<u>TWO-SECTOR KALECKIAN MODELS OF GROWTH</u>

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INTRODUCTION

The models within the Kaleckian tradition have been developed for one economic sector only. The idea is to deal with a typical firm in such a way that it may capture the general behaviour of the economic agents. Kaleckian authors assume that modern industrial economies operate under an oligopoly environment and that this element should be incorporated in the way a typical firm sets its price. The main economic problems undertaken by these models are income distribution and capital accumulation. Particular emphasis is given to the impact of changes in certain key parameters, such as the mark-up of the firm and the propensity to save out of profits, the real wage rate, the effective demand of the economy, the level of output and the rate of growth of the economy.

The purpose of this paper is to extend the standard Kaleckian model of one sector to an economy in which there are two sectors. One of the sectors produces capital goods and the other consumption goods. Most of the assumptions of the one-sector model, that are presented in the first chapter, are assumed to hold in the two-sector model, and the problems that are tackled are also the same. The objective is to verify whether the results of the one sector system still hold when another sector is added to the model.

The literature on two-sector models within the Kaleckian framework is almost non-existent. There are few papers that incorporate two sectors in a growth model. The sectors included are the consumption goods and capital goods sectors. Although the model that is developed here borrows some equations from Dutt (1988), the objectives of the two models are different. Dutt aims to discover under which conditions the rates of profit become equal or different in the long run. The model presented in this paper, on the other hand, seeks to compare the impact of changes in certain parameters with those of the one-sector model. As a result, the solution presented here is distinct from Dutt's solution.

Another point of dissimilarity between the two models is the way in which the firms in the two sectors set their prices. In the model that is presented in chapter two, interdependence between the two sectors and mark-ups is allowed through what is called target return pricing. This element is not considered by Dutt (1988), who uses a simple mark-up pricing procedure. The motive for incorporating this link between the mark-ups arises from the argument presented by Steedman (1992), which states that Kaleckian one-sector models are an oversimplification of the reality. Steedman (1992) claims that if more than one sector is included, the mark-ups of some sectors will affect the prices of other sectors. The concern here is to analyze whether the introduction of one extra sector alters the results of the one-sector Kaleckian model.

This paper is divided into two main chapters. In the first chapter, the standard one-sector model is presented. Its assumptions, equations, solution and some comparative static exercises are presented. Also, a variant of Dutt (1990), incorporating savings by workers is included. At the end of chapter one, the argument by Steedman is briefly presented. In the second chapter, the two-sector model is developed. It follows the basic structure of the one-sector model. Its assumptions, main equations, solution and some comparative static exercises are presented. The conclusion recapitulates the main results of the study and outlines some possibilities for future research.

1. THE KALECKIAN GROWTH MODELS

Many growth models have been developed under the 'Kaleckian' label. The purpose of this section is not to provide an exhaustive survey of them, but to highlight the crucial features that are usually present in these models. The version presented here is adapted from the models developed by Rowthorn (1981) and Lavoie (1992).

The assumptions of these models are the following:

a) the economy produces only one good, which is at once a consumption and capital good;

b) firms are 'sufficiently' homogeneous: the behaviour of the economy can be analyzed by considering a typical firm;

c) the economy is closed and there is no government. The first part of this assumption is relaxed by Blecker (1989) and Sarantis (1990-91). The action of the government in Kaleckian growth models is very briefly presented in Rowthorn (1981);

d) there is no financial system. Where the monetary issue has been incorporated into Kaleckian growth models, it has been through the inclusion of the interest rate (assumed to be exogenously given). Dutt and Amadeo (1989) and Lavoie (1992) incorporate this element.

1.1. The Production Function

There are two factors of production: capital and labour. While capital is usually assumed to be homogeneous, some authors (Rowthorn 1981, Lavoie 1992) distinguish between staff employees and factory workers. The former are usually fixed, with a given level of capacity, while the latter vary with the level of output.

Capital is assumed to be fixed, and there is no circulating capital. However, some authors prefer to use circulating capital, such as Sarantis (1990-91), for example, who uses intermediate imported inputs in his model of an open economy.

There is no substitution between the factors of production. It is assumed that firms operate with idle capacity. Therefore, changes in demand cause changes in output and not in prices. As shown below, prices are sticky when there is available capacity.

The first equation represents the total output of the economy:

$$(1.1) \quad q = (1/a_\ell)\ L_v$$

q is total output, $(1/a_\ell)$ the average product of labour and L_v, variable employment that may go up or down, in line with output. The term a_ℓ is the requirement of variable labour per unit of output. As the assumed production function has fixed coefficients, it is clear that $a_\ell = L_v/q$.

Let q^f be the maximum output that can be produced with the available equipment. It is assumed that the relationship between the stock of capital, K, and the maximum possible output is constant:

$$(1.2)\quad q^f/K = 1/V$$

Let (1/V) be the constant parameter that measures the ratio of the full-capacity output to the capital stock. The variable V specifies the amount of capital stock required to produce a unit of output when the economy is at its full-capacity level. At this maximum stage, the number of variable workers required is stated in equation (1.3):

$$(1.3)\quad L_v^f = a_\ell\, q^f$$

and the ratio of permanent staff to variable workers, in equation (1.4):

$$(1.4)\quad f = L_p/L_v^f$$

The variable L_p indicates permanent employees and f is assumed to be given.

As firms are not operating at full-capacity, a measure for the level of utilization is needed.

$$(1.5)\quad U = q/q^f$$

In equation (1.5) U is the level of utilization of capacity which ranges between zero and one, being one at full-capacity. When making the proper substitutions it is also true that:

(1.6) $U = L_v / L_v^f$

1.2. <u>The price system</u>

Kaleckians assume that firms behave in an oligopolist environment. This allows them to set a mark-up over costs. As Rowthorn (1981) indicates, "... prices are determined by monopolistic factors, and shifts in demand have little effect on the general price level. The average firm responds to shifts in demand by varying the amount it produces, whilst keeping its price roughly constant"[1].

Equation (1.7) formalizes this idea:

(1.7) $P = (1 + \theta)\ UC$

P is the price of the good, UC is the cost of production per unit of output and θ is the mark-up set over these unit costs.

There is a controversy about which sort of unit costs should be considered. If overhead labour is taken into account, unit costs vary with output, and this may alter the price, unless some fixed value of costs is assumed. Lavoie (1992) explains three variants of

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[1] Rowthorn (1981) p. 5.

pricing procedures in Kaleckian models, although he indicates that there is no fundamental difference between them.

The first one is called the "mark-up variant" and assumes that the unit costs considered are average variable costs, mainly labour costs (and intermediate inputs, when included). Since a fixed-coefficient production function is assumed, variable costs are constant[2]. Therefore, it is not necessary to specify at what level of output the average variable costs are being computed. The price remains the same, provided the firm operates below full-capacity.

The second procedure, presented by Lavoie (1992), is the full cost variant. This takes into account not only average variable costs, but also the average of fixed costs. The average variable costs are constant, but the average fixed cost changes according to the level of output at which they are computed. Lavoie (1992) and Rowthorn (1981) affirm that when the full cost pricing procedure is considered, the average (fixed plus variable) costs are calculated over a 'normal' level of output. While the actual cost of production may vary with the level of output, the 'normal' or 'standard' cost does not change, and so firms are not induced to alter their prices. This implies that the price remains the same although output may vary.

The third variant, shown by Lavoie (1992), is target return pricing. According to Lavoie (1992), target return pricing is a

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[2] It is assumed that start-up costs are not included in the model.

specification of full cost pricing, rather than a different procedure. Indeed, target return pricing provides an explanation of how the mark-up is calculated. Given the normal level of output of the firm, this procedure establishes a relationship between the normal or desired level of profits, and the mark-up that is required to achieve this target. In the model that is presented in the next chapter this pricing and mark-up procedure is considered.

Rowthorn (1981) and Lavoie (1992) assume that the only variable costs are labour costs. Therefore,

$$(1.8) \quad UVC = a_{\ell} W$$

where UVC is unit average variable costs. Expression (1.8) shows that unit average variable costs are given by the labour requirement per unit of output, a_{ℓ}, times its cost, i.e., the nominal wage rate.

Finally, the following expression is useful to make some derivations below.

$$(1.9) \quad M = \theta/(1+\theta).$$

M could be considered a measure of the so called "degree of monopoly" in the economy. According to some Kaleckians, the larger the M, the greater the capability of firms to pass costs on to prices.

It should be noted that the above equations only apply for levels of output below full capacity. When the maximum is reached, the only way that an excess of demand can be eliminated is through an increase in prices. At this point, prices are no longer fixed but become flexible, as firms are not able to satisfy excess demand. It is important to reiterate that these models assume firms always produce below full-capacity.

The next section presents the rate of profit of firms which is a key concept in Kaleckian models. In section 1.4 the price equations shown above and the rate of profit from section 1.3 are combined to solve the model.

1.3. The rate of profit

Given the above equations, a relationship between the rate of profit and the level of utilization of capacity can be derived.

Total net profits, in real terms, are given in equation (1.10):

$$(1.10) \quad \Pi = q - W^T/P - \delta K - t_\pi K$$

Π is total net profits, W^T/P is the total wage bill both in real terms, δK measures the amount of depreciation as a constant ratio δ of the stock of capital, and $t_\pi K$ is the amount of taxes paid at a rate of t_π of the stock of capital[3].

[3] The only purpose of the introduction of depreciation and taxes in expression (1.10) is to be consistent with Rowthorn (1981)'s paper. However, depreciation and taxes will not be used later.

Since total employment is given by $L_p + L_v$, and making (W/P) the real wage rate, it follows that:

$$(1.11)\quad W^T/P = (W/P)(L_p + L_v)$$

Substituting (1.11) into (1.10), and dividing the whole expression by the capital stock, the rate of profit is found to be:

$$(1.12)\quad r = \frac{\Pi}{K} = \frac{[q - (W/P)(L_p + L_v)]}{K} - \delta - t_\pi$$

Equation (1.12) shows the standard negative relationship in Kaleckian models between the rate of profit and the real wage rate. More is said about this in the section containing the comparative static exercises.

1.4. <u>The equations and the solution of the model</u>

The following are the main equations of the Kaleckian model: From (1.7) and (1.8)[4]:

$$(1.13)\quad P = (1+\theta)\ a_\ell\ W$$

Solving for (W/P) and using the value of M:

[4] Rowthorn (1981) and Lavoie (1992) consider a simple mark-up procedure. For purposes of setting their prices, firms take into account the unit average variable costs, only.

$$(1.14)\quad W/P = (1-M)/a_{\ell}$$

Substituting equation (1.14) into equation (1.12) and making the proper substitutions from expressions (1.1), (1.2), (1.3), (1.4) and (1.6):

$$(1.15)\quad r = \frac{M}{V} U - \frac{(1-M)}{V} f - \delta - t_{\pi}$$

Equation (1.15) depicts what Rowthorn calls "the profits curve", and what Lavoie (1992) calls "the profits cost curve". According to Rowthorn, it indicates the amount of net profits "created" at each level of utilization of capacity, given the production techniques and the current real wage rate. He argues that, depending on the demand conditions, firms will be able to realize the profits that have been created during the productive process. In fact, if the total output is not sold, all the profits cannot be realized. From (1.15) it is clear that the rate of profit is an increasing function of the level of utilization of capacity. An increase in the mark-up, through M, also affects positively the rate of profit.

Another set of equations that may determine the level of utilization is required. This set is now presented.

In Rowthorn's model, the savings of the economy are given by the ratio of profits, s_p, that are saved. It is assumed that workers do not save, and that the government borrows a certain amount, B, from the private sector[5]. The net savings of the economy, S, are:

$$(1.16) \quad S = s_p \Pi - B$$

In the absence of a foreign sector, these savings must be equal to the full amount of investment in the economy:

$$(1.17) \quad I = S$$

Substituting (1.16) into (1.17), and dividing by K, we find that the growth rate of capital, G, is equal to:

$$(1.18) \quad G = r\, s_p - b$$

where G = I/K and b = B/K. Solving for the rate of profit:

$$(1.19) \quad r = (G + b)/s_p$$

Equation (1.19) is a variant of the "Cambridge Equation". If the

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[5] To simplify the model, Rowthorn assumes away the complications of the interest payments due to the borrowing of government. The savings by workers is included in an alternative model by Dutt (1990), see below.

rate of growth of capital, G, is assumed to be given, as a consequence of past investments, and b is also exogenous, then the rate of profit can be determined directly from (1.19). For the purpose of solving the model, equation (1.18) is kept, with a slight modification.

$$(1.20)\quad G^s = r\, s_p - b$$

The superscript 's' over G, indicates that this expression incorporates the savings side of the rate of growth, as Lavoie (1992) indicates.

