. \tag{2.5}$$

Returns to scale are increasing, constant or decreasing depending on whether the sum of the elasticities, $w_H + w_L$, is greater than, equal to or less than one. The basic Solow model can easily be extended to include any number of other inputs in addition to capital and labour.

Productivity analysts have applied (2.5) in two ways: the calculation approach, also known as growth accounting, and the regression approach. The calculation approach imposes constant returns to scale and perfect competition, and calculates TFPG ($\Delta A/A$) residually. Instead of assuming CRS and perfect competition, the regression approach assumes that the elasticities of inputs with respect to output are constant, and then estimates returns to scale and TFP simultaneously with or without imposing CRS.

The first growth accounting study based on (2.5) was by Solow (1957). First he classified all factor inputs as either H or L, and then assumed that both were paid their marginal products. He further assumed CRS, in which case w_H and w_L are the relative shares of capital and labour and sum to one. They can therefore be approximated by the actual shares of capital and labour. He divided (2.5) through by L and rearranged to obtain per unit (per hour or per person) productivity:

$$\frac{\Delta q}{q}=\frac{\Delta A}{A}+w_H\frac{\Delta h}{h}, \tag{2.6}$$

where q is output per man hour and h is capital input per man hour. Given data on q, h and w_H, Solow calculated the only unknown, $\Delta A/A$ or TFPG, residually using (2.6).

The regression approach assumes that TFPG (or TFP) and the elasticities are constant, say, $w_H=\alpha$ and $w_L=\beta$. Then the general production function can be specified as a Cobb-Douglas production function, $Q=AH^{\alpha}L^{\beta}$. One can then estimate α, β and TFPG or TFP with or without the restriction of CRS.

From the mathematical model (2.3) and the assumption of a Cobb-Douglas production function, we can derive three possible regression models:

$$\ln Q=\ln A+\alpha \ln H+\beta \ln L+\varepsilon_1, \tag{2.7}$$

$$\ln Q = \ln A^0 + bt + \alpha \ln H + \beta \ln L + \varepsilon_2, \tag{2.8}$$

$$\frac{\Delta Q}{Q} = \frac{\Delta A}{A} + \alpha \frac{\Delta H}{H} + \beta \frac{\Delta L}{L} + \varepsilon_3, \tag{2.9}$$

where ε_i represents a random error term. The first model assumes that TFP or A, and thus ln A, is constant. The second and third equations both assume that A is not constant; instead, the growth rate of TFP, $\Delta A/A$ or b, and the initial level of TFP A^0 are constant. The differences between them are that the second equation permits one to estimate A^0 but the third one does not, and the second one assumes that the residual term ε is a function of ln Q while the third one assumes that ε is a function of the growth rate of Q. Therefore, even using the same data set and the same underlying mathematical model, the different regression functions, (2.8) and (2.9), will yield different estimation results.

2.2. The Translog Model

The translog production function was introduced by Christensen, Jorgenson and Lau (1971, 1973), and Gollop and Jorgenson (1980). Since in (2.4) Q, H and L are the aggregates of output, capital input and labour input respectively, to further study the effect of each component in each aggregate, we can take the derivative of each of their components with respect to time, yielding:

$$\frac{d\ln Q}{dt}=\sum w_{Q_i}\frac{dQ_i/dt}{Q_i}, \qquad w_{Q_i}=\frac{\partial\ln Q}{\partial\ln Q_i},$$
$$\frac{d\ln H}{dt}=\sum w_{H_i}\frac{dH_i/dt}{H_i}, \qquad w_{H_i}=\frac{\partial\ln Q}{\partial\ln H_i}, \qquad (2.10)$$
$$\frac{d\ln L}{dt}=\sum w_{L_i}\frac{dL_i/dt}{L_i}, \qquad w_{L_i}=\frac{\partial\ln Q}{\partial\ln L_i},$$

where w_{Qi}, w_{Hi} and w_{Li} are the elasticities of the corresponding aggregate with respect to their components. Under the assumption of perfect competition, the elasticities are the same as the value shares of the corresponding components. If one further assumes market equilibrium, then the total value of output is equal to the total value of inputs, the shares of capital and labour sum to unity, and the value shares of each aggregate sum to unity.

The equations in (2.10) are referred to as the *Divisia indexes of output, capital input and labour input.* According to the Divisia index formula, the growth rate of output is the growth rate of a weighted average of the growth rate of its components, while the growth rate of inputs is calculated in two steps. First the growth rate of inputs is measured by the sum of the growth rates of capital inputs and labour inputs weighted by their corresponding value shares. Next the growth rates of capital and labour inputs are defined to be the weighted averages of the components of each aggregate respectively.

Since the Divisia indexes of output and inputs are defined for continuous data, as in the case of the Solow model an approximation that is applicable to discrete year-to-year data is needed. One can replace d by Δ and set $\Delta t=1$ in the second expression of (2.4) to obtain:

$$\Delta lnQ=\Delta lnA+w_H\Delta lnH+w_L\Delta lnL,$$

or

$$lnA_t-lnA_{t-1} \\ =(lnQ_t-lnQ_{t-1})-w_H(lnH_t-lnH_{t-1})-w_L(lnL_t-lnL_{t-1}). \quad (2.11)$$

If the shares in t and t-1 are different, then w_H and w_L can be neither the shares in t nor in t-1. Christensen, Cummings and Jorgenson (1980) used the two-year average share w* to estimate the average shares, leading to the following formula:

$$TFPG^T=\frac{1}{2}(\Delta lnA_t+\Delta lnA_{t-1}) \\ =(\ln Q_t-lnQ_{t-1})-w_H^*(\ln H_t-\ln H_{t-1})-w_L^*(\ln L_t-\ln L_{t-1}). \quad (2.12)$$

where

$$\begin{aligned} \ln Q_t-lnQ_{t-1}&=\sum w_{Q_i}^*(lnQ_{i,t}-lnQ_{i,t-1}), \\ lnH_t-lnH_{t-1}&=\sum w_{H_i}^*(lnH_{i,t}-lnH_{i,t-1}), \\ lnL-lnL_{t-1}&=\sum w_{L_i}^*(lnL_{i,t}-lnL_{i,t-1}), \\ w_{Q_i}^*&=\frac{1}{2}(w_{Q_i,t}+w_{Q_i,t-1}), \\ w_{H_i}^*&=\frac{1}{2}(w_{H_i,t}+w_{H_i,t-1}), \\ w_{L_i}^*&=\frac{1}{2}(w_{L_i,t}+w_{L_i,t-1}). \end{aligned} \quad (2.13)$$

Christensen, Cummings and Jorgenson (1980, 608) referred to (2.12) as the *translog index of* *technical change.* (2.12) says that at any two discrete points in time, say t and t-1, the average rate of growth of TFP can be expressed as the difference between successive logarithms of output less a weighted average of the differences between successive logarithms of capital and labour input with weights given by average value shares.

To estimate capital input, they further assumed that for the aggregate capital input with n components, each of the components of capital input, $H_{j,t}$, is proportional to the stock of capital at the end of the preceding period, $K_{j,t-1}$:

$$H_{j,t}=\theta_{H_j}K_{j,t-1}, \qquad j=1,2,...n. \tag{2.14}$$

where the constants of proportionality θ_{Hj} vary from one type of capital to another, so that they are taken as measures of the quality of capital input. They assumed that if the quality of capital input changes then the proportionality of its components will change. Then they define an index of the quality of the aggregate capital stock, $\theta_{H,t}$, such that:

$$H_t=\theta_{H,t}\, K_{t-1}. \tag{2.15}$$

where K_t is the aggregate capital stock at the end of the preceding period. Similarly, for the labour input with m components, each of the components of labour input $L_{j,t}$ is assumed to be proportional to the hours worked during the period, say $E_{j,t-1}$:

$$L_{j,t}=\theta_{L_j}E_{j,t-1}, \qquad j=1,2,...m, \tag{2.16}$$

and the index of the quality of labour hours worked is defined as $\theta_{L,t}$, such that:

$$L_t = \theta_{L,t}\, E_{t-1}. \tag{2.17}$$

where E_t is aggregate hours worked during the period t. Then they substitute the logarithm of (2.15) and (2.17), or the translog- quality indexed aggregate capital and labour input, into (2.12), to obtain the translog index of technical change:

$$\begin{aligned} TFPG^T = & (lnQ_t - lnQ_{t-1}) \\ & -w_H^*[(\ln\theta_{H,t} - \ln\theta_{H,t-1}) + (lnK_{t-1} - lnK_{t-2})] \\ & -w_L^*[(\ln\theta_{L,t} - \ln\theta_{L,t-1}) + (lnE_{t-1} - lnE_{t-2})]. \end{aligned} \tag{2.18}$$

In (2.18) the w*s are the two-year average shares. Christensen, Cummings and Jorgenson (1980) argued that for each aggregate of capital and labour input, the quality change is accounted for by the quality indexes. If all components of an input aggregate are growing at the same rate, then the quality index of that aggregate remains unchanged.

Using this framework there are two ways to obtain value shares and $TFPG^T$. One is to assume perfect competition and CRS, and then substitute the actual shares for each period into (2.18), and calculate $TFPG^T$ residually. The other method does not assume perfect competition and CRS, in which case the elasticities may not be the same as the corresponding actual shares. Instead, it assumes that the elasticities and TFPG are constant, so that they can be estimated using a translog production function.

A translog production function is simply a second-order Taylor series approximation to a general twice differentiable production function. The translog production function is specified as follows:

$$\begin{aligned} Q=&\exp[\alpha_0+\alpha_H \ln H+\alpha_L \ln L+\alpha_t t \\ &+1/2\beta_{HH}(\ln H)^2+\beta_{HL}\ln H \ln L+\beta_{Ht}t*\ln H \\ &+1/2\beta_{<}(\ln L)^2+\beta_{Lt}lnL*t+1/2\beta_{tt}t^2], \end{aligned} \tag{2.19}$$

where Q, H and L are aggregate output, capital input and labour input, and t indicates year. They can be aggregated directly, or assumed to be a translog function of their components respectively.[1]

In order to derive the implication of this functional form for equation (2.4), one must first take the logarithm of both sides of (2.19) and then take the total differential with respect to t. A rearrangement and comparison with (2.4) then reveals that:

$$\begin{aligned} w_H&=\alpha_H+\beta_{HH}lnH+\beta_{HL}lnL+\beta_{Ht}t, \\ w_L&=\alpha_L+\beta_{HL}lnL+\beta_{<}lhL+\beta_{Lt}t, \\ \frac{dlnA}{dt}&=\alpha_t+\beta_{Ht}lnH+\beta_{Lt}lnL+\beta_{tt}t. \end{aligned} \tag{2.20}$$

w_H, w_L and $TFPG^T$ can thus be obtained from regression results using (2.20). The translog function is characterized by CRS if and only if the parameters satisfy the conditions:

$$\begin{aligned} \alpha_H + \alpha_L&=1, \\ \beta_{HH} + \beta_{HL}&=0, \\ \beta_{HL} + \beta_{<} &=0. \end{aligned} \tag{2.21}$$

The estimation method is usually used at the individual industry or firm level, while the calculation method is usually used for an industry as a whole or at the national level.

