Uneven-Aged Forest Management

An Analysis and Application Using UNEVEN

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1. INTRODUCTION

Forests once abounded on the Canadian landscape. The resource seemed virtually inexhaustible, and the production of wood, a guarantee to sustainable economic health. But our forests are shrinking. They have demonstrated their finite nature, and their potential disappearance has forced us to acknowledge that forests are not simply producers of timber, but also valuable in their contribution to a healthy environment for life.

The economic importance of Canada's commercial forests is indisputable. They create some 750 000 direct jobs every year, inject close to $30 billion into the economy, and ensure the survival of 300 municipalities (Amyot 1987, p.2). However, our focus on forests as simply another industry has dangerously led us to diminish their value to the timber they produce. This is an overwhelming simplification. Forests provide a habitat for wildlife, they help purify the air and water, protect river basins from erosion, regulate the flow of rivers and streams, reduce the risk of floods, and provide areas for various recreational activities. As they become increasingly sparse, the range of demands made upon our forests are broadening. The concept of the single use forest is clearly inadequate, and is increasingly being rejected in favour of multiple use.

Multiple use forests have generally been addressed in the forest management literature in the context of the even-aged forest, containing trees of approximately uniform age, or the regulated forest, containing as many plots of uniform age as years in the rotation. Uneven-aged management is an alternative that, although seemingly more complex and costly in application, may in fact be more relevant to the economist's goal of maximizing net social benefits. Uneven-aged management has been common practice on privately owned non-industrial forests, but on Crown land and many privately-owned industrial forests, even-aged management with clear-cutting has been the rule.

This paper begins with a look at the management objectives of various classes of forest owners and managers. The literature on multiple use management is discussed, along with the applicability of uneven-aged management in the multiple use context. Finally, UNEVEN (Boothby and Buongiorno 1985), a simulation model for uneven-aged forest management, is considered and applied to the Turkey Lakes Watershed forest in Algoma District, Ontario, near Sault Ste Marie. UNEVEN is a static model that tracks the evolution of a forest or stand under various management scenarios. It is a very simple model that is easily implemented with minimal data requirements, and that provides easily understood and implementable results. Despite its simplicity, UNEVEN includes four management options which recognize the various uses of a forest and the resultant variety in objectives among forest managers.

2. MANAGEMENT OBJECTIVES

A forest in its undisturbed natural state is composed of a distribution of trees of various ages and species. The combination of silvicultural options and intensities available to the forest manager who inherits a natural forest are infinite: clear-cut or selectively cut age classes and species at some time, mine the stand to make way for alternative uses, leave the site undisturbed. The choice of silvicultural method and in turn, its intensity, is a function of the particular objectives of the each forest land owner or manager.

Economic theory states that individuals, and groups of individuals, act and make choices with the goal of maximizing the stream of net benefits that they expect to enjoy. Clearly, if we recognize that a forest provides valuable services in its standing state, as well as in the production of timber, we cannot limit the forest management problem to the latter. The forest manager will choose to maximize the value he or she expects to reap from all of the services that the stand provides.

Forest management objectives, will vary significantly among forest managers with land ownership characteristics, with land characteristics, such as size, location and species, and with the personal characteristics and preferences of the land owner. The list of possible objective functions to the management problem is therefore infinite, encompassing the various single objectives in various combinations and weighted in various ways. Knuchel (1953. p.12 in Hann and Bare 1979, p.2) provides a comprehensive outline of forest management objectives upon which the following discussion is based.

> 1. Maintenance of the health and resistance to damage of the forest; raising the productive capacity of the soil and of the stands to the highest volume possible on the site and maintaining that productive capacity.
>
> 2. Continuous production of the highest possible volume of valuable timber.

These first two objectives correspond to the maximum sustained yield rotation long favoured by foresters. It is a basic biological decision rule which involves harvesting at the maximum mean annual increment. The choice of rotation is such that it maximizes a sustainable periodic flow of timber. Maximum sustained yield policies have attracted criticism on many grounds[1]. The criterion is purely biological, ignoring all economic considerations such as the value to society of the forest resource and the efficient allocation of scarce resources among alternative uses and among lands of varying productivity. It is valid only if prices, costs and interest rates are of no concern and the only objective of management is to produce the largest periodic flow of timber.

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[1] A more thorough discussion of the maximum sustained yield principle is presented in: Conrad and Clark (1987, p.70).

3. Providing the highest possible financial yield.

Combined with objective one, the third leads to the solution to the forest rotation problem that has gained wide acceptance in forest economics. It was presented in principle as early as 1849 by Martin Faustmann. In the Faustmann problem, prices net of harvesting costs (P), regeneration costs (C), interest rates (i), and stand productivity are assumed known and unchanging. The volume of timber is a function, V(T), of stand age; it has the standard form of a growth function, initially increasing in T, then levelling off, and finally decreasing when the trees begin to decay. The forester is assumed to maximize the present discounted value of the land, the returns from a series of plantings and harvests on a stand which is managed in perpetuity. The forest is treated simply as a source of timber saleable, under the assumptions of the model, in a perfectly competitive market, it produces no externalities. Constant returns to timber production are assumed.

The Faustmann problem is described by the equation:

$$(1) \quad \lambda = \max \{[PV(T)e^{-iT}-C]/(1-e^{-iT})\}$$

This equation is based on the convergence of the infinite series representing the sum of all future revenues. The first order conditions give:

(2) $PV'(T) = iPV(T) + i\lambda$

This is the well known Faustmann equation. The interpretation is that harvest should take place at the time when the rate of change of the value of the stand with respect to time is just equal to the interest on the value of the stand plus the interest on the value of the forest land, the marginal user cost. An alternative interpretation is obtained by rewriting equation (2)

(3) $PV'(T)/(PV(T) + \lambda) = i$

Indicating that the stock should be held until the relative rate of growth of the combined asset value of the timber and land just equal the market rate of interest.

The Faustmann rotation maximizes the monetary returns to forest capital, it is 'socially optimal' under the major assumption that the stand provides benefits only in the form of monetary returns from timber production. However, in the broader context of maximizing total net benefits to society, the solution fails. The Faustmann problem fails to consider non-timber benefits, the aesthetic value of a forest stand, its ecological value in the maintenance of wildlife, for recreation, and for proper soil drainage. Forest owners are diverse, they have varying time frames and management objectives. These objectives can not realistically be constrained to include the maximization of value from the flow of timber alone.

Non-timber services provided by the standing trees are the subject of the fourth in Knuchel's list of management objectives:

> 4. Promoting the protective effect of the forest in the widest sense (protection against soil erosion, avalanches and flooding, protection of the scenery, protection of the native flora and fauna).
>
> 5. Provision of regular employment for the local inhabitants, especially during the time when labour is not wanted for agriculture.

Although the fifth objective seems outdated, the provision of stable employment remains an important consideration, especially in region or communities which have little other economic activity. The last in Knuchel's list applies particularly to Crown owned land, in which case forestry may be used to smooth economic downturns.

> 6. The forest should work in a sense like a savings bank, in that in time of need it is in a position to supply a greater yield of material than in normal times, without losing its productive capacity.

Clearly, many of the objectives listed above are in conflict, and satisfying them simultaneously would be an impossible task. Tradeoffs are a necessity. The nature of the tradeoff will depend on the goals of a particular forest manager. These can be expressed by the relative weights assigned to each of the preceding single objectives. This conflict in the provision of the various forest services is the essence of multiple use forest management. The aim is to provide the "best" combination of outputs in each time period and over time. The particular combination will depend on management goals, which are a function of land ownership. A common element, however, in each ownership class is the maximization of net benefits. It is the nature of these benefits that varies, be they private benefits or social.

The broader consideration of maximizing net social benefits is especially relevant in Canada where most forested land is publicly owned. Ninety one percent of Canadian productive forest land is Crown owned, eighty percent by the various provincial governments and eleven percent federally (Canadian Forestry Service 1988). In theory, public agencies are responsible for representing the interests of the citizens they serve. We should therefore expect the majority of our forests to be managed such that the total net benefits to society from all their uses is at, or reasonably close, to its maximum.

A short look at a current newspaper or newscast makes it clear that popular awareness of the role of healthy forests in the preservation of the environment has increased significantly in recent years. The sensitivity of the public to environmental issues is increasing, and the trend can be expected to continue and possibly accelerate through the nineties. The decade has already been declared as the decade of the environment.

Specifically, concern for the preservation of forest lands, and toward the seemingly imminent disappearance of old-growth forests, and of certain wildlife species and ecosystems has been expressed with respect to the Temagami region in Ontario and the Carmanah Valley in British Columbia. The Canadian population has demonstrated its desire to have the provision of non-timber services included in the management plan.

The commitment of the Crown to the management of public forests for multiple uses is also evident. Forestry Canada (1989) states as its objective:

> To promote and enhance the sustained economic utilization of Canada's forest resource through environmentally sound forest management, and to enhance the social and economic benefits derived from publicly and privately owned forests and from forest related activities in Canada.

Similarly, in its annual report, the Ontario Minister of Natural Resources (1987-1988) writes the following, which is representative of most provinces:

> As one of Ontario's most valuable renewable resources, forests provide thousands of jobs and give us a wide range of products which we use every day. Forests are habitat for wildlife. They help prevent flooding and soil erosion. They are locations of recreational enjoyment and contribute to the beauty of the Ontario landscape. MNR's forest management program focuses on balancing the many uses of this renewable resource.

Thus the incorporation of all aspects of the value provided by a forest in practical management models is clearly required.

3. MULTIPLE USE MANAGEMENT

That our forest resources provide various services is indisputable. When these resources were abundant, single use management prevailed -- specific tracts of land were managed for recreation, wildlife, timber production, or set aside as reserves -- but as forests become increasingly scarce, competition between these single uses is escalating. The need to consider multiple use management of single areas is increasing.

What specifically does "multiple-use management" entail? Definitions of the term are abundant and varied. We rely on Evans (1938 in Behan 1967 p.476) and the USDA Multiple Use Sustained Yields Act of 1960 (in Behan 1967 p.478) for ours. Multiple use forest management involves the harmonious and coordinated management of the various resources, the trees, the soil, the waterflow, the forage, the wildlife, and the scenic and climatic values, while preserving the productivity of the land, with consideration being given to the relative values of the various resources, and not necessarily the combination of uses that will give the greatest dollar return or the greatest unit output.