However, Kaleckian authors, in contrast to post-Keynesian authors such as Kaldor and Pasinetti, do not consider an exogenous rate of growth. Instead, they consider it to be endogenous. The two main variables that determine the rate of growth of capital, according to Rowthorn (1981) and Dutt (1984), are the rate of profit and the level of utilization of capacity:

$$(1.21)\quad G^i = \alpha + \beta r + \tau U \qquad \beta,\ \tau > 0$$

In this equation, α is exogenous, and β and τ are behavioural parameters. The reasons for using these variables are the following: an increase in the rate of profit induces the entrepreneurs to expand their activities with the objective of making higher profits in the future; a level of utilization that comes closer to full-capacity causes higher investment, in an attempt to avoid physical constraints to production. The superscript 'i' denotes the inclusion of the investment side of the rate of growth.

There are now three equations: (1.15), (1.20) and (1.21), with three variables: G, r, and U.

In equilibrium: $G^i = G^s$

Combining equations (1.20) and (1.21):

$$(1.22) \quad r = \frac{\tau}{s_p - \beta} U + \frac{b + \alpha}{s_p - \beta}$$

This is what Rowthorn (1981) calls "the realization curve" and what Lavoie (1992) calls "the effective demand curve". According to Rowthorn, for any given level of utilization of capacity, equation (1.22) shows the *realized* rate of profit. It reflects the fact that the amount of investment is equal to the amount of savings. Although Lavoie (1992) is not very explicit here, the label he assigns to equation (1.22) denotes that it reflects the relationship between the actual level of utilization and the realized rate of profit.

Before solving the model, some changes are made to the investment growth function that are useful to understand the stability conditions that follow. Lavoie (1992) assumes that entrepreneurs adjust their investment decisions to a certain expected rate of profit, r^e, and, moreover, to an expected rate of utilization of capacity, U^e. Equation (1.21) can be rewritten as:

$$(1.21') \quad G^i = \alpha + \beta r^e + \tau U^e$$

The expectation functions of the rate of profit, r^e, and the level of utilization, U^e, presented by Lavoie, are dependent on their expected and realized values of the previous period, following an adaptive process:

$$(1.23) \quad r^e_t = \epsilon r^e_{t-1} + (1-\epsilon) r_{t-1} \qquad \epsilon < 1$$

$$U^e_t = \epsilon U^e_{t-1} + (1-\epsilon) U_{t-1} \qquad \epsilon < 1$$

For the solution of the model, the two growth equations are kept. Substituting the value of r from equation (1.15) into (1.20) and (1.21'), the investment and realized savings functions are[6]:

$$(1.24) \quad G^s = s_p \,(M/V)\, U - s_p \,(1-M)\, f/V - s_p \,\delta - s_p \,t_\pi - b$$

$$(1.25) \quad G^i = \alpha + (\tau + \beta M/V) U^e - \beta f \,(1-M)/V - \beta\delta - \beta t_\pi$$

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[6] The expected value of r is not used because the adjustment process will be assumed to take place between the expected and the realized levels of utilization, and the rate of growth. It is also implicitly assumed at this point in Lavoie's model, that the desired rate of accumulation is equal to the realized rate of accumulation.

The savings function depends on the actual level of utilization, whilst the investment function depends on the expected level of utilization of capacity. At this point a stability condition for the model must be established. It will be shown that G^s must be more sensitive to changes in U than G^i for the model to be stable.

Some graphical analysis is useful. On Figure 1.1, the vertical axis measures the rate of growth of capital, and the horizontal axis measures the level of utilization of capacity. The stable case has been drawn, in which:

(1.26) $$s_p - \beta > \tau(M/V)$$

or:

(1.27) $$\tau < (s_p - \beta)(M/V)$$

This expression indicates that the parameter of the investment function related to the utilization rate must be small enough for the model to be stable[7].

Both G^i and G^s are increasing functions of U. U^* and G^* indicate the equilibrium point, A, where the two curves intersect.

[7] The form of this expression has been chosen for convenience. When analyzing the two-sector growth model in the next chapter, a similar condition will be also presented.

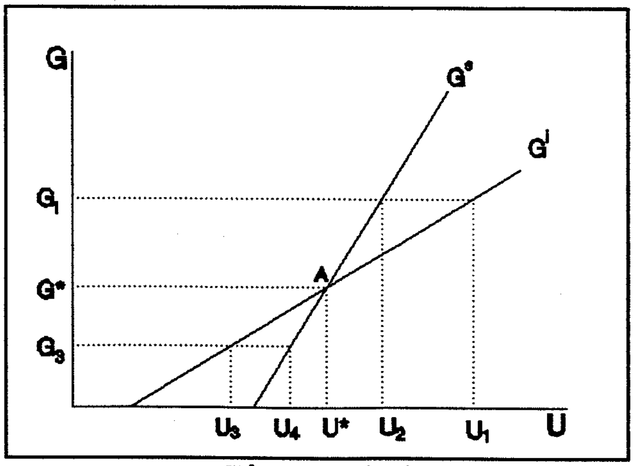

Figure 1.1

Its stability condition can be verified as follows: assume that the expected utilization level is U_1: at this point, the investment growth rate is equal to G_1. However, at this growth rate the realized level of utilization is U_2 only. According to the revision of expectations, given by the expression (1.23), firms reduce their expected level of utilization. This process continues, eventually reaching the equilibrium level, U^*. Starting from below the equilibrium level, if the expected level of utilization is U_3, the actual utilization rate is U_4. Firms change their expectations upwards approaching U^*.

It is worth noting that this equilibrium is achieved because the slope of G^s is greater than G^i. The next graph shows the unstable case where

$$s_p\, M/V < \tau + \beta M/V$$

As in the previous graph, Figure 1.2 shows the rate of growth of capital goods on the vertical axis, and the level of utilization on the horizontal axis. In this case the slope of G^i is greater than G^s. Both curves intersect at point A. However, this is not a stable equilibrium. Assume that the expected level of utilization by the firms is U_1. The rate of growth of investment is equal to G_1. But at that level of growth, the actual level of utilization is equal to U_2, higher than U_1. As firms recalculate their expectations upwards, they move further away from the equilibrium. Eventually they reach the full-capacity level of utilization, but at that point there is a permanent disequilibrium, as the rate of growth of investment is higher than the growth of savings[8].

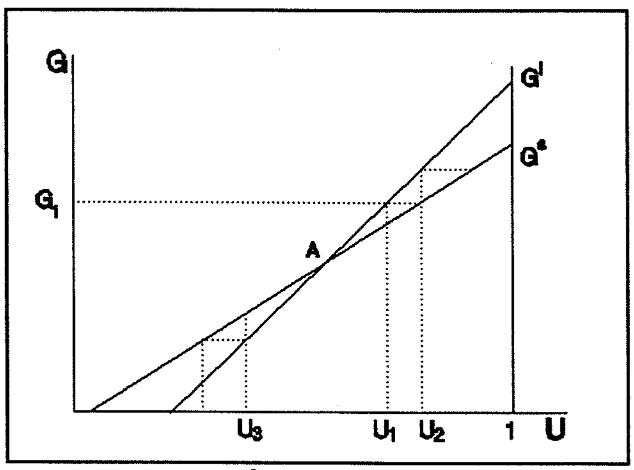

Figure 1.2

On the other hand, when the level of utilization of capacity is below the equilibrium, the actual level of utilization of capacity

[8] In general, Kaleckian models assume that there is excess capacity. The problem when the full-capacity level is achieved is beyond the scope of this paper.

is lower than the one expected by the entrepreneurs. This leads to a reduction in the level of utilization. As the equalization of growth rates cannot be achieved, eventually, firms reduce their output to zero and production does not take place. Therefore, the stable condition is the first case where $G^s > G^i$.

The relationship between the rate of profit and the level of utilization is now presented. In Figure 1.3, the vertical axis measures the rate of profit, and the horizontal axis measures the level of utilization. Equation (1.15) is under the 'PC' label, and equation (1.21) under the 'ED' label. The stable condition set above implies that the 'PC' curve must be more sensitive to changes in U, than the 'ED' curve.

The curves have been drawn such that

$$\tau < (s_p - \beta)(M/V)$$

which is the stable case. The equilibrium, in Figure 1.3, is at point A, with a level of utilization equal to U^* and the rate of profit r^*. The stability of this equilibrium point can be verified as follows: suppose that the level of utilization of capacity is U_1. At this point, there is excess supply, as effective demand is below the PC curve. This induces firms to reduce their output. This process continues until point A, where excess supply is eliminated.

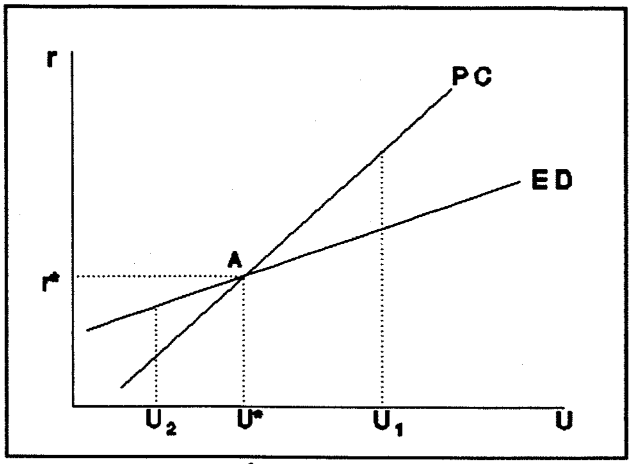

Figure 1.3

Another disequilibrium point is U_2, where the actual level of output is insufficient compared to effective demand. Therefore, firms increase the level of utilization of capacity until point A, where the goods market is cleared. Figures 1.1 and 1.3 are two faces of the same coin. The former links the rate of growth with the capacity level, while the latter relates the rate of profit with the same capacity level.

The equilibrium values for the rate of profit, the level of utilization and the rate of growth are the following:

$$(1.28)\quad U^{*} = \frac{\alpha(V/M) + (s_p-\beta)f(1-M)/M + b(V/M) + (s_p-\beta)(\delta+t_{\pi})(V/M)}{s_p - \beta - \tau(V/M)}$$

$$(1.29)\quad r^{*} = \frac{\alpha - \tau f(1-M)/M + b - \tau(\delta+t_{\pi})(V/M)}{s_p - \beta - \tau(V/M)}$$

$$(1.30)\quad G^* = \frac{\alpha s_p + s_p\tau[f(1-M)/M + (\delta+t_\pi)(V/M)] + b\beta + \tau b(V/M)}{s_p - \beta - \tau(V/M)}$$

Kaleckian authors use the share of gross profits, M, in their algebraic expressions to show its impact over other macroeconomic variables. However, the mechanism through which it actually operates is the real wage rate. Equation (1.14) shows the negative link between M and W/P. An increase in M reduces the real wage rate and this has an impact on the other variables. In order to make this relationship explicit, (1.14) is solved for M, and this value is substituted into (1.28), (1.29) and (1.30).

$$(1.28')\quad U^* = \frac{\alpha V + (s_p-\beta)f\ a_\ell(W/P) + bV + (s_p-\beta)(\delta+t_\pi)V}{[1 - a_\ell(W/P)](s_p - \beta) - \tau V}$$

$$(1.29')\quad r^* = \frac{\alpha[1-a_\ell(W/P)] - \tau f\ a_\ell(W/P) + b\ a_\ell(W/P) - \tau(\delta+t_\pi)V}{[1 - a_\ell(W/P)](s_p - \beta) - \tau V}$$

$$(1.30')\quad G^* = \frac{(\alpha s_p + b\beta)[1-a_\ell(W/P)] + s_p\tau[f\ a_\ell(W/P) + (\delta+t_\pi)V] + \tau bV}{[1 - a_\ell(W/P)](s_p - \beta) - \tau V}$$

Now that the role of the real wage rate is explicit, the impact of changes in the parameters can be evaluated.

1.5. Changes in the mark-up (θ)

Some comparative static exercises that show some important features of Kaleckian models are now considered.

Suppose that the mark-up of the firms increases. Figure 1.4 shows the impact of a higher θ on the rate of profit and the utilization level. Let point A be the initial position, with U^* and r^* being the steady state values. An increase in θ makes the 'profits cost curve' (PC) steeper and closer to the vertical axis. The new steady state is at B with lower values of the level of utilization and the rate of profit.

The economic reason is the following: the higher mark-up leads to an increase in prices. Since the money wage rate is exogenous, the real wage rate decreases. As the real wage rate goes down, the effective demand for goods declines. This negative effect of the wage rate over the level of utilization of capacity can be seen in equation (1.28').

