Rent-Sharing and Expectation Driven Business Cycles

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Abstract
Traditionally, standard business cycle models fail in generating the proper co-movements necessary to conform to empirical observations in response to positive news. This paper explores the potential of adding a new mechanism to the class of models that can generate these co-movements. Specifically, the mechanism that is proposed here is based on the idea of rent-sharing within a labour reciprocity framework. Reciprocity enters the model via the introduction of endogenous effort into the household’s preferences and based on the degree of wage generosity, the household determines its provision of effort to the firm. While the base model slightly improves the results of standard models, it too fails in generating the required co-movements in response to positive news. The model is then extended to incorporate variable capacity utilization and costly capital accumulation. With these added features, the model is able to generate boom in output, consumption, and hours worked, however, it still fails in generating a boom in investment.

1. Introduction
Traditionally, Dynamic Stochastic General Equilibrium (DSGE), in particular, Real Business Cycle (RBC) models are propagated by unexpected changes in Total Factor Productivity (TFP). However, empirical research has recently put into question this assumption. A few studies have shown that stock prices, by their very nature being a forward looking indicator, are highly correlated with long run TFP (Beaudry and Portier, (2005, 2006)). These empirical results imply that agents are anticipating future changes in TFP, which gives credence to the news based view of the business cycle. That is, the view of the business cycle that argues that shocks in TFP are partially anticipated by agents in the economy, and that these anticipations are important for understanding economic fluctuations. Beaudry and Portier (2004) define an Expectation Driven Business Cycle (EDBC) as “a positive co-movement between consumption, investment, and employment induced by a change in expectations while holding the current level of technology constant”. However, most existing business cycle models are unable to replicate these
positive co-movements between aggregate variables, which are required to conform to the empirical observations (Beaudry and Portier, 2007). The key feature, in generating this result is that agents anticipating future increases in productivity feel wealthier, which increases consumption and leisure today. The increase in leisure decreases labour supply, which leads to a decline in output and thus investment. Hence, there must be some additional friction or mechanism that overwhelms the impact that the wealth effect has on labour supply in order to generate EDBC.

Several papers have explored different modifications of the standard RBC model to achieve EBDC. Beaudry and Portier (2004) generated EDBC in a model with a multi-sector production setup. In their model, output, consumption, hours worked and investment all increase on impact of the news. Jaimovich and Rebelo (2008), had three features at the heart of their model: hybrid preferences based on Greenwood, Hercowitz, and Krusell (2000) and King, Plossers, and Rebelo (1998), variable capacity utilization, and capital adjustment cost. With preferences that resulted in a weaker wealth effect, these features were able to incentivize an increase in labour demand and investment on impact of the news. Alternatively, Karnizova (2010) utilizes a combination of capital adjustment costs and the household’s desire to accumulate wealth to make leisure relatively costly on impact of the news leading to an increase in labour supply. The alteration of preferences in Jaimovich and Rebelo (2008) and Karnizova (2010) are important in breaking the negative co-movement in labour for generating EDBC.

In this paper, I explore whether rent-sharing within a labour reciprocity framework can act as alternative mechanism to generate EDBC. Similar to the models above, the household’s preferences are altered. However, in this case, it is to include endogenous effort. The basis of the model is that effort, as is standard in wage efficiency literature, is unobservable and thus not contractible by the firm. However, workers are assumed to receive some psychological benefit from reciprocating a generous wage offer from the firm. Workers thus face an explicit tradeoff between the disutility in providing effort and the psychological utility from reciprocating. As the firm recognizes the household’s reciprocating behaviour, it takes into account the effect their wage offer has on effort and maximizes accordingly.
The key feature in the model is that the traditional Labour Supply schedule has a limited role, with the firm referencing what will be called the Wage Setting schedule in determining the optimal level of effort and labour. Hence, the impact of the wealth effect in our model on labour input is relatively less than in the class of models in a simple neo-classical framework. With this feature, this modification gives the model the potential to generate EDBC. Where the model differs from the previous class of EDBC models is the mechanism through which the alteration of preferences works through. In the EDBC class of models, the mechanism largely works through the household’s responses, whereas in this model, the mechanism is largely driven by the firm’s behaviour in trying to elicit effort from the household.

The basis of the reciprocity hypothesis is well founded in the literature. In the realm of behavioral economics, there is evidence that agents are willing to punish behaviour that is perceived to be unkind/unfair, while rewarding kind/fair behaviour even though these actions are costly. Fehr and Falk (1999) show workers do provide less effort in response to a lower wage offer and more effort in response to a higher wage offer. Kahneman et al. (1986) show that the notion of fairness is intertwined with social and community norms and that firm’s deviation from them can be perceived to be unfair and lead to punishment of that behavior. As a result, firms may deviate from what standard models may deem as optimal to avoid being perceived as unfair. Using these considerations, Danthine and Kurman (2008) introduced rent-sharing considerations within a labour reciprocity framework. They showed that this framework had the potential to solve some outstanding macroeconomic puzzles, in particular the wage-employment puzzle. Furthermore, Danthine and Kurman (2010) extended their model within a DSGE framework and found that rent-sharing considerations are an important determinant in wage setting. Our model here is a simplified version of Danthine and Kurmann’s model, extended to incorporate news shocks.

The first part of the paper starts with the exploration of what will be called the base Rent-Sharing model, denoted by (RS) hereafter, and its implication for EDBC. It is shown that while the RS model prevents the fall of labour input on impact of the news, it is unable to generate EDBC. The second part of
this paper extends the base model to incorporate variable capacity utilization and capital adjustment cost in order to incentivize an increase in hours worked and investment, following Jaimovich and Rebelo (2008).

2. The Base Rent-Sharing Model

This section specifies the household’s problem, defines the reciprocating framework in which households operate in, the firm’s problem and the TFP process. The model assumes a two agent economy: the representative household and the representative firm producing final consumption goods. The RS model has the following key features: 1) preferences and production technology are augmented by unobservable and non-contractible effort, 2) households exhibit reciprocating behaviour towards the firm in response to a generous wage offer and 3) the economy is buffered by news shocks regarding future TFP changes. Rent-sharing considerations are introduced within the labour reciprocating framework through the household’s decision in providing effort to the firm.

2.1 The Household’s Problem

The representative household’s expected lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t e_t),$$  \hspace{1cm} (1)

where $C_t$ is consumption, $1 - L_t$ is hours of leisure with time allotment normalized to 1 in period $t$, and $e_t$ is the level of effort per hour worked. The operator $E_0$ corresponds to the expectations conditioned on the information set available in period 0. The parameter $\beta$ represents the discount factor, with $\beta \in (0, 1)$. The instantaneous utility function is given by

$$U(C_t, 1 - L_t e_t) = \log C_t + \psi \log(1 - L_t) - L_t \left[ \frac{1}{2} e_t^2 - \gamma(e_t, W_t) \right].$$  \hspace{1cm} (2)
The first two terms in the utility function are standard in business cycle literature, with $\psi$ representing the household’s relative preferences weighting placed on consumption and leisure. The last term captures the effect of effort on the household’s utility and is divided into two parts. The first part, $\frac{1}{2}e_t^2$, represents the disutility of effort, as working hard is costly. The second part, $\gamma(e_t, W_t)$, is the reciprocating function, capturing the psychological benefit the household receives from reciprocating a generous wage offer through the provision of effort. Thus, this set-up creates an explicit tradeoff between the material disutility and the psychological utility of providing effort to the firm for the household that is not present in standard models. To facilitate reciprocation in the model, the wage is directly inserted into the utility function in order to introduce the idea of wage generosity to the household. The rational for this will be clearer when the reciprocating function is made explicit in subsection 2.2.

The household faces the following budget constraint

$$C_t + S_t x_t = (D_t + S_t)x_{t-1} + W_t L_t,$$

where $W_t$ is the hourly wage rate, $x_t$ is the share of the representative firm owned by the household at the end of period $t$, $S_t$ is the stock market value of the firm, and $D_t$ is the period $t$ dividends payable to shares owned by the household at the end of period $t - 1$.

