Three Essays on Buyer Power, Market Structure and Government Subsidies

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Abstracts

Chapter 1: Downstream Competition and the Effects of Buyer Power

The first chapter examines the interaction between buyer power and competition intensity in the downstream market in affecting consumer and total welfare. We study a model where oligopolistic retailers compete in quantity in the downstream market and one of them is a large retailer that has its own exclusive supplier. Negotiation between this retailer and its supplier is modeled as a generalized Nash bargaining game. We demonstrate that an increase in the buyer power of the large retailer against its supplier leads to a fall in retail price and consequently an improvement in consumer surplus and this is true even in the extreme case where the downstream market is served by a monopoly. More interestingly, we find that the effects of buyer power are large when the intensity of downstream competition is low, with the effects being the largest in the case of downstream monopoly. This suggests that buyer power and downstream competition are substitutes.

Chapter 2: Subsidy, Product Diversity and Buyer Power

The objective of the second chapter is to analyze the effectiveness of government subsidies in promoting product diversity when the downstream firm (a retailer) has buyer power. We extend the standard Dixit-Stiglitz model of monopolistic competition and compare the effects of subsidies on equilibrium number of product varieties and social welfare in the case where products are sold directly to consumers and the case where they are sold through a monopoly retailer with buyer power. Two types of subsidies are considered, a subsidy on marginal cost and a subsidy on fixed cost. We find that while
the two types of subsides have different effects on the quantity and retail price of each variety, they both raise the number of product varieties and the social welfare. Moreover, a combination of the two types of subsides is able to achieve the social optimum. These results are true even when products are distributed through a downstream monopoly retailer who has all the bargaining power, but the mechanism through which a subsidy increases product varieties is different. In comparison with the case where products are distributed directly to consumers, retailer buyer power reduces product variety and social welfare. Furthermore subsidies become less effective in the presence of buyer power. To be more specific, retailer buyer power has both a level effect and a marginal effect on product diversity. At any given subsidy rate, the equilibrium number of varieties is smaller and a marginal increase in subsidy leads to a smaller increase in the number of varieties.

Chapter 3: Subsidy on Complementary Products in a Model of Monopolistic Competition

The third chapter seeks to re-examine the market provision of product diversity under monopolistic competition and the effects of an infinitesimal subsidy on product variety and social welfare in the case of complementary products. This examination builds on the standard Dixit-Stiglitz model of monopolistic competition but assumes an alternative demand linkage. The results show that, different from the case of substitutable products, demand complementarity leads to multiple equilibriums and the number of product varieties could be higher or lower than the constrained optimum depending on the level of the fixed cost of production. When the fixed costs are small, the market yields too many products and an infinitesimal subsidy exacerbates the problem leading to an over-
supply of product varieties. On the other hand, when the fixed costs are large, there are too few products and in some cases the complementary goods industry becomes non-existent. A subsidy that induces a switch of equilibriums enhances product variety and improves social welfare.
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General Introduction

Large and powerful buyers can be found in many industries. Examples include large retailers such as Walmart and Tesco in their dealings with suppliers; television networks such as CTV and Global in their dealings with Canadian film producers, book chains such as Indigo in their dealings with Canadian book publishers. When buyers exercise the power to influence the terms of trade with suppliers, they are regarded as having buyer power. The past two decades have seen increased concentration at the retail level, which has raised awareness as well as concern over buyer power. Whether buyer power harms consumer and total welfare and what the implication is for enforcement and public policies have been the central aspect of discussion, to name a few, in the three roundtables held by the OECD (OECD 1998, 2004 and 2008), the two symposia by American Antitrust Institute in 2004 and 2007 and the grocery retail industry investigations by UK Office of Fair Trading (1984 and 1999) and Competition Commission (2000 and 2008). Meanwhile, the increased policy interest is matched by litigation and enforcement involving buyer power such as mergers or acquisitions to create or enhance buyer power.  

Buyer power also received considerable attention in the economic literature. One strand of literature (such as Sheffman and Spiller 1992, Chipty and Snyder 1999, Dobson 2005 and Inderst and Wey 2007) focuses on understanding the economics of how and why buyers are able to obtain lower prices (or better terms of trade) from sellers. Although useful in identifying circumstances and conducts that give rise to buyer power, it lends limited

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1 Buyer power has been at issue in cases such as the acquisition of Tuko Oy by Kesko Oy in 1997 in Finland and the merger of Wm Morrisons Supermarkets PLC with Safeway PLC in 2003 in the UK in the retail industry and the acquisition of Advance PCS by Caremakr Rx, Inc in the United States in 2004 in health industry.
theoretical support on whether buyer power warrants scrutiny. To identify the welfare effects of buyer power and address its public policy aspects, an equilibrium analysis that considers the responses by firms both on the upstream and the downstream is required (OECD 2008).

Models trying to accomplish this goal need to have two elements: first it must model the negotiation process between sellers and buyers. The type of upstream contracts used is fundamental in determining the final welfare implication of buyer power. Second it has to model the competition between firms in the upstream and/or downstream. Looking at the lower wholesale prices and possibly larger quantity purchased by the buyers is not sufficient. Indirect effects on welfare may arise as the exercise of buyer power could affect its competitors and sometimes the market structure in the long-run. The final welfare effects depend on how the downstream competition interplays with the exercise of buyer power. Several papers (such as Dobson and Waterson 1997, von Ungern-Sternberg 1997, Chen 2003 and 2004) examine the impact of buyer power using such framework.

The first and second chapter of this thesis sit with this strand of literature and study several aspects of buyer power including the impact of buyer power on retail prices, product variety and social welfare; how buyer power interacts with downstream competition and what is the impact of buyer power on the efficacy of government subsidies. The third chapter is an extension of the second chapter where an alternative demand linkage is assumed.

Galbraith (1952) coined the term of countervailing power and states that the market power of large firms is often not restrained by competition from the same side of the market but by powerful firms on the demand side, who manage to extract better terms of trade which are then passed on to consumers. The hypothesis, however, received little support from the
economic literature. The common theme that emerges from the literature (such as Dobson and Waterson 1997, von Ungern-Sternberg 1997, Chen 2003, Erutku 2005) in fact is that the presence of competition in the downstream retail market is necessary for buyer power to (possibly) improve social welfare which implies that the driving force for the reasonable level of consumer and social welfare is the competition at the retail level instead of buyer power. Moreover, how welfare consequences of buyer power are affected by changes in the intensity of downstream competition is left unexplored. The first chapter re-examines the concept of countervailing power by studying the interaction between buyer power and competition intensity in the downstream market in affecting consumer and total welfare. It contributes to the literature in several ways: first it disentangles the buyer power and the intensity of downstream competition which enables the study of the two factors separately. Second, this paper shows that to the extent that the buyer power is countervailing (i.e., offsets the market power of sellers), it reduces retail prices and improves consumer welfare (at least in the short-run) and this is true even in a case where the downstream is a monopoly. A comparison with the existing literature indicates that the negative welfare effects from the exercise of buyer power may be indirect and arises by affecting competitors in an adverse way so that it substantially reduces competition. Third it identifies a situation where buyer power can be viewed as a substitute for competition as posited by Galbraith. The results in the first chapter have policy implications especially in the analysis of merger cases where the issue of retail concentration has been dealt with and the question of welfare consequence of buyer power needs to be answered given the level of post-merger concentration.

The results from chapter one are based on short-run considerations. Another question is if buyer power can be regarded as a substitute for competition, whether it is a good substitute
in the long-run. In fact concerns have been expressed over the effects of buyer power on product diversity (Competition Commission, 2002 and 2003, OECD 2008). The second chapter contributes to this area of study.

Motivated by the observation of the wide use of government subsidies to promote cultural diversity and the increasingly concentrated retail market distributing the cultural products, this chapter examines the long-run implication of buyer power and the effectiveness of subsidies in promoting product diversity in the presence of such power. It fills the gap in the literature in the following ways. First, product diversity is one important dimension of consumer and total welfare. Although there has been a series of work devoted to examining the impact of buyer power on retail prices, little was done on its effects on product diversity. Second, it has been commonly argued that the holdup problem of suppliers is likely to lead to a lower number of product varieties. This paper identifies a different mechanism through which retailer buyer power may negatively impact product diversity. More specifically, the retailer, acting as the gatekeeper of the downstream market internalizes the negative effects of too “many” products on its profits and would choose to carry a smaller number of products. Third, empirically the ability of competition authority to restrain the exercise of buyer power is rather limited (OECD 2008). Thus the task of sustaining a desirable level of product diversity in the presence of buyer power may better be assigned to a regulatory agent. This paper examines the efficacy of one of the common measures used by regulatory agents, government subsidies, in the presence of buyer power. Two types of subsidies are studied, a subsidy on the unit cost of production and a subsidy on the fixed cost. Our results show that both subsidies enhance product diversity and improve social welfare, but are less effective than the case where the products are sold directly to
consumers. Furthermore, a proper combination of the two types of subsidies can reach the social optimum.

The third chapter is an extension of the second chapter. It has been shown in a series of papers (Dixit and Stiglitz 1977, Spence 1976, Costrell 1990, Broer and Heijdra 2001 and Heijdra 1998) that the market under monopolistic competition yields suboptimal product diversity and a government subsidy could be welfare enhancing. These papers assume that products are substitutes with no complements in existence, while in reality some markets are composed of complementary products. Chapter three seeks to re-examine the market provision of product variety and the effects of a government subsidy under monopolistic competition in the case of complementary products. The results show that, different from the case of substitutable products, demand complementarity leads to multiple equilibriums and the number of product varieties could be higher or lower than the optimum constrained by monopolistic competitive pricing. A subsidy is not necessarily welfare-enhancing, which points out the necessity of investigating the pattern of demand interdependence before choosing a policy. Furthermore, coordination failure in entry decision could occur where mutual gains can be realized but no firm has an incentive to deviate. An infinitesimal subsidy that changes firms’ beliefs can induce a switch of equilibriums and improve social welfare.
Chapter 1

Downstream Competition and the Effects of Buyer Power
1.1 Introduction

Increased concentration in the retail industry and the tremendous success of large retail firms such as Wal-Mart and Tesco has raised awareness and concerns regarding the impact of buyer power. One of the main issues that antitrust authorities in Europe and North America have been trying to understand is whether the success of these retailers lessens or distorts competition at the retail and/or production level. This is reflected through the three roundtables held by OECD to examine the impact of buyer power on competition (OECD 1998, 2004 and 2008). In the UK, there has been an unprecedented level of scrutiny of the retail industry by the Office of Fair Trading (2007) and the Competition Commission (2000 and 2008). Responding to the policy concerns in North America, The American Antitrust Institute held two symposia on buyer power in 2004 and 2007 respectively.

In the literature, a number of authors have analyzed the effects of buyer power on consumer prices and social welfare.\(^2\) In particular, von Ungern-Sternberg (1996) and Dobson and Waterson (1997) show that greater buyer power, as reflected through an increase in concentration at the retail level, leads to reduced consumer prices and consequently higher social welfare only if the competition at the retail level is fierce. More specifically, von Ungern-Sternberg (1996) compares two theoretical models: a Cournot model where an upstream supplier sells to oligopoly retailers and a model with perfect competition in the retail market. He finds that only in the model of perfect competition does a decrease in the number of retailers lead to lower consumer prices. Dobson and Waterson (1997) use a similar model where a monopoly supplier negotiates with oligopoly retailers who offer

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\(^2\) Of direct relevance to the present paper are von Ungern-Sternberg 1996, Dobson and Waterson 1997, Chen 2003, Erutku 2005 and Inderst and Wey 2007. Also of some relevance are the analyses on the relationship between upstream market structure and buyer power (Smith and Thanassoulis 2009, and Mills 2010).
differentiated services. In their model, retailers compete in prices and downstream competition is manifested through the substitutability of retailer services. Their analysis shows that consumer prices fall and consequently social welfare increases with the number of retailers only when retailers are considered as very close substitutes. Since Bertrand competition in the case of homogeneous firms leads to the same equilibrium as perfect competition (with price equalling marginal cost), this finding by Dobson and Waterson can be viewed as a generalization of von Ungern-Sternberg (1996).

As pointed out by Chen (2003), the analysis in von Ungern-Sternberg (1996) and Dobson and Waterson (1997) captures the combined effects of both buyer power and seller power of retailers as increased concentration at the retail level enhances both types of market power simultaneously. To identify more precisely the welfare effects of buyer power, Chen (2003) examines a situation where a single upstream supplier sells to a group of retailers consisting of a dominant retailer and a large number of price-taking fringe firms. Buyer power is modelled as the ability of the dominant firm to obtain a larger share of joint profits with the supplier. He demonstrates that the presence of downstream competition by fringe retailers is crucial in driving welfare effects of buyer power. Specifically, he shows that as the dominant retailer gains more buyer power, the retail price will decrease but the welfare effects depend on the market share of the dominant retailer and the differences in the cost structures among retailers. When the number of fringe retailers is sufficiently large, social welfare improves as a result of increased buyer power.

A common theme in the literature reviewed above is that the presence of competition in the downstream retail market is necessary for buyer power to (possibly) benefit consumers and improve welfare. What is less clear, however, is how the welfare consequences of buyer
power are affected by the intensity of downstream competition. For example, is it true that buyer power is more likely to be beneficial to consumers when downstream competition is more intense? Based on the analyses by von Ungern-Sternberg (1996) and Dobson and Waterson (1997), it would be tempting to answer “yes” to this question. But, as discussed earlier, the effects of increased buyer power in these models are intertwined with those of reduced competition in the retail market. Consequently it is not possible to isolate the effects of downstream competition in these analyses.

In this regard, one issue that is of particular relevance is how competition authorities should deal with cases where a merger between two retailers or conduct by a large retailer that does not significantly increase the concentration at the retail level but enhances the buyer power of the retailer(s). This issue may arise in a situation where the two merging retailers sell in different geographic markets. Even in a case where the merging retailers have some overlaps in geographic markets, a competition authority will either reject the merger or require divestiture by the merging retailers in those geographic markets where post-merger concentration is deemed too high. What this means in practice is that in many instances buyer power becomes the focus of competition analysis only after the issue of retail concentration has already been dealt with. In such cases, a competition authority may want to know the answer to questions such as, “will competition in the retail market ensure that any exercise of buyer power post-merger will not harm consumer and social welfare?” or in a case where the pre-merger concentration in retail markets is high, "should we still be

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3 In 1999, for example, Canada’s Competition Bureau approved two mergers of grocery retail chains after the merging parties agreed to divest certain stores in those local markets where they had significant overlaps. In each case, the two retail chains operated primarily in separate parts of the country and they overlapped in only a small number of local markets before the merger. See Competition Bureau (2009a and 2009b) for details about the Bureau’s review of these two mergers.
concerned about the exercise of buyer power even though the merger does not increase the concentration of each retail market?"