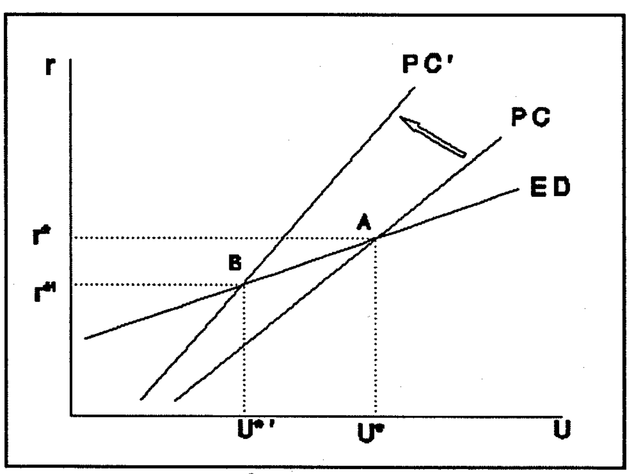

Figure 1.4

At the current level of utilization U^*, the effective demand is lower than the profits cost curve. Firms are no longer selling all their output, and are induced to reduce their production. As the level of output falls, the lower level of utilization causes a decline in the rate of profit (as indicated in both equations 1.15 and 1.22). The rate of growth is also lower, because the level of utilization and the rate of profit decrease.

This is one of the paradoxes of Kaleckian models. Despite the rise in the mark-up of firms the rate of profit decreases. When θ increases, firms achieve more profits per unit of output sold. But the key element is the effective demand issue. The output produced has to be sold. The increase in prices leads to a fall in the real wage rate, and the demand for output decreases. The decline in demand overcomes the higher profits per unit of output.

1.6. Changes in the propensity to save out of profits (s_p)

Suppose that firms become more thrifty, and increase their rate of savings, s_p. Figure 1.5 shows the impact on the rate of profit and the level of utilization.

Suppose the initial level is at A, with r^* and U^* the equilibrium values. An increase in s_p shifts down the effective demand curve, which can be seen in equation (1.22). The result is a lower rate of profit and a reduced level of utilization of capacity.

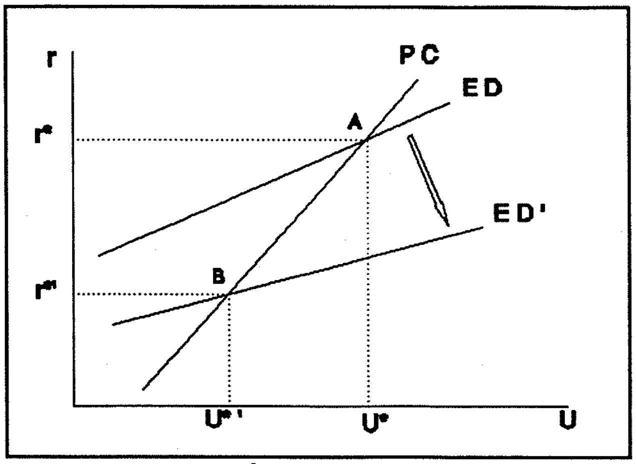

Figure 1.5

Deriving equation (1.28) with respect to s_p indicates:

$$(1.31)\qquad \frac{dU^*}{ds_p} = \frac{-[\delta + t_\pi + (1-M)f/V] - (\alpha + b)M/V}{[(s_p-\beta)M/V - \tau]^2} < 0$$

Thus, the greater rate of savings is inducing more leakages out of the economy. This can be seen more clearly from equation (1.24), by substituting the term (1-M) by a_ℓ (W/P):

$$(1.24')\qquad G^s = s_p\ (M/V)\ U - s_p\ a_\ell(W/P)\ f/V - s_p\ \delta - s_p\ t_\pi - b$$

An increase in s_p reduces demand arising from real wage earners. Although there is a counter-effect coming from the share of profit, M, it does not overcome the first impact, because U is also decreasing.

Figure 1.6 shows the effect on the growth rates. The G^i curve does not move (drawn with a discontinuous line). However, the slope of the G^s curve increases, and the intercept is reduced. The two curves intersect at a point to the southwest of the initial values (U^*,G^*).

The figure in Lavoie (1992), which describes the effect of a change in s_p on the growth rate, shows the G^s curve shifting up in a parallel way. This is not correct as can be verified from equation (1.24). In fact, the G^s curve rotates up over the point $[U,G] = [f(1-M)/M + V/M(\delta + t_\pi), -b]$.

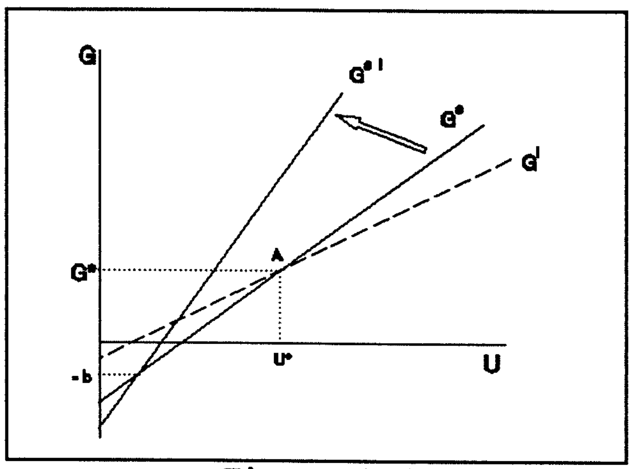

Figure 1.6

The higher s_p also leads to a lower rate of growth. This is the paradox of thrift. Higher rates of savings lead to a lower rate of utilization, a lower rate of profit and a lower rate of growth of the economy. In the end, they even lead to a lower amount of total savings.

1.7. Inclusion of the propensity to save of workers

So far, it has been assumed that only capitalists save a fraction of their earnings. Within the Kaleckian framework, Dutt (1990) builds a model in which the propensity to save by workers is included. In this section, the assumptions, methodology and conclusions of Dutt's model are briefly presented. The notation set above has been used, except where indicated.

Dutt (1990) assumes that workers save a proportion s_w of their income, which is lower than the capitalists propensity to save, s_c. Workers not only earn their wage, but they also receive income from the property of capital. The savings function of the economy, S, is the following:

$$(1.32)\quad S = s_f rK + s_c(1-s_f)rK^c + s_w(1-s_f)rK^w + s_w(q-rK)$$

In this expression, s_f is the propensity to save of the firm on total profits rK ($r = \Pi/K$), which is distributed between capitalists and workers according to their share of capital, K^c and K^w, respectively. The second term is the savings of capitalists' income from their property share of capital, while the third is the savings from workers' property income. Finally the fourth is the savings of wage income by workers.

Dividing this expression by K, the savings function of the economy is equal to:

$$(1.33)\quad G^s = s_f r + s_c(1-s_f)rk^c + s_w(1-s_f)r(1-k^c) + s_w(U-r)$$

where $k^c = K^c/K$. Note that $K^c + K^w = K$. Also, $U = q/K$, is a proxy variable of the level of utilization of capacity.

The rate of growth of capital in Dutt's model is equal to:

$$(1.34)\quad G^i = \alpha + \tau U$$

Note that the difference with the model shown above is that the rate of profit is not present in this investment function.

The last important equation is a simplification of the profits cost curve indicated under (1.15):

$$(1.35)\quad r = M\,U$$

In the short run, with a given k^c, the equilibrium value for the level of utilization can be found by combining equations (1.33), (1.34) and (1.35):

$$(1.36)\quad U^* = \alpha/[s_w - \tau + s_f(1-s_w)M + (s_c-s_w)(1-s_f)Mk^c]$$

The rate of growth of the economy in the short run is given by:

$$(1.37)\quad G^* = \alpha + \alpha\tau/[s_w - \tau + s_f(1-s_w)M + (s_c-s_w)(1-s_f)Mk^c]$$

From these equations it can be seen that an increase in the mark-up of the firms, through M, reduces the level of utilization and the rate of growth of the capital stock.

Although Dutt (1990) does not examine the effect of a change in the propensity to save on profits by firms, it can be calculated from equation (1.36). To know the impact of an increase in the propensity to save on profits by the firms it is necessary to take the derivative of U^* with respect to s_f:

$$(1.38)\quad \frac{dU^*}{ds_f} = \frac{-\alpha M[1 - k^c s_c - (1-k^c)s_w]}{[s_w - \tau + s_f(1-s_w)M + (s_c-s_w)(1-s_f)Mk^c]}$$

Given that k^c, s_c and s_w are less than one, the above expression is negative.

The inclusion of savings and property earnings by workers does not alter the standard results. The paradox of thrift still holds. An increase in s_f reduces the effective demand of the economy. Although the workers, who have a higher propensity to consume, receive part of the new savings, this is not enough to overcome the overall negative effect.

In the long run, the total stock of capital, as well as its share between workers and capitalists, changes. Dutt (1990) shows that, even in the long run, an increase in the mark-up of the firms leads to a lower level of utilization of capacity and lower rate of growth. The derivation of this result incorporates the elements that affect the share of capital property between classes. This is beyond the scope of this paper.

1.8. A critique of Kaleckian models

The basics of Kaleckian models have been presented. In this section, a critique coming from Steedman (1992) is reviewed.

The main criticism made by Steedman against Kaleckians refers to the aggregation of firms in only one industry, that produces one final good, and whose firms apply a unique mark-up. Some Kaleckian authors may justify this assumption by considering that domestic firms are vertically integrated, and that the sub-process of production could be reduced to a complete process carried out by a big firm.

According to Steedman, this simplification leads to a misinterpretation of the reality because interindustry relationships are not considered. He criticizes the vertical integration argument because some "basic" industries[9] supply their products to more than only one sector.

Steedman tries to show how the different industries affect each other. He builds an 'n' sector model, and tries to use the Kaleckian framework, by using simple mark-up pricing and circulating capital. Equation (1.39) shows the price system for Steedman:

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[9] He mentions the steel and chemical industries, for example.

(1.39) $p = (WE + NM_s + pA)(I + \theta)$

where p, W, and N are row vectors of the prices of the domestically produced commodities, the wage rates and the prices of imported inputs, respectively, in local currency. E, M_s, and A are matrices, whose jth columns represent the inputs of labour, imports and locally produced commodities. I is the matrix identity and θ is the diagonal matrix of industry mark-ups.

As the vector $(WE + NM_s)$ is given, for simplification, Steedman makes the following substitution:

(1.40) $Z = (WE + NM_s)$

Replacing Z into (1.39) and solving for p:

(1.41) $p = Z(I + \theta)(I - A - A\theta)^{-1}$

From (1.41) it is clear that p_j is not only a function of θ_j but also of the mark-ups of the other sectors. There are other considerations in his paper but they are beyond the scope of this analysis.

Steedman's model shows that not only the variables of one sector affect the price of that sector, but also that the mark-ups of the other sectors have an impact on that price. Therefore, Steedman indicates that the price of one good does not depend only on the level of concentration of that sector, but that it is indeed affected by the level of collusion in other industries. He concludes that this is an aspect that is not, but should be, considered by Kaleckian authors. The argument by Steedman attacks the isolated way in which firms calculate their prices. He did not comment, however, on whether this change actually affects the conclusions of Kaleckian models.

Steedman builds a model considering circulating capital only. He makes some references to fixed capital, indicating that it does not alter the argument significantly. The result of the inclusion of fixed capital is that the sectors producing capital goods end up "as if" they were vertically integrated.

In the following chapter, a model with two sectors is built, focusing on the issue of fixed capital and whether inclusion of more than one sector alters the results of the models presented in the first section.

2. INTERDEPENDENCE AND GROWTH IN A TWO-SECTOR MODEL

In the previous chapter, the main features of the standard one-sector Kaleckian model were presented, as well as the consequences of certain changes in the parameters. These elements are analyzed in the present chapter, extending the model to a two-sector economy.

Within the Kaleckian framework, the main paper that deals with two sectors in a growth model is the one written by Dutt (1988). Dutt's model aims to explain differences in the rates of profit in the long run. The model that is presented in this chapter is closer to the Kaleckian concern of growth. Although some similarities may be found with Dutt's model, there are important differences between his and the one that is presented here. These elements are made explicit as the model is solved. At this stage it is sufficient to say that in Dutt's model there is no interdependence between the mark-ups of the firms, and that the prices of each sector are affected by their own mark-ups only. Dutt's model is therefore subjected to the critique of Steedman (1992). In the model presented below, this deficiency is corrected by introducing target return pricing.

In the following section, the model is presented by first stating the pertinent assumptions. The pricing system is then described, and the demand and growth equations are presented. Finally, the solutions in the short run and the long run are shown, as well as some comparative static exercises.