The concept of the multiple use forest is not new. However, the management of a forest or other renewable resource when the stock itself has consumption value is the subject only of relatively recent research, first by Gregory (1955) in terms of isocost and isorevenue curves, then by Hartman (1976), Calish et al. (1978) and Samuelson (1976), for the Faustmann model. Multiple use forest management developed mainly as an extension of the even-aged single use forest management problem in which the decision variable is the rotation age, the age at which the entire forest should be harvested. This age is determined by one of several rule-of-thumb principles. For the economist, who treats a renewable resource as a capital asset, this generally takes the form of a profit maximization formula through which the value of the resource to society is maximized. The accepted economic formulation for the determination of the rotation for timber only is the Faustmann model which maximizes soil expectation value. Multiple use management requires, in the maximization of land expectation value, an economic accounting of non-timber yields.

While timber yields occur periodically, at the rotation time, non-timber services are supplied continuously and are generally thought to be related to the stock of the forest, the number, size and species of trees. Hartman (1976) incorporated non-timber outputs of the forest in his model for maximizing land expectation value, the combined present value of timber and non-timber benefits from current and future harvests. His model considers an even-aged stand and assumes that the benefit flow from non-timber services can be expressed as a function, a(n), of the age of the standing stock. Thus $\int_0^T [a(n)e^{-in}]dn$ represents the present value of these amenity services from a single harvest cycle of length T. The problem is:

$$(4) \quad \lambda = \mathrm{Max}_T \ \frac{[PV(T)e^{-iT} + \int_0^T a(n)e^{-in}dn - C]}{(1-e^{-iT})}$$

The first order condition gives:

$$(5) \quad PV'(T) + a(T) = iPV(T) + i\lambda$$

As in the Faustmann problem, the solution in equation (5) requires that harvest take place at the age at which the marginal benefits of delaying harvest just equal the opportunity costs. The benefits of delaying harvest include the increment in value from timber growth (PV'(T)), plus the flow of non-timber benefits during that delay, a(T). The opportunity costs are the same as in the timber only problem, the interest on the value of the stand and the interest on the value of the forest land. Alternatively, the first order condition can be expressed as:

$$(6) \quad \frac{PV'(T)+a(T)}{PV(T)+\lambda} = i$$

Indicating that the stock should be held until the rate of increase in the total value of the land is equal to the rate of interest, where the value of the land is in terms of both timber and non-timber services.

A comparison of equations (5) and (6) with the Faustmann equations (3) and (4) indicates that the presence of non-timber services provided by the standing forest has an impact on the choice of rotation length. The solution depends on the value of the flow of amenity services over the cycle relative to the net timber receipts, and on the separate relative growth rates in the amenity and timber values. If non-timber values are increasing with stand age, then the Hartman rotation exceeds the Faustmann rotation; if they are declining with stand age, then it will be shorter than the Faustmann rotation. Significantly high amenity values may even indicate that the forest should never be harvested. For example, if a logging company were to inherit an old growth forest, the current high flow of non-timber services may justify its preservation. This is precisely the situation under debate in regard to the Carmanah valley in British Columbia, where a logging firm has inherited an old growth forest believed to be in the neighbourhood of one thousand years old. They are facing considerable opposition to their logging plans by several public interest groups concerned that the resultant loss in non-timber values will be enormous.

The Hartman analysis is theoretically correct for the case of an isolated stand, where all non-timber services can be identified and their value expressed as a function of the forest stock.

Under such ideal circumstances, the competitive market system would provide the socially optimal combination of the various services over time, including timber production. In practice, however several problems arise, beginning with the consideration of each stand in isolation. This ignores the undeniable interrelationships among stands which affect their values. The value of a single stand is dependent on the treatment of its neighbouring areas. A popular wilderness hiking area, for example, will loose some of its recreational appeal if an adjacent forest is clear-cut.

In addition to the interactions among stands, the valuation itself of non-timber services presents several problems. Measuring the value of the many individual amenity services, let alone determining their relation with stand characteristics, is a considerable task. Non-timber services are a varied category, some increasing with stand age, some decreasing, and some requiring an irregular age distribution. Consequently, there is no a priori reason to expect them to be either monotonically increasing or decreasing with stand age. In fact their variety indicates that amenity services are an irregular function of stand age, depending upon the particular range of services provided by the stand. A USDA study (Thomas 1979) on forest management for both timber and wildlife objectives recommended a complex array of stands of various characteristics. Calish et al. (1978) derived rotation dependent yield functions for seven non-timber outputs on a largely judgemental basis from relationships and data in several research reports and in consultation with foresters, biologists and engineers. The functions differed significantly. In general, empirical knowledge of the relationship between stand age and the yield of amenity services is very weak, making the specification of a relationship particularly challenging.

For various reasons, most non-timber amenity services provided by the standing forest are not traded in the market and consequently there are no clear market signals indicating value and price. Theoretically, value should correspond to individuals' willingness to pay for particular services. Walter (1977) suggests that the economy fails to efficiently provide the multiple services demanded of the forest due to a combination of precedence and the presence of external economies in forest management. Forests, while abundant, were considered almost worthless for anything but timber production and managed as single use timber holdings. As they became scarce, they acquired value in other uses, but the traditional organization based on single use management has been slow to respond. In addition, these alternative services have been provided to the public free of charge, with a resultant tendency for overuse, and a reluctance to be required to pay.

The essence of the multiple use management problem, and the reason most timber holdings continue to be managed as single use resources, is that the timber stock provides benefits to society which are not entirely captured by the resource owner. The various timber and non-timber services are provided jointly by the forest, where the production of one may hinder or aid in the provision of another. Thus forest management for a particular single use produces externalities, both positive and negative. For example, a stand managed for the production of timber may also provide protection against soil erosion and a habitat for several wildlife species. Under perfect market conditions the recipients of the non-timber services would willingly collectively pay the forest owner to delay a harvest that would reduce the value of such services. Alternatively, the forest owner may pay for the loss of non-timber services. But in the presence of transaction costs and in the absence of clear liability rules in forestry, these externalities are not internalized in the competitive market. Consequently, the actions of independent forest owners maximizing their own net benefits do not lead to the socially desired mix of forest services. This is a common justification for widespread public ownership of forest lands.

In the absence of clear market signals, information on the market values of non-timber forest services can be obtained in several ways. While the number of possible methodologies for evaluating non market forest services is large, the following discussion is limited to some of the more common and more recent contributions. In general, efforts have been directed toward the valuation of recreational services provided by forests. Most early attempts were based on estimates of the total expenditures by the population on these various services. The failure of this approach as an indicator of the value of the actual services was recognized by Zivnuska (1961). In reality, the value of accompanying services such as accommodation and equipment was being measured.

Alternatively, a specific monetary indication of value can be obtained through the derivation of shadow prices for the various multiple use outputs. This is often accomplished on a judgemental basis. The most commonly advocated technique for evaluating shadow prices is through surveys of the various users' willingness to pay for use of the forest[2]. A major problem with this approach, a problem which brings into question the quality of results, is that the respondents are not faced with an actual choice. There is an incentive to underestimate willingness to pay if respondents thinks the survey results will lead to an actual charge, and an incentive to overestimate if they think a service they value may be provided out of public funds. This is the famous 'free rider' problem in the provision of public goods. A second factor complicating the interpretation of results is

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[2] The discussion of shadow prices is based on Walter (1977), pp. 353-355.

that if the users were actually charged the value they declare, the quantity of the good or service consumed would likely change.

Shadow prices can also be evaluated through mathematical programming, or by bargaining between government agencies as to acceptable transfer prices for services or land use.

Evaluation of multiple use services can also be done through artificial markets. For example the government could institute a system of vouchers distributed to consumers, or institute a system of rights, to cut or not cut timber, to camp, which can be put up for bid.

Two other methods, discussed by Pearse (1990) are based on estimates of the fixed travelling costs incurred by recreationists in arriving at a particular site. In the first, the value of a recreation site is expressed as the sum of the maximum access fees or tolls that each consumer is willing to pay, in addition to the fixed costs incurred (Pearse 1990 p.73). These maximum amounts are arrayed, thus determining points on the demand curve. The value of the recreation site is the consumer surplus. It is assumed that the consumer that incurs the highest travel cost is marginal, that is the point where price is equal to cost. The value of the site can then be expressed as the sum of the difference between the cost that each consumer incurs and the travel costs of the marginal consumer. However, this procedure requires some unrealistic assumptions, most significantly, that all consumers of the recreational service are willing to pay the same amount regardless of their income and other characteristics.

An alternative method that avoids this assumption involves estimating consumers' response to these access fees. This estimate is based on a measure of the consumers' sensitivity to the costs they incur to reach the site. This methodology, however, introduces its own assumptions, specifically, that all individuals from a particular geographic zone under consideration, have the same participation rate in recreational activities.

Another method, called the hedonic approach (Pearse 1990 p.76), is based on the response of the consumers of recreation services to the quality characteristics of different sites. This method uses observations on how individuals choose among sites to estimate the value they attach to particular site characteristics. A shadow value is determined for each characteristic which is used to determine willingness to pay for each attribute. Each site can then be evaluated according to its particular characteristics. Problems arise, however, in scaling the quality of site attributes and in calculating their value.

None of the above methods will provide ***the*** correct solution to the valuation of non-timber services. They are subject to judgement, bargaining power, and the distribution of wealth. In the absence of a clear and quantitative definition of the multiple use problem or objective, it seems ludicrous to search laboriously for a precise optimal solution. How are revenues from timber production and the aesthetic value of the standing trees to be compared and weighted? Perhaps we should recognize our lack of knowledge and, without abandoning the quest for better information on non-market values and their relationship with stand age, define the problem in a more realistic and practical context, in the context of our present knowledge. For example, as an alternative to dollar value imputations for non-timber services, constraints or regulations can be imposed to ensure that a threshold beyond which a considerable reduction in certain services will result is not violated.