The household chooses consumption, $C_t$, the number of hours worked, $L_t$, effort level, $e_t$ and financial asset holdings, $x_t$, to maximize their expected lifetime utility subject to the budget constraint. The first order conditions for an optimum in the household’s problem are

$$U_L(C_t, 1 - L_t, e_t) = U_L(C_t, 1 - L_t, e_t)W_t,$$

$$U_C(C_t, 1 - L_t, e_t) = \beta E_t[U_C(C_{t+1}, 1 - L_{t+1}, e_{t+1})R_{t+1}^p],$$

$$C_t + S_t x_t = (D_t + S_t)x_{t-1} + W_t L_t,$$

$$e_t = \gamma(e_t, W_t),$$
where \( R_{t+1}^e = \frac{(d_{t+1} + s_{t+1})}{s_t} \) is the return from holding equities for one period. In addition to the first order conditions, the transversality condition \( \lim_{t \to \infty} \beta^t (U_t(C_t, 1 - L_t, e_t)S_t) = 0 \) is also required for an optimum. Because the value of the firm in equilibrium can also be written as

\[
S_t = p_t k_{t+1},
\]

the transversality condition intuitively indicates that it is not optimal to hold useful capital forever. Equations (4) through (6) are standard conditions representing the household’s leisure consumption choice, the intertemporal choice between consumption today and tomorrow, and the resource constraint, respectively. The only exception is that effort now enters the utility function explicitly. Equation (7) is an additional condition that is the result of including effort in the household’s preferences and will be referred to as the Effort Condition in the remainder of the paper. This condition states that an individual will provide effort up to the point where the marginal benefit of reciprocating a generous wage offer is equal to the marginal cost of providing effort.

2.2 Reciprocity Framework

Using the labour reciprocating framework and notation developed by Danthine and Kurmann (2008, 2010), the reciprocating function, \( \gamma(e_t, W_t) \), equals the product of two components \( d(e_t) \) and \( g(W_t) \). These two components will be called the household’s and the firm’s gifts, respectively. Thus, the reciprocating function takes the form of

\[
\gamma(e_t, W_t) = d(e_t)g(W_t).
\]

The basis of the reciprocating framework is as follows. If the household perceives a generous wage offer from the firm, \( g(W_{t, t}) > 0 \), the utility of the household is increased if they reciprocate with an increased level of effort, \( d(e_{t, t}) > 0 \). Both household’s and firm’s gifts are defined in terms of deviation from some reference or normal level, which could be thought of as being based on societal norms. Due to the
household’s reciprocating behaviour, the firm is able to choose the level of effort the household provides by adjusting the degree of wage generosity (the firm’s gift). That is, the firm is able to substitute between the offer wage and the level of effort it receives from the household, facilitating the optimal choice. A higher (lower) offer wage for a given reference wage results in a higher (lower) level of effort provided to the firm. Thus, the firm’s gift can be interpreted as the price of effort in the input market for effort.

The mechanism that promotes reciprocating behavior within the household’s gift function, \( d(e_t) \), is the explicit assumption that households receive a psychological benefit from rewarding the firm for a generous wage offer with effort above the normal level. Danthine and Kurmann (2010) originally included a non-zero normal effort level in their model. However, this complication did not change the log-linearized dynamics of the model. Thus, I adopt this assumption and normalize the normal level of effort to zero. The household’s gift function is written as follows

\[ d(e_t) \equiv e_t^\alpha, \]  

where \( \alpha \in (0, 1) \) (Danthine and Kurmann (2010)).

The firm’s gift function, \( g(W_t) \), is defined as the difference between the household’s utility obtained from consumption under the actual wage offer and the reference wage. Due to the rent-sharing considerations within the model, the reference wage is determined by the firm’s ability to pay, which in turn is equal to the firm’s output per worker, \( \frac{y_t}{n_t} \). While outside earning opportunities could be included in the reference wage, the task here is simply to explore the ability of rent-sharing considerations in generating EDBC. This assumption may have a meaningful impact on the results; however, I leave this extension for other research. Consequently, I abstract from this complication and define the firm’s gift as

\[ g(W_t) \equiv \log W_t - v \log \left( \frac{y_t}{n_t} \right), \]
where \( \nu \in (0, 1) \). As a result of normalizing effort to zero, the household’s reference wage needs to be restricted to some share of the firm’s output per worker. This can be seen by the fact that if the reference wage was not restricted, there would be no effort provided to the firm in equilibrium, as the firm cannot pay more than \( \frac{W_t}{n_t} \). Thus, the parameter \( \nu \) ensures that effort will be provided in equilibrium.

After substituting equations (9) and (10) into (8), the reciprocating function becomes

\[
y(e_t, W_t) \equiv e_t^{\alpha} \left[ \log W_t - \nu \log \left( \frac{W_t}{n_t} \right) \right].
\]  

(11)

It is important to note that reciprocating behaviour only goes from the household to the firm and not vice versa. Hence, the model’s reciprocating feature is not symmetrical. Firms only observe that households present reciprocating behaviour and optimize with this behaviour in mind. Specifically, the model does not explicitly assume that the firm receives some non-material benefit from offering a generous wage. It is simply maximizing, as is standard in other models, taking into account the behaviour of the household. Thus, any wage generosity that is present in the model is simply because it is optimal and not because it is fair.\(^1\)

### 2.3 The Firm’s Problem

The representative firm produces aggregate output utilizing a Cobb-Douglas production technology, with constant returns to scale and a labour share of income \( \alpha \in (0, 1) \):

\[
Y_t = F(A_t, K_t, n_t, e_t) = (A_t e_t n_t)^{\alpha} (K_t)^{1-\alpha}.
\]  

(12)

Here \( A_t \) is the level of stochastic technology, \( K_t \) is the capital stock, \( n_t \) is employment, and \( e_t \) is the effort level per hour worked. It is further assumed that firms own the capital stock and sell shares of the firm to the household. The firm thus hires labour, \( n_t \), makes investments, \( I_t \), and chooses the level of effort, \( e_t \), that maximizes the firm’s present value for the owners subject to the Effort Condition (7).

---

\(^1\) See footnote 14 in Danthine and Kurmann (2008)
While effort is not directly observable by the firm, the firm does recognize the household’s reciprocating behaviour in response to a generous wage offer in its decision regarding the provision of effort. Consequently, the RS model diverts from the standard setup with the real wage, $W_t$, becoming a choice variable in the firm’s problem, enabling the elicitation of optimal effort.

Ruling out self-fulfilling speculative asset price bubbles

$$\lim_{T \to \infty} \left( \frac{1}{R_{e+T}} \right)^T S_{t+T} = 0$$

and normalizing the firm’s outstanding shares to one, the period $t$ value of the firm is equal to the present value of current and future dividends

$$E_t[D_t + S_t] = D_t + E_t \left[ \sum_{s=t+1}^{\infty} \left( \frac{1}{R_{e+1}} \right)^s D_s \right],$$

where $D_t = Y_t - W_t n_t - I_t$. The firm’s capital stock accumulates in accordance with the accumulation equation

$$K_{t+1} = (1-\delta)K_t + I_t, \quad 0 < \delta < 1$$

where $\delta$ is the capital depreciation rate and the initial value $K_0$ is given.