2.1. Assumptions of the two-sector growth model

The assumptions of the two-sector growth model are the following:

a) there are two sectors in the economy. One produces basic goods and the other non-basic goods;

b) the basic good enters in the production of the other good as well as in its own process. The non-basic good is consumed;

c) the basic good is called the investment or capital good, while the non-basic good is the consumption good;

d) the other factor required for the production process of both goods is labour;

e) there is no fixed (overhead) labour;

f) there is no circulating capital;

g) the factors enter in fixed proportions in the production process.

h) the firms act in an oligopolistic framework, setting mark-up rates over costs; these mark-ups are defined by other parameters, and they may be different in each of the two sectors[10]; Dutt (1988) assumes that the mark-ups of the two sectors are strictly exogenous.

i) the firms set their prices according to a certain standard rate of profit; this assumption allows the inclusion of fixed costs in the calculation of the price, through mark-ups. Dutt does not consider this, and his prices are a function of variable costs only;

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[10] Even when the rates of profit may be equal due to strong competition, mark-ups could be distinct due to different technical coefficients.

j) the nominal wage rate is the same for the two sectors;

k) there are two social classes in the economy: capitalists and workers;

l) capitalists earn all profits and save a proportion s_p of them. Workers do not save;

m) for simplification, it is a closed economy, without government, and without technical progress;

n) capital goods are always valued at their replacement costs, rather than at their historical costs;

o) there is one firm in each sector, in which case the firm in the investment sector has to take into account the opportunity cost of producing its own investment goods; or there are many identical firms in each sector, in which case it is assumed that they behave just like one vertically integrated firm in each sector.

The assumptions relating to the sort of capital in use have the following purposes. Dutt (1988) assumes fixed capital but he does not introduce interdependence. Steedman (1992) developes a model with interindustry flows of inputs, i.e., circulating capital. In the model developed here it is necessary to introduce fixed capital, because the accumulation of capital is a key issue in this paper. Furthermore, it is assumed that firms work with idle capacity. This last feature requires a capital stock variable against which the level of utilization of plants can be computed. A third avenue must then be followed, which is the use of fixed capital, with interdependence given through the way enterprises calculate their profits. Firms calculate their profits considering the value of their capital stock. Accordingly, the sector that produces capital goods influence the profits of the consumption sector through the valuation of their capital, and consequently through the mark-up and the price of this sector. Thus, it is possible to combine the growth issue of Kaleckian authors with the sectorial interdependence of Steedman and the neo-Ricardians.

2.2. <u>The pricing system</u>

It has been assumed that firms require capital goods to produce. Firms set a target rate of return on the value of their capital stocks. This target rate of return will be called standard rate of profit. The standard rate of profit is calculated for a level of utilization of capacity which is at its normal level, also called the standard degree of capacity utilization. This is the third variant of pricing presented by Lavoie (1992), i.e., target return pricing. The normal or standard rate of profit desired by the firms determines the value of the mark-up applied by the firms over their variable costs.

Dutt (1988) does not consider this procedure. Instead, he applies simple mark-up pricing. As it is seen below, the advantage of using target return pricing is that some interdependence between the mark-ups of the two sectors is introduced. This leads to a situation in which the mark-up of one sector affects the price of the other, something that is not present in Dutt's model. This new model is thus an answer to Steedman (1992)'s critique.

In the following, how firms calculate their mark-ups and prices is presented. The methodology follows the target return pricing formula of Lavoie (1992) for a one-sector economy.

The starting point is the profits desired by the firms of the consumption and the capital sectors. If the subscript "c" for the consumption good, and the subscript "i" for the investment or capital good is used, the following can be written:

(2.1) $\Pi_c^s = r_c^s \, P_i \, K_c$

In expression (2.1) Π_c^s is the total amount of profits at its standard or normal level, r_c^s the standard rate of profit, K_c the stock of capital in the consumption sector, and P_i the price of the capital good. The standard rate of profit is related to the level of profits when the level of utilization of the plant is at its normal level.

The following relationships are definitions:

(2.2a) $V_c = K_c/q_c^f$ (2.2b) $U_c^s = q_c^s/q_c^f$

The first value, V_c, is the ratio of the stock of capital K_c, to the full-capacity output, q_c^f, both of the consumption sector. The second one, U_c^s, is a measure of the standard level of utilization of capacity. It is a ratio of the standard level of output in the consumption sector, q_c^s, to its full-capacity output.

Using (2.2), (2.1) can be rewritten as:

(2.3) $\Pi_c^s = r_c^s \; P_i \; V_c \; q_c^s / U_c^s$

The assumption explaining the way in which prices are set by the firms implies that

(2.4) $P_c = (1 + \theta_c) \; UC_c$

where P_c is the price of the consumption good and UC_c is the average unit costs of the firm in the consumption sector[11].

From equation (2.4), the standard level of total profits can be deduced by multiplying it by the standard level of output, q_c^s:

(2.5) $\Pi_c^s = \theta_c \; (UC_c) \; q_c^s$

As circulating capital has been assumed away, the only variable costs are labour costs. If $a_{\ell c}$ is the requirement of labour per unit of output of the consumption good, while "W" is the nominal wage rate, then the unit costs are:

(2.6) $UC_c = a_{\ell c} . W$

Substituting (2.6) into (2.5) yields:

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[11] Recall that it is assumed that there are no fixed labour costs.

(2.7) $\Pi_c^s = \theta_c \; q_c^s \; a_{\ell c} \; W$

Equating (2.3) and (2.7), and solving for θ_c:

(2.8) $$\theta_c = \frac{r_c^s \; V_c \; P_i}{U_c^s \; a_{\ell c} \; W}$$

θ_c is the mark-up obtained through a target return pricing formula for the consumption sector.

In (2.8) it can be seen that the mark-up of the consumption sector depends not only on the normal rate of profit and other parameters related to the same consumption sector but also to the price of the investment good. Any increase in the price of the capital good leads to a higher mark-up in the consumption good sector. In the context of target return pricing, the firms are taking into account the value of their capital stocks to calculate their own standard profits. An increase in this value directly affects their desired profits. These higher desired profits are achieved through a larger mark-up.

Within the Kaleckian framework, the standard way of pricing is through a mark-up over variable costs, as mentioned above. For the investment good this takes the form of:

(2.9) $P_i = (1 + \theta_i) \; UC_i \; = (1 + \theta_i) \; a_{\ell i} \; W$

where θ_i is the mark-up in the investment sector, UC_i is the average variable cost, and $a_{\ell i}$ is the requirement of labour per unit of capital good. As before, the only variable costs are labour costs. Substituting (2.9) into (2.8) the relationship between the mark-ups of the two sectors is:

$$(2.10) \quad \theta_c = \frac{r_c^s V_c}{U_c^s} \frac{a_{\ell i}}{a_{\ell c}} (1+\theta_i)$$

Equation (2.10) indicates the direct relationship between θ_c and θ_i.

Taking the exercise a step further, the rate of profit of the investment sector can be included in this expression.

First, it is necessary to present the way in which firms in the investment sector calculate their mark-ups. The procedure is symmetric to the one of the consumption sector.

It is assumed that firms in the investment sector use a target return pricing method to estimate their total standard profits. This is formalized in equation (2.11).

$$(2.11) \quad \Pi_i^s = r_i^s P_i K_i$$

Π_i^s is the total profits at a normal level, r_i^s the standard rate of profit, and K_i the stock of capital, the three variables related to the investment sector. This standard rate of profit is related to the level of profits when the level of utilization of the plant is at its normal level.

The following ratios are also useful:

(2.12a) $V_i = K_i/q_i^f$ (2.12b) $U_i^s = q_i^s/q_i^f$

In the above expressions, V_i is the ratio of the stock of capital, K_i, to the full-capacity output, q_i^f, of the investment sector. The second relationship, U_i^s, is a measure of the standard level of utilization of capacity in the investment sector. It is a ratio of the standard level of output in the investment sector, q_i^s, to its full-capacity output.

Using (2.12), (2.11) can be rewritten as:

(2.13) $\Pi_i^s = r_i^s \; P_i \; V_i \; q_i^s/U_i^s$

Additionally, the total amount of standard profits can be derived by multiplying equation (2.9) by the normal level of output of the investment sector.

(2.14) $\Pi_i^s = \theta_i \; (UC_i) \; q_i^s$

If expressions (2.13) and (2.14) are equated, and solved for θ_i:

(2.15) $$\theta_i = \frac{r_i^s \; V_i}{U_i^s} \frac{P_i}{UC_i}$$

Solving equation (2.9) for UC_i: $UC_i = P_i/(1+\theta_i)$ and substituting into (2.15) yields

$$(2.16) \qquad \theta_i = \frac{r_i^s\ V_i}{U_i^s}\ \frac{P_i\ (1+\theta_i)}{P_i}$$

Solving for θ_i, the mark-up of the investment sector is equal to:

$$(2.17) \qquad \theta_i = \frac{r_i^s\ V_i}{U_i^s - r_i^s\ V_i}$$

Now, the relationship between the standard rate of profit of the consumption sector with that of the investment sector can be established. Replacing the value of θ_i from (2.17) into (2.10) it can be seen that

$$(2.18) \qquad \theta_c = \frac{U_i^s}{U_c^s}\ \frac{a_{\ell i}}{a_{\ell c}}\ \frac{r_c^s\ V_c}{U_i^s - r_i^s\ V_i}$$

Equation (2.18) indicates explicitly the direct relationship between the mark-up of the consumption sector and the standard rate of profit of the investment sector. For expressions (2.17) and (2.18) to be economically meaningful, the following condition must hold:

$$(2.19) \qquad V_i < (U_i^s/r_i^s)$$

In order to understand the meaning of (2.19) the values of V_i and U_i^s are substituted by the corresponding terms. Thus:

$$K_i < (q_i^s/r_i^s).$$

Multiplying this expression by P_i and rearranging terms shows

$$(2.19') \quad q_i^s\ P_i > P_i\ r_i^s\ K_i$$

Expression (2.19') indicates that the value of output in the investment sector at normal capacity has to be greater than the standard amount of profits. Obviously, this must be the case, because, otherwise, labour costs would have to be negative. Then, it is clear that equation (2.18) is always positive, since equation (2.19) must hold.

2.3. <u>The demand, income and growth equations</u>

The demand equations of the economy are now presented. This part is similar to Dutt (1988). The letter "D" refers to the demand in real terms, and the subscripts "c" and "i" of the previous section continue to hold here.

$$(2.20) \quad D_c = \frac{W}{P_c}(a_{\ell c}\ q_c + a_{\ell i}\ q_i) + (1 - s_p)\left(\frac{P_i}{P_c}\ r_i\ K_i + \frac{P_i}{P_c}\ r_c\ K_c\right)$$

$$(2.21) \quad D_i = \Delta K_c + \Delta K_i$$

The variable s_p is the propensity to save out of profits, and it is assumed to be equal for the capitalists of both sectors. ΔK means a change in the variable, in this case a variation in the capital stock. r_c and r_i are now the realized rates of profit of each sector. Equation (2.20) shows the real demand for consumption goods, which depends on two terms. The first component is the demand coming from wage earners, while the second is the part of profits that are consumed. Equation (2.21) is the demand for capital goods arising from the consumption goods sector, ΔK_c, and from the capital goods sector, ΔK_i.

On the income side, the value of output is distributed to profits and wages. This is shown in equations (2.22) and (2.23), for the consumption and investment sectors respectively.

$$(2.22) \quad P_c\, q_c = r_c\, P_i\, K_c + W\, a_{\ell c}\, q_c$$

$$(2.23) \quad P_i\, q_i = r_i\, P_i\, K_i + W\, a_{\ell i}\, q_i$$

In the expressions (2.22) and (2.23) the first term is the amount of realized profits and the second term is the value of total wages for each sector.

Finally, the growth equations for the two sectors of the economy are presented. It is assumed that the growth function for each sector depends on its level of utilization of available capacity. In equation (2.2b), in section 2.2., a level of utilization based on the full capacity output q_c^f was defined. This level is directly related to the actual stock of capital, K_c, through V_c (equation 2.2a). Therefore, the realized level of utilization of capacity can be written as follows:

$$U_c = V_c \, q_c/K_c$$

Since it is assumed that V_c is a technological parameter, the ratio q_c/K_c can be considered as a proxy variable of the level of utilization.

It is also assumed that there is an exogenous component in the growth function.

$$(2.24) \quad G_c = (\Delta K_c/K_c) = \alpha_c + \tau_c \, (q_c/K_c)$$

$$(2.25) \quad G_i = (\Delta K_i/K_i) = \alpha_i + \tau_i \, (q_i/K_i)$$

In the above equations, the αs represent autonomous investment parts, while the τs, are behavioural parameters. In the growth functions of Dutt's (1988) model, the rate of profit of each sector is also present. The expected rate of profit of each sector has not been included here, mainly for simplification. The inclusion of the rates of profit would have complicated the analysis.

2.4. Solutions in the Short Run

In the short run, output responds to changes either from the supply or the demand side, due to the existence of excess capacity, as Dutt (1988) does. This assumption is formalized under equations (2.26) and (2.27).

$$(2.26) \quad \Delta q_c = D_c - q_c$$

$$(2.27) \quad \Delta q_i = D_i - q_i$$

The next step involves solving for the level of output in the short run given the values of the stock of capital K, the mark-ups θ, the wage rate W, the labor-output coefficients a_ℓ, and the parameters of the growth equations α and τ, for both sectors.