4. UNEVEN-AGED MANAGEMENT

It is undeniable that the flow of value from a forest varies with its characteristics, its age, species and age composition, and the characteristics of the surrounding area. Few will disagree that the aesthetic value of a forest with a more natural appearance, with a wide variety of trees of different ages and species, will be superior to that of a stand of neatly planted trees of uniform age and species. Under such circumstances, the formulation of the multiple use management problem in the even-aged context seems less than ideal. Yet the literature has largely been confined to the even-aged case.

The Hartman and Faustmann problems, and even-aged management in general, neglect the importance of species and age composition in the value that the stand provides to society. Their only decision variable is the rotation length, the 'optimal' time to cut the entire stand. They neglect the impracticality of leaving a stand unharvested for long periods and then felling the entire stand at very specific and extensive time intervals, and the implications of this practice on the intertemporal distribution of the full range of forest services. Although theoretically the present value of the flow of all services is maximized in the Hartman problem, the various services are exogenous. It is reasonable to assume that society will attach a positive value to a more even flow of services rather than for example, ideal hiking conditions this year combined with bad conditions next.

Selective or partial cutting of age classes in an uneven-aged forest is an often neglected option in forest management research and literature. Uneven-aged management leads to a forest with a more natural appearance and structure than the normal and even-aged forests, containing trees of various ages and sizes, and emphasizes the protection and improvement of a stable forest ecosystem, and the production of large-sized, high quality timber (Hann and Bare 1979, p.2). In practice, it is actually more common than clear-cutting, in particular for small non-industrial forests that provide both timber and non-timber services, yet the concept was developed in the literature only after the advancement of the even-aged philosophy (Hann and Bare 1979, p.1). Until the 1970's few contributions were made toward the advancement of uneven-aged forest management. A notable exception is the pioneering paper by Duerr and Bond (1952) in which the determination of the optimal level of growing stock was discussed. The dominance of even-aged management in the forestry economics literature may be explained by its relative simplicity, and the well-known properties of the even-aged approach, which lend themselves to much easier interpretation of analyses.

While the decision variable in the management of the even-aged forest is the rotation length, the case of uneven-aged management decisions is much more complex. The forest management problem requires the choice of the appropriate sustainable volume and distribution of trees among sizes, the species mix, cutting cycle length, and strategy for converting the stand to the sustainable diameter distribution and cutting cycle that best meet management objectives, subject to the constraints of the biology of the forest.

Despite its relative complexity in analysis, uneven-aged management, in practice, may be superior to the even-aged approach. This is particularly true for a forest in which timber production is a secondary or joint consideration, and may also apply to the case of pure timber management. From the economic perspective, the case against uneven-aged management is that harvesting costs per unit volume are generally higher, because a larger area must be covered to remove a given volume of timber than by clear-cutting. Information from the New England states, however, indicates that felling and skidding costs per unit volume are not greatly different between the two forms of management (Filip 1967, in Leak 1985, p.31). There is no reason to believe that the results would differ significantly in reference to other locations. In addition, regeneration costs, which are the most important costs in forestry (Buongiorno and Gilless 1987, p.90) are minimal in uneven-aged management, where regeneration occurs mostly naturally.

The major cost disadvantage of uneven-aged management seems to be the initial fixed costs of building roads to access remote areas where only a relatively small volume of timber is to be removed. Uneven-aged management may also be more costly to administer. In practice, tree marking may be more time consuming and costly than for even-aged management, because more information is required. In addition to stand density, species quality and spacing, diameter distribution must be accounted for in uneven-aged management.

From the biological perspective, tree damage is a more serious problem in uneven-aged management. Logging damage may result as large trees are harvested next to smaller remaining growing stock. The need to take extra care to avoid such damage may put upward pressure on harvesting costs as harvesting becomes a more labour intensive process requiring skilled workers. Windthrow in partially cut stands is another problem of uneven-aged management. It is especially damaging in mixed or softwood stands and may limit the practical application of uneven-aged management in these sites (Leak 1985, p.31).

Uneven-aged management has also been associated with the regeneration of low-value species, and with some species, in the growth of low quality stems as trees crowd each other (Leak 1985, p.28); but these problems can be overcome by careful site selection and silviculture.

In general, because it results in less disturbance to the natural site and less costs in terms of the loss of non-market amenity services, the biological and economic complications of uneven-aged management will not invariably outweigh its advantages. For certain species selective cutting is the only possible silviculture if any regeneration is to occur. The higher fixed costs indicate that it may be more appropriate for the production of large trees leading to high quality timber for which these costs represent a small part of the value of the final product (Buongiorno and Gilless 1987).

5. MODEL AND SIMULATIONS

Given the difficulty of quantitatively expressing many non-timber benefits provided by forest land, the consequent difficulty in representing forest management objectives in the traditional form of an objective function, and in consideration of the intertemporal distribution of the full range of forest services, there is a need to reconsider whether forest management decisions can be dealt with adequately in the context of the even-aged or normal forest. A less restrictive model may be appropriate.

Optimization and simulation are the approaches generally taken in modelling forest management. Optimization methods such as the Faustmann and Hartman problems, mathematical programming, and maximization of sustained yield, schedule harvest flows that maximize the owner's objective, expressed in a well-defined objective function with associated constraints, where applicable. They are designed to identify a particular 'optimal' solution; and force models to conform to very specific forms which present practical difficulties when specific information is not known. This is certainly true of the value of many non-timber services in the case of multiple use management. In addition, specific objective functions often introduce unrealistic assumptions. Linear programming, for example, forces constraints and objective functions to be linear in the variables, which implies constant returns to scale in timber production.

Alternatively, simulation models can predict the behaviour of a forest stand, or any other system, under various management strategies. A single most desirable solution to the problem is not specified by the model. Rather, the decision maker is presented with a menu of alternative outcomes and strategies from which a choice can be made by some method external to the model. Simulation avoids the necessity of assuming the existence of a well-defined objective function to be maximized, and thereby allows for more flexibility in the modelling process. In fact any system that can be represented quantitatively is tractable by simulation.

With the advent of the computer, several models for the management of uneven-aged forests have been developed. The SHAF simulation model (Adams and Ek 1974) is a non-linear programming model. STEMS (USDA Forest Service in Alig and Morris 1979) a forest growth model that describes stand development with or without management activities, and TREES (Johnson et al. 1975 in Alig and Morris 1979), which models aggregate harvest flows, are considered useful for regional or national analyses (Alig and Morris 1979 p.10). Both can be applied to even- or uneven-aged forests.

We considered a simple model, UNEVEN, which applies to the stand or forest level, the level at which most operational problems of harvesting and silviculture are dealt with (Pearse 1990 p. 215).

UNEVEN is a continuous deterministic[3] simulation model for uneven-aged forest management which incorporates both optimization and simulation. It includes four management regimes, the fixed proportion harvest, target distribution harvest, and the economic and regulated economic harvest options which incorporate optimization. It is intended for use by uneven-aged forest managers, and in that vein, is conceptually and operationally simple. Yet UNEVEN maintains reasonable accuracy, and provides the flexibility to consider various management objectives. The program predicts how a particular uneven-aged stand will develop, calculates how much volume the stand will produce, and the net present value of production under various management regimes, including no harvest, and under various economic and biological circumstances. It allows the forest manager to choose the length of the cutting cycle and form of harvest to apply that will best meet management objectives, economic or other, within a specified planning horizon. It also indicates how changes in prices, interest rates, fixed and variable harvesting costs will influence the expected returns from a stand, and is useful in determining what changes should be made in the management regime following or in anticipation of such changes.

UNEVEN requires minimal data which are easily obtainable from the

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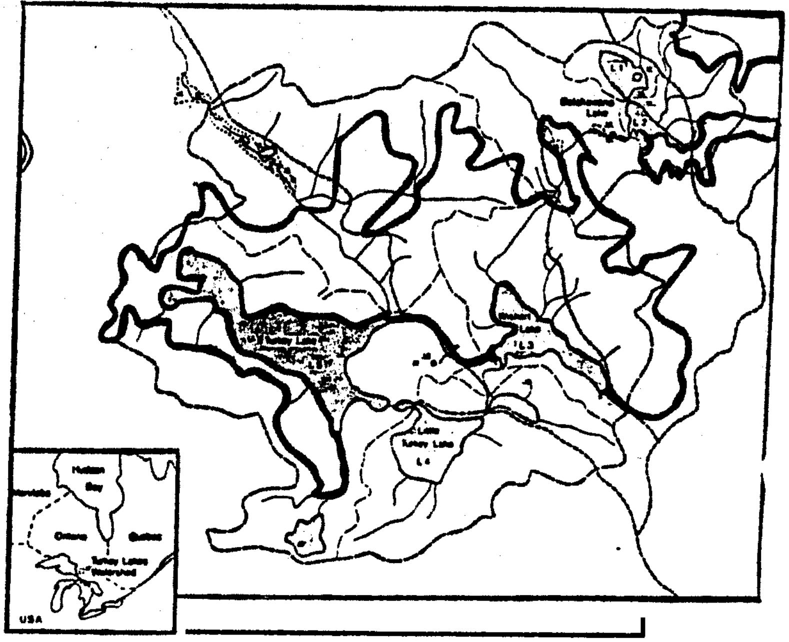
[3] A stochastic simulation model that accounts for uncertainty in stand growth and prices is presented in: Kaya Buongiorno 1987.

stand inventory. Estimation of the growth model requires the allocation of trees into size classes and observations on the number of trees per hectare in each size class at two points in time. This time interval is then used as the time unit for simulations. Basal area per hectare is also required, as is the volume and value of the average tree in each class. Management data include the length of time before the first harvest, length of the cutting cycle, choice of harvest regime, interest rate, and fixed harvesting costs.

FIGURE 1.



To illustrate the application of UNEVEN and determine the accuracy and applicability of its results, we used the model to simulate the evolution of an uneven-aged forest stand in the Turkey Lakes Watershed, located on Crown land in Algoma District, Ontario, under undisturbed and various managed conditions. The inventory data for the stand were originally collected for the

GLFRP-LRTAP Turkey Lakes Watershed Study[4]. The site is an old-growth stand of sugar maple with a minor component of yellow birch. It has been undisturbed, except for a light harvest of yellow birch approximately thirty years ago. The total area, outlined in figure 1, is composed of thirty .10 hectare plots. We selected plots of 90 percent or more sugar maple for the simulation study in order to maintain a uniform species composition throughout the stand.