The firm maximizes (14) with respect to $n_t, W_t (due\ to\ unobservable\ effort), I_t, K_{t+1}$ subject to (15). Recognizing that the level of effort provided to the firm is influenced by the level of labour input the firm utilizes, the capital stock and the firm’s offer wage, the first order conditions in the firm’s problems can be written as

$$F_n(A_t, K_t, n_t, e_t)(1 + \xi_{e,n}) = W_t,$$

$$F_e(A_t, K_t, n_t, e_t) \frac{\partial e_t}{\partial W_t} = n_t,$$
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\[ P_t = 1, \quad (18) \]

\[ P_t = E_t \left[ \frac{1}{n_{t+1}} F_K(A_{t+1}, K_{t+1}, n_{t+1}, e_{t+1}) \left( 1 + \xi_{e,K} \right) + P_{t+1} (1 - \delta) \right], \quad (19) \]

the capital accumulation equation (15) and the Effort Condition (7). The terms \( \xi_{e,n} = \frac{n_t \partial e_t}{e_t \partial n_t} > 0 \) and \( \xi_{e,K} = \frac{K_t \partial e_t}{e_t \partial K_t} < 0 \) represents the elasticity of effort with respect to labour and the elasticity of effort with respect to capital, respectively. The Lagrange multiplier, \( P_t \), represents the shadow price of capital and gives the marginal impact on the lifetime value of the firm discounted to period 0 from an exogenous increase in the capital stock. Utilizing the implicit function theorem and the household’s Effort Condition (7), \( \frac{\partial e_t}{\partial W_t}, \frac{\partial e_t}{\partial n_t} \), and \( \frac{\partial e_t}{\partial K_t} \) can be shown to equal

\[ \frac{\partial e_t}{\partial W_t} = \frac{1}{W_t \left( (2 - \alpha) e_t^{1-\alpha} a^{-1} - \nu a e_t^{-1} \right)} \]

\[ \frac{\partial e_t}{\partial n_t} = \frac{\nu (1 - \alpha)}{n_t \left( (2 - \alpha) e_t^{1-\alpha} a^{-1} - \nu a e_t^{-1} \right)}. \]

\[ \frac{\partial e_t}{\partial K_t} = -\frac{\nu (1 - \alpha)}{K_t \left( (2 - \alpha) e_t^{1-\alpha} a^{-1} - \nu a e_t^{-1} \right)}. \]

At an optimum, equation (16) requires the firm to hire labour up to the point where the marginal product of labour, augmented by the additional benefit a marginal unit of labour has on the level of effort, equals the real wage. Equation (17) reflects the firm’s decision regarding the optimal wage to offer the household. It states that it is optimal for the firm to raise the offer wage such that the marginal product of effort equals the marginal cost of the offer wage. In terms of the firm’s optimal investment decision, equation (18) indicates that the firm will accumulate capital up to the point where the discounted marginal increase in the lifetime value of the firm equals the cost to acquire a unit of capital. On the other hand, equation (19) indicates that the shadow price of capital equals the expected discounted marginal product.
of capital plus the net of depreciation discounted value of the firm in \( t + 1 \), which is standard. However, the firm’s marginal product of capital term in the Euler equation is now augmented by an additional negative effect a marginal unit of capital has on the level of effort.

The key difference of this model versus the standard setup is how the firm’s optimal level of labour input and capital is influenced by the elicitation of optimal effort. To understand the implications of these results, it is important to look at the signs of the elasticity terms in both (16) and (17). The term \( \xi_{e,n} > 0 \) in (16) implies that the optimal level of labour input is relatively higher than in standard models at a given wage rate. The mechanism that is at work here is that firms realize the effect of increasing labour input on effort by influencing the employees’ reference wage. Due to decreasing returns, on the margin, an increase in employment will decrease \( \frac{Y_t}{n_t} \), and thus the reference wage, resulting in a lower offer wage for a given level of effort, all else equal. In contrast, the term \( \xi_{e,K} < 0 \) in equation (17) implies that the economy is relatively less capitalized than in standard models. Similar to the mechanism above, the firm recognizes the impact on effort by influencing the reference wage. In this case, a marginal unit of capital increases \( \frac{Y_k}{n_t} \) and as a result, requires the firm to increase its offer wage for a given level of effort, all else equal. Thus, capital accumulation is relatively more costly than in standard models, resulting in relatively less capital investment. Alternatively, the under capitalization within the RS model could also be interpreted as a consequence of firms no longer being in the position to capture the full return from the marginal unit of capital. Consequently, the firm invests relatively less in maintaining its capital stock.

Combining equations (16) and (17), the optimal rule for the firm in determining the offer wage can be derived to equal

\[
\log W_t = \frac{1 - \nu}{2 - \alpha} + \nu \log \left( \frac{Y_t}{n_t} \right). \tag{20}
\]
Equation (20) defines the wage that will elicit optimal effort from the household for a given output per worker. This equation will be referred to as the Wage Setting schedule and replaces the traditional Labour Supply in setting the optimal level of labour input.

2.4 TFP Process

Technology is assumed to be a stationary auto regressive process of order one:

\[ \ln A_t = \rho \ln A_{t-1} + \epsilon_t, \quad 0 \leq \rho < 1 \]  

where

\[ \epsilon_t = u_t + \zeta_{t-4}. \]  

The two productivity shocks, \( u_t \) and \( \zeta_{t-4} \), are uncorrelated with each other and over time, normally distributed random variables with zero means, variances \( \sigma_u^2 \) and \( \sigma_{\zeta}^2 \). The first shock, \( u_t \), is the traditional unexpected TFP shock that is used in standard models. The second shock, \( \zeta_{t-4} \), introduces the news view of the business cycle, as the shock affects TFP with a delay. This delayed feature, in the realization of a TFP shock, allows agents in the model to anticipate an increase in TFP through two steps. 1) On impact of the shock, news is an initially released signaling that TFP will be higher at some future date by X%. 2) At the future date, the news is realized with an increase in TFP in accordance to the news. Thus, this delayed realization in TFP allows for partial anticipation of a TFP shock, conforming to the empirical observations that changes in long-run TFP are partially anticipated by agents in the economy (Beaudry and Portier (2005, 2006)). In our specification, it is assumed that news is announced four periods prior to the increase in TFP. The basis of selecting the lag in realization is somewhat arbitrary in the literature, as the primary concern is the ability to see the transitory properties of the model. With that said, the literature in this domain traditionally uses a three to four period lag after the news signal (Beaudry and Portier (2004), Jaimovich and Rebelo (2008), Karnizova (2010)). Due to the feature of \( \zeta_{t-4} \), this type of

\[ ^2 \text{Trend technology growth is set to zero to focus on the business cycle properties of the model.} \]
shock will be referred to as a *News Shock*, while \( u_t \) will be referred to as an *Unexpected Shock* for the remainder of the paper.

### 3. Equilibrium

It is assumed that the model’s equilibrium is competitive, consisting of a stochastic sequence of allocations \( \{C_t, L_t, e_t, x_t, n_t, I_t, K_{t+1}, R_t\}_{t=0}^{\infty} \) and prices \( \{S_t, W_t\}_{t=0}^{\infty} \) such that given initial values of \( \{A_0, K_0\} \) markets for goods and assets clear and allocations are optimal given prices. The equilibrium space is formed by the following optimality conditions for the household

\[
\frac{1}{c_t} = \beta E_t \left[ \frac{1}{c_{t+1}} R_{t+1}^e \right],
\]

\[
c_t \left[ \frac{\psi}{1-L_t} + \frac{1}{2} e_t^2 - \gamma(e_t, W_t) \right] = W_t,
\]

\[
\frac{e_t^{2-\alpha}}{\alpha} = \ln W_t - \ln \left( \frac{Y_t}{n_t} \right),
\]

\[
c_t + S_t x_t = (D_t + S_t) x_{t-1} + W_t L_t,
\]

\[
\lim_{t \to \infty} \beta^t E_t \left( \frac{1}{c_t} S_t \right) = 0,
\]

the optimality conditions for the firm

\[
W_t = \exp \left( \frac{1-v}{2-\alpha} \right) \left( \frac{Y_t}{n_t} \right)^v,
\]

\[
\alpha \frac{Y_t}{n_t} \left( \frac{1}{1-v(1-\alpha)} \right) = W_t,
\]

\[
1 = E_t \left[ \frac{1}{R_{t+1}^e} \left[ (1-\delta) + (1-\alpha) \frac{Y_{t+1}}{K_{t+1}} \left( 1 - \frac{\alpha v}{1-v(1-\alpha)} \right) \right] \right],
\]
\[ K_{t+1} = (1 - \delta)K_t + I_t, \quad (31) \]

and the market clearing conditions

\[ Y_t = (A_t e_t n_t)^a (K_t)^{1-a}, \quad (32) \]

\[ Y_t = C_t + I_t, \quad (33) \]

\[ n_t = L_t. \quad (34) \]

Utilizing and rearranging equation (20), the firm’s optimal gift in equilibrium can be shown to be equal to

\[ g^* = \log W_t - \nu \log \left( \frac{Y_t}{n_t} \right) = \frac{1 - \nu}{2 - \alpha}. \quad (35) \]