In this paper we examine the interaction between buyer power and competition intensity in a downstream market. To do so, we construct a model similar to those of von Ungern-Sternberg (1996) and Dobson and Waterson (1997) in that, in the basic version, retailers compete in a downstream market where the competition intensity is measured by the number of retailers. Different from von Ungern-Sternberg (1996) and Dobson and Waterson (1997), however, we separate the buyer power from the intensity of downstream competition by assuming that only one of those retailers is in a bilateral monopoly relationship with its supplier and they negotiate the wholesale price à la generalized Nash bargaining problem (Harsanyi and Selton 1972). The buyer power of this large retailer is then modelled as its bargaining power against the supplier. As such, the retailer’s buyer power is independent of the intensity of downstream competition. This allows us to isolate the effects of retailer buyer power from those associated with a change in the intensity of downstream competition, as well as addressing the questions raised in the preceding paragraph.

Our analysis shows that, with the exception of perfect competition in the downstream market, the wholesale and retail prices indeed fall and consequently consumer surplus improves following an increase in buyer power in the hands of the large retailer. This is true even in the case of downstream monopoly where there is no retail competition. This result is in sharp contrast to von Ungern-Sternberg (1996) and Dobson and Waterson (1997) where

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4 As a point of comparison, we also study the cases where the downstream market structure is monopoly and perfect competition.
stronger buyer power reduces retail prices only if the competition in the downstream market is sufficiently intense. Consistent with the results from the papers cited above, we show that increased downstream competition is beneficial to final consumers in the sense that it forces the large retailer to bargain more rigorously with its supplier and pass on a greater share of savings to consumers. More interestingly, we find that the effects of buyer power are large when the intensity of downstream competition is low, with the effects being the largest in the case of downstream monopoly. This suggests that buyer power and downstream competition are substitutes.

This last result is reminiscent of Galbraith’s countervailing power hypothesis. In his controversial book on American capitalism, Galbraith (1952) argued that buyer power was a substitute for competition. In his own words, “in the typical modern market of few sellers, the active restraint [on the exercise of private economic power] is provided not by competitors but from the other side of the market by strong buyers” (Galbraith 1952 p119). However, Galbraith’s hypothesis has received little support from the existing theoretical analyses of buyer power. Therefore, one contribution of the present analysis is that it identifies a setting where buyer power and competition can indeed be viewed as substitutes.

We also examine the social desirability of buyer power. We find that while buyer power always improves consumer benefits, the net effect on total surplus depends on the cost structure of suppliers and the level of buyer power the large retailer possesses. Given that the supplier to the large retailer has an increasing marginal cost function, very high level of buyer power may cause efficiency loss in production, which could eventually offset the gain in consumer surplus, leading to lower social welfare.
The case where the downstream market is perfectly competitive is an exception in that higher buyer power only affects the share of profits between the large retailer and its supplier and does not have any impact on retail price level and social welfare.

The rest of the paper is organized as follows. Sections 1.2 and 1.3 set out the model and solve it for the case of Cournot competition among retailers. Sections 1.4 and 1.5 extend the analysis to monopoly retailer and perfect retailer competition respectively. Section 1.6 concludes.

1.2 The Model

There are two levels of markets. In the downstream, a group of \( n \) retailers compete in the quantity they sell, denoted by \( q_i \) (\( i = 1, 2, \ldots n \)). From the upstream markets these retailers purchase the supplies of the product. Following von Ungern-Sternberg (1996) and Dobson and Waterson (1997), we consider a simple case of linear market demand. The demand is represented by the function \( p(Q) = a - bQ \), where \( Q \) is the total demand (\( Q = \sum_{i=1}^{n} q_i \)).

To separate the effects of buyer power from those of downstream competition, we assume that one of the \( n \) retailers, retailer \( R_1 \), is in a bilateral monopoly relationship with its supplier, while the remaining retailers obtain their supplies in competitive markets. The idea here is that \( R_1 \) is a large chain store that has its own exclusive source of supply. This exclusive relationship locks \( R_1 \) and its supplier into a bilateral monopoly situation\(^5\). Each

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\(^5\) See Horn and Wolinsky (1988 p408) for a discussion about the sources of bilateral monopoly. In practice, it is not difficult to find examples of such exclusive relationships. For example, Samsung Electronics reached a deal in 2007 to sell its new printers exclusively at Apple retail stores. The Home Depot partnered with Klein
retailer incurs two types of costs: the wholesale price of purchasing the good from its supplier and a constant marginal cost of providing retailing services. The latter is normalized to zero.

On the suppliers’ side, suppose that $R_1$’s supplier’s total cost of production is

$$C_s(q_1) = \left( c_s + \frac{1}{2} d_s q_1 \right) q_1,$$

where $q_1$ is the quantity purchased by $R_1$. On the other hand, the competitive wholesale price at which other retailers, $R_2 \ldots R_n$, purchase their supplies is equal to $c$. We assume that the exclusive relationship between $R_1$ and its supplier generates efficiency gains so that $c_s < c$.

The firms in this model play a two-stage game. At stage one, the large retailer $R_1$ negotiates with its supplier over the wholesale price $w$. We assume that the outcome of the negotiation is determined by generalized Nash bargaining solution (Harsanyi and Selton 1972). At stage two, all retailers compete in quantity in the downstream market.

To solve the model we shall start with the second stage of the game where each retailer chooses simultaneously the quantity to sell given the indirect demand curve $p(Q)$ and the wholesale prices. Their profit-maximization problems can be formulated as:

$$\text{Max } \pi_1 = (a - bQ)q_1 - wq_1$$  \hspace{1cm} (1.1)
for retailer $R_i$ and

$$\max_{q_i} \pi_i = (a - bQ)q_i - cq_i$$  \hspace{1cm} (1.2)$$

for other retailers $R_i (i = 2, 3, \ldots n)$. The first-order conditions of these two optimization problems are:

$$a - bq_i - bQ - w = 0,$$  \hspace{1cm} (1.3)$$

$$a - bq_i - bQ - w = 0.$$  \hspace{1cm} (1.4)$$

Since all retailers other than $R_i$ are identical, they sell the same quantity in equilibrium. Setting $q_i = q$ for $i \neq 1$, we solve the above equations for $q_i$ and $q$ as functions of, among other things, $w$ and $n$:

$$q_i = \frac{a + (n - 1)c - nw}{(n + 1)b},$$  \hspace{1cm} (1.5)$$

$$q = \frac{a - 2c + w}{(n + 1)b}.$$  \hspace{1cm} (1.6)$$

Note from (1.6) that $q > 0$ if and only if $a - 2c + w > 0$. Since the supplier of $R_1$ cannot make a negative profit in equilibrium, we have $w \geq c_s$. Accordingly, we assume $a - 2c + c_s > 0$ to ensure that all other retailers are active in equilibrium.

Using (1.5) and (1.6) we can write the profits of retailer $R_1$ and its supplier as

$$\pi_i = (p - w)q_i = \frac{(p - w)(a + (n - 1)c - nw)}{(n + 1)b},$$  \hspace{1cm} (1.7)$$
\[
\pi_s = wq_i - (c_i + \frac{1}{2} d_i q_i)q_i = \left( n + 1 \right) (w - c_i) b + \frac{1}{2} \left( a + (n - 1)c - nw \right) d_i \left( \frac{a + (n - 1)c - nw}{(n + 1)^2 b^2} \right). \tag{1.8}
\]

At stage one retailer \( R_1 \) plays a generalized Nash bargaining game with its supplier over the wholesale price \( w \). Because of their exclusive relationship, their disagreement payoffs are both zero. The wholesale price is thus determined by the solution to the maximization problem:

\[
\max_w \pi_1^\gamma \pi_s^{1-\gamma}, \tag{1.9}
\]

where \( \gamma \in [0,1] \). Noting that a larger \( \gamma \) implies that retailer \( R_1 \) would obtain a larger share of the profit from the transaction with its supplier, we interpret \( \gamma \) as a measure of the retailer’s buyer power against its supplier. The first-order condition implies

\[
\frac{(2(n + 1)b + nd_s)(a + (n - 1)c)(1 - \gamma) - 2nw + (2(n + 1)b c_s + d_s(a + (n - 1)c))(1 + \gamma)n}{2b^2(n + 1)} = 0. \tag{1.10}
\]

The equilibrium wholesale price thus is

\[
w = \frac{1 - \gamma}{2n} \left( a + (n - 1)c \right) + \left( 1 + \gamma \right) \frac{2(n + 1)b c_s + d_s(a + (n - 1)c)}{2(2(n + 1)b + nd_s)}. \tag{1.11}
\]

Substituting (1.11) into (1.5) and (1.6), we obtain equilibrium outputs for retailers:

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7 A broader definition of buyer power is a buyer’s ability to exploit the surplus from the trade, which can be achieved not only from obtaining a better terms of trade from its supplier (a bigger share of the pie) but also from increasing the joint profits (making a bigger pie) by using for example a more efficient contract. In this paper a narrower definition is used where the buyer power is manifested through a larger share of the profits from the trade.

8 Note that the wholesale price given by (1.11) incorporates the equilibrium retail prices and quantities at stage 2. Therefore, even though \( q_i \) and \( p \) do not appear in this equation explicitly, they have an impact on the wholesale price through (1.7) and (1.8).
\[ q_i = \frac{(1 + \gamma)(a + (n-1)c - nc_s)}{2(n+1)b + nd_s}, \]  
(1.12)

\[ q = \frac{(a-c)}{nb} - \frac{(1 + \gamma)(a + (n-1)c - nc_s)}{n(2(n+1)b + nd_s)}. \]  
(1.13)

Using (1.12) and (1.13) we derive the total quantity and price in equilibrium:

\[ Q = \frac{na - (n-1)c - w}{(n+1)b} = \frac{(n^2 - 1)(a-c)}{n(n+1)b} + \frac{(1 + \gamma)(a + (n-1)c - nc_s)}{n(2(n+1)b + nd_s)}, \]  
(1.14)

\[ p = \frac{1}{n+1} \left( \frac{2n + 1 - \gamma}{2n} (a + (n-1)c) + (1 + \gamma)(2(n+1)bc_s + ad_s + (n-1)cd_s) \right). \]  
(1.15)

The profits of the retailer \( R_1 \) and its supplier are equal to

\[ \pi_i = \frac{(a + (n-1)c - nw)^2}{(n+1)^2b} = b(1 + \gamma) \left( \frac{a + (n-1)c - nc_s}{2(n+1)b + nd_s} \right)^2, \]  
(1.16)

\[ \pi_s = \frac{(1 - \gamma^2)(a + (n-1)c - nc_s)^2}{2n(2(n+1)b + nd_s)}. \]  
(1.17)

### 1.3 Buyer Power and Downstream Competition

In this section we examine how buyer power and the intensity of downstream competition among retailers affect consumer welfare and social welfare. Recall that retailer \( R_1 \)'s buyer power is measured by \( \gamma \). On the other hand, competition in the retail market is more intense the larger the total number of retailers, denoted by \( n \). In this partial equilibrium framework, consumer welfare and social welfare are represented by consumer surplus (CS) and total...
surplus (TS), respectively. The countervailing power hypothesis argues that a retailer with countervailing power will use that power to obtain lower wholesale prices which will be passed on to consumers. In this model it will be shown that, if competition between the retailers is of the Cournot type, an increase in the buyer power of the large retailer \( R_1 \) indeed leads to a decrease in both wholesale and retail price levels.

**Proposition 1.** With downstream Cournot competition, an increase in the buyer power of retailer \( R_1 \) reduces both the wholesale price it pays and the retail price. As a result, the consumer surplus improves.

**Proof.** Differentiating (1.11) and (1.14) with respect to buyer power \( \gamma \) yields:

\[
\frac{\partial w}{\partial \gamma} = -\frac{(n+1)b(a+(n-1)c-nc_s)}{n(2(n+1)b+nd_s)} < 0, \tag{1.18}
\]

\[
\frac{\partial Q}{\partial \gamma} = -\frac{1}{(n+1)b} \left( \frac{\partial w}{\partial \gamma} \right) > 0. \tag{1.19}
\]

Then \( \frac{\partial p}{\partial \gamma} = \left( \frac{\partial p}{\partial Q} \right) \left( \frac{\partial Q}{\partial \gamma} \right) < 0 \). It is straightforward to show that \( \frac{\partial CS}{\partial \gamma} = -D(p) \left( \frac{\partial p}{\partial \gamma} \right) > 0 \). QED

A major departure of our model from von Ungern-Sternberg (1996) and Waterson and Dobson (1997) is that we have separate measures of retailer buyer power and retailer seller power. Accordingly, Proposition 1 shows that an increase in buyer power, in the absence of a simultaneous increase in seller power, benefits consumers. While this conclusion is the
same as Chen (2003) on the surface, the underlying mechanism is different. Here an increase in the buyer power of retailer $R_1$ reduces its wholesale price, making itself a more aggressive competitor in the retail market. Retail price falls because of the intensified competition among retailers.

**Proposition 2.** With downstream Cournot competition, a higher level of buyer power increases the super retailer’s mark-up and profits, but decreases its supplier’s profits. The combined profits of the supplier and super retailer increases with buyer power if

(i) $d_s < 2(1 - 1/n)b$, or

(ii) $d_s \geq 2(1 - 1/n)b$ and $\gamma < \bar{\gamma}$, where $\bar{\gamma} = 2bn/(2b + nd_s)$.