5.1 The Growth Model:

The growth model is the central component of UNEVEN. It is a fixed coefficient matrix model which traces the movement of the forest inventory through size classes, and predicts mortality, and ingrowth into the smallest size class. A major assumption of this type of model is that the stand will evolve in a pattern similar to its past growth. It also requires the use of discrete time. While it is recognized that forest growth is a continuous process, which can in fact be approximated by using very small time units, this growth process is extremely slow. Thus the use of discrete time simplifies the analysis and permits the use of a matrix form without greatly sacrificing the quality of the results.

The first step in the estimation of the growth model is the

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[4] Data collected for Turkey Lakes Watershed Study, Federal Long Range Transport of Air Pollutants Program, Calibrated Watersheds Program.

division of trees into size classes, defined by their diameter at breast height. Thus the state of a stand at any time t can be represented by the column vector **Y**(t).

$$\mathbf{Y}(t) = \begin{bmatrix} y(1,t) \\ y(2,t) \\ \cdot \\ \cdot \\ \cdot \\ y(n,t) \end{bmatrix}$$

Where y(i,t) is the number of trees per hectare in diameter class i.

The data set for the Turkey Lakes site included the diameter at breast height for each tree at two points in time, with a five year interval[5]. The choice of the time unit was therefore determined to be five years. The inventory was divided into 9 diameter classes each with a range of 5cm, beginning with trees of 9.5cm and increasing to the largest class containing trees measuring 49.5cm diameter at breast height and larger. The combination of time unit and diameter class was chosen such that no tree grows more than one diameter class over one time unit. During a specific time period, therefore, trees may either remain in the same class, grow into the next larger class, die, or be harvested. This evolutionary process is described in the model

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[5] The time of measurement was not consistent among all plots. Data were collected in: (1980,85), (1981,86) (1982,87), (1983,88), and combined to estimate the growth model.

by "transition probabilities". These are interpreted as probabilities, however the model is not probabilistic. Where

A probability: a(1), a(2), ..., a(n) are the probabilities that a live tree which is not harvested will be alive and in that same class in T years, the length of the time unit; and

B probability: b(1), b(2), ..., b(n) are the probabilities that it will survive and grow into the next larger class[6].

The probability of mortality in a diameter class, m, is thus 1-a(m)-b(m).

The transition probabilities for the Turkey Lakes stand were estimated by observing the movement of trees between size classes during the five year interval. These estimates are presented in table 1. They were calculated by simple proportion because the data set included, for each plot, the number of trees in each diameter class which, between the two successive inventories, either remained in the same class, moved to the next larger class, or died. There was no harvest over the time period. The estimated transition probabilities are expected to be fairly

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[6] Although it has been suggested that transition probabilities are a function of stand density, they are treated as constants within each diameter class in UNEVEN. Buongiorno and Michie (1980) found stand density to have very little effect on the transition probabilities using data from northern Wisconsin.

accurate for the smaller diameter classes, in which there are a large number of observations, but their accuracy is expected to decrease for larger diameter classes, which have fewer observations.

TABLE 1: Transition Probabilities for the Turkey Lakes Watershed Forest

Diameter Class (cm)	Observations (#)	A Probability[a]	B Probability[b]	Mortality Probability
9.5-14.5	518	0.86	0.10	0.04
14.5-19.5	272	0.85	0.15	0.00
19.5-24.5	207	0.84	0.16	0.00
24.5-29.5	153	0.82	0.18	0.00
29.5-34.5	127	0.85	0.13	0.02
34.5-39.5	114	0.78	0.15	0.07
39.5-44.5	90	0.88	0.08	0.04
44.5-49.5	57	0.84	0.12	0.04
49.5-	41	0.98	0.00	0.02

a:The probability that a live tree in a given diameter class will be alive and in the same class in five years.
b:The probability that a live tree in a given diameter class will be alive and in the next class in five years.

The growth model is the basis from which UNEVEN derives all management plans. Its accuracy is therefore essential in obtaining meaningful simulation results. Ensuring that there is a sufficiently large number of observations in each diameter class is therefore an important consideration in choosing class limits. While no estimate can be expected to be perfectly accurate, large sample populations will ensure desirable properties, which lead to estimates from which inferences can be made. In this case it is expected that the transition probabilities describe reasonably well the actual process.

The ingrowth equation is another component of UNEVEN's growth model. Ingrowth is the number of trees that grow into the smallest diameter class during one time unit. In this case it is defined as the number of trees in each .10 hectare plot attaining a diameter of 9.5cm during the five year interval. Expected ingrowth is described as a linear function of stand basal area and stem density in equation (1) below. This form of ingrowth function accounts for the interdependence of trees within the a stand and thereby allows for the prediction of the stand's response to changes in structure due to harvesting[7].

$$(1) \quad I(t) = B_0 + B_1 \sum^{9}_{i=1}[BA(i)*y(i,t)] + B_2 \sum^{9}_{i=1}y(i,t)$$

Where BA(i) is the basal area of the average tree in diameter class i. The average tree in each diameter class is defined as having a diameter at breast height halfway between its upper and lower diameter limits. For the largest diameter class, the average tree is assigned a diameter at breast height equal to its lower diameter limit plus one half the difference between the upper and lower limits of the preceding class.

The use of an average figure for basal area in each diameter class is adequate here, as the purpose is to determine the general characteristics of the stand. In the smaller diameter classes, trees are expected to be distributed fairly evenly

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[7] A more detailed discussion of this type of ingrowth function is provided in Ek 1974.

throughout the class, because there is a large number of observations, and the average diameter will be at the median. In this case basal area of the average tree will correspond.to average basal area. However, in the largest diameter class, the average tree as defined by the model may underestimate the actual average. In this class, diameter at breast height is not bounded above, leaving very large trees unaccounted for. In the presence of many large trees or few observations, it may be necessary to redefine the average tree in the largest diameter class.

Ingrowth per .10 hectare plot in the Turkey Lakes site was multiplied by 10 to obtain a per hectare figure. The total basal area of each plot was obtained by multiplying the mean basal area (m^2/ha.) by stem density (trees per hectare), and was also converted to a per hectare figure. The ingrowth function was estimated via ordinary least squares (standard errors are in parentheses, t statistics are in square brackets).

$$\begin{array}{lccc} (2)\quad \text{Ingrowth} = & 35.87 & -\ 2.25\ BA & +\ 0.07\ SD \\ & (40.56) & (0.91) & (0.04) \\ & [0.884] & [-2.475] & [1.867] \end{array}$$

$$R^2 = 0.24$$

The adjusted coefficient of determination (R^2) is small, indicating that basal area and stem density explain only 24% of the variation in ingrowth, and consequently, that ingrowth is largely random. This result is not surprising since weather, which is random and cannot be accurately predicted, is an important determinant of tree growth. This ingrowth function was used in the growth model to predict the evolution of the Turkey lakes stand. The standard errors of the coefficients are small relative to the values of the coefficients, and the F statistic (F = 5.606) indicates that with a 99% confidence level, the coefficients jointly differ from zero.

The complete growth model is composed of the ingrowth equation and transition probabilities. It is easily represented in a matrix form.

Rewriting equation (1) in a more compact form:

(3) $I(t) = B_0 + \Sigma^{9}_{i=1}[d(i)*y(i,t)]$

Where $d(i) = [B_1*BA(i)] + B_2$

Combining equation (3) and the transition probabilities leads to an expression for the state of the stand at any future time period. Stocking in the smallest size class in T years is determined by ingrowth plus the number of trees that survive and remain in that class. Operationally:

(4) $y(1,t+T) = I(t+T) + a(1)*y(1,t)$

(5) $y(1,t+T) = B_0 + \Sigma^{n}_{i=1}[d(i)*y(i,t)] + [a(1)*y(1,t)]$

The number of trees in all other size classes is defined as the number of trees that grow from class i-1 to class i plus the number of trees that survive and do not grow out of class i. Operationally:

(6) $y(i,t+T) = [b(i-1)*y(i-1,t)]+[a(i)*y(i,t)] \quad i=2,...,n.$

The growth model, equations (5) and (6), is easily represented in matrix form:

(7) $\mathbf{Y}(t+T) = [\mathbf{G}*\mathbf{Y}(t)] + \mathbf{C}$

where:

$$\mathbf{G} = \begin{bmatrix} a(1)+d(1) & d(2) & d(3) & \dots & d(n) \\ b(1) & a(2) & 0 & \dots & 0 \\ 0 & b(2) & a(3) & \dots & 0 \\ \cdot & & & & \\ & & & & \\ 0 & \multicolumn{2}{c}{0 \dots b(n-1)} & & a(n) \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} B_0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

It is questionable whether a growth model based on observations on the stand over a relatively short time, will be valid over long projection periods. Over the forecast horizon the state of the stand may change significantly. It may be unrealistic to assume that the evolution of a stand over five years can accurately describe its evolution over a fifty or one hundred year horizon. While the issue is recognized and accounted for in the interpretation of simulation results, it is left to the qualified forester to assess. We will concentrate on the economic implications of the various management options. For the purposes of this paper, the growth model is assumed to adequately describe the stand's evolution.

5.1.1 Volume and Value Data:

TABLE 2: Volume and Value Estimates of an Average Hectare

Diameter Class (cm)	Volume (m^3)	Value ($)
9.5-14.5	0.05	4.01
14.5-19.5	0.14	10.81
19.5-24.5	51.8	21.07
24.5-29.5	0.43	35.15
29.5-34.5	0.64	53.36
34.5-39.5	0.89	75.80
39.5-44.5	1.18	102.30
44.5-49.5	1.50	132.70
49.5-	1.85	166.40

UNEVEN also requires volume and value estimates for the 'average' tree in each class to carry out its predictions. These are shown in table 2. Merchantable volume estimates were obtained from the GLFC-LRTAP Project Turkey Lakes Watershed Local Volume Table for sugar maple. Merchantable rather than total volume was used because it is more relevant in calculations of the value of the stand, or value of the timber extracted from the stand, in terms of potential revenue. The use of an average figure for volume estimates in the model is unfortunate in the resulting loss of information (merchantable volume for trees is available for all diameters) but this loss should be minimal if the diameter distribution within each age class is constant. For the smaller classes that have a large number of observations, this assumption is plausible, but for the larger classes with relatively few observations it may be inaccurate. Thus the volume estimates, while not providing detailed information regarding the yield of individual trees, provide a good indication, at the stand level, of the relative timber yield of different diameter classes.