2.2.3 The Productivity Index Model

There are two kinds of productivity index models. One estimates the productivity level or TFP for each year and then calculates the growth rate of TFP or TFPG. The other estimates TFPG by taking the difference between the growth rates of output and input. Although these are also named productivity index models, they are actually alternative versions of the Solow model and the translog model except that they use different weights in calculating the growth rates of output and inputs. For example, the Divisia productivity index can be derived from (2.3) by differentiating output with respect to each of the components of capital and labour input respectively (see Jorgenson and Griliches 1967). Thus in this model, the growth rates of inputs and outputs are the share-weighted sums of the growth rates of their components, while in the Solow model, the growth rates of inputs and outputs are the growth rate of aggregate inputs and outputs which are summed directly without weights. The approximation to the Divisia productivity index for discrete data, the Tornqvist productivity index, is similar to the translog model (2.12) (Diewert, 1991). Since these growth rate models have already been described in detail in section 2.1.1 and 2.1.2, here only the first kind of productivity index model, which is widely used at the industry level, is discussed.

The productivity index model is also based on the framework described by (2.2). "The level of productivity is a ratio between the level of production of some economic units and the quantity of inputs they use." (Statistics Canada 1992, 102). As Jorgenson and Griliches (1967) suggested, (2.2) can be rewritten as:

$$A=\frac{Q}{X}, \tag{2.22}$$

where Q is aggregate output and X is aggregate input, both measured in physical units. Since the qualities of the variables are different for each aggregate and in each period, we need some index to measure these qualities. Under the assumptions of perfect competition and CRS, the quantity of each input or output is usually adjusted by its value share. A productivity index formula is:

$$A^1(t)=TFP^1=\frac{\sum_j w_{Q_j}Q_j}{\sum_i w_{X_i}X_i}, \tag{2.23}$$

where Q_j and X_i are the physical quantities of output j and input i respectively, w_{Qj} is the nominal revenue share of output j and w_{xi} is the nominal cost share of input i, so the adjusted output and input are just *Divisia aggregates of output and input.* Note that a Divisia aggregate is a way to aggregate a variable, not the Divisia productivity index. There are two ways to aggregate inputs and outputs: one is direct aggregation, which involves summing the components of an aggregate without weights; the other is Divisia aggregation, where the aggregate is the share-weighted sum of its components. The variables being aggregated may be continuous or discrete, and may be the growth rates of variables as in the Divisia productivity index, or may be the levels of variables as in the productivity index formula (2.23).

After this step, some measurement problems still remain. Since different goods are measured in different units, we have to measure them in common units to aggregate them. To estimate physical quantities, Statistics Canada takes *constant price values*. A general productivity index formula in terms of constant price values rather than physical quantities is:

$$A(t)=TFP=\frac{\sum_j w_{y_j}y_j}{\sum_i w_{x_i}x_i}, \qquad (2.24)$$

where w_{yj} is the nominal revenue share of output j and w_{xi} is the nominal cost share of input i, and y_j and x_i are the constant price values of output j and input i respectively. Only labour input is directly by measured by its quantity -- manhours -- by Statistics Canada (Johnson 1994, 21). This model is mainly used at the industry level.

2.4 Comparison of the Three Models

There are a number of differences between these three models that will cause estimates of TFGP to differ across models. First, the Solow model produces estimates of the growth rate of TFP -- TFPG, while the translog model estimates a logarithmic approximation to the growth rate of TFP -- $TFPG^T$. If the growth rate of TFP is small, say less than 0.05, then $TFPG^T$ is a good approximation of TFPG. In contrast to these two approaches, the productivity index model estimates the level of TFP first, and then TFPG can be calculated. Second, in the Solow model and the productivity index model, the shares of capital and labour input are the nominal value shares of the current year. In the translog model, the shares are two-year average shares. Third, in the Solow model, the outputs and inputs are aggregated directly by summing the components, while the other two models employ the Divisia aggregation. Lastly, capital input may be measured differently even by researchers using the same model, which is a very important reason for the difference in empirical results.

3. Survey of the Empirical Literature

The relevant empirical studies can be classified into two groups -- growth accounting and the regression approach. Each of these two approaches is discussed below.

3.1 Growth Accounting

Growth accounting is mainly applied to national data for national TFPG measurement and international comparisons. There are two kinds of studies, one of which may be called basic accounting in which the only explicit variables are labour and capital, with all other effects included in the state variable A. The other may be called extended accounting, in which some additional variables, such as education, regulation, weather and crime, are explicitly included as explanatory variables. Since this paper focuses on the U.S. economy, I present in table 1 some of the main results of growth accounting studies by different researchers for the U.S.. Table 1 shows that for the U.S. private economy, the magnitudes of TFPG obtained by different investigators who have used different models are similar.

Table 1. Total Factor Productivity Growth of the United States: Growth Accouting

Author	Period	U.S.			Method		
		TFPG(%) (1) = (2)+(3)	Other effects[a] (2)	Resi-duals (3)	Model	Output measure	Inputs
Solow (1957, 18)	1929-49	1.49			Basic Solow model	GNP	Labour Capital
Wolff (1991, 568)	1880-79 1880-38 1950-79	1.37 0.77 1.36				GDP	
Christensen, Cummings, and Jorgenson (1980, 633)	1947-60 1960-73	1.4 1.3			translog model		
Kendrick (1981, 140)	1960-73 1973-79	1.9 0.6	0.7 0.1	1.2 0.5	Extended Solow model	GDP	Labour Capital Additional variables[b]
Denison (1985, 107)	1929-48 1948-73 1973-82 1929-82	1.25 1.98 -0.37 1.31	0.62 0.58 -0.30 0.45	0.63 1.40 -0.07 0.86		NNP	

a. The other effects are due to the additional variables.

b. Denison's additional variables include improved resource allocation, economies of scale, intensity of demand, legal and human environment, crime and irregular factors. Kendrick additional variables includes, efficiency of resource allocation, economies of scale, intensity of demand and weather variation, crime and government regulations.

Using (2.6), Solow found that TFPG was 1.49% over the period 1909-1949. Using the same Solow model, Wolff (1991) measured TFP in the "Group of Seven" industrial countries for the period 1870-1979. For the period 1880-1938, he found that the TFPG of the U.S. was 0.77%, while the average TFPG for the group (excluding Canada and France) was 0.92% ; for the period 1950-1979, TFPG was 1.36% for the U.S. while the average TFPG of all seven countries was 2.52%. The range of annual average TFPG lies between 1.09%-1.37% for 1880-1979 and 1.36%-4.17% for 1950-1979. Unfortunately he does not report any results for the period 1929-1949, so a direct comparison of his results with those of Solow is not possible.

One problem with Wolff's results is that his measurement of the shares of capital and labour is questionable. Because he used GDP to measure output, the factor shares should be based on GDP, but the factor shares of all countries measured are based on the average ratio of employee compensation to *national income* for the U.K and the U.S over the period 1870-1979, so his measurement may not be reliable.

Using the translog model (2.13), Christensen, Cummings and Jorgenson (1980) estimated TFPG for nine industrialized countries including the "Group of Seven", Korea and the Netherlands for the period 1947-1973. The estimated $TFPG^T$ for the U.S. was 1.4% over the period 1947-1960 and 1.3% for 1960-1973, while the average TFPG of nine countries is 2.83 in 1960-1973. In both periods, the U.S. had the lowest TFPG, even lower than that of the U.K.. This result is hard to accept or explain. As Daly (1980, 694) commented: "most other studies of the same countries show the United Kingdom as the lowest, a result in line with their recurrent balance of payments problems and the assessment of most observers both inside and outside the United Kingdom."

Turning now to the extended growth accounting approach, two famous studies which included some environmental variables were Denison (1985) and Kendrick (1981). They used growth accounting to identify the sources of output growth for the main industrialized countries. Both introduced some additional variables, such as the natural and non-natural environment, economic scale and so on, to identify their contribution to total factor productivity. In these studies, the TFPG of the basic Solow model is split into two parts, one due to the effects of the additional variables, and the other due to the residuals which are not explained by either inputs of capital and labour or the additional variables. These additional variables are similar although not identical, and they also used the same method to adjust labour input by sex, age and education. However, the table shows that there is a great difference between the residuals obtained by Denison and Kendrick. For example, for similar periods (1973-1982 and 1973-1979), the residual estimated by Denison was -0.07 while that found by Kendrick is 0.5. Except for the small difference in their choice of additional variables, most of the difference between the results is probably due to differences in their definitions of output and capital input.

In Solow's study, TFPG is 1.49% for the period 1909-1949, while in Denison's study, TFPG is 1.25% for the period 1929-1948. The smaller TFPG for the latter time period arises because Denison defined output to be NNP while Solow defined output to be GNP, and Denison's labour input is adjusted by man hours while Solow's labour input is the unadjusted number of workers employed. Since the contribution of labour input is larger in Denison's study than in Solow's, a smaller residual is left to be attributed to TFPG as a result of technical change. This comparison of the different studies show that the differences in results are not only due to differences in the underlying models, but also due to differences in the definitions of variables.

In section 4, I will discuss the definitions of variables in more detail.

3.2 The Regression Approach

The regression methodology is widely used in production analysis at the industry or firm level. Relatively few studies have been conducted at the national level, but those that do exist can be divided into two groups. One explicitly includes only labour and capital as explanatory variables while other effects are included in the state variable A. The second includes some additional variables, such as human capital, public goods, R&D and so on.

First consider the studies based on the Solow model. I found only three studies (Levy 1990, Intriligator 1966 and Brown 1968) based on the Solow model that measure long-run returns to scale and TFPG at the national level. All three are for the U.S. economy. The results are presented in Table 2. The range of estimated TFPG obtained by Levy (1990) and Brown (1968) is 0.12-0.77% which is much smaller than that obtained by basic growth accounting for similar period for which the range of estimates of TFPG for the postwar period is 1.3%-1.9%. Intriligator (1966) introduced the unemployment rate as an additional variable and imposed CRS. His estimate of α is 0.14 and β is 0.86, both of which are very different from the actual shares of capital and labour, and the estimated TFPG is 1.67, which is similar to that obtained by growth accounting.