The equilibrium in equations (2.26) and (2.27) is achieved by equating D from (2.20) and (2.21) to q.

$$(2.28) \quad q_c = \frac{W}{P_c}(a_{\ell c}\, q_c + a_{\ell i}\, q_i) + (1 - s_p)\left(\frac{P_i}{P_c}\, r_i\, K_i + \frac{P_i}{P_c}\, r_c\, K_c\right)$$

$$(2.29) \quad q_i = G_c\, K_c + G_i\, K_i$$

In equation (2.29) equations (2.24) and (2.25) have also been used to substitute the values of ΔK_c and ΔK_i.

The values of (W/P_c) and (P_i/P_c) can be obtained from equations (2.4) and (2.9). They are useful to simplify the model.

(2.30) $$W/P_c = 1/(1 + \theta_c)a_{\ell c}$$

(2.31) $$P_i/P_c = (1 + \theta_i)a_{\ell i} \;/\; (1 + \theta_c)a_{\ell c}$$

Equations (2.22) and (2.23) can be solved for the realized rate of profits, r_c and r_i, substituting the values of P_c and P_i.

(2.32) $$r_c = \frac{\theta_c}{(1 + \theta_i)} \; \frac{a_{\ell c}}{a_{\ell i}} \; \frac{q_c}{K_c}$$

(2.33) $$r_i = \frac{\theta_i}{(1 + \theta_i)} \; \frac{q_i}{K_i}$$

Equations (2.32) and (2.33) show that the realized rates of profit depend on the level of utilization of capacity. In the case of the consumption sector, it can be seen that it does not only depend on the mark-up of the same sector but also, inversely, on the one from the investment sector. This is the case shown in Dutt (1988). Although Dutt is not much concerned about this in his paper, he could have argued that the reason for this is that the first effect of an increase in θ_i is an increase in P_i, which, keeping the value of $P_c q_c$ given in the very short run, reduces r_c, through (2.22). This is the result of using simple mark-up pricing.

In the model presented here, however, θ_c and θ_i are connected through (2.10). Substituting (2.10) into (2.32), it can be seen that the realized rate of profit in the consumption sector depends only on variables related to the consumption sector, including its rate of capacity.

$$(2.32') \quad r_c = [(r_c^* V_c)/U_c^*] \quad (q_c/K_c)$$

Although this relationship could be expected from the initial assumptions, the key here is that the connection between θ_c and θ_i sets a different path of change. If θ_i varies, the effect shown above in Dutt's model would not occur because θ_c would also increase. The result, so far, is that the rate of profits of the consumption sector eventually remains intact. The absence of θ_i in (2.32') indicates this.

However, this is not the end of the story. As it is shown below, the level of utilization of the consumption sector does depend on the mark-up of the investment sector, but the only path is through the level of utilization. This is the result of using target return pricing, instead of simple mark-up pricing.

Substituting the values of G_c and G_i from (2.24) and (2.25), W/P_c from (2.30), P_i/P_c from (2.31), and r_c and r_i from (2.32), and (2.33) into (2.28) and (2.29), and rearranging terms:

$$(2.34) \qquad q_c \left[\frac{s_p\ \theta_c}{1 + \theta_c} \right] - q_i \left[\frac{a_{\ell i}}{a_{\ell c}} \quad \frac{1+(1-s_p)\,\theta_i}{1 + \theta_c} \right] = 0$$

$$(2.35) \qquad q_c\ [-\ \tau_c] + q_i\ [1 - \tau_i] = \alpha_c\ K_c + \alpha_i\ K_i$$

This system can be expressed in matrix notation,

(2.36)

$$\begin{bmatrix} -\dfrac{s_p\theta_c}{(1+\theta_c)} & \dfrac{a_{\ell i}}{a_{\ell c}}\ \dfrac{1+(1-s_p)\,\theta_i}{1+\theta_c} \\ \tau_c & -(1-\tau_i) \end{bmatrix} \begin{bmatrix} q_c \\ q_i \end{bmatrix} = \begin{bmatrix} 0 \\ -\ (\alpha_c\ K_c + \alpha_i\ K_i) \end{bmatrix}$$

which will be called $Tq = H$.

The solution for this system is given by the following expressions for q_c and q_i.

$$(2.37) \qquad q_c^* = \frac{(\alpha_c\ K_c + \alpha_i\ K_i)\,(a_{\ell i}/a_{\ell c})\,[1+(1-s_p)\,\theta_i]}{(1-\tau_i)\,s_p\theta_c\ -\ \tau_c\ \ (a_{\ell i}/a_{\ell c})\,[1+(1-s_p)\,\theta_i]}$$

$$(2.38) \qquad q_i^* = \frac{(\alpha_c\ K_c + \alpha_i\ K_i)\ \ (s_p\theta_c)}{(1-\tau_i)\,s_p\theta_c\ -\ \tau_c\ \ (a_{\ell i}/a_{\ell c})\,[1+(1-s_p)\,\theta_i]}$$

As can be seen, the mark-ups of both sectors appear in the two expressions. Therefore, changes in θ_c or in θ_i affect the levels of output in both sectors.

In order to simplify the notation the following simplifications are made:

$$\Omega = (1-\tau_i) s_p \theta_c - \tau_c (a_{\ell i}/a_{\ell c}) [1+(1-s_p)\theta_i]$$

$$C = (a_{\ell i}/a_{\ell c}) [1+(1-s_p)\theta_i]/\Omega$$

$$I = (s_p \theta_c)/\Omega$$

$$\kappa = K_c/K_i$$

So that

$$(2.37') \quad (q_c/K_c)^* = (\alpha_c + \alpha_i/\kappa) \; C$$

$$(2.38') \quad (q_i/K_i)^* = (\alpha_c \kappa + \alpha_i) \; I$$

For these expressions to be economically meaningful, it is required that C, I, and Ω be positive. These restrictions are satisfied if the parameters of the growth functions τ_c and τ_i are 'small enough' to achieve $\Omega > 0$. This is a result similar to that found by Rowthorn (1981) and Lavoie (1992) for their growth models to be stable[12].

[12] Recall the stability condition given by inequality (1.27).

The stability conditions of the system in (2.26) and (2.27) require that the matrix T must have a negative trace and a positive determinant.

This means that

$$- \frac{s_p \theta_c}{(1+\theta_c)} - (1-\tau_i) < 0$$

$$\frac{s_p \theta_c}{(1+\theta_c)} (1-\tau_i) - \tau_c \left(\frac{a_{\ell i}}{a_{\ell c}} \right) \left[\frac{1+(1-s_p)\theta_i}{1+\theta_c} \right] > 0$$

Given the restrictions on τ_c and τ_i these stability conditions are satisfied.

In the short run the values of the rates of profit are:

$$\text{(2.39)} \quad r_c = \frac{1}{\Omega} \left(\frac{\theta_c}{1+\theta_i} \right) \left[1+(1-s_p)\theta_i \right] \left[\alpha_c + \frac{\alpha_i}{\kappa} \right]$$

$$\text{(2.40)} \quad r_i = \frac{1}{\Omega} \left(\frac{\theta_i}{1+\theta_i} \right) \left[s_p \theta_c \right] \left[\alpha_c \kappa + \alpha_i \right]$$

It can be seen that the realized rates of profit of both sectors depend on the mark-ups of both sectors. There is interdependence for both the realized and the standard rates of profit.

2.4.1. Changes in the mark-up of the consumption sector (θ_c)

In this section, some comparative static exercises are presented, showing the effects of changes in the mark-ups over the levels of output and the rates of profit. Since there is a relationship in this model between the mark-ups of the firms, the result differs from Dutt's (1988) model. His model deals with the issue of the values of the rates of profit in the short run and long run, particularly the conditions under which they are equal (in the long run). Here the focus of the analysis is upon the impact on the rate of utilization, the rate of profit and the rates of growth, following changes in the parameters.

First, the case in which θ_c increases is considered. This could happen as result of an increase in the standard rate of profit of the consumption sector, r_c^s. From equations (2.37) and (2.38), the corresponding derivatives can be computed.

To simplify the presentation the following substitutions are made:

$$R = \frac{r_c^s\, V_c}{U_c^s}$$

$$A = \frac{a_{\ell i}}{a_{\ell c}}$$

$$(2.41)\quad \frac{d\ (q_c/K_c)}{d\ \theta_c} = \left(- \frac{\alpha_i}{\kappa^2} (d\kappa/d\theta_c) \right) \left(\frac{A\ [1+(1-s_p\theta_i]}{\Omega} \right) + \left(\alpha_c + \alpha_i/\kappa \right) \left\{ - \frac{A\ (1-\tau_i) s_p [1+(1-s_p)\,\theta_i]}{\Omega^2} \right\}$$

$$(2.42)\quad \frac{d\ (q_i/K_i)}{d\ \theta_c} = \left(\ \alpha_c\ (d\kappa/d\theta_c)\right)\left(\frac{s_p\ \theta_c}{\Omega}\right) + \left(\ \alpha_c\kappa + \alpha_i\ \right)\left\{-\frac{A\ (\tau_c) s_p [1+(1-s_p)\theta_i]}{\Omega^2}\right\}$$

In the short run, changes in the capital stock of both sectors are ruled out, so $d\kappa/d\theta_c$ and $d\kappa/d\theta_i$ are zero. The remaining terms are all positive, with values of s_p and τ_i less than unity. The negative sign in front of them shows that there is an inverse relationship between θ_c and the level of capacity for both sectors. The reason for this is that an increase in θ_c causes a rise in the price of consumption goods (through equation 2.9). As this price goes up, the real wage rate goes down, and, therefore, the demand for consumption goods falls. With the stock of capital fixed in the short run, the level of utilization of capacity diminishes.

In the investment sector, the production of capital goods declines too. Because the firms in the consumption sector see their idle capacity increased, they reduce their current investments, ΔK_c. This reduced demand for capital goods from the consumption sector induces a cutback in their production. Moreover, as capital goods' output declines, there is an increase in the excess capacity of that sector. The demand for new investment goods coming from the capital goods sector falls, leading to a further reduction in the production of capital goods.

The effects on the rate of profit can also be calculated.

$$(2.43)\quad \frac{d\, r_c}{d\, \theta_c} = \left(\frac{1}{\Omega^2} \right) \left[\frac{1+(1-s_p)\theta_i}{1+\theta_i} \right] \left\{ -\, \Omega\, \theta_c \left[\frac{\alpha_c}{\kappa^2} \frac{d\kappa}{d\theta_c} \right] - \left(\alpha_c + \alpha_i/\kappa \right) \left(\tau_c\, A\, [1+(1-s_p)\theta_i] \right) \right\}$$

$$(2.44)\quad \frac{d\, r_i}{d\, \theta_c} = \left(\frac{1}{\Omega^2} \right) \left[\frac{\theta_i}{1+\theta_i} \right] \left\{ s_p\, \Omega\, \theta_c \left[\alpha_c \frac{d\kappa}{d\theta_c} \right] - (s_p) \left(\alpha_c\, \kappa + \alpha_i \right) \left(\tau_c\, A\, [1+(1-s_p)\theta_i] \right) \right\}$$

As before, in the short run, it is assumed that $d\kappa/d\theta_c$ and $d\kappa/d\theta_i$ are zero. In this case, the changes in the realized rates of profit are both negative. The main reason for this is that, as was shown in the previous exercise, the level of output is falling. Hence, despite the increase in the standard rate of profit in the consumption sector, the realized rate of profit is falling in both sectors.

2.4.2. Changes in the mark-up of the investment sector (θ_i)

The impact of a change in the mark-up of the firms of the investment sector on the level of utilization of capacity in both sectors is now considered. It is worth noting that θ_i also affects

θ_c. The link between them is shown in equation (2.10), which has been used here.

$$(2.45) \quad \frac{d\ q_c/K_c}{d\ \theta_i} = \left(\frac{A}{\Omega^2}\right) \left\{ -\ \Omega\ [1+(1-s_p)\,\theta_i] \left(\frac{\alpha_i}{\kappa^2} \frac{d\kappa}{d\theta_i} \right) - \left(\alpha_c + \alpha_i/\kappa \right) \left(R\ A\ (1-\tau_i)\ s_p^2 \right) \right\}$$

$$(2.46) \quad \frac{d\ q_i/K_i}{d\ \theta_i} = \frac{1}{\Omega^2} \left[(\alpha_c s_p \theta_c\ d\kappa/d\theta_i)\,\Omega\ -\ (\alpha_c \kappa + \alpha_i)\, s_p^2\ R\ A^2\ \tau_c \right]$$

Putting aside the $d\kappa/d\theta_c$ and $d\kappa/d\theta_i$ terms, the expressions show negative values. An increase in the mark-up of the investment sector increases the value of capital goods. As was presented in section 2.2., there is a direct relationship between the mark-up of the investment sector and that of the consumption sector. The increased θ_i induces a rise in θ_c with the results shown above.