In other words, the gift of the firm is constant in equilibrium. Because the normal level of effort is normalized to zero, any level of effort provided to the firm will be the household’s gift. With the firm’s equilibrium gift (35) and equation (25), it can be shown that the household’s optimal gift to the firm in equilibrium is also constant and given by

\[ d^* = e^* = \left( \alpha \right)^{\frac{1 - \nu}{2 - \alpha}}, \quad (36) \]

which replaces equation (25) as one of the equilibrium conditions. Thus, it is optimal for the firm to adjust labour input in such a way that the firm’s gift is equal to \( \frac{1 - \nu}{2 - \alpha} \), resulting in the effort level of \( \left( \alpha \right)^{\frac{1 - \nu}{2 - \alpha}} \) for all \( t \). Utilizing the firm’s gift, equation (24) can be re-written as

\[ C_t \left[ \frac{\psi}{1 - L_t} + \frac{1}{2} e_t^2 - e_t^a \left( \frac{1 - \nu}{2 - \alpha} \right) \right] = W_t. \quad (37) \]
4. Steady State and Solution

The model has a unique deterministic steady state that can be solved analytically. In the steady state, stochastic fluctuations are absent. Thus $\varepsilon_t$, $u_t$, and $\zeta_{t-4}$ are all set to equal zero, whereas $A_t$ is set to equal one. Utilizing the constancy of effort in equilibrium condition (36) and the targeted value of hours worked in steady state of $\frac{1}{3}$, the endogenous variables in steady state, represented by $\ast$, can be shown to equal:

$$L^\ast = \frac{1}{3},$$  \hspace{1cm} (38)

$$e^\ast = \left(\frac{1 - v}{2 - \alpha}\right)^{\frac{1}{2 - \alpha}}.$$  \hspace{1cm} (39)

Utilizing the household’s Euler equation, the steady state return from holding equities for one period is

$$\frac{1}{\beta} = R^\ast.$$  \hspace{1cm} (40)

Combining the Wage Setting (28) and the Labour Demand (29) and solving for the steady state value of output results

$$Y^\ast = L^\ast \left(\frac{1}{a} \frac{1}{1 - v(1 - \alpha)}\right)^{-\frac{1}{1 - v}} e^{\frac{1}{2 - \alpha}}.$$  \hspace{1cm} (41)

Rearranging the firm’s output technology, the steady state value of the capital stock can be shown to equal

$$K^\ast = [Y^\ast (A^\ast e^\ast L^\ast)^{-\alpha}]^{\frac{1}{1 - \alpha}}.$$  \hspace{1cm} (42)

From the capital accumulation equation, investment in steady state equals

$$I^\ast = K^\ast \delta.$$  \hspace{1cm} (43)
Substituting the steady state values of output and labour into the Labour Demand curve equation results in the steady state value of the real wage

\[ W^* = \alpha \left( \frac{Y^*}{L^*} \right) \left( \frac{1}{1 - \nu(1 - \alpha)} \right). \quad (44) \]

Lastly, utilizing the economy’s accounting identity, the steady state value of consumption equals

\[ C^* = Y^* - I^*. \quad (45) \]

Since the model cannot be solved analytically, the solution of the model is obtained by log-linearizing the equilibrium conditions around the deterministic steady state. The log-linearization process is done within the MatLab DSGE solving package Dynare.\(^3\)

### 5. Calibration

The model is calibrated to quarterly U.S. data and the parameters used in the simulations are presented in Table 1. The quarterly depreciation rate, \( \delta \), and the persistence of the shock parameter, \( \rho \), are set to 2.5% and 0.979, respectively (King and Rebelo (1999)). The labour share of income, \( \alpha \), and the rent-sharing parameter, \( \nu \), are set such that the quarterly rate of return on capital in steady state is 1.5%, which implies a discount factor 0.9852.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \rho )</th>
<th>( \delta )</th>
<th>( \psi )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>0.5</td>
<td>0.9852</td>
<td>0.979</td>
<td>0.025</td>
<td>1.72</td>
<td>0.2681</td>
</tr>
<tr>
<td>RBC</td>
<td>0.667</td>
<td>0.9852</td>
<td>0.979</td>
<td>0.025</td>
<td>1.68</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

\(^3\) See Annex for Dynare Code
The consumption-leisure weighting parameter, \( \psi \), is set, such that hours worked in steady state equals \( \frac{1}{3} \).

To achieve the parameter value of 1.72 for \( \psi \), equation (4) is solved for \( \psi \) and substitutions of steady values for \( W_t, C_t \) and \( e_t \) are made resulting in

\[
\psi = (1 - l') \left[ \frac{w^*}{c^*} - \frac{1}{2} e^{\ast, 2} + e^{\ast, \alpha} \frac{1 - \nu}{2 - \alpha} \right].
\]  

(46)

Given this calibration, the model implies consumption and investment shares of 74\% and 26\%, respectively.

The calibration of the RBC model largely follows the same strategy as was used in the calibration of the RS model; except the labour share of income is set to its traditional value of \( \frac{2}{3} \) and the preference weighting parameter to achieve \( \frac{1}{3} \) in steady state is based on the following rule

\[
\psi = (1 - l') \frac{w^*}{c^*}.
\]  

(47)

The rent-sharing parameter, \( \nu \), is not present in the RBC model.

6. Effects of Technology Shocks in the RS model

In this section, the RS model is analyzed in response to both a News Shock and an Unexpected Shock in subsection 6.1 and 6.2, respectively. For the News Shock, it is assumed that news corresponding to a future 1% TFP increase is announced in period 1 and realized in period 5. In contrast, the Unexpected Shock is simply a 1% unexpected change in TFP. For comparison purposes, the benchmark RBC model is included in all the results. It should be noted that all simulation begins from the steady state of the respective models.
6.1 News Driven Boom?

Figure 1 demonstrates that the base RS model fails to generate an expectation driven boom in response to positive news regarding TFP. On impact of hearing the news, consumption and the real wage jump up, while output, hours worked, and the capital stock remain unchanged, and investment and the return on equities both fall. In comparison to the benchmark RBC model, the RS model only slightly improves on the results on impact of the news; however co-movements in the macroeconomic aggregates are qualitatively similar in both models. To see what generates these results, it is important to look at the labour market responses.

In the labour market, the Labour Demand and Wage Setting curves determine the equilibrium level of labour input. This is due to the firm referencing the Wage Setting curve instead of the traditional Labour Supply in order to elicit optimal effort. Again, the Wage Setting curve indicates to the firm the wage that will elicit optimal effort for a given level of labour input. On impact of the news, the number of hours worked remains unchanged as neither the Labour Demand nor the Wage Setting schedule shift. To see this, the log-linearized version of the model’s Labour Demand and Wage Setting schedules are presented below:

\[
(1 - \alpha)\hat{L}_t^d = -\hat{\omega}_t + \alpha\hat{A}_t + (1 - \alpha)\hat{K}_t, \tag{48}
\]

\[
(1 - \alpha)\hat{L}_t^{WS} = -\frac{\hat{\omega}_t}{\nu} + \alpha\hat{A}_t + (1 - \alpha)\hat{K}_t, \tag{49}
\]

respectively.\(^4\) As can be seen in (48) and (49), a shift in either curve can only occur if there is a change in \(\hat{A}_t\) or \(\hat{K}_t\), which is not the case as \(\hat{A}_t\) has yet to be shocked and \(\hat{K}_t\) is determined in period \(t - 1\). Even though effort is not present in neither of the log-linearized version of the Labour Demand nor the Wage Setting curves, due to the constancy of effort in equilibrium, it is the firm’s elicitation of effort behaviour that is driving the results in the model. Specifically, it is the elicitation of effort that results in the

\(^4\) Variables with carets are percent deviations from the deterministic steady state values.
introduction of the *Wage Setting* curve, limiting the impact the wealth effect has on hours worked that traditionally affects business cycle models. Thus, the interaction between the *Labour Demand* and the *Wage Setting* curves is the key in generating the different responses in labour input and output relative to the benchmark RBC model. To better understand the influences of the labour market on output in the RS model, it is useful to combine the log-linearized versions of the *Wage Setting* and *Labour Demand*, resulting

\[ \hat{y}_t = \hat{n}_t. \]  

(50)

This condition indicates that deviations in output from steady state are solely driven by deviation in hours worked. Using the result of unchanged labour input in the RS model, equation (50) implies that output is unchanged as well on impact of the news.