Table 2. Total Factor Productivity Growth of the United States: The Regression Approach

Author	Period	Model	R^2	DW	α+ß	TFPG(%)
Brown (1968, 156, Table 10.II)	1890-1906	Δlny=0.0018+0.4156ΔlnK+0.6904ΔlnL (0.5287) (4.5047) (3.0016)	0.867	1.977	1.106	0.18
	1907-1920	Δlny=0.0018+0.7347ΔlnK+0.1272ΔlnL (5.1583) (3.0538) (2.5843)	0.998	2.395	0.8619	0.18
	1921-1939	Δlny=0.0077+0.5046ΔlnK+0.3829ΔlnL (3.0268) (5.0394) (2.3622)	0.928	1.870	0.8875	0.77
	1940-1960	Δlny=0.0059+0.4243ΔlnK+0.5282ΔlnL (3.8121) (2.2497) (2.3381)	0.998	1.944	0.9525	0.59
Intriligator (1965, 70)	1922-1958	ln(y/L)=0.0784-0.605u-1.304u²+ (6.5882) (8.4853) (11.7923) 0.0167t+0.1383ln(K/L) (6.4231) (3.1648)	0.996	2.16	CRS	1.67
Levy (1990,43)	1948-1983	ln(y/L)=0.189+0.0012t+0.416ln(K/L) (-3.643) (1.339) (2.681)	0.58	2.35	CRS	0.12
		ln(y/L)=0.098+0.55ln(K/L) (-0.098) (8.605)	0.35	2.15	CRS	
		lny=-0.250+0.560lnK+0.540lnL (-2.421) (7.887) (3.495)	0.88	2.32	1.10	

a. y is output, L is labour input and K is capital input.

b. The significance of the estimates is represented by their t-ratios (in parentheses).

c. In Intriligator's model, u is the unemployment rate. u=1-e/e*, where e is actual manhours and e* is full employment manhours.

d. In Brown's model, $\Delta lnX = lnX_t - lnX_{t-1}$, where X denotes some variable, e.g. capital input K or labour input L.

It is also worth mentioning that the actual capital and labour income shares are rather stable at around 1/3 and 2/3 respectively for all periods. In the regression approach, the estimates of α and β -- the elasticities of capital and labour input with respect to output -- are very different from the observed capital and labour income shares and also the elasticities vary greatly from one period to the next. For example, Brown obtained α=0.4156 and β=0.6904 for 1890-1906, but α=0.7347 and β=0.1272 for 1907-1920. Even though the period 1907-1920 includes WW I, it seems unlikely that WW I caused a reversal of the income shares. Furthermore, none of the estimates of elasticities in the regression approach are consistent with the income shares in the corresponding periods. Therefore the estimation results suggest that the two assumptions of CRS and perfect competition did not hold over the period of estimation.

The objective of the extended regression model is to distinguish TFPG from the contributions of factors such as human capital, R&D and public goods. Various researchers have done so by including these variables as additional factors of production. In this section, the studies involving human capital are discussed in detail, because this paper focuses on the basic production function which includes only labour and capital. Human capital can be regarded as a variable related to the measurement of the quality of labour input. In contrast, the cost of public goods and R&D are considered to be already incorporated in labour input and capital input, so only the main results of studies that include these variables are mentioned.

Most investigators consider human capital to be an important determinant of modern economic growth and a critical factor in explaining the convergence in growth rates across countries. Mankiw, Romer and Weil (1992) offer an extended Solow model that includes a human capital variable. Rather than estimating the Cobb-Douglas production function directly, they solve for the steady-state capital-labour ratio which is a function of the growth rates of labour and technical progress, and the rates of saving and depreciation. They then substitute the ratio into a Cobb-Douglas production function to derive the regression model. Capital is split into two parts -- human capital and physical capital. Human capital is approximated by the percentage of the working-age population that was in secondary school in 1960, while the dependent variable is real GDP per capita in 1985. They use the 1960 value of the human capital variable, because they assume that human capital is an important determinant of *future* GDP growth. The estimation results from a cross-sectional data set of 98 countries showed that the income shares of human capital, physical capital and labour are all about 1/3.

Holtz-Eakin (1993) applied a similar regression model to all states of the U.S. and found that human capital plays a substantial role in productivity growth. Output is the average gross state product per worker of 1973-1986 and human capital per worker is approximated by the percentage of individuals, aged 25 or older, having completed 4 or more years of college. Under the CRS constraint, returns to capital, human capital and labour are 0.15, 0.18 and 0.67 respectively. It is noteworthy that all the regressions that include human capital as an explanatory variable are based on cross-sectional data for one year or the average of several years, so the growth rate of TFP cannot be estimated.

Public goods such as roads, education, or police and fire protection are provided by government and have the property of non-excludability in consumption. Some articles examined the role of public goods in the growth of the economy, by allowing government expenditures on public goods to enter into the production function as another factor of production. The results show that public goods have been an important determinant of productivity. Using data for the period 1929-1989, Peden (1991) found that the maximum productivity growth occurs when government expenditure represents 20% of GDP over the period 1929-1986. Cullison (1993) found that not only investment in production but also investment in human capital, such as education, training, hospitals and structures by government have a positive statistically (and numerically) significant effect on the future growth of real private GDP.

Extended Solow models have also been used to estimate the direct and indirect effects of R&D in TFP growth. R&D is typically measured by real gross investment on R&D. Most researchers at the industry level and firm level have found that there exists a strong link between R&D and the TFP growth rate: the range of estimated rates of returns to R&D is from 0.2 to 0.5.[2] However, the only study of the effect of R&D on GDP at the national level that I can find concluded that the direct rate of return on R&D is not significantly different from zero (Postner and Wesa, 1983).

3.3. Summary

The preceding literature survey shows that every method has its strengths and weaknesses. Growth accounting assumes perfect competition and imposes CRS. The main advantage of this approach is that under the assumptions of perfect competition and CRS, returns to scale is just the sum of the income shares of capital and labour, which can be relatively precisely measured. The variability of estimated TFPG is due to the definition of capital input, so that growth accounting produces less variability than the regression approach in estimates of TFPG. However, perfect competition and CRS may not hold in the real world. If so, the estimates of TFPG obtained using growth accounting are biased. The disadvantage is that there is no way to estimate returns to scale using growth accounting.

The regression approach is better than growth accounting, since it can estimate returns to scale and TFPG simultaneously without imposing some subjective assumptions. However, it yields greater variability in the estimates of returns to scale and TFPG than growth accounting, so that it will be difficult to compare the results with each other. Within the regression approach, the most appropriate regression model depends on the objectives of the research. There are two possible objectives of the regression approach at the national level. One is to distinguish the contributions of some important factors from total factor productivity; the other is to estimate TFPG and returns to primary inputs as accurately as possible in order to compare the efficiency of the economy over time or among countries. This paper focuses on the latter objective. Human capital may be a good variable for estimating the quality of labour; however, there is no operational definition of human capital. Some investigators have used education to measure human capital. Even if it is a good proxy, the data available in the census are discontinuous, since censuses are conducted only every five to ten years in most countries. Time series estimation is thus impossible.

The state variable A can reflect all the effects of changes in the qualities of inputs and their environment. It includes not only human education levels but also other forms of human capital, training and health levels; not only public goods but government policy and institutions and so on. In short, in the sense of accuracy in the measurement of TFPG as a whole, the basic Solow model may be better or at least as good as the extended models.

4. Problems in Measuring Returns to Scale and TFPG Objectively

Regression estimates of returns to scale and TFPG depend on the definitions of output and inputs, and the functional form of the estimating equation. This section gives a detailed discussion of these issues.

4.1 Output

In the production function (2.3), output is defined as aggregate physical output. As outputs of different goods are measured in different physical units, we have to convert them into the same units to aggregate them. We usually assume that the quantity and quality of outputs are proportional to their value. For any specific good, holding its quality constant, its value is proportional to its quantity; while holding its quantity constant, its value is proportional to its quality. Therefore, aggregate physical output can be estimated by its value; thus the most widely used method of measuring aggregate output is to simply sum the real value of the various outputs produced in the economy.

Two conceptually different measures of the value of output at the national level are NNP and GDP. GDP or Gross Domestic Product is defined as the sum of the money values of all final goods and services that are produced in an economy during a specified time period; GNP or Gross National Product equals GDP plus incomes earned abroad minus foreign incomes earned in the domestic country. NNP or Net National Product, on the other hand, is GNP minus the depreciation of the capital stock.

There has been a long-standing controversy regarding the use of net versus gross measures of national product in accounting for economic growth. Some investigators have preferred NNP to GNP (or GDP). Indeed, Solow (1957) and Denison (1967, 14) suggested that TFPG measures should be based on the cleanest measure of aggregate output -- net national product.

Other investigators prefer gross output as the appropriate concept to measure productivity, "since the depreciation and discarding of capital are unequivocally part of the cost of production" (Norsworthy 1984, 312). We should care about the consumption of capital input since it is a major cost of production. First, for identical net outputs and the same cost of labour, lower depreciation implies lower consumption of fixed capital, and thus a more efficient use of fixed capital. Second, improvements in TFP tend to increase depreciation, so depreciation is relevant to TFP.

Therefore most investigators agree that gross product is the correct output concept for estimating TFP or TFPG and returns to scale, and studying the structure of production, while net product is the correct concept for measuring income and the welfare due to economic growth. That is why most studies that measure TFP are based on the gross value of output.

4.2 Inputs

Inputs can be classified into two broad categories: primary inputs and intermediate inputs. At the national, industry and firm levels, the primary inputs are labour, capital and land. At the industry or firm levels, intermediate inputs are the inputs provided by sectors other than the one being measured; e.g., materials and energy are intermediate inputs for the manufacturing industry. But for a national economy as a whole, intermediate inputs are just intermediate outputs, that is, an intermediate step in the production process rather than a final end, so intermediate inputs are not taken into account in national accounting. Furthermore, at the national level, the quantity of land is fixed. Although any improvement in the quality of land will increase output, the cost of such a quality change can be included in the cost of capital and labour inputs. Thus at the national level, only the two primary inputs -- labour and capital -- are taken into account in estimating TFP or TFPG. The next two sections provide a detailed discussion of how to measure labour and capital inputs respectively.

4.2.1 Labour Input

As Jorgenson and Griliches (1967) pointed out, the stock of labour and labour input are two different concepts. They suggested that the stock of labour is measured by the number of workers including unemployed workers, while the flow of labour services is measured by the man hours used during the period of measurement. The value of the labour stock is measured by the sum of discounted lifetime incomes while the value of the flow of labour services is the sum of labour incomes received by workers during the period of measurement. They defined labour input as the flow of labour services measured in manhours.