The next exercise is to study the impact of an increase in the mark-up of the investment sector on the rates of profit.

$$(2.47) \quad \frac{d\ r_c}{d\ \theta_i} = \frac{1}{\Omega^2} \left\{ -\ \frac{(\Omega)\,(R)\,(A)\,[1+(1-s_p)\,\theta_i]\,(\alpha_i)}{\kappa^2}\,(d\kappa/d\theta_i) - (1-\tau_i)\,(s_p^2)\,(\alpha_c + \alpha_i/\kappa)\,(R\ A)^2 \right\}$$

$$(2.48)\quad \frac{d\ r_i}{d\ \theta_i} = \frac{1}{\Omega^2}\Big[\ (\Omega)\ R\ A\ (s_p\theta_i)\ \alpha_c\ (d\kappa/d\theta_i) + (\alpha_c\ \kappa + \alpha_i)(s_p\ R\ A^2)[s_p\ R(1-\tau_i) - \tau_c]\Big]$$

This exercise shows results which are different from those of the previous case, shown by equations (2.45) and (2.46), where both variables move in the same direction. In the present case, the rate of profit of the consumption sector goes down, but the sign of the change of the rate of profit in the investment sector is indeterminate.

In the short run, the term $(d\kappa/d\theta_i)$ is assumed to be zero. The sign of the remaining term depends on whether $s_p\ R(1-\tau_i) - \tau_c$ is positive or negative. However, this cannot be determined. This expression will be compared with Ω, which is required to be positive.

In order to make them comparable, the following manipulations are made. First, the $(d\kappa/d\theta_i)$ term is cancelled. Then, the squared A is divided and introduced inside the brackets.

$$\frac{d\ r_i}{d\ \theta_i} = \frac{1}{\Omega^2}\ (\alpha_c\ \kappa + \alpha_i)(s_p\ R\ A)[s_p\ RA(1-\tau_i) - A\tau_c]$$

From equation (2.10) it can be seen that $\theta_c = R\ A\ (1+\theta_i)$. Solving for R and substituting this result in the term inside the brackets:

$$\frac{d\, r_i}{d\, \theta_i} = \frac{1}{\Omega^2} \quad (\alpha_c\, \kappa + \alpha_i)\,(s_p\, R\, A)\,[s_p\,(1-\tau_i)\frac{\theta_c}{1+\theta_i} - A\tau_c]$$

Factoring out the term $1/(1+\theta_i)$ from the brackets:

$$\frac{d\, r_i}{d\, \theta_i} = \frac{1}{\Omega^2} \quad (\alpha_c\, \kappa + \alpha_i)\,\left(\frac{s_p\, R\, A}{1+\theta_i}\right)[s_p\,(1-\tau_i)\,\theta_c - A\tau_c(1+\theta_i)]$$

Operating the multiplication inside the brackets:

$$[(1-\tau_i)\, s_p\, \theta_c - \tau_c\, A - \tau_c\, A\, \theta_i]$$

The expanded expression for Ω is:

$$[(1-\tau_i)\, s_p\, \theta_c - \tau_c\, A - \tau_c\, A\, \theta_i + s_p\, \tau_c\, A\, \theta_i]$$

It can be clearly seen that Ω has one more term that is positive. Hence, it cannot be clearly stated whether the term inside the brackets is positive or negative. Therefore, $dr_i/d\theta_i$ is indeterminate.

The source of the difference can be seen through the initial expressions for r_c and r_i, in equations (2.32) and (2.33). The variable r_c depends basically on two terms. The first one is a ratio between θ_c and θ_i, and the second one is q_c/K_c. When θ_i changes, the effects on θ_c and θ_i cancel out because they are directly related.

Therefore, the only effect that remains is the negative effect of q_c/K_c, leading to the negative result shown above. On the other hand, r_i depends on two expressions, too. The first one depends on θ_i only, while the second is q_i/K_i. When θ_i increases, the first term rises. But the second part, q_i/K_i goes down, and the result cannot be determined.

The economic reason for this difference is that, when the mark-up of the investment sector goes up, the firms of the consumption sector are induced to increase their mark-up, too. Therefore, the final effect on the rate of profit is related only to the change in the level of utilization of capacity. As was shown above, the utilization rate is reduced.

In the investment sector, when raising the mark-up, firms are trying to increase not only their total profits, but also the share of profit per unit of output. In fact, this is what happens, as $\theta_i/(1+\theta_i)$ increases. But the second impact is that the level of production, and utilization of capacity, is reduced because the demand for investment goods is falling. Accordingly, the impact on the rate of profit of the investment sector is indeterminate.

This can be seen in Figure 2.1. The rate of profit of the investment sector is shown by the vertical axis, and its level of utilization of capacity is shown on the horizontal axis. The straight line starting from the origin is equation (2.33), with a slope equal to $\theta_i/(1+\theta_i)$. The vertical line is equation (2.38).

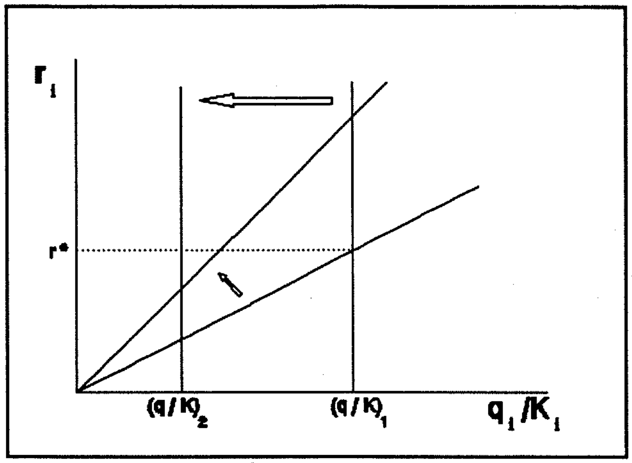

Figure 2.1

When the mark-up of the investment sector increases, the curve from the origin rotates up, leading to a higher rate of profit per unit of output. But the second effect is that the level of utilization falls, from $(q/K)_1$ to $(q/K)_2$. The final effect cannot be clearly stated. In the case of the rate of profit of the consumption sector, the straight line starting from the origin does not move, and the only effect is the reduction in the level of output. This mechanism can be seen through equation (2.32'). An increase in θ_i, would only affect the rate of profit of the consumption sector by reducing the utilization rate of the consumption sector.

In the short run, each sector is growing at different rates, once the level of utilization of capacity has been set. In section 2.5 the long run issue is tackled. Before that, the short run impact of a change in the propensity to save out of profits is shown in the next section.

2.4.3. Changes in the propensity to save out of profits (s_p)

The last exercise refers to a change in the propensity to save on profits by the firms. Deriving equations (2.37) and (2.38) with respect to s_p, it is found that:

$$(2.49) \quad \frac{d(q_c/K_c)}{ds_p} = \frac{1}{\Omega^2}\left\{ [-(\alpha_i/\kappa^2)d\kappa][A][1+(1-s_p)\theta_i][\Omega] - [\alpha_c+\alpha_i/\kappa][1-\tau_i][A\theta_c][1+\theta_i]\right\}$$

$$(2.50) \quad \frac{d(q_i/K_i)}{ds_p} = \frac{1}{\Omega^2}\left\{ [\alpha_c d\kappa][s_p\theta_c][\Omega] - [\alpha_c\kappa+\alpha_i][A][\theta_c]^2[1+\theta_i]\right\}$$

Equations (2.49) and (2.50) indicate that in the short run, when $d\kappa$ is zero, the levels of utilization of capacity of both sectors are diminishing. This should not be a surprising result, since the effective demand coming from the firms is falling. The higher propensity to save causes a reduction in the consumption of goods, and, consequently, the level of output in the consumption sector falls. Firms in this sector notice that the level of utilization of capacity is moving further from the full capacity limit and reduce their demand for capital goods. Therefore, the output in the investment sector also falls.

From equations (2.32) and (2.33) it is clear that a reduction in the levels of output leads to decreases in the rates of profit of the consumption and the investment sectors. Accordingly, the increase in s_p causes a reduction in the rates of profit.

In the next section, the solution of the model, as well as these comparative static exercises in the long run, are presented.

2.5. Solutions in the long run

The difference between the short run and the long run is that, in the latter, the rates of growth of both sectors must be equal. This condition aims to assure that the economy grows steadily in the long run, and that no bottlenecks appear during the process. Another difference is that the capital stocks for both sectors can change. These two conditions imply that a combination between the stocks of capital of both sectors must be found.

The previous statement implies that $G_c = G_i$. Equating expressions (2.24) and (2.25):

$$G_c = \alpha_c + \tau_c \ (q_c/K_c) = \alpha_i + \tau_i \ (q_i/K_i) = G_i$$

Substituting the short run equilibrium values of q_c/K_c and q_i/K_i, using the simplified expressions for C, I, and κ, and solving for κ :

$$\kappa^2[-\tau_i \ I \ \alpha_c] + \kappa[\alpha_c + \tau_c\alpha_c C - \alpha_i - \tau_i\alpha_i I] + [\tau_c\alpha_i C] = 0$$

The only economically meaningful answer for κ is:

$$\kappa^* = \frac{\begin{array}{l}[C^2 \tau_c^2 \alpha_c^2 + \alpha_c^2 + 2C \tau_c \alpha_c^2 + I^2 \tau_i^2 \alpha_i^2 + \alpha_i^2 \\ + 2I \tau_i \alpha_i^2 - 2C \tau_c \alpha_c \alpha_i - 2I \tau_i \alpha_c \alpha_i + 2CI \tau_c \tau_i \alpha_c \alpha_i \\ - 2 \alpha_c \alpha_i]^{1/2} + [\alpha_c (C\tau_c + 1) - \alpha_i (I\tau_i + 1)]\end{array}}{2 (\alpha_c \tau_i I)}$$

This combination of K_c/K_i grants that the rates of growth of both sectors are equal in the long run. It also satisfies the stability conditions stated in the short run.

In the long run, the rate of growth for both sectors, and for the economy is equal to[13]:

(2.51) $$G = G_c = G_i = \alpha_i + \tau_i(\alpha_c\kappa^* + \alpha_i) I$$

2.5.1. <u>The rate of growth in long run equilibrium</u>

Some analysis of comparative statics in the long run are now presented.

In section 2.4.2. the short run impact of a change in θ_i on various variables was presented. The main difference now is that κ

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[13] The growth function of the investment sector was chosen only for simplicity.

changes. Changes in both, in the short run and the long run, over the rates of growth, are dealt with first[14].

The rates of growth for each sector are given by equations (2.24) and (2.25), computed at the equilibrium values of q_c/K_c and q_i/K_i, from equations (2.37') and (2.38').

Considering the case in which θ_i changes, the result is the following:

$$(2.52) \quad \frac{d\ G_c}{d\ \theta_i} = \left(\frac{\tau_c\ A}{\Omega^2}\right) \left\{ -\ \Omega\ [1+(1-s_p)\,\theta_i] \left(\frac{\alpha_i}{\kappa^2}\ \frac{d\kappa}{d\theta_i} \right) - \left(\alpha_c + \alpha_i/\kappa \right)\left(R\ A\ (1-\tau_i)\ s_p^2 \right) \right\}$$

$$(2.53) \quad \frac{d\ G_i}{d\ \theta_i} = \frac{\tau_i}{\Omega^2} \left[(\alpha_c s_p \theta_c\ d\kappa/d\theta_i)\,\Omega\ -\ (\alpha_c \kappa + \alpha_i)\, s_p^2\ A^2\ R\ \tau_c \right]$$

In the short run, the case in which $d\kappa$ is zero is considered. In both sectors there are negative effects. The increase in the mark-up of the investment sector leads to a reduction in the rate of growth of both sectors. The higher mark-up, θ_i, induces a proportional increase in the mark-up of the consumption sector. As

[14] It will seem that the exposition may become disordered at this point. One might expect the long run results of the levels of output, and later of the rates of growth. However, it is necessary to present the latter before in order to understand the opposite effects that will appear when dealing with $d\kappa$ different than zero.

was shown, this led to a reduction of output, as well as in the level of utilization of capacity. As the level of utilization of capacity decreases, entrepreneurs reduce their demand for investment goods, and the growth rates of both sectors fall.

In the long run, the capital stocks do change, so κ may be allowed to vary. Also, the rates of growth of both sectors are the same, either may be used.

From equation (2.51), it can be derived that:

$$(2.54)\quad dG/d\theta_i = \alpha_i \tau_i \ (dI/d\theta_i) + \alpha_c \tau_i \kappa \ (dI/d\theta_i) + \alpha_c \tau_i I \ (d\kappa/d\theta_i)$$

The value of $dI/d\theta_i$ is given by:

$$dI/d\theta_i = -\ (s_p/\Omega)^2 \ (\tau_c) \ (A)^2 \ R \ < 0$$

The term $d\kappa/d\theta_i$ cannot be determined, however. It may be positive, negative, or even zero depending on the particular values of all the parameters.