Value estimates are much more complex. Ideally, and perhaps unrealistically, an estimate of the net total value to society of harvesting each individual tree would be used. In the context of this ideal scenario, the maximization of net present value would provide the 'optimal' management plan in the sense that no other plan could provide a larger net benefit to society. The marginal benefits of harvesting the last tree would equal the total marginal cost of its harvest. However, defining, let alone quantitatively expressing all the benefits provided by a stand would prove a difficult, if not impossible task. On a more practical note, and in the context of the model under consideration, value will reflect the price that a buyer is willing to pay.

Still, our definition of value remains unresolved. The value of a cubic meter of standing timber is by no means a straightforward concept. It varies by species, quality, region, time, according to its prospective use, and according to the perspective of the investor[8]. To the landowner, value is the price that a buyer is

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[8] Nautiyal (1981) provides an analysis of stumpage prices for trees of various species, qualities and diameters in the Niagara district of Ontario.

willing to pay for the right to cut the timber. To the logging firm it is the difference between log sales and fixed and variable production costs, including Crown dues. To the government, value includes not only the employment, demand for capital, tax revenues and multiplier effects provided by dependent industries, but also the environmental and aesthetic costs resulting from felling the trees.

The UNEVEN Users' Manual defines the value of the average tree in each class as its value net of the cost of harvesting a tree of that size, which corresponds roughly with stumpage prices. It is fixed, as are costs, productivity, and the rate of return, over the planning horizon. This assumption is contrary to actual observations which indicate that prices vary substantially over time[9]. Thus present value estimates will be valid only under the assumption of static prices, and intertemporal decisions accounting for possible increases or decreases in the value of the product over the planning horizon will be impossible under UNEVEN[10]. However, the user can very simply examine the effects of actual or anticipated price changes at any time by editing the inventory data, and restructuring management plans accordingly.

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[9] Data compiled by Dr. O.J. Menezes from company returns of Statistics Canada annual Census of Manufacturers questionnaires, indicate that prices in Ontario and Northern Ontario varied substantially over time, and that price trends differ for different species and uses.

10 Kaya and Buongiorno (1987) account for uncertainty in prices in their model of uneven-aged forest management.

For the purpose of this analysis, value was initially set at the estimated mill delivered value of hard maple sawlog lumber, $85 per cubic meter[11], adjusted for tree size to recognize the higher value per cubic meter of larger diameter trees[12]. As diameter increases, the variable harvesting costs per cubic meter are reduced, quality is generally higher, and the versatility in the use of trees increases (Nautiyal, 1983 p.50). Value can also be expected to decrease beyond a certain age or diameter as the tree begins to decay.

Value was then changed to the Canada-Ontario FRDA values for hard maple, $198/m^3 for veneer and $91/m^3 for sawlogs[13]. All logs larger than 45cm in diameter were considered suitable for veneer;

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[11] Source: Ontario Ministry of Natural Resources (1988), p.01.

[12] Price adjustments were recommended by Stig Andersen, Analyst, Socio-Economic Analysis and Planning Unit, Great Lakes Forestry Centre, Sault Ste Marie, Ontario. Their arbitrariness is recognized, but the adjustments are useful in their recognition of relative price differences per cubic meter between size classes.

Class	**Price**
1	$85x.90
2	85x.92
3	85x.94
4	85x.96
5	85x.98
6	85x1.00

Class	**Price**
7	85x1.02
8	85x1.04
9	85x1.06

[13] Source: G. Raines, Algoma Central Railways, Sault Ste Marie, Ontario. Personal conversation

and smaller logs, suitable for sawlogs only[14].

The UNEVEN users' manual states that "value estimates should reflect average quality of the trees in each class" (Boothby and Buongiorno 1985)[15]. This generalization is unfortunate and has been the main complaint from the model's users[16]. It is damaging to the analysis because value varies extensively with tree quality, which in turn varies extensively within size classes and stands. A large tree, for example, may be suitable for veneer and command a high price, or may yield a low price if it is suitable only for pulp. Distinguishing low from high quality trees would certainly enrich the analysis. Investing in a low quality tree will yield a lower return, and may not be economically warranted. Removal of the poorer quality trees, while yielding a relatively lower immediate return, will reduce congestion and allow the surrounding trees to mature uninhibited, and thereby yield higher future returns.

5.2 Management Options:

UNEVEN has five management options to account for varying management objectives. The Turkey Lakes forest was simulated

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[14] Stig Andersen, Analyst, Socio-economic Analysis and Planning Unit, Great Lakes Forestry Centre, Sault Ste Marie, Ontario. Personal conversation.

[15] UNEVEN is currently undergoing improvements to incorporate tree quality.

[16] Stig Andersen, personal conversation.

under various scenarios for all harvest rules, including the undisturbed condition where no harvest occurs. The effects on the net present value of harvests themselves was examined with different cutting cycles, initial harvest delays, interest rates, fixed harvesting costs and prices.

When a harvesting rule is specified, management data must be specified. These include the length of delay before the first harvest, length of the cutting cycle, interest rate and fixed harvesting costs. Fixed costs ($/ha.) include all costs that are independent of the number of trees harvested. Certain fixed costs, the initial investment in road construction and capital, occur only in the first period, while others occur at each harvest. UNEVEN does not recognize the former in the calculation of present value. Although these initial fixed costs will not affect the harvest regime, they are instrumental in the decision to manage a stand previously left idle. A positive present value, net of initial investment costs, indicates an economically wise investment opportunity, unless other non market costs are significant enough to outweigh the value of the stream of monetary returns from timber production.

Fixed harvesting costs were assumed to be zero in this analysis, although the impact on the management regime and present value of increasing them was studied. The zero fixed cost assumption is not unrealistic for the Turkey Lakes site. Roads extending throughout the forest were built for research purposes and as a result, do not constitute a cost for any proposed forestry operation, assuming that the harvesting activity occurs once the research function is terminated. The problem does arise, however, in the multiple use context, of allocating fixed costs to specific uses. Similarly, the cost of existing roads, as in this case, but built for a competing use, should be allocated to the various uses. However the allocation of fixed costs to different uses of the forest has no simple solution.

Other fixed costs which occur at every harvest, such as transporting labour and machinery to the site, are also expected to be minimal, and a zero assumption is not very damaging. The site is close to both the city and main highway, and there are generally several people at the location.

The discount rate or the alternative rate of return that is applied to the flow of future harvests should reflect the real riskless rate of return, net of inflation, or the opportunity cost of tying up capital in the forestry operation. It should correspond with the social rate of return, itself a rather ambiguous concept. The value commonly used in forestry and other economic studies ranges from 2% to 5%. We initially set the interest rate at 5% and the effects of its variation on the present value of harvests and the economic harvest levels themselves was simulated.

The delay before the initial harvest and the length of cutting cycle are policy variables that the forest manager can set to best accommodate various operational and economic factors.

5.2.1 The Natural Forest:

The natural forest option simulates the development of the forest assuming that no harvest or natural disturbances occur. UNEVEN uses the growth model to predict the diameter distribution that characterizes this steady state. It is defined where:

$$(8) \quad y(i,t+1) = y(i,t) = y^*(i) \quad \text{for all } i$$

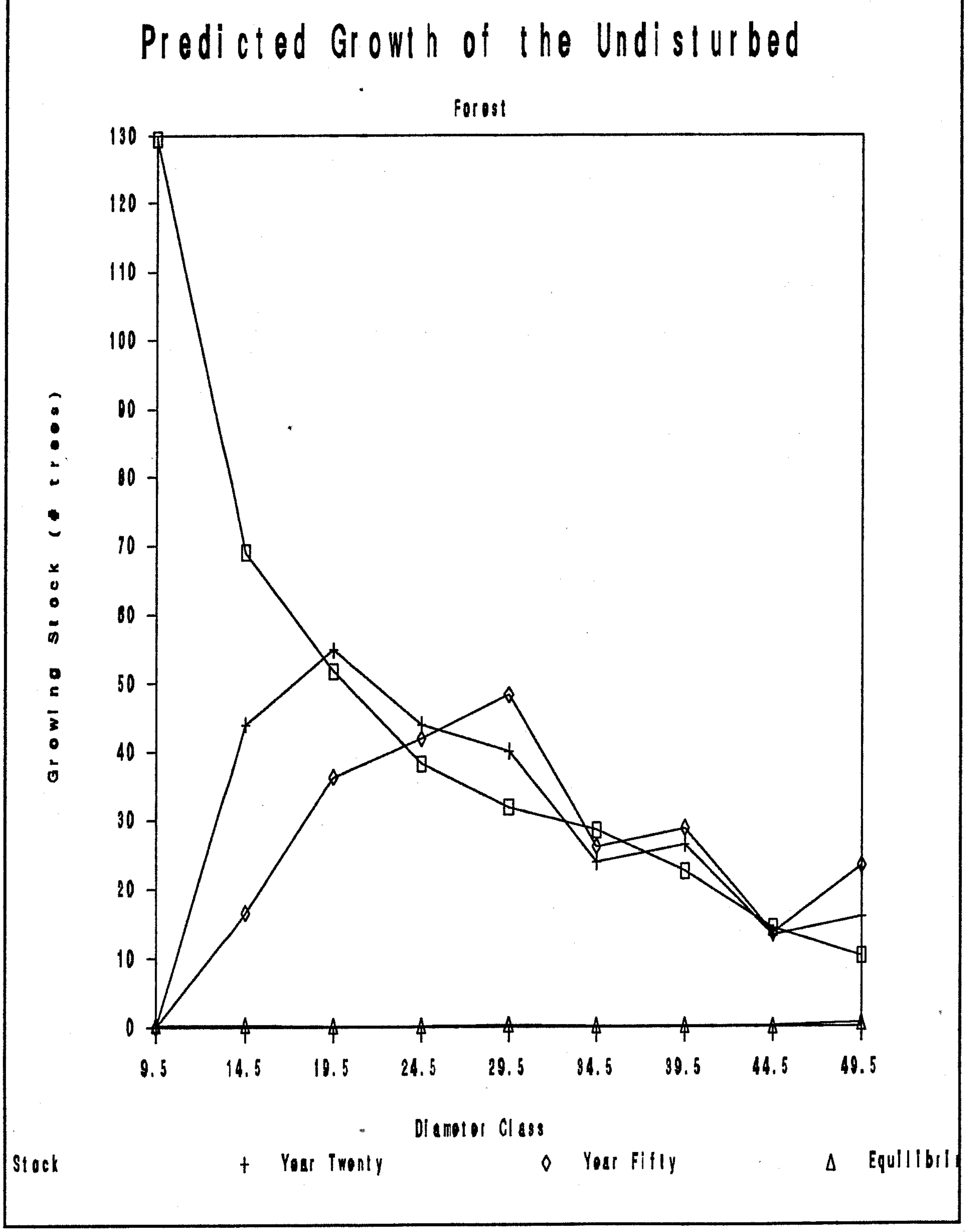
And where $y^*(i)$ is the equilibrium number of trees in diameter class i.