The news shock in the RS and RBC models results in co-movements between output, consumption, real wage, hours worked, investment, and the capital stock that are qualitatively similar. Both models predict a recession following a positive TFP news shock and stay in recession until the shock is realized in period 5. However, the main difference between the RS and RBC models is the behaviour of output and hours worked on impact of the news. Referring to the output and hours worked plots in figure 1, it can be seen that hours worked and output fall on impact of the news in the RBC model, while remaining unchanged in the RS model. These differing results in the RS model is sourced from the fact that the firm finds it optimal to have a relatively higher level of employment than in standard models in order to keep the household’s reference wage relatively low. Without this feature, the benchmark RBC model is unable to overcome the wealth effect, which pushes down hours worked and output on impact of the news.

The representative household’s decisions are qualitatively similar in the RS and RBC models with both increasing consumption and decreasing their willingness to supply labour at a given wage rate on impact of hearing the news. Since effort in the model is constant, the *Labour Supply* schedule in the
RS behaves largely the same as in the RBC model. The log-linearized Labour Supply schedule in the RS model is given by

\[ \hat{L}_t^z = -\hat{w}_t + \hat{c}_t + \frac{l^*}{1 - l^*} \left( 1 + \frac{1}{(1 - l^*)(\frac{1}{2}e^{2\gamma} - \gamma(e^*, W^*))} \right) \hat{t}_t. \] (51)

The labour and consumption decisions work through the wealth effect, and are reinforced by the intertemporal substitution of labour for leisure and current for future consumption. Since consumption and leisure are both assumed to be normal goods, anticipation of higher future productivity leads to an increase in anticipated wealth, which in turn increases both consumption and leisure today. Higher future productivity also implies that the real wage and the return on equities are expected to be relatively higher in the future as well, which makes leisure today relatively cheaper than in the future. Thus, the intertemporal substitution of leisure moves in the same direction of the wealth effect resulting in an unambiguous fall in the willingness of the household to supply labour at a given wage. Similarly, the relatively lower return from holding equities for one period makes consumption today relatively cheaper than in the future, reinforcing the increase in consumption that results from the wealth effect. All these factors tend to increase consumption and decrease the willingness to provide labour at a given wage.

Since the firm needs to maintain the same labour input ex-post the news, combined with the decrease in the household’s willingness to work, the response in the real wage is relatively stronger than it is in the RBC model. In contrast, the response in the real wage is tempered in the RBC model by the fact hours worked fall on impact of the news. This stronger response in the wage implies that anticipated wealth is relatively higher in the RS model than in the RBC model, which results in a relatively stronger response in consumption.

The boom in the economy does not occur until after the realization of the improvement TFP in period 5. The boom is fueled by an increase in the marginal product of labour which causes an expansion
of Labour Demand (29) and the Wage Setting (28), resulting in an increase in hours worked. While investment does jump up as result of the improvement of the marginal product of capital, it jumps relatively less in the RS model than in the RBC model (see investment in figure 1). This is due to the additional negative impact that a marginal unit of capital has on the level of effort provided to the firm. The increase in the capital stock further pushes the firm to increase its labour input in order to sterilize the increase in the household’s reference wage that results from the increase in labour productivity. This sterilization of the capital stock gives the model an improved internal propagation mechanism. To see this, it is easier to look at an unexpected shock, which is what will be presented next.

6.2 TFP Driven Boom

Figure 2 illustrates the impulse response functions for both the RS and RBC models. An interesting feature of the RS model is the ability to generate humped shaped responses in output and hours worked that is present in the data (Cochrane (1994)). The Danthine and Kurmann (2010) model was able to generate the humped shaped responses in output and hours worked. Their model featured, in addition to rent-sharing, a wage entitlement and external earning opportunities. In conjunction with our results, this implies that rent-sharing features appear to improve the internal propagation mechanism within DSGE models. Conversely, the simple RBC model is unable to replicate these results, which is fairly well established in the literature (Cogley and Nason (1995)). This can be seen by the fact that the response function of output, hours worked, etc… all return to their steady state largely in conjunction with the level of technology. As Cogley and Nason (1995) point out, the RBC model suffers from a weak internal propagation mechanism that relies on exogenous factors to enable it to generate the stylized facts of the business cycle.

The key in generating the humped shaped impulse responses in hours worked and output is sourced in the firm sterilizing the increases in the household’s reference wage that results from increases in the capital stock. This sterilizing behaviour is due to the fact that firms find it optimal to maintain a
constant output to labour ratio over time, as the output to labour ratio is the household’s reference wage. This can be seen by referring to the firm’s Wage Setting curve (20). If the firm deploys an additional unit of capital, the wage to maintain optimal effort will increase for a given level of labour input. To combat this wage pressure, the firm needs to hire more labour to offset the increase in labour productivity. This action essentially sterilizes the household’s reference wage to changes in productivity. Thus, the sterilization of the reference wage enables the shape of the impulse response function of the capital stock to be partially transmitted to the impulse response function of hours worked. By equation (50), the hours worked humped shape is then transmitted to the impulse response function of output as well.

7. Rent Sharing with Extension

The previous results indicate that in order to generate the needed co-movements necessary for EDBC, the RS model needs a mechanism that will increase hours worked as well as investment on impact of agents hearing the news. Jaimovich and Rebelo (2008) suggest that models with a weak wealth effect, variable capacity utilization and capital adjustment cost can generate EDBC. Following their example, the base RS model is extended to incorporate both endogenous capacity utilization as well as costly capital accumulation. In the next subsection, I describe the amendment made to the base model. The subsection ends with how these new features impact the firm’s first order conditions.

7.1 Description of the Modified Model

To incorporate capital adjustment cost, the capital accumulation equation (15) is amended as follows

\[ K_{t+1} = \left(1 - \delta(x_t)\right) K_t + Q \left(\frac{l_t}{K_t}\right) K_t, \]  

(52)
where $\delta(\chi_t)$ is an increasing convex function capturing the cost to the firm of lowering the capital stock from more intensive utilization through higher depreciation and $Q \left( \frac{I_t}{K_t} \right)$ is the capital adjustment function. The last $K_t$ term on the right of equation (52) is added so the steady state properties of the model are invariant to the addition of adjustment costs.\(^5\) The depreciation function is given by

$$\delta(\chi_t) = \frac{\theta}{\mu} \chi_t^\mu,$$ (53)

where $\chi_t$ is the variable capacity utilization. The parameters $\theta$ and $\mu$ are selected on the basis of achieving a normalized utilization rate of 1 in steady state. To achieve this result, the formula that $\theta$ must follow is

$$\theta = R^* - 1 + \delta^*,$$ (54)

where the variables $R^*$ and $\delta^*$ represent the rate of return on capital and the depreciation rate in steady state, respectively.

The implied value of $\mu$ is

$$\mu = \frac{\theta}{\delta^*}.$$ (55)

The capital adjustment cost function $Q \left( \frac{I_t}{K_t} \right)$ is

$$Q \left( \frac{I_t}{K_t} \right) = \phi_1 + \frac{\phi_2}{1 - \Omega} \left( \frac{I_t}{K_t} \right)^{1-\Omega}.$$ (56)

The two parameters $\phi_1$ and $\phi_2$ are chosen to guarantee that the steady state properties of the model are invariant to the degree of adjustment costs, $\Omega$. To achieve this steady state invariant property, the $\phi_1$ and $\phi_2$ must be set in accordance to the formulas as follows

\(^5\) In the case of no capital adjustment cost, $Q \left( \frac{I_t}{K_t} \right) = \frac{I_t}{K_t}$ which requires $Q \left( \frac{I_t}{K_t} \right) K_t$ such that $Q \left( \frac{I_t}{K_t} \right) K_t = I_t$, reverting the capital accumulation equation back to its base state as in equation (15).
\[ \phi_1 = -\frac{\Omega}{1 - \Omega} \delta, \]  

(57)

\[ \phi_2 = \delta^n, \]  

(58)

respectively. The no capital adjustment cost case corresponds to parameter values of 0, 0.025, and 1 for \( \Omega, \phi_1, \) and \( \phi_2, \) respectively. The adjustment cost function has the property \( Q'' \left( \frac{l_t}{k_t} \right) > 0 \) with respect to \( l_t. \) This property implies that the larger the size of the marginal capital adjustment in period \( t, \) the more costly that adjustment will be. Thus, firms are incentivized to smooth out their capital adjustment plans over time.