Instead of manhours, Levy (1990) used the number of employees as the measure of labour input for the U.S. over the period 1948-1988. This measure is somewhat flawed, since the growth rate of labour input measured in terms of the number of employees is the same as the growth rate of labour input measured in manhours only if average work hours are constant over time. However in the real world, average work hours per week have decreased from 41.6 hours in 1947 to 34.0 hours in 1988 with the growth of the economy (Denison 1985, 117).

Brown (1968), like Jorgenson and Griliches (1967) used manhours as a measure of labour input. However, although manhours is an appropriate unit for measuring the quantity of labour input, it does not reflect the quality of labour. The same number of manhours of different qualities will produce different amounts of output. Ideally, then, labour input measured in manhours should be adjusted by some index to reflect its variable quality.

Denison (1979, 1985) and Kendrick (1981) in their growth accounting studies adjusted manhours by age, sex and education indexes. For example, to obtain education indexes, the measures of labour input for each education group were weighted by the average earnings of otherwise similar individuals who differ in their levels of education. Annual changes in the indexes for the two sexes were combined using total earnings as weights. Basically, Denison's method is to adjust labour input by earnings; a manhour that recieves a higher payment is assumed to have higher productivity and thus is converted into more low-productivity manhours.

In the translog model, a similar method of adjusting for labour quality is used. Labour inputs are divided into different groups according to five factors: age, sex, education, occupation, and class of worker (employee or self employed/unpaid family work and industry). Labour inputs are then aggregated using a Divisia index, that is, the aggregate labour input is the sum of each group weighted by its corresponding compensation share. In the productivity index model, labour input is manhours and is aggregated in the same manner as in the translog model.

The fact that the Divisia aggregate labour input relies on the assumptions of CRS and perfect competition leads to some qualifications. If either CRS or the perfect competition assumption or both do not hold, then the elasticities of output with respect to each type of labour input are not equal to the corresponding compensation shares. Then the Divisia aggregate labour input measure must be different from its true value, even if we do not know what the true value is. Despite this problem, the Divisia aggregate remains a popular method of aggregating not only labour inputs, but also outputs and other inputs.

4.2.2 Capital Input

The most difficult problem in measuring TFPG and returns to scale is the measurement of capital input. This section first studies the relations between the capital stock and the flow of capital services, and then examines the empirical application of measures of capital input.

4.2.2.1 Definition

Before the study of Jorgenson and Griliches (1967), the total net or gross capital stock was regarded as the appropriate measure of capital input. Jorgenson and Griliches (1967, 267) pointed out that one should distinguish between the stock and the flow of services:

> *In converting estimates of capital stock into estimates of capital services we have disregarded an important conceptual error in the aggregation of capital services. While investment goods output must be aggregated by means of investment goods or asset prices, capital services must be aggregated by means of service prices. The prices of capital services are related to the prices of the corresponding investment goods; in fact, the asset price is simply the discounted value of all future capital services. Asset prices for different investment goods are not proportional to service prices because of differences in rates of replacement and rates of capital gain or loss among capital goods.*

They then suggested precisely analogous definitions of capital and labour: the stock of capital is measured by the number of machines including unemployed machines, while the flow of capital services is measured by the machine hours used during the period of measurement. The value of the capital stock is the value of the discounted asset price of the capital stock, while the value of the flow of capital services is the rental value of machine hours. The point of these definitions is that the user cost of capital goods in period t must be distinguished from the purchase cost of capital goods. Most investigators accept these definitions.

Theoretically, capital input should be measured in physical units. Unfortunately, it is not possible, since the capital stock has too many components of different kinds and different functions. To help identify a good estimate of the physical capital input, consider the relationship between the capital input cost and capital stock K, $p_H \bullet H = r \bullet K$, where H is the machine hours used and p_H is the rental rate of machine hours, so $p_H \bullet H$ is the rental cost; while K is the capital stock measured in constant dollars and r is the rental rate of the capital stock, so $r \bullet K$ is the rental cost of the capital stock. This identity states that the rental cost of capital services is equal to the rental cost of the capital stock.

Intuitively, the value of capital services, $p_H \bullet H$, seems like a good estimate of the physical capital input -- machine hours, since it is proportional to the quality and quantity of physical capital input. For simplicity I refer to the physical capital input unit as machine hours, athough capital input includes more than just machines. Holding the quantity of machine hours H constant, $p_H \bullet H$ is proportional to the quality of the machine. Larger values of $p_H \bullet H$ imply a higher price per machine hour. Under perfect competition, the higher rental cost implies a higher marginal product or a higher productivity. While holding the quality of machines p_H constant, then, $p_H \bullet H$ is proportional to the quantity of machine hours H.

Jorgenson and Griliches (1967) derive this conclusion more formally. Using the identity $p_H \bullet H = r \bullet K$, they derived the rental cost or implicit rental value of capital services as the value of capital input from either of two well-established models. The two models are based on optimal firm behaviour and the assumption of a long-run equilibrium characterized by CRS and perfect competition. The first model is a dynamic optimization model. In this model, the firm maximizes the present value of its net cash flow by choosing the optimal path for both control variables, labour and gross investment, and the state variable, net capital stock, subject to the firm's technology and investment equation. In the second model, the firm chooses its optimal level of investment at the point at which the marginal product of capital is equal to its cost. For asset or capital stock A, in the absence of taxation, the rental price of capital services or the value of capital input is the sum of three components: the return to capital services, depreciation and capital gains caused by the rise or fall in capital goods prices (Norsworthy and Jang 1992, 94; and Jorgenson and Griliches 1967, 257). Alternatively:

$$\begin{aligned} P_A &= p_H * H = r * K \\ &= \gamma K_{t-1} + \delta K_{t-1} - (K_t - K_{t-1}), \end{aligned} \tag{4.1}$$

where P_A is the cost of capital input, γ is the rate of return, K_t and K_{t-1} are the values of asset A at t and t-1 respectively, and δ is the rate of depreciation. The aggregate total rental price of all assets can be computed in a variety of ways, including a direct aggregation or a summation weighted according to the Divisia index formula as described in section 2.2.

Now consider how to estimate the capital stock using the expression for the value of capital input in (4.1). A standard formula for estimating the capital stock is:

$$K_t = I_t + (1-\delta) K_{t-1} \tag{4.2}$$

where I_t is the level of investment at time t. The problem now is how to measure K and δ. Since gross capital stock is composed of different vintages that were purchased under different regimes of technology, its components have different productivities. To estimate the value of the capital stock, the following methods have been suggested:

> Method 1: The capital stock is measured by the amount it would have cost in the base period to produce the actual stock of capital goods existing in the given year (not its equivalent in ability to contribute to production).
>
> Method 2: The capital stock is measured by its current productive capacity. For example, if the new capital good has a marginal product twice as large as the old one, then it is measured as contributing twice as much to the capital stock.
>
> Method 3: The capital stock is measured by its long-run productive capacity or the asset price which is simply the discounted value of all future capital services.

These three methods are listed in Usher (1980, 13-18), Denison (1989), Jorgenson and Griliches (1967), and several other papers. Among them, the most widely used is method 3.

Not only do researchers differ in their choice of method of estimating the capital stock, they also differ in their definitions of what constitutes the capital stock. In the framework of Jorgenson and Griliches (1967), the calculation of the returns to capital is based only on tangible physical assets including equipment, construction assets and inventory assets. While some other economists (Hulten and Wykoff 1981) specified capital goods as asset type, others such as Diewert (1980, 481) pointed out that capital input should include both physical capital and working capital.

In addition, researchers differ in their calculations of the rate of return to capital services. Some investigators think of the rate of return as the short-term interest rate (Arrow 1968), while others think of it as the external standard rate of return over the long term, or the internal rate of return for the short term (Norsworthy and Jang 1992). Finally, as is the estimation of the capital stock, the estimation of depreciation is complicated. Economic depreciation is the loss in the value of an asset as it ages. Depreciation depends on the price of new capital goods, the price of used capital goods, taxation, inflation, etc. A detailed discussion of how it should be estimated is beyond the scope of this paper.

However, regardless of the differences in specific methods, most investigators agree that the value of the capital input is the cost of capital services derived from the capital stock, which includes both physical and financial assets, and the rate of return is assumed to be the same for all assets (Norsworthy and Jang 1992, Ch.4).

4.2.2.2 Empirical Applications

Now we are ready to compare some methods of estimating capital input that have been used in empirical studies. In all the econometric studies that I have found, capital input is measured by the total net or gross capital stock. Levy (1990) uses the net capital stock as the capital input. Intriligator (1965) calculated the capital stock using a dynamic formula which includes capital-embodied technical change, an exponential depreciation rate, returns to capital and the growth rate of capital. Brown (1968) measured capital input as the product of the two-year moving average of the available net capital stock and a utilization factor which he estimated.

Turning now to growth accounting, Denison (1985) and Kendrick (1981) both correctly defined capital input as the flow of capital services and assumed that it was proportional to the capital stock. Thus the growth rate of capital inputs is assumed to be the same as the growth rate of the capital stock. They both aggregated the capital stock directly by summing the values of different kinds of capital and calculated capital's share of income as the ratio of nonlabour payments to output. However, due to differences in their definitions of the capital stock and output, there is a big difference between their results, as discussed below.

In the Solow model on which both Denison and Kendrick base their work, the contribution of the capital input to the growth of output is measured by the product of capital's share in income and the growth rate of the capital input. Denison (1985) defined output to be net national product, so depreciation is excluded from both output and inputs or factor cost. Thus Denison's estimate of the capital share is much smaller -- around 20% -- than the approximately 34% obtained by Kendrick (1980, 1981), whose calculation is based on gross domestic product. Denison and Kendrick also differ in their choice of definition of the capital stock. Denison's capital stock measure is the sum of the gross capital stock weighted by 0.75 and the net capital stock weighted by 0.25, where the net capital stock is the gross stock less accumulated depreciation computed by the straight-line depreciation formula. Kendrick, on the other hand, simply uses the gross capital stock.

These differences in definition lead to big differences in estimates of the contribution of capital input. For the period 1973-1981, Denison (1985) obtained an estimate of 0.11 while Kendrick (1980) obtains 0.5 for the period 1973-1979. Consequently, Denison concluded that the post-1973 slowdown in productivity is a "mystery", while most other empirical studies found that the major explanation was the slowdown in capital investment. As mentioned in section 4.1, Denison's method can be criticized on the grounds that it is inconsistent with economic theory, since he ruled out a major part of production cost -- depreciation.

In applying the translog model, Gollop and Jorgenson (1980) calculated capital input according to (2.12) which also involves the Divisia aggregation. To calculate the shares of different capital inputs for the Divisia aggregation, they calculated the rental price or cost for each type of capital based on (4.1) and extended the formula by adding some factors -- such as corporate taxes, indirect business taxes and tax credits -- to (4.1).