**Proof.** Differentiate ($p-w$), $\pi_1$, and $\pi_s$ with respect to $\gamma$:

\[
\frac{\partial}{\partial \gamma}(p - w) = b \frac{a + (n - 1)c - nc_s}{2(n + 1)b + nd_s} > 0, \tag{1.20}
\]

\[
\frac{\partial \pi_1}{\partial \gamma} = 2(1 + \gamma)b \left(\frac{a + (n - 1)c - nc_s}{2(n + 1)b + nd_s}\right)^2 > 0, \tag{1.21}
\]

\[
\frac{\partial \pi_s}{\partial \gamma} = -2\gamma \frac{(a + (n - 1)c - nc_s)^2}{2n(2(n + 1)b + nd_s)} < 0. \tag{1.22}
\]

Differentiate the sum of (1.16) and (1.17) with respect to $\gamma$:

\[
\frac{\partial (\pi_1 + \pi_s)}{\partial \gamma} = \frac{(a + (n - 1)c - nc_s)^2}{n(2(n + 1)b + nd_s)^2} \left(\frac{2bn - 2b\gamma - nd\gamma}{2b + nd_s}\right), \tag{1.23}
\]
which is positive if and only if \( \gamma < \tilde{\gamma} \). Since \( \gamma \leq 1 \), \( \gamma < \tilde{\gamma} \) always hold if \( \tilde{\gamma} > 1 \), which is equivalent to \( d_s < 2(1-1/n)b \). QED

Proposition 2 shows that greater buyer power leads to a higher profit margin for retailer \( R_1 \) as it retains part of the savings through the lower wholesale price. This, combined with higher sales brings about a higher level of profits. The effect of buyer power on the combined interests of the supplier and retailer, on the other hand, depends on the initial level of buyer power in the hands of retailer \( R_1 \) and its supplier’s costs. As retailer \( R_1 \) gains greater buyer power, the wholesale price decreases and its sales expand which tend to increase the joint profits from the trade. However with \( d_s > 0 \), \( R_1 \)’s supplier moves up its marginal cost curves as its sales to \( R_1 \) expand which results in efficiency loss. Hence a greater level of buyer power increases the joint interests of both the retailer and its supplier if the marginal cost of the supplier increases at a smaller rate (i.e., condition (i) in proposition 2) or the initial level of buyer power is relatively small if the supplier’s costs increase at a greater rate (i.e., condition (2) in proposition 2). Condition (i) also implies that a greater level of buyer power is always beneficial to the combined interests of the supplier and the retailer \( R_1 \) if the supplier has a constant marginal cost (i.e., if \( d_s = 0 \)).

Propositions 1 and 2 suggest that an increase in buyer power raises consumer surplus, increases the large retailer’s profits, but reduces its supplier’s profits. Based on a typical welfare trade-off analysis, one might conjecture that the loss in the supplier’s profits should be compensated by the gain in consumer surplus and the large retailer’s profits. Such a
conjecture does not turn out to be always right. The net effect on total surplus in fact depends on the cost structure of suppliers and the level of buyer power the large retailer possesses.

Proposition 3. With downstream Cournot competition, an increase in the buyer power of retailer \( R_1 \) enhances total surplus if (i) \( d_s < \eta b \), or (ii) \( \eta b \leq d_s < \mu b \) and \( \gamma < \theta \), where

\[
\begin{align*}
\eta &= 2 \left( 1 - \frac{(n-1) \left( 2 - \frac{(c - c_s)}{(a-c)} \right)}{2n - 1 + n^2 \frac{(c - c_s)}{(a-c)}} \right), \\
\mu &= \frac{1}{n-1} \left( 2 + \frac{1}{n} \frac{(c - c_s)}{(a-c)} (2n^2 + 2n - 1) \right), \\
\theta &= \frac{(2n+1)b(a+(n-1)c-nc_s) + 2n(n^2-1)b(c-c_s) + n(n-1)d_s(c-a)}{(b+n^2d_s)(a+(n-1)c-nc_s)}. 
\end{align*}
\]

Otherwise, it lowers total surplus.

Proof. Total surplus in this market can be written as

\[
TS = \int_0^{q^*} (a - bQ)dQ - (n-1)cq - (c_s + \frac{1}{2}d_s q_1)q_1.
\]  

(1.24)

Using the equilibrium conditions we rewrite (1.24) as

\[
TS = (a - c_s)q_1 - \frac{1}{2}(b + d_s)q_1^2 + (n-1)(a - c)q - \frac{1}{2}(n-1)^2bq^2 - (n-1)bq_1
\]  

(1.25)

Noting, from (1.12) and (1.13), that
\[
\frac{\partial q_t}{\partial \gamma} = \frac{a + (n-1)c - nc}{2(n+1)b + nd}, \quad (1.26)
\]

and that

\[
\frac{\partial q}{\partial \gamma} = -\frac{a + (n-1)c - nc}{(2(n+1)b + nd)n}, \quad (1.27)
\]

we differentiate (1.25) to obtain

\[
\frac{\partial TS}{\partial \gamma} = \frac{(a+(n-1)c-nc)[(n+1)b][2n+1]b-(b+nd)\gamma(a+(n-1)c-nc)+2n(n^2-1)b(c-c)-n(n-1)d(a-c)]}{(2n+1)b+nd)^2},
\]

(1.28)

which is positive if and only if \(d_s \leq \mu b\) and \(\gamma < \theta\). Since \(\gamma \leq 1\), \(\gamma < \theta\) holds if \(\theta > 1\). It can be shown that \(\theta > 1\) if and only if \(d_s < \eta b(\mu b)\). QED

Proposition 3 suggests that the effect of buyer power on social welfare depends on the initial level of buyer power in hands of retailer \(R_1\) and cost differences between \(R_1\)’s supplier and other suppliers. Intuitively, an increase in the buyer power of \(R_1\) leads to higher total sales and lower retail price, which tends to improve social welfare. On the other hand, it increases the super retailer’s sales at the expense of other retailers. With \(d_s > 0\), the supplier of \(R_1\) moves up its marginal cost curve as its sales to \(R_1\) expand. This would be a source of efficiency loss if this supplier’s marginal cost ends up higher than \(c\). The latter point is further substantiated through a variation of equation (1.28):
\[
\frac{\partial TS}{\partial \gamma} = \frac{a + (n-1)c - nC}{bq + nc - nMC(q_1)}.
\] (1.29)

That is, more buyer power in the hands of retailer \( R_1 \) improves social welfare when the marginal cost of its supplier remains below \( c \) and no efficiency loss is incurred. Hence, total welfare would increase with \( R_1 \)’s buyer power as long as the marginal cost of \( R_1 \)’s supplier increases at a sufficiently low rate (i.e., condition (i) in Proposition 3). And this condition implies that buyer power would definitely improve welfare if \( R_1 \)’s supplier has a constant marginal cost (i.e., if \( d_s = 0 \)). Alternatively if the rate of increase in the costs of \( R_1 \)’s supplier is relatively high, one way to ensure that welfare rises with buyer power is that the initial buyer power is not too large in a sense that \( \gamma < \theta \) (i.e., condition (ii) in Proposition 2). Of course when the rate of increase surpasses a certain threshold \( (d_s > \mu b) \), a higher level of buyer power always reduces welfare.

The main results on the effects of buyer power from this section are summarized in the first row of Table 1.1. As a point of comparison, Sections 1.4 and 1.5 study the cases where downstream market structure is monopoly and perfect competition. The results are also shown in this table.

**Table 1.1: The impact of an increase in the buyer power of retailer \( R_1 \)**

<table>
<thead>
<tr>
<th>Models</th>
<th>Wholesale Price ((w))</th>
<th>Retail Price ((p))</th>
<th>Consumer Surplus ((CS))</th>
<th>Total Surplus ((TS))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cournot</td>
<td>(\downarrow)</td>
<td>(\downarrow)</td>
<td>(\uparrow)</td>
<td>(\uparrow) if certain conditions are met</td>
</tr>
<tr>
<td>Monopoly</td>
<td>(\downarrow)</td>
<td>(\downarrow)</td>
<td>(\uparrow)</td>
<td>(\uparrow) if certain conditions are met</td>
</tr>
<tr>
<td>Perfect Competition</td>
<td>(\downarrow)</td>
<td>Unchanged</td>
<td>Unchanged</td>
<td>Unchanged</td>
</tr>
</tbody>
</table>

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Turning to the effects of downstream competition, recall that in Dobson and Waterson (1997), a reduction in the number of retailers can lead to lower consumer prices and higher social welfare if the competition at the retail level is fierce. By measuring buyer power separately from the concentration in the retail market, our analysis yields different conclusions.

**Proposition 4.** With downstream Cournot competition, a more concentrated retail market (i.e., a smaller number of retailers) raises mark-up and wholesale price of the super retailer. The retail price is increased and consumer surplus is reduced as a result of the higher retail price.

**Proof.** Differentiating (1.11) with respect to $n$, we obtain:

$$
\frac{\partial w}{\partial n} = \frac{2(1-\gamma)(n+1)^2(c-a)b^2 + n^2(c-a)d_2^2 + 2(1-\gamma)n(c-a)bd_n - n^2bd_n(2(a+c_s-2c) + (1-\gamma)(a-c_s))}{n^2(2(n+1)b + nd_n)^2} \tag{1.30}
$$

which has a negative sign because $a + c_s - 2c > 0$. Furthermore,

$$
\frac{\partial (p-w)}{\partial n} = -\frac{(1+\gamma)b(2b(a+c_s-2c) + (a-c)\delta)}{(2(n+1)b + nd_n)^2} < 0, \tag{1.31}
$$

$$
\frac{\partial Q}{\partial n} = \frac{1}{b(n+1)^2} \left( -\frac{\partial w}{\partial n} (n+1) + (a-2c+w) \right) > 0. \tag{1.32}
$$

From (1.32) we know that $\partial p/\partial n = -b\partial Q/\partial n < 0$. It is then straightforward to show that

$$
\frac{\partial CS}{\partial n} = -D(p)(\partial p/\partial n) > 0. \text{ QED}
$$
Proposition 4 indicates that in our model, increased concentration in the retail market has the usual effect on consumers despite the presence of buyer power. Intuitively, in a Cournot model, each retailer takes the quantity set by others as given, evaluates its residual demand and then behaves as a monopoly. The market price is set at a level such that demand equals the total quantity sold in the downstream market. As $n$ decreases, a retailer faces higher residual demand and reacts with higher sales volume. As a result, the retail price increases. Reduced competition intensity in the retail market allows the super retailer to pay a higher wholesale price to its supplier and earn a larger profit margin for itself. This confirms that the (sometimes) positive effect of increased retail concentration on consumers in Dobson and Waterson (1997) is really caused by mingling the buyer power and seller power. The following proposition confirms what happens to the sales of all retailers and the profits of super retailer $R_1$ and its supplier.

**Proposition 5.** With downstream Cournot competition, a more concentrated retail market increases sales by all retailers. The profits of the super retailer and its supplier are both increased.

**Proof:** Differentiate (1.12) and (1.13) with respect to $n$:

\[
\frac{\partial q}{\partial n} = \frac{1}{b(n+1)^2} \left( \frac{\partial w}{\partial n} (n+1) - (a-2c+w) \right) < 0, \tag{1.33}
\]

\[
\frac{\partial q_i}{\partial n} = -(1+\gamma) \left( \frac{2b(a+c_s-2c)+(a-c)d_s}{(2(n+1)b+nd_s)^2} \right) < 0. \tag{1.34}
\]
Furthermore, differentiate $\pi_1$ and $\pi_s$ with respect to $n$:

$$\frac{\partial \pi_1}{\partial n} = \frac{-2(a + (n-1)c - nw)}{b(n+1)^3} \left( \frac{\partial w}{\partial n} n(n+1) + (a - 2c + w) \right) < 0, \quad (1.35)$$

$$\frac{\partial \pi_s}{\partial n} = \frac{1 - \gamma}{2n(n+1)b} \left( \frac{(a-c)(a+(n-1)c-nw)}{n} + \frac{(a+(n-1)c-nc)}{n+1} \left( \frac{\partial w}{\partial n} n(n+1) + (a - 2c + w) \right) \right) < 0. \quad (1.36)$$

QED

The above shows that increased concentration in the downstream market is always beneficial to the interests of both the super retailer and its supplier.

It is commonly thought that reduced competition is detrimental to social welfare. It can be shown in our framework that this is not necessarily true.

**Proposition 6.** With downstream Cournot competition, a more concentrated retail market reduces total surplus if $MC(q_t) > c$ in equilibrium.

**Proof.** Differentiating total surplus with respect to $n$ and use (1.32) and (1.34), we obtain:

$$\frac{\partial TS}{\partial n} = \frac{\partial Q}{\partial n} (p - c) + \frac{\partial q_t}{\partial n} (c - MC(q_t)), \quad (1.37)$$

which is positive if $c - MC(q_t) < 0$. QED
Despite its definite effect on consumer welfare, reduced competition intensity has an ambiguous effect on social welfare. Proposition 6 leaves open the possibility that social welfare may be higher in a more concentrated retail market if the supplier of the retailer $R_1$ has a lower marginal cost than other suppliers (i.e., $MC(q_1) < c$). Intuitively, as the number of retailers decreases, the large retailer sells more units (as can be seen from (34)). In the case where $R_1$’s supplier is more efficient than other suppliers (i.e., $MC(q_1) < c$), this brings about a gain in production efficiency, which may outweigh the loss in consumer surplus caused by a more concentrated retailer market.  

One main purpose of our paper is to examine the effects of buyer power and downstream competition separately. It has been shown that greater buyer power reduces retail prices and improves consumer surplus while a more concentrated retail market raises retail prices and reduces consumer surplus. One might wonder how buyer power interacts with the intensity of downstream competition. The following proposition shows that when the downstream market is less competitive, an increase in buyer power in fact brings greater decrease in retail prices and consequently more gain in consumer surplus. The large retailer, on the other hand, achieves greater increase in its mark-up.

**Proposition 7.** With downstream Cournot competition, the smaller is the number of downstream retailers, the greater is the reduction in the wholesale and retail prices in

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9 In the literature, other authors have also shown that reduced competition does not necessarily decrease welfare. For example, Symeonidis (2008) analyzes the effects of downstream competition when downstream firms engage in bargaining with upstream agents over a linear tariff or a two-part tariff. He shows that if the bargaining is over a linear tariff, there are circumstances where reduced competition may raise social welfare. Furthermore, consumer and social welfare decrease with the intensity of downstream competition if the bargaining is over a two-part tariff.
response to an increase in buyer power. Meanwhile, the smaller is the number of retailers, the larger is the increase in retailer $R_i$’s mark-up.