The firm’s output technology in (12) is amended to incorporate utilizations:

\[ Y_t = F(A_t, \chi_t, K_t, n_t, e_t) = (A_t n_t)\alpha (\chi_t K_t)^{1-\alpha} \]  

(59)

The firm thus maximizes (59) with respect to \( n_t, W_t, l_t, \chi_t, \) and \( K_{t+1} \) subject to (52). Recognizing again that the level of effort provided to the firm is influenced by the level of labour input, the capital stock, the level of capacity utilization and the firm’s offer wage, including equations (16) and (17), the first order conditions for the firm’s problem become

\[ P_t = \frac{1}{Q' \left( \frac{l_t}{k_t} \right)} \]  

(60)

\[ P_t = E_t \left[ \frac{1}{p_{t+1}} \left( F_{K} (A_{t+1}, K_{t+1}, n_{t+1}, e_{t+1}) (1 + \xi_{e, K}) \right) \right. \]

\[ + \left. P_{t+1} \left\{ 1 - \delta (\chi_{t+1}) - Q' \left( \frac{l_{t+1}}{k_{t+1}} \right) \frac{l_{t+1}}{k_{t+1}} + Q \left( \frac{l_{t+1}}{k_{t+1}} \right) \right\} \right] \]  

(61)

\[ (1 - \alpha) \frac{\chi_t}{\chi_t} (1 + \xi_{e, \chi}) = P_t \delta_{\chi} (\chi_t) K_t \]  

(62)

where \( Q' \left( \frac{l_t}{k_t} \right) = \phi_2 \left( \frac{l_t}{K_t} \right)^{-\Omega} \quad \forall \ t \)
The term $\xi_{e, x} = \frac{\chi_t}{e_t} \frac{\partial e_t}{\partial \chi_t} < 0$ represents the elasticity of effort with respect to capacity utilization. Using the implicit function theorem and the Effort Condition (7), $\frac{\partial e_t}{\partial \chi_t}$ can be shown to be equal to

$$\frac{\partial e_t}{\partial \chi_t} = -\frac{\nu(1 - \alpha)}{\chi_t \left( (2 - \alpha)e_t^{1-\alpha} \alpha^{-1} + \nu \epsilon_t^{-1} \right)}.$$

Equations (60) and (61) are restatements of (18) and (19) respectively, while equation (62) is an additional equation representing the optimality condition for capacity utilization. Both equations (60) and (61) have similar interpretations as in the base RS model. Specifically, in equation (60), the cost to acquire a unit of capital has been augmented by the addition of the marginal adjustment costs term $Q' \left( \frac{h_t}{k_t} \right)$, while the increase in the expected discounted lifetime value of the firm from an extra unit of capital in period $t + 1$ in equation (61) is now net of depreciation and capital adjustment costs. The optimality condition for capacity utilization, equation (62), indicates that it is optimal for the firm to increase capacity utilization as long as the marginal benefit from the increase in output is at least equal to the decrease in the marginal value of the capital stock. Similarly to the case of capital, utilization positively affects the household’s reference wage. Thus, the firm faces an additional cost to increase utilization beyond the increase in the depreciation rate through its impact on effort. This implies that firms will utilize its capital stock relatively less intensively than in standard models. The household’s first order conditions remain unaffected.

### 72. Calibration of Model with Extension

The parameters that are used in the simulation of the extended model are presented in the table 2. The parameters $\alpha, \beta, \rho, \psi, \nu, \mu, \theta, \Omega, \phi_1, \phi_2$ remain unaffected in the extension.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\rho$</th>
<th>$\psi$</th>
<th>$\nu$</th>
<th>$\mu$</th>
<th>$\theta$</th>
<th>$\Omega$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>0.5</td>
<td>0.9852</td>
<td>0.979</td>
<td>1.72</td>
<td>0.2681</td>
<td>1.6</td>
<td>0.04</td>
<td>0.99</td>
<td>-2.475</td>
<td>0.026</td>
</tr>
<tr>
<td>RBC</td>
<td>0.667</td>
<td>0.9852</td>
<td>0.979</td>
<td>1.68</td>
<td>N.A</td>
<td>1.6</td>
<td>0.04</td>
<td>0.99</td>
<td>-2.475</td>
<td>0.026</td>
</tr>
</tbody>
</table>
The adjustment cost parameter is selected on the basis of providing the best odds for incentivizing an increase investment on impact of the news. The remaining parameters, $\theta$, $\mu$, $\phi_1$ and $\phi_2$ are selected based on the equations (54), (55), (57) and (58), respectively.

8. Effects of a News Shock in the Extended Model

To examine the impact of variable capacity utilization and costly capital accumulation, the results section is divided into three subsections to explore each component in isolation before exploring both components in conjunction. The order of the subsections are as follows: 1) model with only capacity utilization, 2) model with only capital adjustment cost, 3) both components, capital adjustment cost and capacity utilization are put together to form the full extended model.

8.1 Capacity Utilization

Figure 3 presents the impact of adding capacity utilization in the base RS model. Again, news regarding a future TFP change is announced in period 1 and is realized in period 5. On impact of the news, output, hours worked, and investment all fall, while consumption and the real wage rise. As can be seen, the introduction of capacity utilization is working in the wrong direction of achieving the necessary co-movements in generating EDBC.

The story here is largely the same as the base RS model for the household, with the wealth effect reinforced by the intertemporal substitution effects. This leads to an increase in consumption and a decrease in the willingness of the household to work. What is different is the impact of endogenous capacity utilization on the firm’s decisions, allowing the firm to intertemporally allocate its capital stock to periods of high productivity. With this added flexibility, the firm reduces its capacity utilization on impact of the news. This result is based on the expected return of a unit of capital being relatively higher in the future. The fall in capacity utilization in turn leads the firm to invest relatively less than in the
constant capacity utilization case, as the investment required to maintain a given level of capital in the economy is now less due to the fall in the depreciation rate. Hence, the firm is conserving its capital stock for use when productivity is expected to be relatively higher. To better see the impact of utilization in the labour market, the log-linearized version of the Labour Demand and Wage Setting curves are presented below

\[(1 - \alpha)\hat{L}^d_t = -\hat{\omega}_t + \alpha\hat{A}_t + (1 - \alpha)\hat{K}_t + (1 - \alpha)\hat{\gamma}_t, \quad (63)\]

\[(1 - \alpha)\hat{L}^{WS}_t = -\frac{\hat{\omega}_t}{v} + \alpha\hat{A}_t + (1 - \alpha)\hat{K}_t + (1 - \alpha)\hat{\gamma}_t, \quad (64)\]

The fall in capacity utilization results in a shift in both the Labour Demand and Wage Setting curves, leading to the fall in hours worked on impact. With both hours worked and capacity utilization falling on impact, output must also fall. Thus, the addition of capacity utilization makes the model worst in producing an expectation driven boom than the base case.

To increase hours worked and thus output on impact of the news, utilization must increase. To see this, notice in equation (63) and (64) \(\chi_t\) must increase. This can be seen by the fact that \(\hat{A}_t\) and \(\hat{K}_t\) do not respond to the news as \(\hat{K}_t\) is determined in period \(t - 1\) and \(\hat{A}_t\) has yet to be shocked. This leaves utilization as the only path through which hours worked and output can rise. To incentivize an increase utilization on impact, the value of a unit of capital must fall. To understand this result, refer to equation (62), which is rewritten here for convenience

\[(1 - \alpha)\frac{\hat{y}_t}{\chi_t} (1 + \xi_{\epsilon,\chi}) = P_t \epsilon_{\chi_t} K_t. \]

As can be seen, the only way that utilization can increase on impact of the news is a fall in the Lagrange Multiplier, \(P_t\). However, in the variable capacity utilization version only, \(P_t\) is constant and equal to 1 for all \(t\). Thus, to have \(P_t\) fall, the capacity utilization only version of the model needs to be extended to incorporate capital adjustment cost, as the addition of costly capital adjustment allows for the the
Lagrange Multiplier to vary in equilibrium. In the next subsection the model with capital adjustment cost only will be explored.