Rewriting equation (7) leads to an expression for the steady state diameter distribution of the stand.

$$(9) \quad \mathbf{Y}^* = (\mathbf{I}-\mathbf{G})^{-1}*\mathbf{C}$$

Equation (9) indicates that the equilibrium distribution is independent of the initial stand conditions. It depends only on the growth of the stand as defined by **G** and **C**.

Our first simulation predicted the evolution of the stand over five decades, under the assumption of no harvest. This exercise is useful for validation of the growth model, to determine



whether simulated conditions of the forest evolve according to what is known about the growth of uneven-aged stands. The diameter distribution of uneven-aged stands has generally been observed to have an inverse J shape (Buongiorno and Michie 1980, p.616).

Figure 2 illustrates the initial distribution of trees among diameter classes, the distribution at twenty and fifty years, and the equilibrium. Over the forecast horizon, the composition of the stand changes gradually, initially in favour of larger trees. The long term equilibrium is characterized by very few trees, with a distribution that is slightly U shaped. It can't be ignored, however, that only trees exceeding 9.5cm diameter were counted, and therefore it is expected that the number of small trees is underestimated. The actual long term equilibrium distribution, in keeping with the theory, is likely J shaped. These simulations, and specifically the drastic changes between the initial and subsequent diameter distributions, indicate that the initial stand is not a mature stand as previously indicated.

5.2.2 The Fixed Proportion Harvest:

Under forest management options that involve periodic timber harvesting, UNEVEN's growth model is modified. Each harvest is assumed to take place at the beginning of the cutting cycle and to take a length of time short enough that growth over the harvest period is negligible. This is not an unrealistic assumption and fairly standard in forest management models. It greatly simplifies the analysis without introducing any conceptual problems.

As the undisturbed stand, a harvested stand will tend over time toward an equilibrium steady state. At the steady state, the volume cut at a particular time is equal to the amount by which the stand has grown since the last harvest. It is assumed in this model that mortality is eliminated through harvesting. It is also assumed that an objective of forest management is to bring the stand to a steady state where a constant harvest can be maintained indefinitely, assuming no natural disturbances occur. UNEVEN predicts the harvest level and growing stock that will maintain the steady state equilibrium. Although practically unattainable, the steady state harvest and growing stock are useful as targets for a management plan. Once a desirable combination of harvest and stock is determined, that combination remains desirable unless external economic variables change.

Under the fixed proportion harvest option, the harvest regime is defined by the proportion of trees to be cut from each diameter class. UNEVEN simulates the physical and economic consequences of the continued application of the specified harvest and cutting cycle over time.

The fixed proportion harvest rule is represented in the model by a diagonal matrix. The harvest matrix, **H**, is added to the growth model in the years that the forest is scheduled to be harvested.

$$\mathbf{H} = \begin{bmatrix} p(1) & 0 & 0 \ldots 0 \\ 0 & p(2) & 0 \ldots 0 \\ \cdot & \cdot & \cdot \quad\; \cdot \\ 0 & 0 & 0 \ldots p(n) \end{bmatrix}$$

Where each p(i) is the fraction of trees in diameter class i that is to be harvested during the time interval T.

The growth model with harvest included becomes:

$$(10)\quad \mathbf{Y}(t+T) = [\mathbf{G}*(\mathbf{I}-\mathbf{H})*\mathbf{Y}(t)] + \mathbf{C}$$

The conditions of the forest every T years during a cutting cycle of length mT are:

$$(11)\quad \mathbf{Y}(t+T) = [\mathbf{G}*(\mathbf{I}-\mathbf{H})*\mathbf{Y}(t)] + \mathbf{C}$$

$$(12)\quad \mathbf{Y}(t+2T) = [\mathbf{G}*\mathbf{Y}(t+T)] + \mathbf{C}$$

$$= \{\mathbf{G}*[\mathbf{G}*(\mathbf{I}-\mathbf{H})*\mathbf{Y}(t) + \mathbf{C}]\} + \mathbf{C}$$

$$\vdots$$

$$(13)\quad \mathbf{Y}(t+mT) = [\mathbf{G}*\mathbf{Y}(t+mT-T)] + \mathbf{C}$$

$$= [\mathbf{G}^{m}*(\mathbf{I}-\mathbf{H})*\mathbf{Y}(t)] + \Sigma^{m-1}{}_{i=1}[\mathbf{G}^{i}*\mathbf{C}]$$

The steady state is determined analytically by substituting

$\mathbf{Y}(t+mT)=\mathbf{Y}(t)=\mathbf{Y}^*$ into equation (13), which yields:

$$(14)\quad \mathbf{Y}^* = [\mathbf{I}-\mathbf{G}^m(\mathbf{I}-\mathbf{H})]^{-1}*\sum^{m-1}{}_{i=0}[\mathbf{G}^i*\mathbf{C}]$$

The vector $\mathbf{Y}^*$ describes the diameter distribution of the stand in the steady state. Note that the equilibrium situation of the stand, as under undisturbed conditions, is independent of the initial conditions, and dependent only on the growth function (G), the length of the cutting cycle (mT), and the harvest matrix (H). The forest manager will choose the combination of cutting cycle and harvest that best meet his or her objectives.

The fixed proportion harvest option can be used to represent several cases, such as the natural forest option where growth is undisturbed, $p(i)=0$ for all i, or large tree silviculture, where all trees exceeding a certain diameter are marked for harvest, $p(i)=1$ for $i \geq n$ and $p(i)=0$ otherwise.

We chose a fixed proportion harvesting regime fairly arbitrarily as an illustration. It is described under 'current conditions' in table 3. The effects of the continued application of our chosen harvest regime to the Turkey Lakes forest were simulated. Fixed costs were assumed to be zero, the discount rate was set at 5 percent, and we specified no delay before the initial harvest.

TABLE 3: Long Term Equilibrium of an Average Hectare Under a Fixed Proportion Regime
(F=0, delay=0, r=5%)

Diameter Class	Current Conditions			Long Term Equilibrium							
	Stock	Harvest		Cutting Cycle							
	# trees	%	#	5		10		15		20	
				stock	harvest			stock	harvest	stock	harvest
9.5-14.5	129.5	50	64.7	22.2	11.1	3.8	1.9	8.6	4.3	0.5	0.2
14.5-19.5	69.0	50	34.5	1.9	1.0	4.0	2.0	3.3	1.7	3.3	1.6
19.5-24.5	51.8	50	25.9	0.2	0.1	0.8	0.4	1.3	0.7	1.5	0.7
24.5-29.5	38.3	50	19.1	0.0	0.0	0.2	0.1	0.5	0.2	0.6	0.3
29.5-34.5	31.8	50	15.9	0.0	0.0	0.0	0.0	0.2	0.1	0.3	0.2
34.5-39.5	28.5	50	14.2	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.1
39.5-44.5	22.5	50	11.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0
44.5-49.5	14.3	70	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
49.5-	10.3	90	9.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Net Present Value				$153		$26		-$5		-$24	

Our harvesting rule was simulated with a cutting cycle of 5, 10, 15, and 20 years. Under the 10, 15, and 20 year cutting cycles, the system had not yet converged to the equilibrium sustained yield at the 50 year mark. In fact, equilibrium for the 15 year cutting cycle was reached only at 180 years. In addition, the present value of the perpetual harvest is negative for the 15 and 20 year cutting cycles. Under the 5 year cutting cycle, the sustained yield was reached within 45 years, still a rather long transition period. The net present value of an average hectare is positive but small at $153. The low present value, combined with the slow convergence of the system to equilibrium and the small equilibrium growing stock indicate that the harvesting rule or cutting cycle should be altered. A combination with a higher present value of timber production, or with a resultant residual growing stock which is conducive to non-timber values of the stand, and which converges to equilibrium more rapidly should be found.

The implications on the stand of large tree silviculture were also examined using the fixed proportion harvest option. All trees in the two largest diameter classes were marked for harvest in strategies with five and twenty year cutting cycles. In both cases the regime proved to be disastrous. The sustained yield conditions were characterized by zero growing stock and necessarily, zero harvest. The present value of the harvests was negative, -$132 in the case of the five year cutting cycle, and -$129 for 20 years. These results indicate that the proposed management plan is unacceptable and again, that it should be modified.

The fixed proportion harvest option may be appropriate for a forest managed for multiple uses. The manager can choose the harvest and cutting cycle that best meet joint objectives, by observing the equilibrium steady state and present value of the harvests under various scenarios. A negative net present value, as for all management options, does not invariably indicate an unwise cutting regime. The cost associated with the investment in growing stock should be reduced by an amount equal to the present value of the non-market benefits it provides. While this is not always a straightforward calculation, the manager can use his or her expertise to judge the value of non-timber services. An educated guess is certainly a step above assigning a zero value to valuable services. It is this final net present value that is relevant to the decision-maker.

In a multiple use context, the fixed proportion harvest regime chosen may in fact be appropriate. While the net present value of timber production for each cutting cycle is small or negative, this should be increased if it is to represent the net present value of a forest which also provides non-timber services. In such cases, the residual growing stock is a key decision variable, along with the net present value of timber production, in the choice of a harvest rule.