**Proof:** Differentiate (1.11) with respect to $n$:

$$\frac{\partial^2 w}{\partial n \partial \gamma} = \frac{2(n+1)^2(a-c)b^2 + 2n(a-c)bd + n^2bd(a-c)}{n^2(2(n+1)b + nd)^2} > 0. \tag{1.38}$$

Since $\frac{\partial p}{\partial \gamma} = (\frac{\partial p}{\partial Q})(\frac{\partial Q}{\partial \gamma}) = -b \frac{\partial Q}{\partial \gamma}$, we obtain

$$\frac{\partial^2 p}{\partial n \partial \gamma} = -b \frac{\partial^2 Q}{\partial n \partial \gamma} = \frac{1}{(n+1)^2} \left( (n+1) \frac{\partial^2 w}{\partial n \partial \gamma} - \frac{\partial w}{\partial \gamma} \right) > 0. \tag{1.39}$$

Then differentiate (1.31) with respect to $\gamma$ to obtain

$$\frac{\partial^2 (p-w)}{\partial n \partial \gamma} = -\frac{2b^2(a+c-2c)+(a-c)bd}{(2(n+1)b + nd)^2} < 0. \tag{1.40}$$

QED

Proposition 7 states that when the downstream market is less competitive, an increase in buyer power brings about a greater decrease in retail price and consequently a larger gain in consumer surplus. This result is surprising because the pressure for a retailer to pass on the cost savings from a lower wholesale price to consumers is stronger when there is more intense competition among retailers (see Proposition 4). Intuitively, this would seem to suggest that the reduction in retailer price resulting from the exercise of buyer power should be smaller when the retail market is less competitive. However, what this intuition misses is the fact that the profit margin between retail price and the marginal cost of production is also
higher when the retail market is less competitive. Hence, there is more room for the retail price to fall in response to increased buyer power, and Proposition 8 shows that this is indeed what happens in equilibrium.

**1.4 Downstream Monopoly**

As discussed in section 1.1, a common theme in the existing literature on buyer power is that the presence of competition in the retail market is necessary for buyer power to (possibly) benefit consumers and increase welfare. It is then worthwhile to investigate in our model the effects of buyer power in the situation where competition is absent in the retail market. Will the effects of buyer power found in section 1.3 still hold in the case of a monopoly retailer?

We modify our model such that the downstream market has no other retailers but $R_1$. All other aspects of the model remain the same as before. Having agreed upon the wholesale price with the supplier, the retailer $R_1$ maximizes its profits by equating its marginal revenue to marginal cost. The equilibrium sales volume is equal to

$$q_i^m = \frac{a - w}{2b}, \quad (1.41)$$

and the equilibrium price is

$$p_i^m = \frac{a + w}{2}. \quad (1.42)$$
Stage one remains as a Nash bargaining game over the wholesale price \( w^m \) between retailer \( R_1 \) and its supplier. The equilibrium wholesale price, retail price and sales are given by

\[
w^m = \frac{2b(1-\gamma) + c_s (1+\gamma) + ad_s}{4b + d_s},
\]

(1.43)

\[
p^m = \frac{2ab + ad_s + ab(1-\gamma) + bc_s (1+\gamma)}{4b + d_s},
\]

(1.44)

\[
q^m_i = \frac{a-w}{2b} = \frac{(1+\gamma)(a-c_s)}{4b + d_s}.
\]

(1.45)

The profits of retailer \( R_1 \) and its supplier are equal to

\[
\pi^m_1 = \left( \frac{a-w}{2} \right)^2 \frac{1}{b} = \left( \frac{(1+\gamma)(a-c_s)}{4b + d_s} \right)^2 b,
\]

(1.46)

\[
\pi^m_s = \left( w-c_s - \frac{d_s}{2} \frac{a-w}{2b} \right) \frac{a-w}{2b} = \frac{(1-\gamma^2)(a-c_s)^2}{2(4b + d_s)}.
\]

(1.47)

**Proposition 8:** In the case of a monopoly retailer, an increase in its buyer power

a) reduces wholesale price;

b) reduces equilibrium retail price level and improves consumer surplus;

c) raises total surplus if and only if \( \gamma < 3b/(b + d_s) \).
Proof: Differentiating (1.43) to (1.45) with respect to $\gamma$, we obtain

$$\frac{\partial w^m}{\partial \gamma} = \frac{2b(c_s - a)}{4b + d_s} < 0,$$

(1.48)

$$\frac{\partial p^m}{\partial \gamma} = \frac{b(c_s - a)}{4b + d_s} < 0,$$

(1.49)

$$\frac{\partial q_1^m}{\partial \gamma} = \frac{a - c_s}{4b + d_s} > 0.$$

(1.50)

Total surplus in this case can be written as

$$TS = \int_0^{q^m} (a - bq_1^m)dq_1^m - (c_s + \frac{1}{2}d_s q_1^m)q_1^m.$$

Differentiating total surplus with respect to $\gamma$ and noting (1.50)

$$\frac{\partial TS}{\partial \gamma} = \left(\frac{(a - c_s)(3b - \gamma b - \gamma d_s)}{4b + d_s}\right)\frac{\partial q_1^m}{\partial \gamma} > 0,$$

(1.51)

if and only if $\gamma < 3b/(b + d_s)$. QED

A comparison of Proposition 8 with Propositions 1 and 3 indicates that the effects of buyer power in the case of a monopoly retailer are qualitatively the same as those in the oligopolistic retail market. This conclusion is in sharp contrast to von Ungern-Sternberg (1996) and Dobson and Waterson (1997) in which increased buyer power in a highly
concentrated retail market is detrimental to consumer and social welfare. Results from propositions 9 and 10 are summarized in the second row of Table 1.1.

Of course, there are quantitative differences between a monopolistic retail market and an oligopolistic retail market. As one would expect, both retail price and retailer mark-up are higher under monopoly than under oligopoly. The magnitudes of the buyer power effects are also different, as the following propositions shows.

**Proposition 9:** Both the wholesale and retail prices are higher in the case of downstream monopoly than the case of downstream Cournot competition. Also retailer \( R_1 \) and its supplier earn higher profits in the case of downstream monopoly than in the case of downstream Cournot competition.

**Proof:** Use (1.12) and (1.43) and assume \( n = 2 \)

\[
 w^m - w = \frac{(1-\gamma)[2b^2(a-c)+2d_1^2(a-c)+bd_1(3(1-\gamma)(a-c)+4(a+c_1-2c)+2(1-\gamma)(a-c_1))]}{4(3b+d_1)(4b+d_1)} > 0 . \tag{1.52}
\]

Since \( \frac{\partial w}{\partial n} < 0 \) for all \( n \) in the case of Cournot competition, \( w^m > w \) for all \( n \)

Subtracting (1.15) by (1.44), we can easily show that

\[
p^m - p = \frac{(n-1)a-2(n-1)c+(n+1)w^m-2w}{2(n+1)} > \frac{(n-1)(a-2c+w)}{2(n+1)} > 0 . \tag{1.53}
\]

To examine the relative magnitude of profits for retailer \( R_i \), we compare its sales and profit margin. Subtracting (1.45) by (1.12) and letting \( n = 2 \) to obtain
\[ q_i^m - q_i = \frac{a - 2c - 3w^m + 4w}{6b} = \frac{6b^2(1 + \gamma)(a + c_s - 2c) + 3bd_s(1 + \gamma)(a - c) + 6abd_s}{6b} > 0. \quad (1.54) \]

Meanwhile using (1.11) (1.15) (1.43) and (1.44) and let \( n = 2 \), we have

\[ (p^m - w^m) - (p - w) = \frac{a - 2c - 3w^m + 4w}{6} > 0. \quad (1.55) \]

Since \( \frac{\partial q_i}{\partial n} < 0 \) and \( \frac{\partial (p - w)}{\partial n} < 0 \) for all \( n \) in the case of Cournot competition, \( q_i^m - q_i > 0 \) and \( (p^m - w^m) - (p - w) > 0 \) for all \( n \). As a result, \( \pi_i^m - \pi_i = (p^m - w^m)q_i^m - (p - w)q_i > 0 \).

The mark-up of the retailer \( R_i \)'s supplier is \( w - c_s - \frac{1}{2}d_s q_i \). Using (1.11) (1.12) (1.43) and (1.45) and with \( n = 2 \), we obtain

\[ \left( w^m - c_s - \frac{1}{2}d_s q_i^m \right) - \left( w - c_s - \frac{1}{2}d_s q_i - \frac{b(a - c)(1 - \gamma) + 4\gamma b c_s}{4b} > 0. \quad (1.56) \right. \]

It can be shown that

\[ \frac{\partial}{\partial n} \left( w - c_s - \frac{1}{2}d_s q_i \right) = -\frac{(1 - \gamma)(a - c)}{2n^2} < 0. \quad (1.57) \]

As a result, \( \left( w^m - c_s - \frac{1}{2}d_s q_i^m \right) - \left( w - c_s - \frac{1}{2}d_s q_i \right) > 0 \) for all \( n \). Since \( q_i^m - q_i > 0 \) for all \( n \) and \( \pi_s = \left( w - c_s - \frac{1}{2}d_s q_i \right) q_i \), \( \pi_i^m - \pi_i > 0 \) for all \( n \).

\[ \text{QED} \]
**Proposition 10:** An increase in buyer power causes greater decrease in wholesale and retail price in the case of downstream monopoly than in the case of Cournot competition. Meanwhile, the large retailer’s mark-up under downstream monopoly shows greater increase than under Cournot competition.

**Proof:** Subtracting (1.18) from (1.48)

\[
\frac{\partial w}{\partial \gamma} - \frac{\partial w^m}{\partial \gamma} = (n-1)b \frac{(n+1)(a-c)(4b+d_s)+n(a-c)d_s}{n(4b+d_s)(2(n+1)b+nd_s)} > 0. \tag{1.58}
\]

Using (1.19), (1.50) and the above inequality, we obtain

\[
\frac{\partial Q}{\partial \gamma} - \frac{\partial q_i^m}{\partial \gamma} = b \left( \frac{1}{2} \frac{\partial w^m}{\partial \gamma} - \frac{1}{(n+1)} \frac{\partial w}{\partial \gamma} \right) < \frac{1}{b} \frac{\partial w}{\partial \gamma} \left( \frac{1}{2} - \frac{1}{(n+1)} \right) < 0. \tag{1.59}
\]

Using the demand function and (1.49) we find

\[
\frac{\partial p}{\partial \gamma} - \frac{\partial p^m}{\partial \gamma} = b \left( \frac{\partial q_i^m}{\partial \gamma} - \frac{\partial Q}{\partial \gamma} \right) > 0. \tag{1.60}
\]

Substituting (1.20) (1.48) and (1.49), we have

\[
\frac{\partial (p-w)}{\partial \gamma} - \frac{\partial (p^m-w^m)}{\partial \gamma} = \frac{(n-1)bd_s(c-a)-(n-1)2b^2(a-2c+c_s)}{(4b+d_s)(2(n+1)b+nd_s)} < 0. \tag{1.61}
\]

QED

Therefore, under downstream monopoly, although the retail price level is higher than downstream oligopoly, the benefit of increased buyer power for consumers is the largest
when competition in the retail market is the weakest. This, in conjunction with Proposition 7, suggests that buyer power and downstream competition are substitutes in terms of their effects on consumers.

1.5 Perfect Competition in Downstream Market

In von Ungern-Sternberg (1996), buyer power is manifested through both the bargaining power and the number of downstream retailers. He shows that under a downstream Cournot model, higher level of concentration in the downstream market is detrimental to consumer benefits. However, within a framework of downstream perfect competition, a decrease in the number of retailers can reduce retail price and improve consumer surplus. Hence, we consider a downstream market that is of perfect competition where all retailers act as price takers.

We consider two different specifications of the negotiation process between the large retailer and its supplier. The first specification is the same as in the cases of downstream Cournot competition and monopoly; that is, the super retailer and its supplier negotiate over the wholesale price $w$ only. The equilibrium under this specification can be easily determined by noting the following. Perfect competition among retailers in the downstream market drives the retail price down to $c$. Accordingly, a price-taking retailer would want to sell an infinite number of units if its unit cost is below $c$ and would want to sell nothing if its unit cost is above $c$. Given that the supplier of the super retailer faces rising marginal cost of production, the only possible equilibrium wholesale price between the super retailer and its supplier is $w^e = c$. Consequently, the equilibrium in this case is characterized by $p^e = w^e = c$. 
The level of buyer power possessed by the large retailer has no influence on the equilibrium whatsoever.

An alternative, and more interesting specification is that the large retailer and its supplier negotiate over both the wholesale price and the quantity purchased by the retailer. The generalized Nash bargaining problem then becomes:

$$\max_{w,c} \pi_i^r \pi_i^{1-r} = \left((p^c - w^c)q_i^c\right)^\gamma \left(\left(w^c - c_s - \frac{1}{2}d_sq_i^c\right)q_i^c\right)^{1-\gamma}.$$  \hfill (1.62)

The first-order conditions with respect to $w^c$ and $q_i^c$ respectively are

$$\gamma(p^c - w^c)^{\gamma-1} \left(w^c - c_s - \frac{1}{2}d_sq_i^c\right) = 1 - \gamma,$$  \hfill (1.63)

$$w^c - c_s - d_sq_i^c \left(1 - \frac{1}{2}\gamma\right) = 0.$$  \hfill (1.64)

It can be shown that the second-order conditions $\partial^2 \pi_i^r \pi_i^{1-r} / \partial w^c < 0$ and $\partial^2 \pi_i^r \pi_i^{1-r} / \partial q_i^c < 0$ are all satisfied. Using the above first-order conditions, we obtain the equilibrium values of the following variables:

$$w^c = \frac{1}{2}c_s + \left(1 - \frac{1}{2}\gamma\right)p^c = \frac{1}{2}c_s + \left(1 - \frac{1}{2}\gamma\right)c,$$  \hfill (1.65)

$$p^c = c,$$  \hfill (1.66)

$$q_i^c = \frac{p^c - c_s}{d_s} = \frac{c - c_s}{d_s}.$$  \hfill (1.67)
Proposition 11. With downstream perfect competition, an increase in buyer power decreases wholesale price, but does not affect retail price and quantities sold by retailers. It has no impact on consumer surplus and only affects the division of profits between the large buyer and its supplier.

Proof: Differentiate (1.65) with respect to $\gamma$ to obtain

$$
\frac{\partial w^c}{\partial \gamma} = \frac{1}{2} (c_s - c) < 0.
$$

(1.71)
It is straightforward from (1.66) to (1.68) that a higher $\gamma$ won’t affect retail price, quantities and consumer welfare. As a result, $\partial CS/\partial \gamma = -D(p)(\partial p/\partial \gamma) = 0$.

Differentiating (1.69) and (1.70), we have:

\[
\frac{\partial \pi^r}{\partial \gamma} = \frac{1}{2} \frac{(c-c_s)^2}{d_s} > 0, \quad (1.72)
\]

\[
\frac{\partial \pi_s}{\partial \gamma} = -\frac{1}{2} \frac{(c-c_s)^2}{d_s} < 0. \quad (1.73)
\]

QED

Intuitively, when all retailers act as price-takers, the decision of retailer $R_1$ no longer influences the market price. As a result, the level of buyer power has no effect on the retail price. Given the increasing marginal cost of $R_1$’s supplier, the large retailer and its supplier decide on the amount of quantity that will maximize the joint profits. The buyer power only impacts the share of profits that $R_1$ obtains.