8.2 Capital Adjustment Cost

Figure 4 presents the results of adding capital adjustment cost to the base RS model. On impact of the news, as was the case in the base RS model, output, hours worked, and the capital stock all remain unchanged, real wage and consumption increase and investment, the price of capital, and the return on equity all fall. This specification only slightly improves the results from the base model as the addition of adjustment cost largely prevents the decline in output and hours worked in the transitionary period between the news and the realization of TFP. However, the fall in investment still plagues the model on impact, suggesting that capital adjustment cost will not provide a solution to the negative correlation issue that is present in the base RS model.

The reasoning behind these results are largely the same as was presented in the base case. The addition of capital adjustment cost incentivize an increase in investment relative to the base RS model at the expense of consumption on impact of the news, as firms prefer to maintain a largely stable level of capital in order to smooth its accumulation plans over time. The effect of this change is that responses in aggregate variables are not only smaller on impact but remain largely unchanged until the TFP shock is realized. Since the response in consumption is relatively smaller in this specification, the Labour Supply curve (51) shifts relatively less, resulting in a relatively less response in the wage. In terms of the transitionary period between the news signal and the realization of TFP, the key driver in the dynamics in aggregate variables was the decline in capital in the base model, which is largely eliminated in this specification.

The point of interest here, though, is that even with capital adjustment cost set to its maximum in the adjustment cost function, the RS model is still unable to incentivize an increase investment on impact of the news. In contrast, the RBC model is able to generate an increase in investment on impact. The key
point of difference comes back to the result that the RS model has relative less investment than does the RBC model through the negative impact a marginal unit of capital has on effort. While the smoothing of capital accumulation overtime does has a marginally positive effect in the transitory period, the incentive is not strong enough to overcome the negative impact of effort for investment to rise rather than fall on impact of the news. This result implies that the RS model will be unable to generate the needed co-movements to achieve EDBC in the full model.

### 8.3 Full Model

Figure 5 presents the results from combining all the components together to create the full model. As was done previously, the impulse response function in figure 5 are based on a 1% news shock realized in period 5, announced in period 1. As can be seen in the figure, output, hours worked, consumption, and utilization all increase, while investment, the price of capital, and the return from holding equities all fall on impact of the news. As a result, the model does produce a boom in the economy with output, consumption and hours worked increasing, but due to the fall in investment, it fails to generate EDBC.

The combination of capital adjustment cost and endogenous capacity utilization play a critical role in generating these results. As discussed in section 8.1, the addition of capital adjustment cost allows for the fall in the price of capital, which in turn leads to an increase in capacity utilization, as the cost to replace a unit of capital decreases in the future. The rise in utilization shifts the Labour Demand and the Wage Setting curve through its impact on the marginal product of labour. This leads to an increase in hours worked. Following an increase in hours worked, equation (50) implies that output must increase as well. The household’s decisions largely remain unaffected from the base analysis, with the wealth effect increasing consumption and decreasing the willingness to work. This leads to an increase in the real wage. While all the above results support the generation of EDBC within the RS model, the firm’s unwillingness to raise its level of investment on impact, due to its relative aversion to investment in its capital stock, prevents the generation of EDBC.
The model’s boom in output, consumption, hours worked, and the real wage all continue after the realization of TFP in period 5. The increase in technology increases the marginal product of capital, leading to an increase in investment, the price of capital, and the return on equities. The increase in the price of capital results in a fall in capacity utilization, as the cost to replace unit of capital has increased. With the increase in the level of technology, the marginal product of labour increases, despite the fall in capacity utilization, leading to a further shift in Labour Demand and the Wage Setting curves, increasing hours worked. The increase in hours worked further increases output.

In contrast, the RBC model is able to generate an increase in investment on impact of the news. However, the rise in investment is at the cost of consumption falling and the failure of the RBC model to generate EDBC. This can be seen in figure 5 by noticing that output is unchanged on impact of the news. Thus, by the accounting identity of the economy, a rise in investment must mean a fall in consumption. This result follows from the fact that capacity utilization remains largely unchanged on impact of the news, and without an increase in utilization, the model cannot generate an expansion in hours worked and output.

As a result of these observations, both models fail in generating EDBC. However, The RS model does have some improved features relative to that of the RBC model. Specifically, the RS model does generate a boom in hours worked, output and consumption. However, the rent-sharing features of the RS model while in one way helps in generating EDBC through its impact on labour input and output, it also prevents EDBC through its impact on investment.

9. Conclusion

Most standard business cycle models fail in generating EDBC, predicting a negative correlation between consumption, hours worked and investment on impact of positive TFP news. This paper has explored the potential for generating EDBC through the use of a labour reciprocity framework. Even though the model features constant effort in equilibrium, elicitation of effort by the firm lays at the heart
of the mechanism that improves the results relative to standard models. This mechanism has two main effects: 1) limits the impact the wealth effect has on labour supply and 2) limits the firm’s desire to accumulate capital. While the first effect works in the direction of generating EDBC, the second effect works in the opposite direction, preventing EDBC. The mechanism embedded in the labour reciprocity framework working in conjunction with capacity utilization and capital adjustment cost does, however, lead to a boom in output, hours worked, and consumption on impact of the news even though it does fail in generating a boom in investment. The boom in hours and output is largely driven by the demand side of the labour market, with the marginal product of labour increasing on impact of the news via higher capital utilization. Thus, the model does break the negative correlation between consumption and hours worked that troubles standard models.

In addition to the effect of labour reciprocity on the news views of the business cycle, the elicitation of effort has an impact on the ability of the model to propagate an unanticipated exogenous change in the level of technology. This elicitation of effort improves the propagation mechanism relative to the benchmark RBC case, enabling it to generate the humped shaped impulse responses in output and hours worked that are present in the data. The key in generating this result is sourced from the firms sterilizing behaviour, preventing the household’s reference wage from changing in equilibrium. This is a nice feature of the model that the benchmark RBC model cannot replicate.

While the pure rent-sharing model is unable to generate EDBC, there are some potential extensions to the model that could be useful in generating a positive co-movement between all macroeconomic aggregates in response to news shocks. One interesting question is whether a specification in which effort varies over time would change the results of the model. In other words, will variable effort improve the likelihood of generating EDBC? Alternatively, what is the impact on EDBC from reducing the importance of rent-sharing in the household’s reference wage by incorporating an external reference as well? I leave these questions for further research in this domain.
References


Annex A: Figures

Figure 1: News Shock: RS vs. RBC (Base)

Note: TFP is announced in period 1 that it will higher by 1% in period 5.
Figure 2: Unexpected Shock: RS vs. RBC (Base)

Note: TFP is unexpectedly increased in period 0 by 1%.
Figure 3: News Shock: RS vs. RBC (Capacity Utilization)

Output

Hours Worked

Effort Level

Capital

Capacity Utilization

Price of Capital

Consumption

Real Wage

Investment

Return on Equity

Depreciation

Technology

Note: TFP is announced in period 1 that it will higher by 1% in period 5.
Figure 4: News Shock: RS vs. Base RBC (Capital Adjustment Cost)

Note: TFP is announced in period 1 that it will higher by 1% in period 5.
Figure 5: News Shock: Full RS vs. Full RBC

Note: TFP is announced in period 1 that it will higher by 1% in period 5.
Annex B: Dynare Code

Annex B.1 Rent Sharing Model Code

```matlab
var y, c, l, w, e, i, k, R, chi, delta, P, Q, H, DQ, a, z;

/* Two type of shocks: */
/* (1) u: An unexpected TFP Shock */
/* (2) news: A TFP Shock Learned About 4 periods Prior */

varexo u, news;

parameters beta, theta, mu, phi_1, phi_2, Omega, alpha, sigma, rho, psi, v;

/* Steady State Rate of Return */
Rss = 1.01499977831;

/* Steady State Rate of Depreciation*/
Delta_SS = 0.025;

/* Discount Factor */
beta = 1/Rss;

/* Labour Share of Income */
alpha = 0.5; // 2/3;

/* Size of the Technology Shock */
sigma = .01;

/* Level of Persistence of a Technology Shock */
rho = 0.979;

/* parameter to set hours work in steady state 0.33 */
psi = 1.72173507827348;

/* Adjustment Cost Parameter */
Omega = 0.99;

/* Model without Adjustment Cost Parameter (Set = 0 to transform model without Adjustment Cost) */
/* Omega = 0; */
```
/*Selected to make Steady State invariant to Adjustment Cost */
phi_1 = -Omega*Delta_SS/(1-Omega);