The Divisia aggregate capital input is subject to the same criticisms that apply to the Divisia aggregate labour input. It will not be equal to its true value if either CRS or perfect competition does not hold. Furthermore, although (4.1) is a theoretically correct formula, there are some problems with implementing it in practice. The formula includes several unknown variables, and the estimation of each of these variables may involve some problems of definition and measurement as mentioned in 4.2.2.1. A good theoretical method may not be a good operational method, and a good method for the firm level may not be a good one for the national level. For example, Christensen, Cummings and Jorgenson (1980, 633) calculated values for the capital input based on (4.1) and found that the TFPG of the U.K. was 2.1% while the TFPG of the U.S. was 1.3% for the period 1960-1973. These results are hard to accept and explain as mentioned in section 3.1.

Statistics Canada (1994) calculated the TFP of the Canadian manufacturing industry using the productivity index model. As mentioned in section 2.3, physical capital input is estimated by the real cost of capital services. Statistics Canada defined the capital input as the *real value of the residual between output and all other input costs.* That is, the current value of capital input, $p_H \cdot H$ or $r \cdot K$, is estimated "based on what the industry would charge itself for using its own capital assets. This is assumed to be the income generated from those capital services, which is the residual income after paying for all other input costs." (Statistics Canada 1994, 21). The current value of the capital input is converted into its constant price value using a price index. To construct a price index for capital input $p_H \cdot H$, one needs the price p_H for each year.

Unfortunately one only knows that the value of $p_H \bullet H$ is equal to the value of $r \bullet K$. Since one does not know H, it is not possible to identify p_H nor is it possible to construct an index for p_H. Therefore Statistics Canada constructs a price index that is actually based on r, not p_H. That is, r is regarded as an estimate of p_H. r is obtained by dividing the estimate of rK by K, where K is the net capital stock of the previous year measured in constant dollars. Therefore, to construct the price index of capital input one still needs to define and measure the capital stock K. For further details see Statistics Canada (1994, 19-21 and 138).

Therefore, the capital input measured using Statistics Canada's operational definition is consistent with a definition of capital input defined by user cost. It includes the costs of both physical and financial assets -- not only the rental cost of equipment, construction and inventory, but also the rental cost of natural resources and working capital, and the rate of return is assumed to be the same for all assets.

4.2.2.3 Summary

Section 4.2.2 deals with two things: first, what the correct conceptual definition of capital input is; and second, which method or operational definition of capital input is best. As mentioned above, it is commonly accepted that instead of the purchase cost of the capital stock, the user cost of capital services is the correct conceptual definition of capital input. However, the agreement does not mean Jorgenson's equation (4.1) is the best operational definition for capital input. Two points need to be mentioned. First, in Jorgenson's framework the price of capital is based on the price of the firm's newly invested capital, while Mohr (1986, 115 and 153) found that:

> *...the rental price of the firm's newly invested capital is not the appropriate rental price to impute to the firm's aggregate stock of capital. ... careful examination of the practical and conceptual considerations underlying the components of the rental practice shows that the correct aggregate-capital rental price (for use with conventional perpetual inventory capital stock aggregates) is measured as a weighted average of the vintage-specific rental prices of the several vintages embedded in the firm's aggregate capital stock.*

Thus there exists a significant gulf between theory and practice. The gulf may be due to the unrealistic assumptions of perfect competition and CRS made by Jorgenson. Therefore a good theoretical method may not be the best operational method.

Second, in Jorgenson's framework, the calculation of the returns to capital is based only on tangible physical assets including equipment, construction assets and inventory assets. As Diewert (1980, 481) pointed out, "An individual firm will generally hold an 'inventory' of financial capital (or working capital, as it is sometimes called) ... and the cost of holding this 'inventory' is just as real a cost to the firm as a payment to labour." Thus Jorgenson's measure of capital input should also include working capital.

Therefore the agreement on the theoretical definition of capital input does not lead to an agreement on the operational definition. It is hard to say which operational definition is best. The evaluation of these methods depends on whether they are theoretically correct, practically operational and proper for measurement at the national, industrial or firm levels. The measurement of capital input is still an open issue which requires further research. In my opinion, the operational definition of capital input of Statistics Canada is a better one than the others mentioned for the *regression approach*. First, it is consistent with theoretical definition that the capital cost is an appropriate estimate of physical capital input. Second, this method does not require the assumption of CRS; it requires only the assumption that the economy is in long-run perfectly competitive equilibrium.

4.3 Functional Form

Regression estimates of returns to scale and TFPG depend not only on the definitions of output and inputs, but also on the functional form of the estimating equation. For example, Kim (1992) regressed capital, labour, energy and materials used by the U.S. manufacturing industry on output using a translog production function. The estimates of returns to scale were 1.2836, 1.1970, and 1.1517, for the cases of nonhomotheticity, homotheticity and homogeneity of production functions respectively. When Fare and Nijinkru (1989) applied both nonparametric and parametric linear programming models to a data set for the Japanese agricultural sector, they found that the number of firms with DRS, CRS and IRS were 4, 24 and 72 for the nonparametric model, and 4, 0 and 96 for the parametric model respectively.

Thus the scale characteristics estimated are method-dependent. Even for the same basic mathematical model, different regression equations can yield different estimates. As shown in table 2, Levy (1990) found that returns to capital were 0.56 and returns to labour were 0.54 for the period 1948-1983. He then concluded that the long term aggregate production function of the post-war U.S. economy exhibits slightly increasing returns to scale.[3]

However, I think that Levy (1990) misspecified the regression equation. In section 2.1, it was shown that we can derive four different regression equations -- (2.6), (2.7), (2.8) and (2.9) -- from the same production function $y=AH^{\alpha}L^{\beta}$. Levy estimated α and β based on equation (2.7), which assumes that the level of technology A is constant over time. This is a very unrealistic assumption, since in the real world, technology does change over time. As long as there is an improvement in TFPG, then the estimation results derived from this model will overestimate the degree of returns to scale, since some of the contribution due to growth of TFP was treated as returns to scale.

Given the same data and the same theoretical model, CRS, DRS and IRS may all get empirical support from different regression equations. Therefore, the scale character crucially depends on the regression equation selected even though they are based on the same basic model. In my opinion, equation (2.9) is the best of the four equations, since it does not assume that technology is fixed.

5. The Empirical Work

5.1 The Regression Model

For the reasons mentioned in section 3.3, I choose as my model the basic Solow model, since my objective is not to try to distinguish the contribution of additional variables from TFPG, but to estimate the returns to primary inputs and measure TFPG. For the reasons mentioned in section 4.3, I choose the growth rate equation (2.9) as my regression model. Two versions of (2.9) will be estimated:

$$ZHL \quad model: \qquad z=a+\alpha h+\beta l+\varepsilon_1,$$

$$ZHLT \quad model: \qquad z=a+bt+\alpha h+\beta l+\varepsilon_2,$$

where z is the growth rate of output; h is the growth rate of capital input; l is the growth rate of labour input, and t is a time trend. The difference between the two models is that the ZHLT model allows one to test whether TFPG has slowed down over time. The production function associated with the ZHLT model is

$$y=A^0 e^{at+\frac{1}{2}bt^2} H^\alpha L^\beta,$$

where y is the level of output. If the sign of b is negative, then TFPG has slowed down.

Previous studies on growth accounting suggest that after 1973 there was a substantial slowdown of both the world and the U.S. economies, due to a slowdown in TFPG and the growth of capital input. In order to compare the results of this study with those of others, I divide the time series into two segments. I estimate the ZHL and the ZHLT models three times, first for the period 1951-1973, then for 1974-1988, and then for the entire period 1951-1988. The first objective of my empirical work is to test the assumptions of CRS and derive an estimate of returns to scale. The second objective is to measure TFPG and test whether TFPG has decreased. If TFPG has decreased, then the coefficient of the time trend in the ZHLT model for the period 1951-1988 will have a negative sign, and the estimates of TFPG in the two sub-periods would be different.

5.2 Variable Definitions and Data

Let us now define the variables. Output is defined as real GDP at factor cost which is the difference between real GDP and indirect business taxes. Labour input is manhours employed. In defining capital input, I assume that the U.S. economy is in long-run perfectly competitive equilibrium during the period 1951-1988. Under this assumption, there is no profit, and nonlabour payments -- the difference between the current value of GDP at factor cost and the current value of labour compensation -- by definition, equal the capital cost. Further I assume that the physical capital input is proportional to the *real value of nonlabour payments.*[4]

The first step in measuring capital input is to calculate the current value of capital input, which is the difference between the current value of GDP at factor cost and the current value of labour compensation. Labour compensation is the sum of wages and salaries and supplements to wages and salaries. These data are obtained from the *National Income and Product Accounts of the United States* (1993) and the *Statistical Abstract of the United States* (1994).[5]

The second step is to convert the current value of capital input into a real value. The price index I used comes from a series of producer price indexes (PPI) available in the *Statistical Abstract of the United States (114th)*. The specific price index I used is the *Producer Purchasing Power Index (PPPI)* constructed by the U.S. Bureau of the Census with the base year 1982.[6] (see Appendix 1 for a more detailed description). One component of PPPI -- the price index for capital goods -- is constructed based on the selling price of the capital goods. For the reasons I mentioned in section 4.2.2.1 the purchasing price of capital goods is not the proper one for constructing a price index for capital goods. However I cannot find an other *readily available* price index that is better than this one.

5.3 Estimation Results

Table 3 reports some regression results for the U.S. economy for the period 1951-1988, and the two subperiods 1951-1973 and 1974-1988. Since for each period several regression equations are estimated, I select the best equation to use for the analysis of TFPG based on overall performance.

First I checked for autocorrelation, since if there is autocorrelation, the t ratios for the parameter estimates will not be reliable. For the period 1951-1988, Ordinary Least Squares (OLS) estimation of the ZHL equation yields a DW statistic of 0.9239, which indicates a serious autocorrelation problem or a misspecification of the model. I attempted to correct the problem by assuming an AR(1) error structure, and re-estimating the equation using the Cochrane-Orcutt iterative procedure[7]. Since the DW test is no longer valid after this type of correction, the Durbin h test was used to verify that the autocorrelation had been eliminated. The Durbin h statistic for the ZHL (AUTO) equation is -0.9619. Since its absolute value is less than the critical value of 1.96 at the 5% significance level, one can conclude that the autocorrelation has disappeared.