It follows naturally that an increase in buyer power would not have any impact on social welfare.

*Proposition 12.* With downstream perfect competition, a change in buyer power or the number of downstream retailers has no impact on total surplus.
Total surplus can be formulated as

\[
TS = \int_0^\gamma (a - bQ^e) dQ^e - (n - 1)cq^e - (c_s + \frac{1}{2}d_s q_i^e)q_i^e. \tag{1.74}
\]

Using (1.67) and (1.68), it can be shown that

\[
(n - 1)q^e = Q^e - q_i^e = \frac{d_s(a - c) - b(c - c_s)}{bd_s}. \tag{1.75}
\]

Differentiate total surplus with respect to \(\gamma\) and \(n\) separately and note (1.67) and (1.68) and (1.75):

\[
\frac{\partial TS}{\partial \gamma} = p^e \frac{\partial Q^e}{\partial \gamma} - c \frac{\partial((n - 1)q^e)}{\partial \gamma} - (c_s + d_s q_i^e) \frac{\partial q_i^e}{\partial \gamma} = 0, \tag{1.76}
\]

\[
\frac{\partial TS}{\partial n} = p^e \frac{\partial Q^e}{\partial n} - c \frac{\partial((n - 1)q^e)}{\partial n} - (c_s + d_s q_i^e) \frac{\partial q_i^e}{\partial n} = 0. \tag{1.77}
\]

QED

Therefore, when the downstream market is perfectly competitive, the level of buyer power becomes irrelevant to social welfare. It only impacts the share of profits between the supplier and retailer. The main results are shown in the third row of Table 1.1.

A comparison of the framework of downstream perfect competition with those formulated in sections 1.3 and 1.4, namely the downstream Cournot and downstream monopoly, illustrates the importance of downstream competition.
Proposition 13. The retail price is the lowest and the consumer surplus is the highest in the case of perfectly competitive downstream market when compared to the cases of Cournot competition and monopoly.

Proof: Subtracting (1.68) from (1.14), we obtain

\[ Q^\gamma - Q = \frac{b((a-c)(2n+1-\gamma)-n(1+\gamma)(c-c_\gamma))}{b(2(n+1)b+nd_\gamma)n} > 0. \]  \hspace{1cm} (1.78)

Using the demand function we have

\[ p^\gamma - p = a - bQ^\gamma - (a - bQ) = b(Q - Q^\gamma) < 0. \]  \hspace{1cm} (1.79)

Since the retail price under downstream Cournot is lower than that of downstream monopoly \((p < p^m)\), it is straightforward to show that \(p^\gamma < p < p^m\). It follows then that \(CS^\gamma > CS > CS^m\), where \(CS^\gamma\) denotes the consumer surplus under downstream perfect competition. QED

1.6 Concluding Remarks

Using a model that isolates the effects of buyer power from those of downstream competition, we demonstrate that, with the exception of downstream perfect competition, an increase in buyer power leads to a fall in retail price and consequently enhances consumer surplus. And this is true even in the case where the downstream market is a monopoly. In this regard, our results are in sharp contrast to those obtained by von Ungern-Sternberg (1996) and Dobson and Waterson (1997), who show that higher buyer power (as reflected through an increase in concentration at the retail level) leads to reduced consumer prices only if the competition at the retail level is fierce. Moreover, it is demonstrated that more intense
competition downstream forces the large retailer to bargain more rigorously with the supplier and passes a greater share of cost savings to consumers. Surprisingly, though, the beneficial effect of an increase in buyer power on consumer welfare is larger when the intensity of downstream competition is lower, with the effect being the largest in the case of downstream monopoly.

In this model, we assume that retailer $R_i$’s supplier and other suppliers have different production costs because they face different trading environments. We can change this assumption in a couple of ways. First, we can modify the baseline model of downstream Cournot competition so that the large retailer’s supplier also has a constant marginal cost $c_s$ (i.e., $d_s=0$). It is easy to see that all propositions in section 1.3 hold except those related to social welfare. In this case, an increase in buyer power always improves social welfare since an expansion of retailer $R_i$’s sales no longer causes efficiency loss. On the other hand, a more concentrated downstream market is always harmful as there will not be any possibility of efficiency gain if more products are produced by the retailer $R_i$’s supplier at a lower cost. Second, we can modify the model further and use a more general form of cost functions for suppliers. A higher level of buyer power manifests itself through a lower wholesale price paid by the large retailer. With a lower cost, the retailer would always want to increase its sales, which intensifies downstream competition and improves consumer surplus even with a more general form of supplier cost functions. The social welfare on the other hand depends on the trade-off between the gain in consumer surplus and the possible loss in efficiency as a result of the change in market share of suppliers and consequently the costs of production.

Another important assumption in our baseline model is that the contract between retailer $R_i$ and its supplier takes the form of a linear wholesale price, which implies that the gain of
the retailer is always at the expense of its supplier. An alternative assumption is an efficient contract where the interests of the retailer and its supplier are perfectly aligned. One example is a two-part tariff where the retailer and its supplier have two parameters (a wholesale price and a fixed fee) to decouple profit sharing from joint profit maximization. In this case, the equilibrium would be qualitatively different and an increase in buyer power only has distributional effects and does no impact retail price and welfare. A linear wholesale price and an efficient contract are two extremes of contracting in terms of the ability of the negotiating parties to solve their conflicting interests over the gains from the trade. In reality, the inability to write a complete contingent contract and the incentives to ex post take opportunistic advantage of circumstances can make it almost impossible to put together an efficient contract. On the other hand, a contract between retailer and supplier often involves more than a simple unit wholesale price. Whether a contract is more efficient or inefficient in nature is an empirical question. Our results in section 1.3 hold qualitatively as long as the contract does not lead to perfect alignment of the interests between the retailer and its supplier.

The analysis provides an answer to the policy questions posed in the Introduction. First, it suggests that the traditional approach to merger reviews, under which the focus of a competition authority is on maintaining competition in the local retail markets, can actually work reasonably well even in a situation where the merger enhances the buyer power of the merged entity. By preventing the downstream market from becoming more concentrated, the competition authority would ensure that the post-merger retail price will not rise and may possibly fall. Second, the competition authority does not necessarily have to be more concerned about the effects of buyer power in a more concentrated retail market. A
concentrated retail market is not desirable, but a merger that enhances the buyer power of the merged entity without increasing concentration in such a retail market can partially offset the negative effects of high concentration. Third and finally, it is not necessarily the case that an increase in buyer power is more likely to be beneficial to consumers and social welfare when downstream competition is more intense. In our model, the opposite is true; that is, the beneficial effects of an increase in buyer power are larger when downstream competition is less intense.
Chapter 2

Subsidy, Product Diversity and Buyer Power
2.1. Introduction

The economic rationale for the public support of cultural activities has been rigorously examined in the literature (Dixit and Norman 1980, 273-281; Francois and van Ypersele 2002 and Aubert, Bardhan and Johnson 2003). In practice, the use of public subsidies to promote cultural diversity is widespread in many countries. In Canada there are various funds such as the grants given to book publishers to ensure access to a diverse range of Canadian-authored books\(^\text{10}\), the Canada Feature Film Fund intended to encourage diversity in Canadian feature film production, and the Canadian Film or Video Production Tax Credit (CPTC) created to support the growth of a viable indigenous film and video production industry. Given the fact that these cultural goods are commonly distributed through downstream firms\(^\text{11}\) and the downstream markets are increasingly dominated by large, powerful organizations such as the national book store chain Chapters-Indigo\(^\text{12}\) and the major Canadian television network ownership groups BCE, Shaw and Rogers\(^\text{13}\), one natural question to ask is: how effective are the subsidies in achieving the objective of promoting product diversity when they could potentially be squeezed out of the hands of the producers and into the pockets of the downstream firms through the exercise of buyer power?

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\(^{10}\) In 2010-11 Canada Book Fund approved 235 applications and granted a total amount of $30 million to publishers. Source: Canadian Heritage.

\(^{11}\) For example, books are mainly distributed through chain bookstores, independent bookstores and non-traditional retail market such as mass market and discount outlets. The made-for-TV films and videos are usually distributed directly through television networks such as CTV, Global and Citytv.

\(^{12}\) The national bookstore chain, Indigo, held 44% of domestic book sales in 2006 according to a study conducted by Canadian Heritage in all regions except for Quebec, where two regional chains—Archambault and Renaud-Bray—hold the major share.

\(^{13}\) According to CRTC (Canadian Radio-television and Telecommunications Commission) Communications Monitoring Report 2012, the English-language private conventional television sector includes three major ownership groups: BCE (CTV & CTV Two) with a revenue share of 46%, Shaw (Global) with 29%, and Rogers (Citytv & Omni) with 17%.
The efficiency of market equilibrium in the provision of product diversity and the implication of public subsidies have been analyzed in a framework of monopolistic competition. Spence (1976) and Dixit and Stiglitz (1977) posit a representative consumer utility function, which is defined over the quantities of a number of differentiated products and an outside numeraire good, and find that, when the utility function is separable between the numeraire good and the differentiated products and when the utility over the differentiated products takes the CES form, the free-entry market equilibrium gives a lower number of products than the social optimum under competitive conjecture\textsuperscript{14}. Further by comparing the market equilibrium to the constrained optima, Spence (1976) concludes that a subsidy to induce additional varieties is welfare-improving.

Costrell (1990) allows a variable elasticity of substitution among the differentiated products and shows that market equilibrium could yield a larger or smaller number of product varieties and an infinitesimal input subsidy always improves social welfare. Droge and Schroder (2005) apply the Dixit-Stiglitz model of monopolistic competition to investigate the effectiveness of environmental policies and show that, when initially the dirty sector is important and the policy target is moderate, a subsidy on the green sector increases the share of clean products and is a superior tool in improving social welfare.

A common assumption shared by the studies of product diversity cited above is that producers sell products directly to consumers. In reality, however, most of the cultural goods are distributed through the downstream markets (such as the book retail market or television networks), which have become increasingly concentrated. Concerns have been expressed

\textsuperscript{14} Koenker and Perry (1981) demonstrate that the equilibrium number of products could be higher or lower than the social optimum when allowing firms to have conjectural variations on the output responses of their rivals.
over the long-term effects of such power on the product diversity (Competition Commission 2002 and 2003, OECD 2008). Particularly, it is argued that by exercising buyer power, these downstream firms are able to suppress the purchasing price and extract a larger share of profits from producers\textsuperscript{15}. In the long run, this may reduce producers’ incentives to invest in new products and even induce exit. Putting this in a framework of monopolistic competition where the failure of revenues to cover the costs of socially desirable products is the contributing factor to the insufficient product diversity (Spence 1976), a squeezed profit margin by downstream firms with buyer power is likely to further exacerbate the problem and render the government subsidies ineffective.

Although there is a large volume of literature examining the effect of buyer power on consumer prices (for example, von Ungern-Sternberg 1996, Dobson and Waterson 1997, Chen 2003, Erutku 2004), very few have studied the effects on product diversity and none in a framework of monopolistic competition. One related analysis is by Chen (2004) who uses a model where a monopolistic manufacturer produces differentiated products and compares the effects of buyer power on product diversity in the case where products are sold directly to consumers with those in the case where they are sold through a retailer. He finds that an increase in a retailer’s countervailing power reduces the manufacturer’s marginal gain of producing an extra product and exacerbates the problem of insufficient variety. Further, although countervailing power leads to lower consumer prices, overall welfare decreases as the adverse effect from reduced product diversity dominates the positive effects of reduced retail prices.

\textsuperscript{15} There is some empirical support for the negative correlation between downstream concentration and upstream profitability. See Scherer and Ross (1990) for a survey, and also more recently Ellison and Snyder (2002), Sørensen (2003) and Normann, et al. (2005).
Also related is the paper by Inderst and Shaffer (2007) which demonstrates that a retailer, following a cross-border merger may find it more profitable to delist a product to enhance its buyer power. Moreover, suppliers, in anticipation of such a merger, would optimally manage its product profile and reduce the degree of product differentiation, which further reduces product diversity. The treatment of product diversity in the analysis is rather limited as there are only two products and the overall effects of the merger on social welfare cannot be measured.

The objective of this paper is to combine the different strands of literature and analyze the effects of government subsidies on product diversity and social welfare when the downstream firm (a retailer) has buyer power. To this end, we extend the Dixit-Stiglitz model of monopolistic competition and compare the case where products are directly sold to consumers and the case where the products have to sell through a retailer that has all the power. Two types of subsidies, a subsidy on the marginal cost of production and a subsidy on the fixed costs are studied.

We find that while these two types of subsidies have different effects on the quantity and retail price of each variety, they both increase the number of product varieties and enhance social welfare. Moreover, a combination of the two types of subsidies is able to achieve the social optimum. These results are true even when products are distributed through a downstream retailer who has all the bargaining power, but the mechanism through which a subsidy increases product varieties is different. In comparison with the case where products are distributed directly to consumers, retailer buyer power reduces product variety and social welfare. Furthermore the subsidies become less effective in promoting product diversity. To
be more specific, a marginal increase in subsidy leads to a smaller increase in the number of varieties when retailer buyer power is present.

The rest of the paper is organized as follows. Section 2.2 sets out the benchmark model where products are sold directly to consumers and solves for the market equilibrium. Section 2.3 studies the unconstrained and constrained social optima and identifies possible market failures. Section 2.4 examines the effects of government subsidies on product diversity and social welfare and section 2.5 extends the model to the case where products are sold through a retailer and examines the implication of retailer buyer power and the effectiveness of government subsidies in the presence of such power. Section 2.6 discusses the assumptions used in this model and section 2.7 concludes.

2.2 The Benchmark Model

Before studying the effects of government subsidies on product diversity and social welfare when the downstream firm (the retailer) has buyer power, we consider first the situation where products are sold directly to consumers by producers. When compared with the social optimum which will be delineated in the next section, it helps us to analyze whether the market equilibrium leads to a suboptimal number of varieties and, if it does, whether a government subsidy is a useful instrument in addressing the issue. Also it serves as a benchmark which can be used to investigate whether the existence of retailer buyer power indeed exacerbates the situation and diminishes the effectiveness of government remedies.

A general equilibrium model of monopolistic competition is used. The structure of monopolistic competition provides a consistent framework for modelling the cultural goods
market where consumers value product diversity, production technology often involves sizable fixed costs and economies of scale matter. In addition, since it is a general equilibrium model, the full consequences of redistributing subsidies can be studied and the measure of social welfare is readily available.