/*Set = 0 to transform model without Adjustment Cost */
/*phi_1 = 0;*/

/*Selected to make Steady State invariant to Adjustment Cost */
phi_2 = Delta_SS^Omega;

/*Set phi_2 = 1 to transform model without Adjustment Cost */
/*phi_2 = 1;*/

/*Selected to make Capacity Utilization = 1 in steady state */
theta = Rss - 1 + 0.025;

/*Set theta = 0.025 to transform model without variable capacity utilization */
/*theta = 0.025;*/

/*Selected to make Capacity Utilization = 1 in steady state */
mu = theta / 0.025;

/*Set mu = 1 to transform model without variable capacity utilization */
/*mu = 1;*/

/*Selected to make Steady State Return of Equity = 1.5% quarterly */
v = 0.2681;

model;
/*Euler Equation*/
(1)*
1/c = beta * ((1 / c(+1)) * R(+1));

/*Labour Supply*/
w = c * (psi / (1-l) + (1/2)*e^2 - (e^alpha)*((1-v)/(2-alpha)));

/*Labour Demand = Wage Setting Curve (Wage that elicits optimal effort)*/
alpha * (y / l)*(1 /(1 - v * (1 - alpha))) = exp((1 - v) / (2 - alpha)) * (y / l)^v;

/*Effort Condition*/
(4)*
e = (alpha*((1 - v) / (2 - alpha)))^(1 / (2 - alpha));
/*Place holder
(5)*/
\[ H = \frac{i}{k(-1)}; \]

/* Capital Adjustment Cost Function
(6)*/
\[ Q = \phi_1 + (\phi_2 / (1 - \Omega)) * H^{(1 - \Omega)}; \]

/*Place Holder for the first part of the partial derivative of Q (dQ/d(i/k))
(7)*/
\[ DQ = \phi_2 * H^{(-\Omega)}; \]

/*Variable Capital Depreciation Function
(8)*/
delta = (\theta / \mu) * \chi^\mu;

/*In the case of no capacity utilization, depreciation is set 0.025
*/
/*delta = 0.025;*/

/*Evolution of Capital
(9)*/
\[ k = (1 - \delta) * k(-1) + Q * k(-1); \]

/*Shadow Price of Capital equal the cost to acquire a unit of capital
(10)*/
P = \frac{1}{DQ};

/*MB of capital = the shadow price capital
(11)*/
P(-1) = \left(\frac{1}{R}\right) * ((1-\alpha) * (y / k(-1) - v * w * l / k(-1)) + P * (1 - delta - DQ*H + Q));

/*Optimality Condition for the level Utilization
(12)*/
\[ (1-\alpha) * (y - w * l * v) = P * \theta * k(-1) * \chi^\mu; \]

/*In the case of no capacity utilization, chi is set 1
*/
/*\chi = 1;*/

/*Production Function
(13)*/
y = ((a * l * e)^\alpha) * (\chi * k(-1))^{(1-\alpha)};

/*Accounting Identity
(14)*/
y = i + c;

/*Technology Process
(15)*/
a = z;
/*Technology Shock Process
(16)*/
z = z(-1)^\rho \ast \exp(u + \text{news}(-4));
end;

/*Steady State*/

initval;
a=1;
z=a;

l=1/3;
delta=0.025;
R=1/beta;

chi=1;

e = (alpha \ast ((1 - v) / (2 - alpha)))^{(1 / (2 - alpha))};
y = 1 * ((alpha / (1 - v \ast (1 - alpha)))^{(-1 / (1 - v))) \ast \exp(1 / (2 - alpha))};
k = (y \ast (a * e * l)^{-alpha})^{(1 / (1 - alpha))};
w = alpha * (y / l) * (1 / (1 - v * (1 - alpha)));
i = k \ast delta;
c = y - i;

H=1/k;
Q=H;
DQ = phi_2 \ast H^{-\Omega};
P = 1 / DQ;

u=0;
news=0;
end;

steady;

cHECK;

shocks;
//var u; stderr sigma;
var news; stderr sigma;
corr u, news = 0;
end;

stoch_simul(irf=10,nograph);
Annex B.2 RBC Model Code
/*RBC Model with Firms owning the Capital Stock and household owning the firm */

var y, c, l, w, i, k, R, chi, delta, P, Q, H, DQ, a, z;

/*Two type of shocks: */
/* (1) u: An unexpected TFP Shock */
/* (2) news: An TFP Shock Learned About 4 periods Prior */

varexo u, news;

parameters beta, theta, mu, phi_1, phi_2, Omega, alpha, sigma, rho, psi;

Rss = 1.015; //SS rate of return
Delta_SS = 0.025;

beta = 1/Rss; //discount factor
alpha = 2/3; //Labour Income Share
sigma = 0.01; //Size of the Technology Shock
rho = 0.979; //Level of Persistence of a Technology Shock
psi = 1.68421053;//parameter to set hours work in steady state 0.33

/*Adjustment Cost Parameter */
Omega = 0.99;

/*Model without Adjustment Cost Parameter (Set = 0 to transform model without Adjustment Cost) */
/*Omega = 0;*/

/*Selected to make Steady State invariant to Adjustment Cost */
phi_1 = -Omega*Delta_SS/(1-Omega);

/*Set = 0 to transform model without Adjustment Cost */
/*phi_1 = 0;*/

/*Selected to make Steady State invariant to Adjustment Cost */
phi_2 = Delta_SS^Omega;

/*Set phi_2 = 1 to transform model without Adjustment Cost */
/*phi_2 = 1;*/

/*Selected to make Capacity Utilization = 1 in steady state */
theta = Rss - 1 + 0.025;
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/*Set theta = 0.025 to transform model without variable capacity utilization*/
/*theta = 0.025;*/

/*Selected to make Capacity Utilization = 1 in steady state*/
mu = theta / 0.025;

/*Set mu = 1 to transform model without variable capacity utilization*/
/*mu = 1;*/

model;
/*Euler Equation (1)*/
1/c = beta * ((1 / c(+1)) * R(+1));

/*Labour Supply*/
w = c * psi / (1 - l);

/*Labour Demand*/
w = alpha * y / l;

/*Place holder (5)*/
H = i / k(-1);

/* Capital Adjustment Cost Function (6)*/
Q = phi_1 + (phi_2 / (1 - Omega)) * H^(1 - Omega);

/*Place Holder for the first part of the partial derivative of Q (dQ/d(i/k)) (7)*/
DQ = phi_2 * H^(-Omega);

/*Variable Capital Depreciation Function (8)*/
delta = (theta / mu) * chi^mu;

/*Evolution of Capital (9)*/
k = (1 - delta) * k(-1) + Q * k(-1);

/*Shadow Price of Capital (10)*/
P = 1 / DQ;

/*MB of capital = the shadow price capital (11)*/
P(-1) = (1 / R) * ((1-alpha) * y / k(-1) + P * (1 - delta - DQ*H + Q));

/*Optimality Condition for the level Utilization (12)*/
(1-alpha) * y = P * theta * k(-1) * chi^mu;
/*Production Function*/
y = ((a * l)^alpha) * k(-1)^(1-alpha);
/*Accounting Identity*/
y = i + c;
/*Technology Process*/
a = z;
/*Technology Shock Process*/
z = z(-1)^rho * exp(u + news(-4));
end;

initval;
z=1;
a=z;
l=1/3;

chi = 1;
delta = 0.025;

R = 1 / beta;
k = ((1/(1-alpha))*(1/beta -1 + delta))^(1/(-alpha))*a*l;
i = k*delta;
y = ((a * l)^alpha) *(chi* k)^(1-alpha);
c = y - i;
w = alpha * y / l;

H=i/k;
Q=H;
DQ = phi_2 * H^(-Omega);
P = 1 / DQ;

u=0;
news=0;
end;

steady;

check;

shocks;
var u; stderr sigma;
var news; stderr sigma;
corr u, news = 0;
end;

stoch_simul(irf=10, nograph);