5.2.3 Target Distribution Harvest:

Under the target distribution harvest option, the user must specify the desired residual stocking following harvest. UNEVEN simulates the development of the forest, applying the harvest necessary to leave a residual stock which most closely approaches that specified. This option is appropriate for a forest managed for multiple uses, and for which timber production is a joint objective or an objective secondary to aesthetics, wildlife or species preservation, or other environmental goals, where a certain minimal residual diameter distribution may be required in order to provide such benefits. The stand can be managed by choosing a cutting cycle that, given the specified residual stocking levels of each hectare, maximizes the present value of the perpetual flow of timber.

Once again, we chose a residual distribution at random to illustrate the application of this option on the Turkey Lakes forest. It is represented in table 4 along with the equilibrium conditions for the five year cutting cycle. The results indicate that the specified stocking levels cannot be maintained with cutting cycles ranging from five to fifty years. Delaying the initial harvest by twenty years did not improve the results. Expectedly, the equilibrium remained the same, while the net present value of timber production increased marginally, although remaining negative. As the cutting cycle was lengthened, the number of trees in the residual growing stock decreased, reflecting the higher opportunity cost of delaying harvest for longer periods.

The results in table 4 indicate that the desired residual diameter distribution should be changed. If the continued provision of non-timber benefits prohibits this, the landowner will need to weigh the alternatives of managing the land for timber production or leaving it undisturbed. The option that best meets management objectives will be chosen. Theoretically, if the present value of non-timber benefits accruing to the owner, or to the public exceed the present value of the stream of income from the harvest, the economic choice is to forego harvesting. The difficulty once again lies in assessing the present value of non-timber benefits, and has no objective and practicable solution.

TABLE 4: Equilibrium Condition of an Average Acre - Target Distribution Harvest (cutting cycle=5 years, r=5%, delay=0, F=0)

Diameter Class	**Target Distribution of Residual Stocking**	**Sustained yield Harvest**	**Residual Growing Stock**
9.5-14.5	50.0	0.0	0.0
14.5-19.5	50.0	0.0	0.0
19.5-24.5	40.0	0.0	0.0
24.5-29.5	30.0	0.0	0.1
29.5-34.5	30.0	0.0	0.4
34.5-39.5	20.0	0.0	0.5
39.5-44.5	15.0	0.0	2.1
44.5-49.5	10.0	0.0	1.9
49.5-	5.0	0.3	5.0

Net Present Value = -$1213

5.2.4 Economic Harvest:

Under the economic harvest option, the harvest itself is an endogenous variable. UNEVEN uses linear programming to determine the structure of the steady state forest and corresponding harvest that, given the length of the cutting cycle, maximize the present value of the returns to the owner from timber production net of fixed and variable harvesting costs, and net of the investment in growing stock. The investment in growing stock, or the residual stock left after harvest, represents a cost to the landowner, the foregone opportunity of cutting the stock and investing its returns elsewhere. However, it also represents a benefit to the owner, or to society, for the various services it provides. UNEVEN does not explicitly consider this aspect.

The objective function is:

$$Z = \frac{(\mathbf{V}*\mathbf{h}^{*}-F)}{[(1+r)^{mT}-1]} - \mathbf{V}*(\mathbf{Y}^{*}-\mathbf{h}^{*}) \quad (14)$$

Where: $V = [V(1)\ V(2)\ \ldots\ V(n)]$ $$h^{*} = \begin{bmatrix} h^{*}(1) \\ h^{*}(2) \\ \cdot \\ \cdot \\ h^{*}(n) \end{bmatrix}$$

$V(i)$ ≡ value of the average tree in size class i, net of variable costs;

$V{*}h^{*}$ ≡ value of the sustained yield harvest;
F ≡ fixed harvesting costs per hectare;
r ≡ interest rate, or alternative rate of return;
mT ≡ length of the cutting cycle in years.

The objective function, Z, is maximized subject to two sets of constraints:

(1) Sustained yield conditions: $[\mathbf{I}-\mathbf{G}^{m}]{*}\mathbf{Y}^{*} + [\mathbf{G}^{m}{*}h^{*}] = \Sigma^{m-1}_{i=0}[\mathbf{G}^{i}{*}\mathbf{C}]$

(2) Harvest from each diameter class cannot exceed stocking:

$\mathbf{Y}^{*}-h \geq 0$.

As is seen upon examining equation (14), the choice of h^{*} and y^{*} that maximizes net present value to the owner is independent of fixed costs, given the cutting cycle. But fixed costs are important in the choice of cutting cycle that maximizes Z.

UNEVEN's economic option does not address the problem of choosing a strategy for converting the stand to the 'optimal' steady state. It simulates the conversion to the sustained yield condition that maximizes present value, by harvesting all trees in excess of the economic growing stock. However, the conversion strategy is an important decision variable in uneven-aged forest management, and ignoring it is a large oversight. The value of the harvests in the conversion period is an important element in the calculation and maximization of net present value. In fact, these harvests are even more important than the perpetual steady state harvest, because they occur in the earlier periods of the analysis and are, as a result, not discounted as heavily. Land expectation value is the present value of harvests during a conversion from the existing stand structure to an economically superior stand, plus the discounted sum of all future yields once conversion has been completed (Michie 1985). Ignoring the first element results in a harvest strategy that does not maximize the value of the timber producing land.

Michie (1985) extended the mathematical model underlying UNEVEN to the analysis of conversion from a given initial stand to the economic steady state. He outlined the conversion method that maximizes the present value of harvests during the conversion, which is to be completed in a predetermined number of years. He found that allowing three or more conversion harvests leads to a mathematical program that is difficult to solve. Haight et al. (1985) and Haight (1985) also worked on the conversion problem, using optimal control models with a free terminal state. These methods were also complex. Perhaps this justifies the absence of conversion strategies in UNEVEN, which was formulated on the premise of simplicity.

We used the economic harvest option to determine the optimal cutting cycle for the Turkey Lakes forest under various economic conditions. In all simulations, the steady state harvest that maximized present value, given the cutting cycle, was to cut all stock, and was reached in the first period. These results suggest that leaving any standing timber after harvest is not economically warranted, and that if maximizing timber yield is the sole objective of the forest manager, then perhaps the stand should be converted to an even-aged stand.

Tables 5 to 9 show the present value of the perpetual stream of harvests. Again, in all cases the shortest cutting cycle maximized present value. It is expected that present value would continue to rise if the cutting cycle were further shortened. It is interesting to note that present value declined in the ten year cutting cycle, then rose in the fifteen year cycle and declined again as the cutting cycle was lengthened.

The effect of increasing fixed costs, as expected, was to decrease the present value. Present value declined slowly as fixed costs increased and reached zero at the unrealistically high fixed cost level of $141 per hectare, with the rate of return set at 5 percent.

Postponing the first harvest resulted in no changes in net present value or cutting regime.

TABLE 5: Present Value of the Forest in Function of Year Before First Harvest and Cutting Cycle
(Economic Harvest, r=5%, F=0)

Cutting Cycle	Year Before First Harvest 0	10	15	20
5	520	520	520	520
10	98	98	98	98
15	116	116	116	116
20	58	58	58	58

TABLE 6: Present Value of Forest in Function of Year Before First Harvest and Cutting Cycle
(Economic Harvest, r=5%, F=$10)

Cutting Cycle	Year Before First Harvest 0	10	15	20
5	484	484	484	484
10	82	82	82	82
15	107	107	107	107
20	52	52	52	52

Higher interest rates also had the predicted effect of decreasing present value, which was still positive, although small, for all cutting cycles at 19 percent, the maximum interest rate that the program will accept. Results are presented in table 7. The combined effect of varying fixed costs and interest rates was also studied and presented in table 8. Increasing fixed costs had the effect of decreasing present value by the same percentage regardless of the interest rate; and increasing interest rates decreased present value by the same percentage at all values of fixed costs.

TABLE 7: Present Value of the Forest in Function of Interest Rate and Cutting Cycle (Economic Harvest, delay=0, F=0)

Cutting Cycle	Interest Rate (%) 1	3	5	7	10	15	19
5	2819	903	520	357	236	142	104
10	589	179	98	64	39	20	13
15	776	224	116	71	39	18	10
20	433	118	58	33	17	6	3

TABLE 8: Present Value of Forest in Function of Interest Rate and Fixed Costs (Economic Harvest, delay=0, Cycle=5)

Fixed Costs	Interest Rate 1	5	10	15
10	2623	484	219	132
20	2427	448	203	122
50	1839	340	154	93
100	859	159	72	43

Price changes to the Canada-Ontario FRDA values were also examined. Again, the steady state equilibrium was to clear cut, and this was achieved in the first period. The five year cutting cycle maximized present value. Table 9 shows the present value of the economic harvest under the initial MNR prices and the FRDA prices, under various cutting cycles.

TABLE 9: Present Value of Forest Under Economic Harvest Option In Function of Prices

Cutting Cycle	Net Present Value (FRDA prices)	Net Present Value (MNR)
5	618	520
10	115	98
15	136	116
20	67	58
25	61	-
30	41	-

Table 10 shows, for various cutting cycles, the effects of substantially decreasing the value of a cubic meter of timber. The present value does not change substantially. The steady state remains to clear cut and the optimal cutting cycle is five years.

TABLE 10: Present Value of the Forest in function of price/m^3 and Cutting Cycle Under the Economic Harvest Option

Cutting Cycle	Price/m^3 .5	 10	 40
5	3	61	245
10	1	11	46
15	1	14	55
20	0	7	27

Given that the economic harvest option indicates that the optimal steady state harvest is to cut all of the standing timber over very short cutting cycles, the addition of constraints to linear program would be useful, perhaps to bound the minimum residual distribution. The results obtained in this exercise may be particular the stand under consideration, but they indicate that, in its present form, UNEVEN's economic harvest does not adequately deal with the objective of maximizing net social benefits. The multiple use forest manager would be better served by using an alternative option, such as the fixed proportion or the target distribution harvest. The cutting cycle that maximizes net present value subject to the specified harvest regime could then be selected. An alternative is the regulated economic harvest, to which attention is now turned.