In this model, the economy consists of two industries. One industry is characterized by monopolistic competition where \( N \) producers produce products (brands) \( x_i \) (\( i=1, 2...N \)) that are viewed by consumers as differentiated goods, whereas the other is assumed to be perfectly competitive where firms produce a homogenous good, denoted as \( Z \).

Following Dixit and Stiglitz (1977) and Spence (1976), we model the benefits of variety through strictly convex preferences of a representative household over \( N \) differentiated products. We assume the preferences take the functional form

\[
U = Z + \left( \sum_{i} x_i^{ap} \right)^{1/\rho}.
\]  

(2.1)

For products \( x_i \) and \( x_j \) to be substitutes, we need \( \partial^2 U / \partial x_i \partial x_j < 0 \), which requires \( \rho > 1 \). In the case where the representative household consumes the same quantity of \( N \) differentiated products (\( x_i = x_j \)), the utility function can be re-written as

\[
U = Z + N^{1/\rho} x^a.
\]

(2.2)

---

16 Our paper is not the first to use the framework of monopolistic competition to model the cultural goods market. For example, Spence and Owen (1977) model the television program market using the monopolistic competitive framework. So do Canoy, et al. (2006) in modelling the book market.
To ensure that the preferences are convex over \( x_i \) in both the case where \( x_i = x_j \) and the case where \( x_i \neq x_j \), we need \( 0 < \alpha < 1 \) and \( 0 < \alpha \rho < 1 \). Equation (2.2) shows that utility increases with the number of products, indicating the household’s preference for diversity.

The representative household is endowed with \( H \) units of labor, which is supplied inelastically. Labor is the only factor of production and is perfectly mobile across the two industries. For simplicity, we normalize the wage rate to 1. Thus the household’s budget constraint is \( p_x Z + \sum_i p_i x_i = I \), where \( I \) is the household disposable income and equals the wage the household receives (\( H \)) plus the profits of the firms (\( \sum_i \pi_i \)) and minus lump-sum tax (\( T \)) by the government, as the case may be. The household maximizes its utility subject to the budget constraint, which is formulated as follows

\[
\begin{align*}
\max_{x_i, Z} & \quad U = Z + \left( \sum_{i} x_i^{\alpha p} \right)^{\rho} \\
\text{s.t.} \quad & \quad p_x Z + \sum_i p_i x_i = I
\end{align*}
\]

To solve this problem, we set up the Lagrangian 

\[
L = Z + \left( \sum_{i} x_i^{\alpha p} \right)^{\rho} - \lambda (p_x Z + \sum_i p_i x_i - I),
\]

where \( \lambda \) is the Lagrangian multiplier. The first-order conditions are:

\[
\frac{\partial L}{\partial Z} = 1 - \lambda p_x = 0,
\]

\[
\frac{\partial L}{\partial x_i} = \alpha \left( \sum_{i} x_i^{\alpha p} \right)^{\rho-1} x_i^{\alpha p-1} - \lambda p_i = 0,
\]

50
\[ \frac{\partial L}{\partial A} = p_z Z + \sum_i p_i x_i - I = 0. \]  

(2.7)

Solving the above equations for \( x_i \) and \( Z \) and defining a price index

\[ q = \left( \alpha^{1-\alpha} \left( \sum_j p_j^{\alpha p_{j-1}} \right)^{1-\rho} \right)^{1-\alpha p} \]

as standard in the literature of monopolistic competition, we have the demand function

\[ x_i = q^{1-\alpha p} p_i^\frac{1}{1-\alpha p}, \]  

(2.8)

\[ Z = \left( I - \alpha^\rho \ q^{\frac{1}{\rho-1}} \right) / p_i. \]  

(2.9)

It can be clearly seen from (2.8) and (2.9) that with this type of consumer preferences, the demand for goods \( x_i \) is independent of income, and any income left is then spent entirely on \( Z \). Further the cross elasticity of substitution (\( \sigma \)) within the differentiated goods industry equals \( 1/(1 - \alpha \rho) \). When \( N \) is reasonably large so that a change in any \( p_i \) has a negligible impact on the price index \( q \), the elasticity of demand for \( x_i \) is \( 1/(1 - \alpha \rho) \), which is the elasticity of the Chamberlin’s (1950) \( dd \) curve, i.e., the curve relating the demand for each product type to its price holding all other prices constant. If all prices in the group move together on the other hand, considering the symmetric situation where \( x_i = x \) and \( p_i = p \), the demand function becomes

\[ x = \alpha^{1-\alpha} N^{1-\rho} \left( \frac{1-\rho}{\rho(1-\alpha)} \right) p^\frac{1}{1-\alpha}, \]  

(2.10)
which corresponds to the Chamberlinian $DD$ curve and the elasticity of demand is $1 / (1 - \alpha)$. Moreover equation (10) implies that the demand for each product variety decreases as the number of product varieties increases. This is due to the representative household having to allocate income over more product choices.

On the production side, suppose that one unit of labor produces one unit of $Z$. Since the market is perfectly competitive, the price of good $Z$ equals its marginal cost, which is the wage rate $1$. On the other hand, production of the differentiated products involves a fixed cost $F$ in units of labor and a marginal cost of $a_x$ units of labor. Hence the level of output produced using $L_i$ units of labor is $x_i = \max\{0, (L_i - F) / a_x\}$. This implies that the total cost of production is $C_i(x_i) = L_i = a_x x_i + F$.

Firms play a two-stage game. At stage one, each firm decides whether to enter the market. At stage two, firms that have entered the market choose $p_i$ to maximize their own profits.

To solve for the market equilibrium, we start from stage two where each firm maximizes its profits

$$\max_{p_i} \pi_i = (p_i - a_x) x_i - F \quad (2.11)$$

Substituting in the demand function (2.8) and solving the optimization problem, we have

$$p_i = p = \frac{1}{a \rho} a_x. \quad (2.12)$$

---

17 Following Dixit and Stiglitz model of monopolistic competition, we assume $N$ is sufficiently large so that the influence of an individual price on price index is ignored. This implies that each producer disregards the cross elasticity of demand and overestimates its own price elasticity of demand.
In equilibrium, entry occurs until the marginal firm can just break even

$$\pi_i = (p_i - a_s)x_i - F = 0$$  \hfill (2.13)

Thus the representative household’s disposable income is $I = H + \sum \pi_i = H$. Considering a symmetric situation where $x_i = x$ and $p_i = p$ for all $i$ and using (2.8), (2.9), (2.12) and zero profit function, we are able to derive the equilibrium quantities and varieties, which are summarized in the following table. We also present in the following table the results from unconstrained and constrained optima from section 2.3 for the ease of comparison.

### Table 2.1
Comparison of Market Equilibrium, Unconstrained and Constrained Social Optimum

<table>
<thead>
<tr>
<th>Market Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^e = \frac{1}{\alpha \rho} a_s$</td>
</tr>
<tr>
<td>$x^e = \frac{\alpha \rho}{(1 - \alpha \rho)a_s} F$</td>
</tr>
<tr>
<td>$Z^e = H - (1 - \alpha \rho)^{\frac{1-\alpha \rho}{\rho-1}} \left( \alpha \rho \right)^{\frac{\alpha}{\rho-1}} a_s^{\frac{\alpha}{\rho-1}} F^{\frac{1-\alpha \rho}{\rho-1}}$</td>
</tr>
<tr>
<td>$N^e = \left( (1 - \alpha \rho)(\alpha \rho)^{\frac{1}{\rho-1}} a_s^{\frac{\alpha}{\rho-1}} \right)^{\frac{\rho(1-\alpha)}{\rho-1}}$</td>
</tr>
</tbody>
</table>

18 With the imposition of symmetry in the differentiated goods industry, although the number of product varieties is still relevant, any group of $N$ products is assumed to be as good as any other group of $N$. The literature has shown that the market does not automatically produce the right products. For example, it may bias against those with low price elasticity of demand (Dixit and Stiglitz, 1977 and Spence, 1976) as the revenue generated by these products is relatively small compared to the surplus it brings to consumers. Since the focus of this paper is the number of product varieties with and without the presence of buyer power, we maintain the assumption throughout the analysis.
### 2.3 Social Optima and Market Equilibrium

One question we are interested in is whether the equilibrium number of products is optimal. In this section, we compare the outcomes of the market equilibrium derived in the previous section with the optimal social planning mechanism. In addition, we study the second-best regulation mechanism that is often examined in the literature and see whether it provides a public policy alternative.

#### 2.3.1. Unconstrained Optimum

In the case of unconstrained optimum, the firms’ profits no longer have to be above zero. The social planner chooses $N$ and $x$ to maximize consumer utility subject to the economy’s constraints.

<table>
<thead>
<tr>
<th>Geometric Optimum</th>
<th>$p^U = a_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^U = \frac{\alpha p}{(1-\alpha p)a_s} F$</td>
<td></td>
</tr>
<tr>
<td>$Z^U = H - N(a_s x + F) = H - (1 - \alpha p) \frac{1}{\rho - 1} (\alpha p)^{\frac{1}{\rho-1}} \alpha^{\frac{\rho}{\rho-1}} F^{\frac{1}{\rho - 1}} a_s^{\frac{\alpha}{1-\alpha}} a_{\frac{\alpha}{1-\alpha}}^{\frac{\rho(1-\alpha)}{\rho-1}}$</td>
<td></td>
</tr>
<tr>
<td>$N^U = \left( (1 - \alpha p)(\alpha p)^{\frac{1}{\rho - 1}} \alpha^{\frac{\rho}{\rho-1}} F^{\frac{1}{\rho - 1}} a_s^{\frac{\alpha}{1-\alpha}} a_{\frac{\alpha}{1-\alpha}}^{\frac{\rho(1-\alpha)}{\rho-1}} \right)^{\rho-1}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constrained Optimum</th>
<th>$p^c = \frac{1}{\alpha p} a_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^c = \frac{\alpha p}{(1-\alpha p)a_s} F$</td>
<td></td>
</tr>
<tr>
<td>$Z^c = H - (1 - \alpha p) \frac{1}{\rho - 1} (\alpha p)^{\frac{1}{\rho-1}} \alpha^{\frac{\rho}{\rho-1}} F^{\frac{1}{\rho - 1}} a_s^{\frac{\alpha}{1-\alpha}} a_{\frac{\alpha}{1-\alpha}}^{\frac{\rho(1-\alpha)}{\rho-1}}$</td>
<td></td>
</tr>
<tr>
<td>$N^c = \left( (1 - \alpha p)(\alpha p)^{\frac{1}{\rho - 1}} \alpha^{\frac{\rho}{\rho-1}} F^{\frac{1}{\rho - 1}} a_s^{\frac{\alpha}{1-\alpha}} a_{\frac{\alpha}{1-\alpha}}^{\frac{\rho(1-\alpha)}{\rho-1}} \right)^{\rho-1}$</td>
<td></td>
</tr>
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</table>
resource constraint. Since firms are symmetric, the maximization problem can be written as follows:

\[
\max_{N,x} U = Z + N^{\frac{\rho}{\nu}} x^\alpha
\]  

(2.14)

*Subject to:*  \[ Z = H - N(a \cdot x + F) \]  

(2.15)

Here we treat \( N \) as a continuous variable, which allows us to derive the following first-order conditions:

\[
-a \cdot N + \alpha N^{\frac{\nu}{\rho}} x^{a-1} = 0, \\
-(a \cdot x + F) + \frac{1}{\rho} N^{\frac{\nu-1}{\rho}} x^\alpha = 0.
\]  

(2.16) 

(2.17)

Solving (2.16) and (2.17), we obtain the social optimal number of firms \( N^U \) and output \( x^U \). Then using the consumer demand function (2.8) and letting \( x_i = x \) and \( p_i = p \) for all \( i \), we can derive the price which is equal to marginal cost. Finally from the economy’s resource constraint, we obtain the optimal consumption in \( Z \). The results are summarized in Table 2.1.

The fact that firms are charging a price that is equal to the marginal cost of production implies that in unconstrained optimum, firms in the differentiated goods industry incur a loss in the amount of \( F \). Thus, lump-sum transfers to the firms are needed to cover the fixed costs if the planner is to implement the unconstrained optimum in a decentralized economy.
2.3.2. Constrained Optimum

Considering the practical difficulty of implementing the unconstrained optimum, it is of interest to explore a second-best regulatory mechanism where firms make at least non-negative profits. In this case, the aim of the social planner is to maximize utility while keeping each firm’s profits non-negative. Again, due to the assumption of symmetric firms, the firms have the same output and earn zero profits in equilibrium. Using the demand function (2.8) and setting \( x_i = x \) and \( p_i = p \) for all \( i \), the planner’s optimization problem is simplified as follows:

\[
\max_{N, p} U = H + (1 - \alpha)\alpha^{1-\alpha} N^{1-\alpha p} p^{1-\alpha} \tag{2.18}
\]

Subject to:

\[
(p - a_x)\alpha^{1-\alpha} N^{1-\alpha p} p^{1-\alpha} - F = 0 . \tag{2.19}
\]

We can set up the Lagrangian

\[
L = H + (1 - \alpha)\alpha^{1-\alpha} N^{1-\alpha p} p^{1-\alpha} - \lambda ((p - a_x)\alpha^{1-\alpha} N^{1-\alpha p} p^{1-\alpha} - F),
\]

where \( \lambda^c \) is the Lagrangian multiplier. The first-order conditions are:

\[
-N = \lambda^c \left( H - \frac{p - a_x}{(1 - \alpha) p} \right), \tag{2.20}
\]

\[
\frac{N}{\lambda^c} = \frac{p - c}{p} \frac{\alpha (1 - \rho)}{(1 - \alpha) (1 - \alpha p)}, \tag{2.21}
\]

\[
(p - a_x)\alpha^{1-\alpha} N^{1-\alpha p} p^{1-\alpha} = F . \tag{2.22}
\]
The above three equations yield the price and variety. Substituting them into the demand function \((2.8)\), we solve for the quantities. The results on constrained optimum are also summarized in Table 2.1.