When a time trend is introduced into the model, there is no longer an autocorrelation problem. The ZHLT(OLS) equation shown in Table 3 has a DW statistic of 1.722, which lies within the acceptance region for the null hypothesis of no autocorrelation at the 5% significance level. In the same manner, I tested for and corrected all autocorrelation in the two subperiods using the Cochrane-Orcutt iterative procedure. The results in Table 3 do not involve auotcorrelation problems.

The next step in evaluating the regression results is to look at the overall performance of the regression equations. First I will discuss the estimation results for the two regression equations for the entire sample period. Both equations have quite a high adjusted R^2, 0.9457 and 0.9595 respectively, which means that approximately 94% of the variation in the growth of output can be explained by the growth of TFP, capital input, labour input and a time trend. Furthermore, all of the coefficient estimates have quite high t ratios; thus all the coefficients are statistically different from zero at the 1% significance level.

From an econometric point of view the ZHLT(OLS) equation is better than the ZHL(AUTO) model, because the autocorrelation in the ZHL(OLS) equation may be a sign that an important variable -- the time trend -- has been omitted. The sign of the coefficient of t in the ZHLT(OLS) equation is negative, which means that the TFPG of the U.S. has decreased over time.

Now consider the overall performance of the equations for the two subperiods 1951-1973 and 1974-1988. Table 3 shows that the t ratios of the time trend t in the ZHLT equation for both subperiods 1951-1973 and 1974-1988 are not statistically different from zero at the 5% significance level. This result suggests that the time trend has no significant effect in either subperiod. Thus in both subperiods, the ZHL model seems to be superior to the ZHLT model. A correction for autocorrelation proved to be necessary for the 1951-1973 subperiod, but not for the 1974-1988 subperiod. As was the case for the 1951-1988 period, the two equations reported in rows (5) and (7) of Table 3 have a high adjusted R^2 and coefficients that are statistically different from zero at a 1% significance level.

Next I tested model stability by applying the CHOW test and the recursive CUSUM and CUMSUMSQ tests to the OLS estimation results. These tests examine whether the coefficients are statistically identical for the full estimation period. That is, they test whether there were any structural breaks during the period 1951-1988. The CHOW test is performed by splitting the sample into two parts. If the test statistic is less than the critical value from an $F(k, N_1+N_2-2k)$ distribution,[8] then there is no evidence for a structural break. This test was performed at every possible point in the sample. The CUSUM and CUSUMSQ tests are derived from the results of recursive estimation, which involves running a series of regressions in which one observation is added to the sample for each new regression. The CUSUM test is based on the cumulative sum of the recursive residuals, and the CUSUMSQ test is based on the cumulative sum of squares of the recursive residuals.[9] Both tests are carried out by constructing confidence bounds for the CUSUM and the CUSUMSQ at the 5% significance level. If the values of the CUSUM or CUSUMSQ test statistics stray outside of the bounds, then the null hypothesis -- that the coefficients are constant over the sample period -- is rejected.

For the period 1951-1988, 55% of the CHOW test statistics for the ZHL equation are greater than the critical value, and the CUSUM strays outside of its bounds from 1982-1988 and 1961-1951 (for backward CUSUM recursive estimation); only the CUSMSQ test is passed. After the introduction of the time trend, the CUSUM and CUSUMSQ test statistics always lie inside in the upper and lower bounds, and all the F-statistics for the CHOW test are less than the critical value. Thus, both the CHOW test and the recursive residuals tests (CUSUM and CUSMSQ) consistently suggest that the coefficients of the ZHL equation are not constant during the period 1950-1988. However, all the coefficients of the ZHLT equation are very stable. Furthermore, the value of the CHOW test that divides the sample into the subperiods 1951-1973 and 1974-1988 is 0.3254, which is less than its critical value 2.96 at the 5% significance level. Therefore there appears to have been no structural break in 1973 or during the entire period 1951-1988.

Model stability within the two subperiods was also tested. For the period 1974-1988, the ZHL(OLS) equation passes all tests -- CHOW, CUSUM and CUSMSQ. However, for the period 1951-1973, the first two (1954 and 1955) F-statistics out of 15 do not pass the CHOW test, and the CUSUMSQ is above the upper bound in 1953 and 1954. Thus there appears to be some instability in the coefficients prior to 1973.

Table 3. The Regression Results for the Aggregate U.S. Economy

Period	Model	Result	R^2	DW[a]	DH[b]	$\alpha+\beta$
1951-88	ZHL (AUTO)(1)	z=0.01231+0.2162h+0.7134l (6.225) (9.586) (16.85)	0.9457		-0.96	0.93
	ZHLT (OLS)(2)	z=0.8207-0.00041t+0.2118h+0.7484l (5.534) (-5.456) (10.16) (19.95)	0.9595	1.722 (1.66)		0.96
	ZHLT (OLS)(3)	z=0.8223-0.00041t+0.2147h+0.7852l (5.515) (-5.443) (10.32) (37.73)	0.9591	1.902 (1.66)		**CRS imposed**
1951-73	ZHL (AUTO)(4)	z=0.0152+0.2149h+0.7330l (8.032) (6.147) (14.82)	0.9487		-0.39	0.95
	HLT (AUTO)(5)	z=0.5487-0.00027t+0.2174h+0.7359l (1.436) (-1.395) (6.275) (15.20)	0.9507		0.59	0.95
1974-88	ZHL (OLS)(6)	z=0.0068+0.1908h+0.7496l (4.293) (6.938) (10.95)	0.9628	2.061 (1.54)		0.95
	ZHLT (OLS)(7)	z=0.6823-0.00034t+0.2020h+0.7498l (1.007) (-1.067) (6.893) (11.01)	0.9632	2.209 (1.79)		0.95

a. DW is the Durbin-Watson statistic. The values in parentheses are the corresponding critical values -- the upper limit d_u at the 5% significance level. In each equation, if the DW statistic is greater than d_u and less than $4-d_u$, then we can not reject the null hypothesis that there is no autocorrelation.

b. DH is the Durbin h statistic. The critical value is 1.96 for a two-tailed test at the 5% significance level. If the absolute value of the DH statistic is less than 1.96, then we can not reject the null hypothesis that there is no autocorrelation.

c. The values in this column are F-statistics for testing CRS. The corresponding critical values are in parentheses. If an F-statistic is less than its critical value, then we can not reject the null hypothesis of CRS.

Together these tests suggest that the ZHLT (OLS) equation performs best for the period 1950-1988, while the ZHL (AUTO) and ZHL(OLS) equations best explain output growth from 1950-1973 and 1973-1988 respectively. From an econometric point of view all these three equations perform well, but there is an inconsistency between them. The ZHLT(OLS) equation suggests that the rate of TFPG for the period 1951-1988 has decreased over time, while the equations for the two subperiods suggest that the rate of TFPG is constant over time. According to equations (4) and (6) of Table 3, the estimated rate of TFPG is 1.52% for the period 1951-1973, and 0.68% for 1974-1988. In Table 3, the coefficients of the time trend have the same negative sign and the same magnitude in both subperiods as in the whole period, but they are significantly different from zero only at the 20% and 30% significance levels respectively. When the two subperiods are combined the time trend is significantly different from zero at the 1% level. Thus I think that the ZHLT(OLS) equation may be the best equation to describe the entire period.

5.4 Returns to Scale

The results of the test for CRS are shown in Table 3. All F statistics are less than their corresponding critical value, thus we accept the null hypothesis that the sum of α and β is not statistically different from one at the 5% significance level, which implies that CRS holds. Therefore the ZHLT(OLS) equation was re-estimated with CRS imposed for the period 1951-1988. The results, presented in row (3) of Table 3, show that once CRS is imposed, the coefficients and the t ratios of the constant term, the time trend and capital input α are quite stable, while the coefficient of labour input jumps from 0.7484 to 0.7858. Both of their t ratios increase (the t ratio of ß increases greatly from 19.95 to 37.73). In addition, the adjusted R^2 decreased only very slightly, from 0.9595 to 0.9591. The DW statistic is 1.9, which is also better than before. The equation also passes all model stability tests. Thus the equation with CRS imposed has the best overall performance, and it is therefore chosen for the further analysis of TFPG.

It should be pointed that CRS holding at the national level does not imply that CRS holds at the industry and firm level. The Solow growth model just assumes that CRS holds at the national level, which does not imply CRS for *every* firm, so it does not rule out the possibility of increasing returns to scale at the firm level. As long as competition exists, there exists an incentive for innovation since new goods can bring about an extra return to compensate inventors. In the real world, at the firm level, CRS, IRS and DRS all exist, since technologies are not identical for all firms.

5.5 Total Factor Productivity Growth

Now I turn to the analysis of TFPG. For an easy comparison between the regression approach and growth accounting, I constructed a table for each. In Table 4, based on the regression approach TFPG is computed as follows:

$$TFPG^{m} \cong z - \alpha h^{m} - \beta l^{m},$$

where $TFPG^{m}$, α and ß are obtained from the ZHLT(OLS) equation with CRS imposed as reported in row (3) of Table 3. The subscript m denotes the mean of the variable. Table 5 is based on growth accounting:

$$TFPG^m \cong z - w_H^m * h^m + w_L^m * l^m .$$

The results provided in Table 4 and Table 5 identify the sources of growth. According to Table 4, during 1951-1988 and the two sub-periods 1951-1973 and 1974-1988, the average annual growth rate of GDP in the U.S. was 3.20%, 3.65% and 2.50% respectively, of which 2.13%, 2.24% and 1.95% was due to an increase in factor inputs and the remaining 1.07%, 1.41% and 0.55% was attributable to TFPG respectively. From these results, it is easy to see that TFPG has slowed down: the average TFPG during the period 1974-1988 is only 40% of that obtained during the period 1951-1973.

The calculated rates of TFPG shown in Table 5 are 0.83%, 1.14% and 0.37% for the periods 1951-1988, 1951-1973 and 1974-1988 respectively. The differences between the two approaches are due to the differences between the elasticities of output with respect to inputs and the actual cost shares. For the whole period, the average capital cost share is 0.34 and the average labour cost share is 0.66. Since the growth rate of capital input is twice as large as the growth rate of labour input, and the actual share of capital input (0.34) is much bigger than the estimated elasticity of capital input (0.22), then in growth accounting, the contribution of capital input is 38%, which is 1.6 times larger than the contribution estimated by the regression approach, namely 24%. Labour's contribution in the growth accounting approach, however, is less than that estimated by the regression approach. In total, the factor inputs in growth accounting explained more economic growth than in the regression approach -- 2.4% out of 3.2%. Despite these differences, the rates of TFPG obtained from the two approaches have the same tendency, since the elasticities and shares change in the same direction. After 1973, the rate of

TFPG decreases by 61% and 68% according to the regression and growth accounting approaches respectively.