Therefore, there are two effects on the rate of growth of the economy: one of them is negative, and the other is indeterminate. However, the analysis can be presented graphically, and this will help to understand the final effect.

In Figure 2.2, G_c and G_i are presented. On the horizontal axis the level of κ is measured, while the rates of growth are shown by the vertical axis.

The curves represent the equations (2.24) and (2.25), evaluated at the equilibrium values of the level of utilization given by expressions (2.37') and (2.38').

(2.55) $G_c^* = \alpha_c + \tau_c \, [(\alpha_c + \alpha_i/\kappa) \; C]$

(2.56) $G_i^* = \alpha_i + \tau_i \, [(\alpha_c\kappa + \alpha_i) \; I]$

From here it is easy to check that G_c is a hyperbola, while G_i is a straight line, with a positive slope $\tau_i \, \alpha_c$ I.

The intuitive reasons for the slope of the curves are the following: when κ increases, it indicates that the capital stock of the consumption sector is growing (relatively faster than that of the investment sector). This higher capital stock in the consumption sector expands its capacity of production. As this capacity augments, the level of utilization of capacity, q_c/K_c, reduces and the motivation for higher investment lessens. Thus, the growth rate of the consumption sector slows down. In the investment sector, as κ rises, its capital stock falls (relative to that in the consumption sector). This is related to a level of utilization of capacity which is coming to its full-capacity limit, and therefore more investment is needed. Accordingly, higher levels of κ are related to higher growth rates in the investment sector.

Let the point A be the initial level of long run equilibrium, with the economy expanding at the rate of G^*. If θ_i increases, both G_c and G_i go down. In the short run, the value of κ does not change, and the two sectors generally grow at two different rates. In the short run, the new rates of growth are 'c' for the consumption sector, and 'i' for the investment sector. In Figure 2.2, it is assumed that c > i. In the long run, κ changes. To return to an equilibrium level, in which both sectors must grow at the same rate, the ratio of capital stocks must increase to $\kappa^{*'}$. At this point, both growth rates are equal, but less than the starting level G^*.

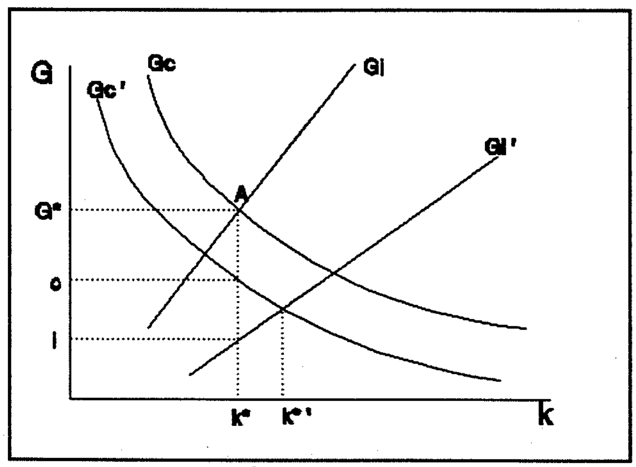

Figure 2.2

The stability of the model can be clearly seen if the growth rate of κ is calculated.

$$\kappa = K_c/K_i$$

$$\hat{\kappa} = \hat{K}_c - \hat{K}_i$$

where ^ means the growth rate of the variable.

In this case $\hat{K}_c$ is equal to 'c', and $\hat{K}_i$ is equal to 'i'. Given that c > i, κ increases.

In the above case, c > i, which means that the growth rate of the consumption sector is less responsive than the investment sector to the change in θ_i. In Figure 2.3, the case where the impact on the consumption sector is greater than the change on the investment sector is presented. Assume that A is the starting long run equilibrium point, with κ^* being the ratio of capital stocks and G^* the rate of growth of the economy. An increment in θ_i causes a fall in both curves. As the consumption sector is more responsive than the investment sector, i > c, in the short run.

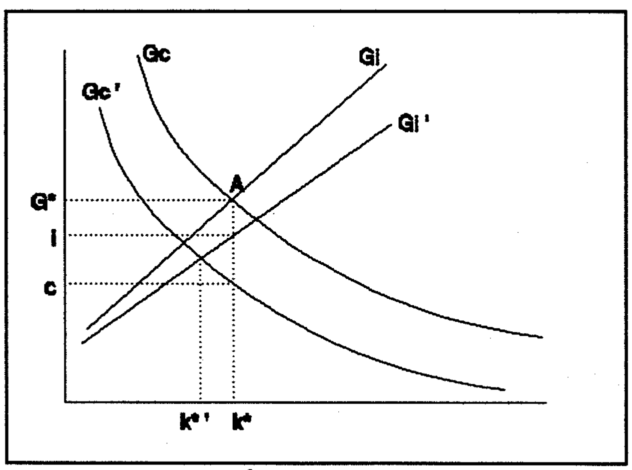

Figure 2.3

In the long run, κ changes. In order to reestablish the equilibrium, it falls to $\kappa^{*\prime}$. In this case, the ratio of capital stocks, κ, was reduced. But, again, a lower overall rate of growth is achieved.

It can be seen that the change in κ depends on the sensitivity of the rate of growth of each sector, in the short run. If the values of equations (2.52) and (2.53), evaluated at κ^* are compared, it can be determined which curve shifts down more. Graphically, the distances G^*i and G^*c are being compared (see Figure 2.2). If the former is greater than the latter, then κ increases in the long run.

This implies that $d\kappa > 0$ if:

$$|dG_i/d\theta_i| > |dG_c/d\theta_i|$$

$$(\tau_i/\Omega^2)(\alpha_c\kappa + \alpha_i)[s_p^2\ A^2\ \tau_c\ R] > (\tau_c/\Omega^2)(\alpha_c + \alpha_i/\kappa)[s_p^2\ A^2\ (1-\tau_i)\ R]$$

Simplifying

$$(\tau_i)(\alpha_c\kappa + \alpha_i) > (\alpha_c + \alpha_i/\kappa)(1-\tau_i)$$

Factoring out κ from the first term

$$\kappa(\tau_i)(\alpha_c + \alpha_i/\kappa) > (\alpha_c + \alpha_i/\kappa)(1-\tau_i)$$

Simplifying again, and reordering

$$\kappa > (1-\tau_i)/\tau_i$$

Briefly, if $\kappa > (1-\tau_i)/\tau_i$, then in the long run κ increases, when θ_i rises.

2.5.2. Long run impact of changes in θ_c

It has been assumed that θ_i was changing. An increase in θ_c would have given the same results, since both parameters move in the same direction. In any case, whether κ goes up or down, the rate of growth of the economy decreases when the mark-up of any sector increases.

In section 2.4.1. some comparative statics exercises were done, assuming a short run horizon. Up to that point, it was assumed that κ was not changing. Those results are recalled, taking into account that κ does change in the long run.

The capital stocks ratio may increase or decrease. In the following exercises, the case in which $d\kappa$ is positive is considered. Therefore, some of the short run effects may be strengthened, while others may be weakened.

Analyzing equation (2.41), it can be seen that the level of utilization of the consumption sector falls even further in the long run. This can be seen in Figure 2.4, where the values of κ are measured on the vertical axis, and the levels of utilization of the consumption sector are measured on the horizontal axis. Assume that the initial equilibrium is point A, where the ratio of capital stocks is κ^*, and the corresponding level of utilization is $(q_c/K_c)^*$. The hyperbola represents equation (2.37′).

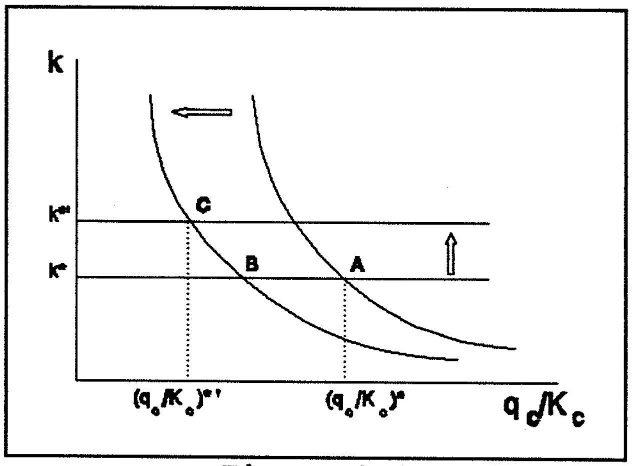

Figure 2.4

An increase in θ_c shifts the hyperbola backwards to the origin. In the short run κ does not change, so the level of utilization decreases only to point B. In the long run κ may go up or down depending on the values of the parameters. Suppose that κ increases in the long run ($d\kappa > 0$). Then, q_c/K_c goes back even further, to point C. Therefore, the level of utilization, $(q_c/K_c)^{*\prime}$, is lower in the long run.

This result is reflected in the realized rate of profit of the consumption sector. The lower level of utilization of capacity in the long run leads to an even lower rate of profit of that sector. This is reflected in equation (2.43).

On the other hand, the results of the investment sector are not strengthened, but weakened in the long run. Again, the case where κ rises is being considered. Equation (2.42) shows that in the short run the level of utilization of capacity in the investment sector decreases. In the long run, however, this utilization ratio increases with respect to its short run level, due to the increase in κ. This may not overcome the short run negative effect, however[15]. Figure 2.5 illustrates this situation. As in the previous case, κ and q_i/K_i are drawn on the vertical and horizontal axes, respectively. The upward sloping line is equation (2.38').

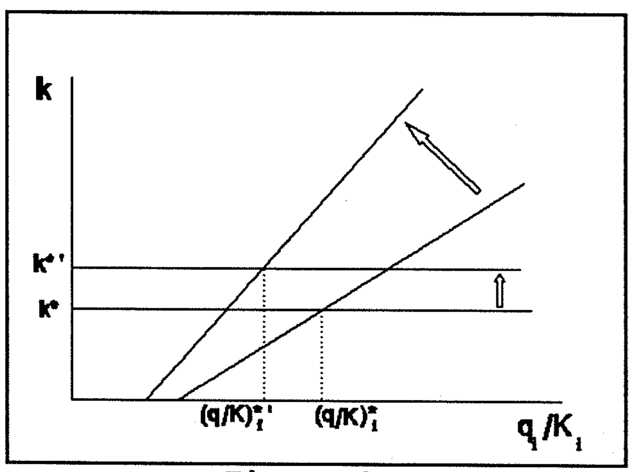

Figure 2.5

[15] This statement can be done once the short run and long run impacts on the growth rates are analyzed. Cf. Footnote 14.

When θ_c rises, this curve becomes steeper. With a given κ in the short run, the level of utilization in the investment sector falls. But in the long run, when $d\kappa$ is positive, it recovers from the short run negative effect, but it does not overcome it.

The clue to this is given in Figure 2.2, where it can be seen that the rate of growth of the investment sector decreases in the short run to 'i'. This rate of growth is related to the fall in the short run level of utilization given by equation (2.42). In the long run, as κ^* increases to $\kappa^{*'}$ the rate of growth of the investment sector also increases, but it does not overcome the initial G^* level. This recovery reflects what is happening to the utilization level. In the long run, q_i/K_i rises, but to a level lower than its initial level. In Figure 2.5, the final utilization level $(q/K)_i^{*'}$ is lower than $(q/K)_i^*$[16].

These opposite forces are reflected in the rate of profit of the investment sector. As indicated by equation (2.44), in the short run, the increase in the mark-up of the consumption sector reduces the rate of profit of the investment sector. In the long run, there is a positive effect coming from the capital stocks ratio that improves the short run negative result. However, in the end, the rate of profit of the investment sector is still lower than the starting equilibrium level.

[16] The mathematical analysis of equation (2.42) shows these opposite effects. However, in this case, the algebra is not useful to shed light on which effect is greater.

2.5.3. Long run impact of changes in θ_i

The effects of changes in θ_c, in the short run and long run, presented above, are similar to those of θ_i.

It was shown that, in the short run, an increase in the mark-up of the investment sector reduces the level of utilization of capacity of the consumption sector. This result is presented in equation (2.45). In the long run, this negative effect is strengthened by the change in the capital stocks ratio, since the case where the term $d\kappa/d\theta_i$ is no longer zero but positive is being considered. With the negative sign in front, it indicates that the fall in the level of utilization of the consumption sector is even greater in the long run. Figure 2.4, above, shows the effects of a change in θ_c. The same adjustment mechanism applies to changes in θ_i. An increase in θ_i moves the hyperbola backwards to the origin, leading to a lower q_c/K_c. In the long run κ increases, and the decline in q_c/K_c is deeper.

Accordingly, as the level of utilization decreases, the fall in the rate of profit of the consumption sector is also more pronounced in the long run. This relationship is formalized under equation (2.47).