2.3.3 Comparison

Now it is easy to compare the magnitudes of the variables in the market equilibrium, unconstrained optimum and constrained optimum\(^{19}\). It can be shown that

\[ p^U < p^c = p^e \]

\[ N^U > N^c = N^e \]

\[ x^U = x^c = x^e \]

\[ Z^U < Z^c = Z^e \]

Similar to Dixit and Stiglitz (1977), our model of monopolistic competition shows that the market equilibrium coincides with the constrained optimum. Further, a comparison of the market equilibrium to the unconstrained social optimum shows that it exhibits two market

\(^{19}\)Recall that we use the common approach in a DS model and assume that an individual price change has no effect on the aggregate price index. This solution underestimates the number of product varieties while the solution to the constrained and unconstrained social optima is not affected. Thus it is not difficult to see that, after relaxing the assumption, the equilibrium number of product varieties \(N^e\) becomes larger than the constrained social optimum \(N^c\). On the other hand, the magnitude between \(N^e\) and the unconstrained social optimal number of product varieties \(N^U\) is no longer determinate. Recall that the assumption supresses the impact of own price change on the aggregate price index and thus the cross elasticity of demand is not accounted for. Since \(N^U > N^e\) when the cross elasticity of demand is zero, by the principle of continuity, we know that this still holds when the cross elasticity of demand is relatively small, or equivalently when products are sufficiently differentiated. This conjecture is consistent with Yang and Heijdra (1993) who use Cobb-Douglas utility function to aggregate the utility from the differentiated goods industry and the rest of the economy and show that the relative magnitude of \(N\) under market equilibrium and the social optimum depends on the elasticity of substitution among the differentiated goods.
failures: insufficient resources allocated to the differentiated goods industry and too few product varieties. Just as Spence (1976) has pointed out, when the revenue fails to cover the costs of a socially desirable product because of the setup costs, there tend to be a lower than optimal number of varieties. In the following section, we investigate whether government subsidies make sensible policy remedies.

2.4 Effects of Government Subsidy

Input subsidies such as grants and tax credits given to book publishers and film and video producers are commonly used by the government. Since subsidies reduce a firm’s costs, one would expect it to attract more resources to the industry and increase the product diversity where the entry decision hinges on the level of profits. In this section, we study the effects of government subsidies on product variety and social welfare when products are sold directly to consumers. We consider two types of subsidies, a subsidy on marginal cost and a subsidy on fixed cost as they lead to different pricing and output behaviors. The case where the products are sold through a retailer is examined in the next section.

Suppose that the government subsidizes producers at a rate of $s$ on the unit cost of production, a rate of $\tau$ on the fixed costs or both so that a producer’s total cost becomes

$$C_i(x_i) = (1-s)a_xx_i + (1-\tau)F.$$  \(^{20}\)

The subsidy is financed through a lump sum tax on the representative household’s endowment. Using the demand function (8) derived in section 2.2 and multiplying $a_x$ by $(1-s)$ and $F$ by $(1-\tau)$ in the producers’ profit maximization problem

\(^{20}\) In the case where the government subsidizes only the unit cost (fixed cost), we let $\tau=0$ ($s=0$).
(2.11) and zero-profit condition (2.13), we are able to solve for the equilibrium price, quantities and varieties:

\[ p_i = \frac{(1 - s)a_x}{\alpha p}, \quad (2.23) \]

\[ x = \frac{\alpha p (1 - \tau)F}{(1 - \alpha p)(1 - s) a_x}, \quad (2.24) \]

\[ Z = H - (1 - \alpha p) \frac{1 - \alpha p^{1 - \rho}}{\rho - 1} \frac{\alpha p}{\rho} \alpha^\rho \frac{\rho}{1 - \rho} \frac{1}{(1 - \tau)F} \frac{1 - \alpha p^{1 - \rho}}{1 - \rho} \left[ 1 + \frac{\tau}{1 - \tau} (1 - \alpha p) + \frac{s}{1 - s} \alpha p \right], \quad (2.25) \]

\[ N = \left( 1 - \alpha p \right) \frac{\alpha}{1 - a} \alpha^{\frac{1}{1 - \alpha}} \left( \frac{1 - \alpha p}{\alpha} \right)^{\frac{1}{1 - \alpha}} \left( (1 - \tau)F \right)^{\frac{1}{1 - \rho}} \left( (1 - s a_x) \right)^{\frac{\alpha}{1 - \alpha}} \frac{\rho^{\left( 1 - \alpha \right)}}{\rho - 1}. \quad (2.26) \]

Deducting the amount of lump sum tax from the household’s endowment, social welfare can thus be written as

\[ W = H - \sum_i \left( s a_x x_i + \pi F + p_i x_i \right) + \left( \sum_i x_i^\alpha \right)^{\frac{1}{\rho}} \]

\[ = H + (1 - \alpha p) \frac{1 - \alpha p^{1 - \rho}}{\rho - 1} \frac{\alpha p}{\rho} \alpha^\rho \frac{\rho}{1 - \rho} \frac{1}{(1 - \tau)F} \frac{1 - \alpha p^{1 - \rho}}{1 - \rho} \left[ 1 - \alpha - \frac{s}{1 - s} - \alpha (1 - \alpha p) \right]. \quad (2.27) \]

2.4.1 Subsidy on the Unit Cost of Production

We first examine the case where the government subsidizes producers on their unit cost of production and let \( \tau = 0 \) in (2.23) to (2.27).
Proposition 1. When the government subsidizes the unit cost of production, an increase in the rate of subsidy $s$ reduces the retail price and profit margin of each firm and increases the output and product diversity. In equilibrium, more resources are allocated to the differentiated goods industry.

Proof. Differentiating equations (2.23) to (2.26) with respect to the subsidy $s$, we have

$$\frac{\partial p_{m}^{\alpha}}{\partial s} = -\frac{1}{\alpha \rho} a_x < 0, \quad (2.28)$$

$$\frac{\partial N_{m}^{\alpha}}{\partial s} = \frac{a_x}{\rho - 1}(1 - \alpha \rho) \frac{\rho(1-\alpha)}{1-\rho} (\alpha \rho)^{\frac{\alpha}{1-\rho}} \frac{\rho}{1-\rho} F \frac{\rho(1-\alpha)}{1-\rho} ((1-s)a_x)^{\frac{\alpha}{1-\rho}-1} > 0, \quad (2.29)$$

$$\frac{\partial x_{m}^{\alpha}}{\partial s} = \frac{\alpha \rho}{1 - \alpha \rho} \frac{1}{(1-s)^2} F a_x > 0, \quad (2.30)$$

$$\frac{\partial z_{m}^{\alpha}}{\partial s} = -(1-\alpha \rho)^{\frac{1-\alpha}{\rho-1}} (\alpha \rho)^{\frac{\alpha}{\rho-1}+1} \rho \frac{\rho}{\rho-1} F \frac{1-\alpha}{\rho-1} ((1-s)a_x)^{\frac{\alpha}{\rho-1}} \left(1 - s + s\alpha \rho \frac{1}{\rho-1} \right)^2 < 0, (2.31)$$

Using (2.23) to derive the profit margin and differentiating it with respect to $s$, we obtain

$$\frac{\partial \left(p_{m}^{\alpha} - (1-s)a_x \right)}{\partial s} = -\frac{1-\alpha \rho}{\alpha \rho} a_x < 0. \quad (2.32)$$

QED

Proposition 1 points out that government subsidy can be used as an instrument to attract more resources and more product varieties to the industry. An increase in the rate of subsidy reduces the marginal cost of production and motivates producers to reduce retail price and expand output. Although the profit margin is squeezed, the profits of each firm increase due
to the higher sales. As a result, more firms enter, leading to a higher number of product varieties. Compared to the social optimum, an increase in the rate of subsidy brings the retail price closer to the marginal cost of production, which stimulates demand and production and attracts more resources to this industry. However, since the laissez faire equilibrium output per product is the same as the social optimum, the increased output as a result of the subsidy indicates that the resources that should have been used to introduce more products are used to expand production and exploit economies of scale. That is, the increased resources are spread over too few products. The following proposition confirms this point.

**Proposition 2.** The rate of government subsidy on the unit costs to achieve the socially optimal number of varieties is $s^{mil^*} = 1 - (\alpha \rho)^{\frac{1}{\alpha}}$. With this subsidy, the retail price falls below the marginal cost of production and the resources allocated to the differentiated goods industry become higher than the social optimum.

**Proof.** Letting $\tau = 0$ and equating (2.26) to the number of unconstrained optimal number of product varieties to solve for $s$, we obtain

$$s^{mil^*} = 1 - (\alpha \rho)^{\frac{1}{\alpha}}.$$  \hfill (2.33)

Substituting (2.33) in (2.23) and (2.25) respectively, we can derive the retail price and quantity for $Z$ at $s^{mil^*}$

$$p_{i}^{mil^*} = (\alpha \rho)^{\frac{1-\alpha}{\alpha}} a_s < a_s,$$ \hfill (2.34)
\[ Z^{m^*} = H - (1 - \alpha \rho) \frac{1 - \alpha \rho}{\rho - 1} \left( \alpha \rho \right)^{\frac{\rho}{\rho - 1}} F \frac{1 - \alpha \rho}{\rho - 1} \alpha \rho \left( \alpha \rho \right)^{\frac{\rho}{\rho - 1}} \left( 1 - \alpha \rho + (\alpha \rho)^{\frac{1 - \alpha}{\alpha}} \right). \quad (2.35) \]

Subtracting \( Z^{m^*} \) from \( Z^U \), we have

\[ Z^U - Z^{m^*} = (1 - \alpha \rho) \frac{1 - \alpha \rho}{\rho - 1} \left( \alpha \rho \right)^{\frac{\rho}{\rho - 1}} F \frac{1 - \alpha \rho}{\rho - 1} \alpha \rho \left( \alpha \rho \right)^{\frac{\rho}{\rho - 1}} \left( (\alpha \rho)^{\frac{\rho}{\rho - 1}} (1 - \alpha \rho) + (\alpha \rho)^{\frac{\rho}{\rho - 1}} (\alpha \rho)^{\frac{1 - \alpha}{\alpha}} - 1 \right) > 0. \quad (2.36) \]

QED

This proposition shows that, to achieve the socially optimal number of product varieties, the subsidy has to be so high that excess resources are allocated to the differentiated goods industry. With the market mechanism, the increased resources are used not only to generate new products but to expand output.

Since there are conflicting forces at work to social welfare, one might wonder whether a subsidy is indeed welfare-enhancing. A higher rate of subsidy ameliorates the problem of insufficient variety and resources in the differentiated goods industry, which moves the market equilibrium towards the social optimum. On the other hand, the subsidy leads to excessive output of each firm so that resources that should have been used to generate more products in the differentiated goods industry and to spend on the products in the other industry (\( Z \)) are used to exploit the economics of scale. This is a force pulling the market equilibrium away from the optimum. In the following proposition, the outcomes of these conflicting forces are analyzed.
Proposition 3. When the government subsidizes the unit cost of production, an increase in the rate of subsidy $s$ increases social welfare if $s < \frac{1 - \alpha \rho}{1 - \alpha (1 - \alpha \rho)}$. Otherwise, welfare decreases with the rate of subsidy.

Proof. Differentiating equation (2.27) with respect to the rate of subsidy $s$, we obtain

$$
\frac{\partial W_{m1}}{\partial s} = \frac{a_s}{\rho - 1} (1 - \alpha \rho)^{1-\alpha} \alpha^{1-\alpha} F^{1-\rho} ((1 - s) a_x)^{\alpha \rho} \left(1 - \alpha \rho - \frac{s}{1 - s} \alpha (\alpha \rho - 1 + \rho)\right),
$$

(2.37)

which is positive if and only if $s < \frac{1 - \alpha \rho}{1 - \alpha (1 - \alpha \rho)} < 1$. QED

Therefore, when the subsidy is kept at a moderate level, a higher rate of subsidy always increases social welfare, and welfare reaches the maximum when $s = \frac{1 - \alpha \rho}{1 - \alpha (1 - \alpha \rho)}$. The underlying mechanism can be better understood through the following functional form

$$
\frac{\partial W_{m1}}{\partial s} = \frac{1}{\rho} (N^{m1})^{1-\alpha} (x^{m1})^\alpha \frac{\partial N^{m1}}{\partial s} + \alpha (x^{m1})^{1-\alpha} (N^{m1})^\alpha \frac{\partial x^{m1}}{\partial s} + \frac{\partial Z^{m1}}{\partial s}. 
$$

(2.38)

From proposition 1, we know that the first and second terms are positive while the third is negative. That is, a higher rate of subsidy increases output per product and promotes product diversity, which tends to improve social welfare. In addition, given the fact that the production technology exhibits increasing return to scale, a higher output means a lower average cost and this would further improve social welfare. However, the expansion of the differentiated goods industry is at the expense of a lower spending in the other industry $Z$. 
Using the consumer budget constraint and the zero profit function, the term for \( Z \) in equation (2.38) can be re-written in terms of \( x \) and \( N \)

\[
\frac{\partial Z^{ml}}{\partial s} = -F \frac{\partial N^{ml}}{\partial s} - a_s \left( x^{ml} \frac{\partial N^{ml}}{\partial s} + N^{ml} \frac{\partial x^{ml}}{\partial s} \right). 
\] (2.39)

This shows that the reduced spending on \( Z \) is due to the increase in resources devoted to the fixed costs of adding more products and the variable costs associated with producing more output and more varieties in the differentiated goods industry. When \( s < \frac{1 - \alpha \rho}{1 - \alpha(1 - \alpha)} \), the beneficial effect outweighs the negative impact and an increase in the government subsidy improves social welfare. However, when \( s \) surpasses the threshold, the large amount of investment in fixed costs, the excess expansion in the output of each product as well as the shrinking of the industry \( Z \) eventually lead to a decrease in social welfare.

### 2.4.2 Subsidy on the Fixed Cost of Production

Having studied the effect of a government subsidy on the unit cost of production, it is of interest to also explore whether a subsidy on the fixed cost of production presents a better alternative since the two subsidies provide different incentives for producers and consequently lead to different output and pricing behaviors. Letting \( s=0 \) in (23) to (27), we examine the effects of this subsidy on equilibrium price, quantities, varieties and social welfare.
Proposition 4. When the government subsidizes the fixed cost of production, an increase in the rate of subsidy $\tau$ increases the product variety but reduces the output of each firm. Overall, more resources are allocated to this industry and it has no impact on retail prices.