Neither of these results are directly comparable to the estimates of TFPG obtained by other studies for the post-war period. There are some important differences between this study and the other studies mentioned in Section 3. In the regression approach, the capital input is usually measured by the capital stock. In growth accounting, capital input was distinguished from the capital stock, but capital input was assumed to be proportional to the capital stock, so that the growth rate of capital input was approximated by the growth rate of the capital stock. In this study, the capital input is assumed to be proportional to its cost and measured by the real value of the difference between GDP and labour compensation. However, the growth rate of capital cost has the same trend as the growth rate of the capital stock. The average annual growth rate of capital input for 1951-1988 in this study is 3.56%, which is similar to the growth rate of the capital stock in other studies; for example, the Denison and Jorgenson methods yielded 2.87% and 3.44% for the period 1948-1979 (Norsworthy 1984, 324).

A comparison of the above regression results with those of other studies shows that after World War II and before 1973, the estimate of the rate of TFPG obtained by this study is 1.41%, which lies within the range of estimates of previous studies, 1.37%-1.98%, as shown in Table 1. After 1973, the rate of TFPG is 0.55%, which is also within the range of estimates obtained by Denison, -0.37%, and Kendrick, 0.6%, as shown in Table 1.

Table 4. The Contributions of TFPG, Capital Input and Labour Input to the U.S. Economic Growth (Calculated by Regression Result)

Period	TFPG (1)	Output z (2)	Capital			Labour			Percentage Contribution		
			α (3)	h (4)	α*h (5)	ß (6)	l (7)	ß*l (8)	TFPG/z	α*h/z	ß*l/z
1951-88	1.07	3.20		3.56	0.77		1.73	1.36	0.33	0.24	0.43
1951-73	1.41	3.65	0.215	3.82	0.82	0.785	1.80	1.41	0.39	0.22	0.39
1974-88	0.55	2.50		3.17	0.68		1.63	1.28	0.22	0.27	0.51

a. α and ß are based on 1951-88 ZHLT(OLS) with CRS imporsed. Other variables are defined in section 5 except the column of percentage contribution, all values are their mean.

Table 5. The Contributions of TFPG, Capital Input and Labour Input to the U.S. Economic Growth (Calculated by Growth Accounting)

Period	TFPG (1)	Output z (2)	Capital			Labour			Percentage Contribution		
			w_H (3)	h (4)	w_H*h (5)	w_L (6)	l (7)	w_L*l (8)	TFPG/z	w_H*h/z	w_L*l/z
1951-88	0.83	3.20	0.34	3.56	1.21	0.66	1.73	1.14	0.27	0.38	0.36
1951-73	1.14	3.65	0.35	3.82	1.34	0.65	1.80	1.17	0.31	0.37	0.32
1974-88	0.37	2.50	0.32	3.17	1.01	0.68	1.63	1.11	0.15	0.42	0.44

a. All variables are defined in section 5 except the column of percentage contribution, all the values are their mean.

Now I investigate the reason for the post-1973 slowdown in the U.S. economy. Table 4 shows that all three sources of economic growth decreased after 1973. The decreases in the rate of TFPG, capital's contribution and labour's contribution are 0.86, 0.14 and 0.13 percentage points respectively. The relative percentage contributions of the three sources of output growth may reveal the reason for the slowdown. The contribution of TFPG decreases by 17 percentage points, while the contributions of capital and labour input increase by 2 and 10 percentage points respectively.

Some investigators found that capital plays a major role in the slowdown in labour productivity growth (Norsworthy 1984), but the slowdown in capital investment may be due to the slowdown in TFPG, since there have been fewer important innovations in technology after 1973 than before (Lewis 1979), resulting in fewer profit opportunities and thus less incentive for investment. Therefore, the capital share decreased from 0.35 to 0.32. The growth rate of capital input decreased by 17% while the growth rate of labour input only decreased by 9%. The percentage contribution of labour input to output increased from 39% to 51% while that of capital input only increased from 22% to 27%.

6. Conclusion

This study estimates returns to scale and TFPG for the U.S. economy over the period 1951-1988, and the sub-periods 1951-1973 and 1974-1988. The regression is based on the Solow model with primary inputs capital and labour. The difference between this study and others is that the capital input is measured differently. In previous studies, capital input is measured using the capital stock. In the regression approach, capital input is defined as the capital stock. In growth accounting, capital input was distinguished from the capital stock, but it was assumed to be proportional to the capital stock. This study is based on the well-established Jorgenson capital input theory, which measures capital input as the capital cost. Based on the evaluation and comparison of different methods of capital input measurement, this study estimates capital input by the Statistics Canada operational definition rather than by Jorgenson's formula.

The Statistics Canada method has some advantages. First, the method is consistent with the conceptual definition based on economic theory in that it takes the user cost rather than the purchase cost as the estimate of capital input. The user cost is the value of the flow of capital services, while the purchase cost is the value of the capital stock. Second, the method is operational.

The main findings of this study are the following:

(i) In the post war U.S. economy, the asumption of constant returns to scale holds. The equation with CRS imposed performs the best overall.

(ii) For the post war period, TFPG has decreased at an annual rate of 0.00041. The average TFPG during the period 1974-1988 is 0.55 which is only 40% of average TFPG during 1951-1973. TFPG plays a major role in the slowdown of the U.S. economy; the contribution of TFPG decreased from 42% to 27%. The model with a time trend is very stable over the whole sample period of 1951-1988, and there appears to be no structural break in 1973.

The main weaknesses of this study are that the price index for capital input is an imperfect proxy. The combination of the Solow model with the Statistics Canada capital input measurement has some advantages. First, the method is consistent with the conceptual definition based on economic theory. Second, it is more objective than growth accounting, since it does not require one to impose CRS.

Notes

1. Aggregate output, capital input and labour input can be represented as the sum of the their components, or assumed to be a translog function of their components respectively. For example, the translog aggregate of output Q is defined as

$$Q=\exp[\alpha_1 \ln Q_1+\alpha_2 \ln Q_2+...+\alpha_m \ln Q_m$$
$$+1/2\beta_{11}(\ln Q_1)^2+\beta_{12}\ln Q_1 \ln Q_2+...$$
$$+1/2\beta_{mm}(\ln Q_m)^2].$$

If we replace Q by H or K in the above equation, we can get the translog function for capital or labour. This specification is very difficult to apply at the national level, since at the national level each aggregate has too many components.

2. See Minasian (1969), Griliches (1980, 1980b), Mansfield (1980), Griliches and Lichtenberg (1984) and Link (1978).

3. Levy (1990) found that the F statistic for testing the hypothesis of CRS is 3.9 with DF1=1 and DF2=139. The critical value for F(1,139) is not directly available. The critical values for F(1, 60), F(1, 120) and F(infinite) are 4.00, 3.92 and 3.84 respectively, which suggests that he could not reject CRS.

4. In the real world, there is a profit. Thus nonlabour payments can be divided into two parts: capital costs and profit. A better way to estimate capital input would be to assume that the physical input is proportional to the capital costs. However, in order to do so one must be able to distinguish capital cost and profits. In the national accounts, corporate profits include both the rental cost due to corporations using their own capital assets, and economic profits. The first part is really a capital cost just like the rental cost incurred for the working capital borrowed from a bank. Unfortunately, I am unable to distinguish between the two parts so I cannot assume that the capital input is proportional to the capital cost.

I also applied regression approach to Denison's (1985) data, under the assumption that capital input is proportional to the capital stock. The results, which seem unreasonable, see appendix A.

5. Further details on data sources are provided in appendix 1.

6. The real capital input is calculated using this producer price index measured in 1982 constant dollars, and then the value is then approximately converted into 1987 constant dollars, since the base year of GDP deflator used is 1987.

7. SHAZAM uses Cochrane-Orcutt iterative procedure with a convergence criterion of 0.001.

For more detailed information on this procedure see chapter 9 of Judge et al (1989).

8. N_1 and N_2 are the number of observations in sub-periods 1 and 2 respectively, and k is the number of parameters to be estimated.

9. See Green (1993, 216-220) for a definition of the recursive residuals and the formulas for the test statistics.

Appendix 1

Source: *National income and product Accounts of the United States U.S. (1993)*

GDP (current $) and Labour Compensation: Table 1.9 Gross domestic product and labour compensation (Billions of dollars)

GDP (1987$) and Indirect Business Tax: Table 1.10 Gross domestic product and indirect business tax and nontax liability plus business transfer payments less subsidies plus current surplus of government enterprises (Billions of 1987 dollars)

Labour Input: Table 6.9B Labour input hours (Millions of hours);

Source: *Statistical Abstract of the United States* (1994) 114th Edition.

Producer Purchasing Power Index: Table 746 Purchasing Power of the Dollar:1950-1993. Producer prices based on finished goods index. Obtained by dividing the average price index for the 1982=100, PPI (Producer Price Indexes). Finished Goods index is the weighted average of consumer goods index and Capital equipment index. The weights used in those indexes represent the total net selling value of commodities produced in the U.S. See pages 485, 486 and page 496 table 753 Producer Price Indexes, by Stage of Processing: 1960 to 1993.

Appendix 2

I also applied the regression approach to Denison's (1985) data including his measure of capital input. The ZHL and the ZHLT models described in section 5.1 are applied to different subperiods. In order to show that the capital stock, either net or gross, is not a good measure of capital input in the growth rate regression model, different capital stocks are used. For the periods 1948-1982 and 1951-1973, the growth rate of capital input is defined as the growth rate of the net capital stock including both fixed capital stock and inventories; for the period 1951-1982, the net capital stock is replaced by the gross capital stock. The estimation results, data source and description are shown in Table A.

It is easy to see that the overall performance of the nine regressions is not as good as that of the regressions that use capital costs as capital input given in Table 3. First, the adjusted R^2s are not as high as those in Table 3. Half of the estimated rates of TFPG are not significantly different from zero at the 5% significance level. None of the estimated coefficients of the growth rate of capital input (i.e., the growth rate of net or gross capital stock) for different subperiods pass the t-test, and some of them are negative. Three coefficients of the rate of growth of the gross of capital stock are statistically different from zero at the 20% significance level; however, two of them are negative, and α is positive only in the ZHL(OLS) equation for 1948-1982 (net capital stock). But when a time trend was introduced, in the ZHLT(OLS) equation, the estimate of α did not pass the t test (t=0.33), and when the capital input was dropped, in the ZLT(OLS) equation, the adjusted R^2 became even higher than that in the ZHLT(OLS) equation, from 0.80 to 0.81, which implies that the variable -- the growth rate of the net capital stock -- has no contribution to explain the growth rate of output.