The effects of a change in θ_i on the variables of the investment sector show different results. Equation (2.46) shows the negative impact in the level of utilization of capacity of the investment sector in the short run, when $d\kappa$ is zero. In the long run, however, if κ increases, q_i/K_i recovers from the first effect. Using Figure 2.2 once more, it can be checked that the rate of growth of the investment sector rises in the long run, but it does not recuperate the level before the rise in θ_i.

This reflects the changes in the level of utilization of the investment sector. The level of utilization of capacity cannot be higher than its initial level because, otherwise, the rate of growth of this sector would also be higher. This is not ocurring, as can be seen from Figure 2.2.

The results presented so far are quite clear. Nevertheless, the final effect on the rate of profit of the investment sector is uncertain. When the short run effect was analyzed, it was said that the result was indeterminate. Equation (2.48) shows the effect of an increase in the mark-up of the investment sector on the rate of profit of the same sector. In the long run, if the capital stocks ratio increases, there is a positive effect on the rate of profits of the investment sector. In terms of Figure 2.1, this means that the level of utilization (q_i/K_i) recovers from its fall to $(q/K)_2$. It cannot be ascertained that the rate of profit of the investment sector increases in the long run when the firms increase their mark-up. However, this could be the case, depending on the change in the mark-up, θ_i, and the final level of utilization of capacity, q_i/K_i in the long run.

The reader may wonder why the firms from either the consumption or the investment sector are not able to increase their realized rate of profit, with certainty, even in an oligopoly framework. The economic reason for this is that when the mark-up of the consumption sector rises, there are two opposite effects on the total amount of profits of that sector. On one hand, the share of profits on the price of consumption goods, $\theta_c/(1+\theta_c)$, is increasing. On the other hand, the quantity of output sold, q_c, is falling. This can be seen through equation (2.5), where θ_c is going up, while q_c is falling down. However, the impact of the lower output is higher and leads not only to a lower rate of profit, but also to a lower overall level of profits in the consumption sector.

The case of the investment sector is quite different. When the mark-up of the investment sector goes up, due to an increase in the normal rate of profit, for example, there are three effects. First, the rise in θ_i causes a direct increase in the amount of total profits. Second, it also leads to an increase in the price of the capital good, P_i. This increases the replacement value of the capital stocks. Since there is a given desired rate of return, higher profits are required. These higher desired profits induce an even higher mark-up of the investment sector. This 'second round' on the mark-up causes a positive effect on total amount of profits, too. The third effect is that the higher θ_i leads to an increase in the mark-up of the consumption sector and, consequently, in the price of the consumption good. As has been seen, this causes a fall in the real wage rate, in the output of the consumption sector and, ultimately, in the output of the investment sector. Thus, the third effect is a smaller number of capital goods sold, which affects in a negative way the amount of profits of the investment sector.

Therefore, there are two positive effects and one negative effect over the rate of profits, as well as over the whole amount of profits of the investment sector. The difference with the consumption sector is that the two rounds of the mark-up of the investment sector have a stronger impact than the sole increase in the mark-up of the consumption sector. Therefore, they may balance the negative effect of the utilization level.

The source of the different results for the two sectors is the way of pricing. If simple mark-up pricing would have been adopted, the impact over the rate of profits and the overall amount of profits would have been the same for both sectors. They both would have been negative.

2.5.4. Long run impact of changes in s_p

Finally, the long run impacts of a change in s_p are presented. Equations (2.49) and (2.50) can be recalled to allow $d\kappa > 0$, as has been assumed. The level of utilization of capacity of the consumption sector falls even further in the long run, while the output in the investment sector recovers from its short run lower position.

These effects on the rates of profit of both sectors are also present because they depend directly on the level of utilization of capacity.

Figure 2.2 showed the impact on the rates of growth when the mark-up of the investment sector increased. A rise in s_p produces the same effect, shifting both curves down. In the short run, they may end up with different rates of growth. However, in the long run, if the capital stocks ratio increases, the rate of growth of the consumption sector decreases even further. The rate of growth of the investment sector recovers from the short run fall, although it does not overcome the initial level.

Figure 2.6 shows the relationship of the growth functions with the levels of utilization of capacity.

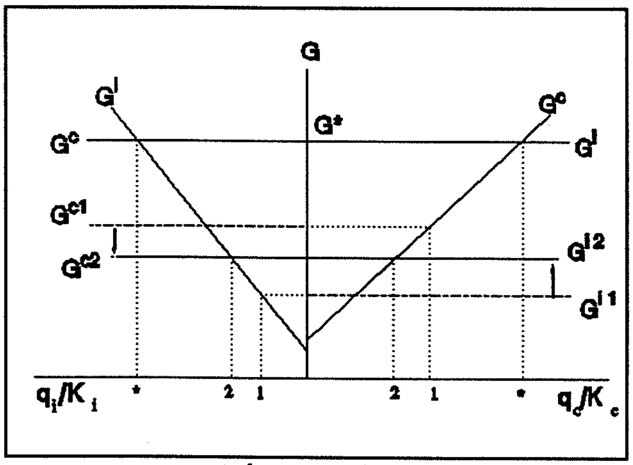

Figure 2.6

On the right hand side, the rate of growth of the consumption sector is measured by the vertical axis, and the level of utilization of the same sector, by the horizontal axis.

The upward sloping curve, G^c, is equation (2.24), while the flat curve, G^i, is equation (2.25). The left quadrant represents the analogous variables for the investment sector. The increasing curve, G^i, is the investment sector growth function, given by equation (2.25), while the flat line is equation (2.24).

Assume that initially the steady state output of the economy is growing at a rate of G^*. As has been stated, the growth rate of both sectors must be equal. The asterisks on the horizontal axis denote the initial level of utilization of capacity for both sectors. Now, consider an increase in the propensity to save on profits, s_p. The level of utilization of the consumption sector falls, say to '1' on the q_c/K_c axis. Given the growth function of the consumption sector, G^c, this corresponds to a rate of growth equal to G^{c1}. In the second quadrant, the flat G^c falls to G^{c1} (drawn with a discontinous line). On the other hand, the increase in s_p also reduces the level of output in the investment sector to a level like '1' on the q_i/K_i axis. According to the growth function, this level of utilization corresponds to a rate of growth of G^{i1}. In the first quadrant, G^{i1} is the new rate of growth of the investment sector (drawn with a discontinuous line), which is below G^i.

In the short run, the rates of growth of the consumption and investment sectors can be different. In the long run, however, they must be equal. Assuming that the capital stocks ratio, κ, increases, the level of utilization of capacity in the consumption sector falls further in the long run, to a level like '2' on the q_c/K_c axis. In the investment sector, however, there is a recovery, and the level of utilization increases to '2' on the q_i/K_i scale. In the end, both sectors grow at the same rate (drawn with a continuous line).

The negative effect of a higher propensity to save on profits is also present in this model. The rates of utilization, the rates of profit, and the rates of growth of both sectors end at lower values. Although, a savings function has not been presented, it is implicit in equation (2.20). The savings function is introduced here for the only purpose of checking the paradox of thrift.

$$(2.57) \quad S = s_p \; (P_i/P_c)\,(r_i \; K_i + r_c \; K_c)$$

where S is total savings in real terms. Factoring out K_i from (2.57):

$$(2.58) \quad S = K_i \; (P_i/P_c) \; s_p \; (r_i + r_c \; \kappa)$$

Substituting the values of r_c from (2.39) and r_i from (2.40) into (2.58), and taking the derivative of S with respect to s_p:

$$(2.59) \quad \frac{dS}{ds_p} = \frac{K_i(P_i/P_c)}{\Omega^2}\left\{ [\alpha_c d\kappa]\,[s_p\theta_c]\,[\Omega] - [\alpha_c\kappa+\alpha_i]\,[A]\,[\theta_c]^2\,[1+\theta_i]\right\}$$

Expression (2.59) indicates that, in the short run when $d\kappa$ is zero, the amount of savings falls. In the long run, savings recover but without overcoming the initial level. To prove that dS/ds_p is negative, even in the long run, some algebraic manipulation is needed. It has been shown that the equilibrium rate of growth falls when s_p rises. A link between this growth rate and the total savings is now presented so that the result can be clearly stated.

Substituting the values of r_c and r_i into (2.58) and simplifying

$$(2.60) \quad S = K_i \ (P_i/P_c) \ s_p \ \theta_c \ (\alpha_c \kappa + \alpha_i)/\Omega$$

From the steady growth rate in (2.51) the value of $(\alpha_c \kappa + \alpha_i)$ can be found

$$(2.51') \quad (\alpha_c \kappa + \alpha_i) = (G - \alpha_i)/\tau_i I$$

Substituting (2.51') into (2.60)

$$(2.61) \quad S = K_i \ (P_i/P_c) \ (s_p \ \theta_c) \ (G - \alpha_i)/(\tau_i \ I \ \Omega)$$

Replacing the value of I and simplifying

$$(2.62) \quad S = K_i \ (P_i/P_c) \ (G - \alpha_i)/\tau_i$$

Equation (2.62) establishes a direct relationship between the rate of growth of the economy and the total amount of profits. It was seen that the rate of growth falls when s_p rises. Expression (2.62) confirms that, when G decreases, the total savings also go down. Therefore, the paradox of thrift is also present in this model.

CONCLUSIONS

This paper dealt with the issue of extending the standard Kaleckian growth model of one sector to a two-sector economy. In the first chapter the one sector model was shown. The results of the Kaleckian model presented were mainly that an increase in the mark-up by firms led to a reduction in the real wage rate and consequently in effective demand. Within the Kaleckian framework of sticky prices, lower demand induced firms to reduce their production. Finally, the fall in output was reflected in a lower rate of profit and a lower rate of accumulation. This same outcome resulted from the desire of profit recipients to become more thrifty. An increase in the propensity to save out of profits reduced effective demand, the level of output, the rate of profit and the rate of growth fell. The economy altogether deteriorated. Even when the propensity to save by workers was introduced, the overall situation worsened.

At the end of chapter one, the argument made to Kaleckians by Steedman (1992) was presented. He criticized the simplified assumption of Kaleckian models that firms of different sectors were not linked. Particularly, he commented on the issue that the price of one sector depended on the mark-up of that sector only. He argued that in reality prices of some sectors were indeed affected by the prices and mark-ups of other sectors.

In the second chapter, a two-sector growth model was developed with Kaleckian assumptions. The main objective was to include dependency between the two sectors, through the pricing system, and to analyze whether the results of the one-sector model still held. Some important results were the following. First, increments in the mark-ups of the consumption or the investment sectors caused a reduction in the output levels of the economy in the short run. Although in the long run, some recovery was present for one of the two sectors (depending whether the capital stocks ratio increased or decreased) the final result for both of them was always negative. Second, as the rates of profit depended directly on the level of output, the higher mark-ups led to lower rates of profit. The exception was the rate of profit of the investment sector, when the mark-up of that sector increased. The result was indeterminate.

The main source of difference in the behaviour of both sectors was the way they set their prices. It was assumed that firms use target return pricing to calculate their desired profits, as well as the mark-up over costs. On one hand, this created inter-dependence between the sectors through the mark-ups. On the other hand, it created a mechanism to improve the profits of the investment sector by making their capital stocks more valuable, and requiring higher profits. The effect of an increase in the mark-up of the investment sector over their own profits depended on the particular values of the parameters.

The third impact of higher mark-ups was lower rates of growth for both sectors in the short run. In the long run, one of them recovered, but not enough to overcome the overall negative effect.

It should be noted that this outcome was a result of the form of the growth functions that were adopted. The growth functions of both sectors were only functions of the level of utilization of capacity, plus an exogenous term. In these functions the rates of profit were not included. Had the rates of profit been incorporated, the obtained results would have been, not only more complicated, but also more uncertain. For example, the behaviour of the investment growth function would have been almost impossible to determine. The share of profits would have been going up, while the utilization ratio would have been going down. This indefinite outcome would have eventually affected other variables and indefinite signs would have spread to all parts of the model.

Finally, the paradox of thrift of the one-sector model was also present in the two-sector model, in the short run and the long run. An increase in the propensity to save out of profits pushed down the effective demand, leading to a lower level of output in the consumption goods sector. Accordingly, the fall in the level of utilization of this sector led to a reduction in their demand for capital goods and the output of the investment sector also fell. These lower levels of output affected negatively the rates of profit and ultimately the total amount of savings. Thus, a higher propensity to save caused lower savings. Another consequence of a change in this parameter was a slowdown in the rates of growth of both sectors.

The results presented above indicate that, when a two-sector economy is built within the Kaleckian frame, most of the results of the one-sector model hold. The key point here is the pricing system that is being used and the form that may take the different equations of the model. With respect to the latter, it would be interesting to consider the case where the growth rates are also a function of the rates of profit or when savings by workers are present. Presumably, the model would become difficult to handle and it would be almost impossible to find clear cut results.

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