5.2.5 Regulated Economic Harvest:

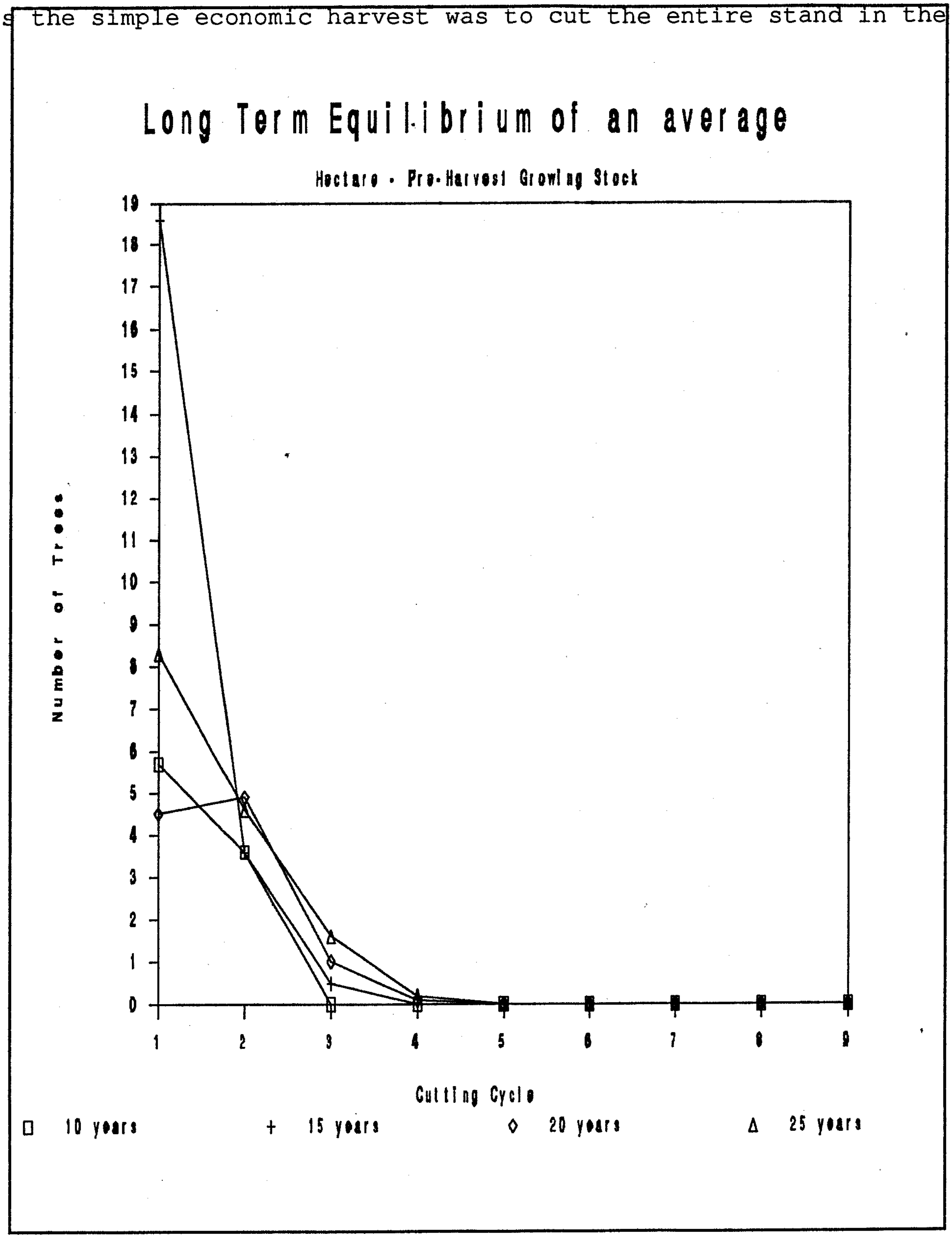
In the regulated economic harvest option, the forest is divided into a minimum of two equal areas to allow for continuous harvesting during the cutting cycle. Every period, a fraction, m, of each hectare is harvested, leaving the remaining area undisturbed.

The objective function is more complex than in the case of the simple economic harvest. It takes the following form:

$$\text{(15)} \quad Z = \frac{V}{m} * \left\{ \frac{h^*}{[1+r]^T} + \Sigma^{m-1}{}_{i=0}[\mathbf{G}^i * h^*] \right\} - \frac{V}{m} * \left\{ \Sigma^{m-1}{}_{i=0}[\mathbf{G}^i * y^*] + \Sigma^{m-1}{}_{i=1}\Sigma^{i-1}{}_{j=0}[\mathbf{G}^j * \mathbf{C}] \right\}$$

The first term is the present value of an infinite stream of harvests occurring every T years from one m^{th} of a hectare. The second term is the investment in growing stock per hectare, where the hectare is composed of m equal areas, each at a different stage of development in the cutting cycle.

This option is more conducive to multiple use management than the simple economic harvest because at each time interval, in this case five years, it leaves at least one half of every hectare undisturbed. The consequence is that it may leave more growing stock and therefore result in a smaller net present value. In the case of the present site, this is certainly to be expected, as the simple economic harvest was to cut the entire stand in the



earliest time period.

The evolution and harvest of the Turkey Lakes stand was simulated under the regulated economic harvest, with various cutting cycles and interest rates. The results in table 11 indicate that a fifteen year cutting cycle should be chosen to maximize net present value, and hence the stand should be divided into three equal parts.

TABLE 11: Present Value Under the Regulated Economic Harvest Option in Function of Interest Rate and Cutting Cycle (delay=0, F=0)

Cutting Cycle	Interest Rate .01	.05	.10
10	122	40	-21
15	193	82	0
20	67	4	-44
25	-180	-6	-44

Alternatively, the sustained yield growing stock can be examined along with the present value, to choose a combination that best meets particular multiple use objectives. Figure 3 illustrates the diameter distribution of the steady state pre-harvest stock of an average hectare of the Turkey Lakes forest for various cutting cycles.

UNEVEN cannot be described as a rigorous economic model. Its four management options, however, provide the forest manager with a means of predicting the outcome of various harvest regimes and intensities, and of balancing the various uses of the stand in such a way that its total value is judged to be at its maximum.

The appropriate management option will depend on the characteristics of the stand, and on the particular objectives and the priorities assigned by the individual, corporate, or Crown owner.

The simplicity and practicality of the model, however, are provided at a cost. The main weakness of UNEVEN is its failure to account for variations in tree quality, and hence of market value, within each diameter class. However, Buongiorno and Lu are currently developing an improved version of UNEVEN that includes tree quality and species as part of the inventory data. Other weaknesses are the assumption of perfect knowledge regarding future timber price levels and costs, and perfectly competitive stumpage and input supply markets. UNEVEN also fails to consider price and cost changes over the planning horizon. Overall the simplicity of the model is well worth the cost.

6. CONCLUSION

UNEVEN combines the convenience of linear programming with a recognition of the subjective nature of many forest management objectives and decisions, to provide a useful tool for the manager of an uneven-aged forest, in particular a forest managed for multiple uses. In the development of economic models, the tendency has been to demand a single optimal solution to the forest management problem, and to design models that will supply it. The question of whether such a solution exists is often not considered, perhaps dismissed. A consequence of such neat and tidy economic models is the introduction of sometimes unrealistic assumptions regarding human behaviour, requiring that all 'rational' decision makers maximize utility or profits according to their membership in one of two distinct groups of economic agents: consumers and producers. In addition, markets are often assumed to be perfect.

Forest owners, however, are a diverse class of agents with diverse objectives. Their behaviour cannot be entirely explained through economic efficiency models. In addition, markets are incomplete, and as a result, a wide variety of forest services are not exchanged in the market. The value of these services is not determined through conventional market forces. But the absence of market-determined prices does not imply the absence of value. In fact, it is undeniable that the aesthetics of a forest, its role in soil drainage, in the provision of wildlife habitat and recreational areas, are valued services.

Traditional economic models, the well-known Faustmann model in particular, have not accounted for all of the relevant returns to the landowner or to the population at large from forested land. Specifically, the nonmonetary benefits and costs of alternative uses have been neglected. While the value of these models in illustrating the basics of the economic theory of forest management is acknowledged, their usefulness remains illustrative. They simply do not approximate reality within reasonable limits to be applicable and practical tools for the forest manager. Perhaps it was not their intention.

UNEVEN succeeds in providing a practical tool for the forest manager through its combination of objectivity and subjectivity. While non-timber services are not considered explicitly in UNEVEN, the forest manager is provided with a means of evaluating alternative management plans, and on that basis, of choosing the strategy that best meets his or her particular objectives.

We ran several simulations of Turkey Lakes stand using various harvest regimes and cutting cycles. Each of UNEVEN's harvest options proved to be useful for determining the evolution of the uneven-aged stand under various conditions. The fixed proportion and target distribution harvest cases serve as an illustration of the experimentation process that is required to determine the appropriate harvest for the stand, the harvest that most closely meets the forest manager's objectives. It was found that the prescription of a harvest, and in particular of a desired residual distribution requires a certain amount of knowledge of forestry principles, of how forest stands, in general, develop over time. The target distribution harvest option was found to be most useful for multiple use management because the residual distribution, which is a decision variable, is prescribed by the forest manager. The model then determines the harvest. Neither the residual distribution nor the harvest quantity are exogenous in the fixed proportion harvest option. This option was found to be useful only in determining the consequences of continually harvesting a certain proportion of the growing stock.

The economic and regulated economic harvests use a defined objective function, maximization of the value of timber production, and hence only one harvest solution exists, given the cutting cycle. These options are both useful for multiple use forest management. The value of non-timber services will depend on the residual growing stock. Thus the cutting cycle can be chosen that results in the best combination of residual growing stock and value of timber.

The ultimate choice of harvest option, harvest regime, and cutting cycle length will depend on experimentation with all options. While no attempt was made to identify or evaluate non-timber uses of the Turkey Lakes stand, such an evaluation is necessary in choosing the harvest rule and cutting cycle that maximize net present value.

The model simulation exercises indicate that the target distribution harvest, the regulated economic harvest, and the economic harvest with the addition of a constraint on the residual growing stock are the most applicable options for the management of a stand for multiple uses. Under these three harvest options, the harvest, growing stock and cutting cycle combination can be chosen which best meet the purpose of management. If the objective is the production of timber only, then the economic harvest option, which maximizes the discounted present value of timber production, is the most appropriate.

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