The estimates of both α and ß vary greatly in the equations with or without including a time trend, and they both vary greatly in the different samples. α and ß also differ greatly from their actual shares of capital and labour, and thus are difficult to interperet. Therefore, from the viewpoint of econometrics, neither the gross nor the net capital stock seem to be appropriate measures of capital input using Denison's data.

Table A. The Regression Results for the U.S. Private Economy

Period	Model	Result	R^2	DW[a]	WH[b]	$\alpha+\beta$	Test $\alpha+\beta=1$	Capital input
1948-82	ZHL (OLS)	z=0.00933+0.3282h+1.0816l (1.037) (1.392*) (7.852)	0.75	1.668 (1.57)		1.41	4.21 (4.17)	growth rate of net capital stock
	ZHLT (OLS)	z=1.4824-0.00074t+0.0762h+1.2033l (2.966) (-2.948) (0.337) (9.228)	0.80	1.934 (1.66)		1.28	2.29 (4.17)	
	ZLT (OLS)	z=1.5472-0.00078t+1.2289l (3.04) (-3.361) (11.82)	0.81	1.890 (1.57)				
1951-73	ZHL (OLS)	Z=0.02985-0.1758h+1.2334l (2.744) (-0.603) (7.354)	0.81	1.712 (1.54)		1.06	0.077 (4.35)	
	ZHLT (OLS)	Z=0.1346-0.00005t+0.1620h+1.2279l (0.1651) (-0.1285) (-0.5106) (6.924)	0.80	1.720 (1.66)		1.07	0.088 (4.35)	
	ZHL (OLS)	z=0.03245-0.2427h+1.2427l (5.927) (-1.511*) (7.734)	0.82	1.712 (1.54)		**CRS**		
1951-82	ZHL (AUTO)	z=0.02925-0.3040h+1.2763l (1.764) (-0.6478) (8.807)	0.78		1.48	0.97	0.005 (3.33)	growth rate of gross capital stock
	ZHLT (OLS)	z=1.1972-0.0006t-0.02757h+1.2038l (2.221) (-2.171) (-0.068) (8.18)	0.76	1.684 (1.66)		1.18	0.297 (4.18)	
	ZHL (AUTO)	z=0.02814-0.27249+1.2725l (5.904) (-2.053*) (9.589)	0.78		1.47	**CRS**		

a. Definition: z is the growth rate of GDP of the private economy in the U.S., h is the growth rate of the nonresidential business capital stock (net or gross) measured in 1972 dollars, l is the growth rate of manhours in the private economy, others are defined as the in Table 3.

b. Sources: z and l are based on Denison (1985, Table 2-1 p77), h is based on Denison (1985, Table 4.1 p92).

c. Superscript * in t ratio indicates that the coefficient is significantly different from zero at the 20% significance level.

References

Arrow, K.J. (1968) "Optimal Capital Policy With Irreversible Investment," *Papers in Honour of Sir John Hicks, Value, Capital and Growth*, Editor, J.N. Wolfe, Edinburgh.

Baumol, W., A.S. Blinder and W.M. Scarth (1991) *Macroeconomics, Economics, Principles and Policy*, Third Canadian Edition, Harcourt Brace Jovanovich, Canada.

Bhagwati, J.N. (1983) *Lectures on International Trade,* The MIT Press, Massachusetts.

Brown, R.S. (1968) *On the Theory and measurement of Technological Change,* Cambridge University Press, New York.

Christensen, L.R; D. Cummings and D.W. Jorgenson (1980) "*Economic Growth 1947-73; An International Comparison,*" in Kendrick and Vaccara (ed) 1980: 595-691.

Christensen, L.R; Jorgenson, D.W.; and Lau, L.J. (1971) "Conjugate Duality and the Transcendental Logarithmic Production Function," *Econometrica* 39: 255-56.

---- (1973) "Transcendental Logarithmic Production Functions," *Review of Economic and Statistics* 55: 28-45.

Cullison, W.E. (1993) "Public Investment and Economic Growth," *Federal Reserve Bank of Richmond Economic Quarterly* 79(4):19-33.

Daly, D.J. (1980) "Comment," in Kendrick and Vaccara (ed.) (1980): 691-98.

Denison, E.F. (1969) "Some Major issues in Productivity Analysis: an Examination of Estimates by Jorgenson and Griliches," *Survey of Current Business* 49 1-27.

---- (1967) *Why Growth Rates Differ, Postwar Experience in Nine Western Countries*, Washington, D.C., The Brookings Institution.

---- (1979) *Accounting for Slower Growth*, Washington, D.C., The Brookings Institution.

---- (1985) *Trends in American Economic Growth, 1929-1982*, Washington, D.C., The Brookings Institution.

---- (1989) *Estimates of productivity Change by Industry*, The Brookings Institution, Washington, DC.

Diewert, W.E. (1980) "Aggregation Problems in Measurment of Capital," in Usher, D. (ed) (1980), 433-582.

Diewert, W.E. (1991) *The Measurement of Productivity in Regulated Industries,* Discussion Paper No. 91-20, Department of Economics, University of British Columbia, Vancouver, Canada V6T 1W5.

Fare, R. and Nijinkru, D. (1989) "Computing Returns to Scale," *Economics Letters* 30: 55-59.

Green, W.H. (1993) *Econometric Analysis*, second edition, Macmillan Publishing Company, New York.

Gollop, F.M. and D.W. Jorgenson (1980) "Productivity Growth by Industry, 1947-73," in Kendrick and Vaccara (ed) 1980.

Griliches, Z. (1980) "Returns to Research and Development Expenditures in the Private Sector," in Kendrick and Vaccara (ed) 1980.

---- (1980b) "R&D and the Productivity Slowdown," *American Economic Review* 70 (2), 343-348.

---- and F. Lichtenberg (1984) "Inter-industry Technology Flows and Productivity Growth: A Reexamination," *Review of Economics and Statistics* 61, 324-29.

Hicks, J. (1989) "The Assumption of Constant Returns to Scale," *Cambridge Journal of Economics 13:* 9-17.

Holtz-Eakin, D. (1993) "Solow and the States: Capital Accumulation, Productivity, and Economic Growth," *National Tax Journal* 46(4): 425-39.

Hulten, C.R. and Wykoff, F.C. (1981) "The Measurement of Economic Depreciation," in Hulten, C.R., ed., *Depreciation, Inflation and the Taxation of Income capital*, papers from a conference held by The Urban Institute, 81-132.

Intriligator (1965) "Embodied Technical Change and Productivity in The United States 1929 1958," *Review of Economics and Statistics* 47: 65-70.

Johnson, J. (1994) "Analyzing Canadian Manufacturing Using the KLEMS," in *Statistics Canada*, Cat. No. 15-204E, *Aggregate Productivity Measures*. Ottawa: Ministry of Science and Technology.

Jorgenson, D.W. and Griliches, Z. (1967), "The Explanation of Productivity Change, " *Review of Economic Studies* 249-82.

Jorgenson, D.W., Gollop, F.M and Fraumeni, B.M. (1987), *Productivity and U.S. Economic Growth*, Harvard University Press, Cambridge, Massachusetts, and London.

Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lutkepohl and T.C. Lee (1990) *Introduction to the*

Theory and Practice of Econometrics, 3rd ed. New York, Wiley.

Kendrick, J.W. (1980) *Productivity in the United States, Trends and Cycles*. Johns Hopkins University Press, Baltimore and London.

---- (1981) "International Comparisons of Recent Productivity Trends," In W.Fellner (ed), *Essays In Contemporary Economic Problems: Demand, Productivity and Population,* Washington, D.C.: American Enterprise Institute.

---- and Vaccara, B.N. (ed.) (1980) *New Developments in Productivity Measurement and Analysis*, The University of Chicago Press, Chicago and London.

Kim, H.Y. (1992) "The Translog Production Function and Variable Returns to Scale," *Review of Economics and Statistics* 74(3): 546-52.

Levy, D. (1990) "Aggregate Output, Capital, and Labour in the Post-War U.S. Economy," *Economics Letters* 33(1): 41-45.

Lewis, W.A. (1979) "The Slowing Down of the Engine of Growth," *American Economic Review*, 70 (4) 1980.

Link, A.N. (1978) "Rates of Induced Technology from Investments, Research and Development," *Southern Economic Journal* 55: 370-79.

Lucas, R. (1988) "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22: 3-42.

Mankiw, N.G., Romer, D. & Weil, D.N. (1992) "A Contribution to the Empirics of Economic Growth ", *The Quarterly Journal of Economics* 107: 407-435.

Mansfield, E., (1980) "Basic Research and Productivity Increase in Manufacturing," *American Economic Review* 70: 863-73.

Minasian, J.R. (1969) "Research and Development, Production Function and Rate of Return," *American Economic Review* 59: 80-5.

Mohr M. F. (1986) "The Theory and Measurement of the Rental Price of Capital in Industry, Specific Productivity Analysis: A Vintage Rental Price of Capital Model," *Measurement Issues and Behaviour of Productivity Variables*, edited by A. Dogramaci, Kluwer. Nijhoff Publishing, Boston.

Norsworthy, C.J. (1984) "Growth Accounting and Productivity Measurement," *Review of Income and Wealth* 30: 309-29.

----, and Jang, S.L. (1992) *Empirical Measurement and Analysis of Productivity and technological Change: Applications in High-Technology and Service Industries.* Amsterdam: Elsevier Science Publisher B.V.

---- (1990) "Endogenous Technological Change," *Journal of Political Economy* 98: s71-103.

Peden, E.A. (1991) "Productivity in the United States and Its Relationship to Government Activity: An Analysis of 57 Years, 1929-1986," *Public Choice* 69(2): 153-73

Postner H.H. and Wesa, L. (1983) *Canadian Productivity Growth: An Alternative Analysis*, Canadian Government Publishing Centre, Supply and Services Canada, Ottawa, Canada.

Romer, P.M. (1986) "Increasing Returns and Long-Run Growth," *Journal of Political Economy* 94: 1002-37.

Solow, R.M. (1957) "Technical Change and Aggregate Production Function," *Review of Economics and Statistics* 39: 312-320.

Statistics Canada (1992) *Aggregate Productivity Measures*, Cat. No. 15-204E

Statistics Canada (1994) *Aggregate Productivity Measures*, Cat. No. 15-204E

U.S. Bureau of the Census (1994) *Statistical Abstract of the United States*, 114th Edition, , Washington D.C.

U.S. Department of Commerce Bureau of Economic Analysis (1993) *National Income and Product Accounts of the United States,* Washington DC, U.S. Government Office 1993.

Usher, D. (ed) (1980) *The measurement of Capital*. The University of Chicago Press, Chicago and london.

Wolff, E. N. (1991) "Capital Formation and Productivity Convergence Over the Long Term," *The American Economic Review* 81: 565-79.

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