Proof. From (2.23), it can be clearly seen that a subsidy on the fixed cost of production has no impact on retail price. Also differentiating equation (2.23) to (2.26) with respect to the subsidy $\tau$, we have

$$
\frac{\partial x^{m2}}{\partial \tau} = -\frac{\alpha \rho}{(1-\alpha \rho) a_s} F < 0,
$$

$$
\frac{\partial Z^{m2}}{\partial \tau} = -(1-\alpha \rho)^{\rho-1} (\alpha \rho)^{\rho-1} \alpha^{\rho-1} a_s^{\rho-1} (1-\tau) F^{\rho-1} \left(1-\alpha \rho\right) \frac{1-\alpha \rho}{(\rho-1)(1-\tau)^2} < 0,
$$

$$
\frac{\partial N^{m2}}{\partial \tau} = \frac{\rho(1-\alpha)^{\rho-1}}{\rho-1} (1-\alpha \rho)^{\rho-1} \alpha^{\rho-1} a_s^{\rho-1} F(1-\tau) F^{\rho(1-\alpha)} > 0.
$$

QED

Similar to the effects of a government subsidy on the unit cost of production, an increase in the rate of subsidy on fixed costs increases the number of product varieties and attracts more resources to this industry. What is different, however, is that the retail price is unchanged and the output of each firm is reduced. When the government subsidizes the fixed cost of production, the reduction in total costs raises the profits and attracts more firms to enter the market. As $N$ increases, each producer faces reduced demand and reacts with lower
output; and this effect is greater the higher is the elasticity of substitution$^{21}$. The price remains unchanged because the marginal cost of production and the elasticity of demand have not changed. In this scenario, the subsidy acts as a vehicle that brings more resources into the differentiated goods industry, which is solely used to generate more varieties. Recall that the market equilibrium output of each firm is the same as that under the social optimum. The reduced output by each firm as a result of the government intervention thus indicates that the added resources to the differentiated goods sector are not optimally utilized from a welfare point of view. In fact producers operate at a higher average cost, which causes efficiency loss.

Proposition 5. An increase in the rate of government subsidy $\tau$ on the fixed costs increases social welfare if $\tau < \frac{1-\alpha \rho}{1-\alpha^2 \rho}$. Otherwise, welfare decreases with the rate of subsidy.

Proof. Differentiating equation (2.27) with respect to the rate of subsidy $\tau$, we have

$$\frac{\partial W^{m^2}}{\partial \tau} = \frac{F}{\rho - 1} \left(1 - \alpha \rho\right)^{\frac{\rho(1-\alpha)}{1-\rho}} \alpha^{\frac{\alpha}{1-\rho}} a_{x_{m}} \left(1 - \alpha \rho \right)^{\frac{\rho(1-\alpha)}{1-\rho}} \left(1 - \alpha \rho - \frac{\tau}{1-\tau} \alpha \rho (1 - \alpha)\right). \tag{2.43}$$

which is positive if and only if $\tau < \frac{1-\alpha \rho}{1-\alpha^2 \rho} < 1$. QED

$^{21}$ Equation (2.24) can be rewritten in terms of the elasticity of substitution $\sigma$

$$\frac{\partial x^{m^2}}{\partial \tau} = -(\sigma - 1) \frac{F}{a_{x}}.$$

Differentiating with respect to $\sigma$, we have

$$\frac{\partial x^{m^2}}{\partial \partial \sigma} = -\frac{F}{a_{x}} < 0.$$
Therefore, overall a subsidy on the fixed cost of production has qualitatively similar effects on social welfare as the one on the unit cost of production: It improves social welfare as long as it is at a moderate level and the welfare reaches the maximum when \( \tau = \frac{1 - \alpha \rho}{1 - \alpha(1 - \alpha \rho)} \).

However, the underlying mechanism is different. The effects of a subsidy on social welfare can be decomposed into the following three terms

\[
\frac{\partial W^{m2}}{\partial \tau} = \frac{1}{\rho} \left( N^{m2} \right)^{1/\rho} \left( \tau^{m2} \right)^{1 - \alpha} \frac{\partial N^{m1}}{\partial \tau} + \alpha \left( N^{m2} \right)^{1/\rho} \left( \tau^{m2} \right)^{1 - \alpha} \frac{\partial \tau^{m2}}{\partial \tau} + \frac{\partial Z^{m2}}{\partial \tau}. \tag{2.44}
\]

From proposition 4 we know that the first term is positive while the second and the third are negative. That is, a higher rate of subsidy increases the number of products, which, given the representative consumer’s preference for variety, tends to improve social welfare. On the other hand, since all producers operate at a smaller scale and bear higher average costs, the subsidy results in efficiency loss. This, combined with lower consumption of all products in both sectors tends to reduce social welfare. The term for \( Z \) in equation (2.44) can be re-written in terms of \( x \) and \( N \)

\[
\frac{\partial Z^{m2}}{\partial \tau} = -(F + a \tau^{m1}) \frac{\partial N^{m2}}{\partial \tau} - a \tau^{m1} \frac{\partial \tau^{m2}}{\partial \tau}. \tag{2.45}
\]

The first term shows that the lower spending in \( Z \) is due to the fixed and variable costs involved in adding more products in the diversified industry. The second term indicates that although the reduction in the consumption of products \( x \) reduces the consumer’s utility, it frees up resources to be used to produce \( Z \), which could improve social welfare. Overall when \( \tau < \frac{1 - \alpha \rho}{1 - \alpha(1 - \alpha \rho)} \), the beneficial effect from increased variety outweighs the negative
impact and an increase in subsidy improves social welfare. However, when $\tau$ surpasses the threshold, the lower consumption of all products and the efficiency loss eventually reduce social welfare.

2.4.3 Effectiveness of the Two Types of Subsidies

Compared to the case where the government subsidizes the unit cost of production, the additional resources attracted to the differentiated goods industry through the use of a subsidy on the fixed cost are fully used to generate additional products (proposition 4). As a result a subsidy on fixed cost tends to be more effective in increasing product diversity and social welfare. On the other hand it causes efficiency loss as firms are operating at a smaller scale and thus more resources are needed to produce the same amount of output. This tends to limit the number of products that can be introduced and reduce social welfare. The following two propositions compare the effectiveness of the two types of subsidies. Also a policy that combines the two types of subsidies is explored.

Proposition 6. An infinitesimal subsidy on the unit cost of production increases product variety more than a subsidy on the fixed costs if and only if $\alpha > \frac{1}{2}$.

Proof. Divide (2.29) by (2.42) and let $s=0$ and $\tau=0$

\[
\frac{\partial N^{m_1}/\partial s}{\partial N^{m_2}/\partial \tau} = \frac{\alpha}{(1 - \alpha)},
\]

(2.46)

which is greater than 1 if and only if $\alpha > \frac{1}{2}$. QED
Proposition 6 shows that a subsidy on the unit cost of production is more effective at promoting product diversity when the elasticity of the Chamberlinian $DD$ demand curve $1/(1 - \alpha)$ is high. Otherwise, a subsidy on fixed cost is more effective. It is clear from the previous analysis that a government subsidy on the unit cost of production encourages entry through influencing producers’ pricing behaviors. Given the number of firms, the more elastic is the $DD$ demand curve, the greater is the expansion in sales and consequently the increase in profits in response to a decrease in prices. In addition, higher sales and output by each firm means lower average costs, which would raise profits further. Since firm entry will not stop until the marginal firm earns zero profits, in equilibrium there tends to be a higher number of active firms when $\alpha$ is large. On the other hand, a subsidy on the fixed cost stimulates entry through directly influencing the profit level of firms. The retail prices remain unchanged and the elasticity of demand has no impact on the effectiveness of government policy in this case. Recall from proposition 4 that with a subsidy on the fixed costs, all added resources to the differentiated goods industry are used to generate additional products. This, with other things equal, tends to increase the number of varieties more than the case where the government subsidizes the unit costs. Moreover, a lower $\alpha$ means a lower cross elasticity of substitution $1/(1 - \alpha \rho)$ and less competition within the differentiated goods industry. As a result, firms experience a smaller reduction in output and a smaller rise in average costs as entry occurs. This, in turn, leaves more room for entry. Thus a subsidy on fixed costs becomes more effective when $\alpha$ is relatively small.

Note that since the government subsidizes all firms at the same rate $s$, the prices would change simultaneously and thus it is the elasticity of $DD$ demand curve that matters.
Proposition 7. An infinitesimal subsidy on the unit cost of production is more effective in improving social welfare than an infinitesimal subsidy on the fixed cost if and only if \( \alpha \rho > 1/2 \).

Proof. Subtracting (2.38) by (2.43) and let \( s=0 \) and \( \tau = 0 \)

\[
\frac{\partial W^{m1}}{\partial s} - \frac{\partial W^{m2}}{\partial \tau} = \frac{1}{\rho - 1} (1 - \alpha \rho)^{\rho/(1-\rho)} (\alpha \rho)^{1/(\rho - 1)} \alpha^{\rho - 1} \alpha^{1-\rho} a^{1-\rho}_x F^{1-\rho} (2\alpha \rho - 1),
\]

which is greater than 0 if and only if \( \alpha \rho > 1/2 \). QED

Therefore, a subsidy on the unit cost of production is more effective in improving social welfare when products are close substitutes. Consider first the case of a subsidy on the unit cost of production. When \( \alpha \rho \) is higher, the subsidy leads to a greater increase not only in the number of product varieties (from proposition 6) and but also the output of each variety in the differentiated goods industry. Further, the greater output in \( x \) brings efficiency gain and frees up more resources to be used in the production of \( Z \). All these forces beget greater improvement in social welfare when \( \alpha \rho \) is higher. On the other hand, in the case of a subsidy on the fixed cost of production, a lower elasticity of substitution means less competition in the differentiated goods industry. There is a smaller reduction in output and less efficiency loss. As a result, more resources are saved to be used in the industry of \( Z \). Both tend to increase social welfare more. Therefore when \( \alpha \rho \) is lower, welfare increases more when the government subsidizes the fixed cost of production.
Having analyzed the relative effectiveness of the two types of subsidies, one might wonder whether a combination of these two would achieve an optimal result. Recall from propositions 1 and 4 that although both types of subsidies increase product varieties and attract resources to the differentiated goods industry, each changes the output decisions of producers in a way (opposite ways) that deviates from the social optimum. One natural question to ask is whether a mix of the two would have the negative effects on output cancelled out and brings the market equilibrium to the social optimum. The following proposition shows that it is indeed the case.

**Proposition 8.** A combination of a subsidy on the unit cost of production and a subsidy on the fixed cost can achieve the social optimum. The optimal subsidizing rates are

\[ s^m = \tau^m = 1 - \alpha \rho. \]

**Proof.** The government uses both types of subsidies to achieve the maximum social welfare. It maximizes (2.27) by choosing \( s \) and \( \tau \), which is formulated as follows

\[
\max_{s, \tau} W = H + (1 - \alpha \rho) \alpha \rho^{-1} \left( \alpha \right) \alpha \rho^{-1} \left( 1 - \tau \right) F \alpha \rho^{-1} \left( (1 - s) a \right) \alpha \rho^{-1} \left( \tau - 1 \right) \left( 1 - \alpha - \alpha^2 \rho - s - \alpha (1 - \alpha \rho) \frac{\tau}{1 - \tau} \right). \tag{2.48}
\]

The first-order conditions are

\[
(1 - \alpha \rho) \alpha \rho^{-1} \left( \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( 1 - \tau \right) \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( (1 - s) a \right) \alpha \rho^{-1} \left( \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( \tau - 1 \right) \right) \right) \frac{1 - \alpha - \alpha^2 \rho - s - \alpha (1 - \alpha \rho) \frac{\tau}{1 - \tau}}{\alpha \rho^{-1} \left( \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( 1 - \tau \right) \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( (1 - s) a \right) \alpha \rho^{-1} \left( \tau - 1 \right) \right) \left( 1 - \alpha - \alpha^2 \rho - s - \alpha (1 - \alpha \rho) \frac{\tau}{1 - \tau} \right)} = 0. \tag{2.49}
\]

\[
(1 - \alpha \rho) \alpha \rho^{-1} \left( \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( 1 - \tau \right) \frac{1 - \alpha \rho}{\alpha \rho^{-1}} \left( (1 - s) a \right) \alpha \rho^{-1} \left( \tau - 1 \right) \right) \left( 1 - \alpha - \alpha^2 \rho - s - \alpha (1 - \alpha \rho) \frac{\tau}{1 - \tau} \right) = 0. \tag{2.50}
\]
Solving (2.49) and (2.50) we obtain

\[ s^m = r^m = 1 - \alpha \rho . \]  (2.51)

Substituting (2.51) in (2.23) to (2.27), it can be shown that with these subsidies, the prices, quantities and varieties are the same as those under the unconstrained social optimum. QED

2.5 Retailer Buyer Power

A common assumption shared in the literature of monopolistic competition is that producers sell products directly to consumers. In the previous sections, we have shown that under this assumption, the market mechanism yields a suboptimal number of product varieties and a small subsidy on either the unit cost of production or the fixed cost increases product diversity and social welfare. Further, a combination of the two types of subsidies is able to achieve the social optimum. In this section we study the market provision of product diversity and effectiveness of government subsidies in the presence of buyer power. In particular, we want to verify two conjectures. First, there have been concerns over the effects of buyer power on product diversity. It has been argued that powerful retailers extract a larger share of profits from producers which may reduce producers’ incentives to invest or even induce exit. We want to examine whether buyer power does reduce the number of product varieties in a framework of monopolistic competition and if it does, what is the underlying mechanism. Second, also an extension of the first point, with greater power, would the buyer extract the gains associated with the subsidy from the producers, leaving no changes to producers’ incentives and rendering the government subsidies ineffective?
We modify our model such that the differentiated products are sold through a single retailer $R^{23}$. That is, the retailer purchases the products from producers and sells them to consumers. To simplify the mathematical deduction, we assume that the retailer has all the bargaining power and makes a take-it-or-leave-it offer to each producer. In addition to the wholesale cost of purchasing the supplies, $R$ incurs constant marginal retailing costs, normalized to zero. All other aspects of the model remain the same as the benchmark model.

The firms in this model play a three-stage game. At stage one, producers compete to become the suppliers of the retailer $R$ and the retailer selects the producers whose products it will carry$^{24}$. At stage two, the retailer makes a take-it-or-leave-it offer to each selected producer. The offer consists of a wholesale price, denoted by $w_i$ and a fixed fee, denoted by $k_i$. At the third stage, retailer $R$ sets the retail prices and sells the products to the end consumers.

To solve the model we shall start with stage three of the game where the retailer sets the retail prices. The profit maximization problem of the retailer is as follows:

\[
\text{Max } \pi_R = \sum_{i}^{N} \left( (p_i - w_i)x_i - k_i \right). \tag{2.52}
\]

The first-order condition of this optimization problem is given by$^{25}$

---

$^{23}$ The buyer acts as a gate-keeper, which has been alleged to be important in retailing (OECD 2008). In this case, the retailer has significant market power in the downstream market because there are no other options for distribution. Without selling through the downstream retailer, producers cannot access the downstream market efficiently. 

$^{24}$ The role of gatekeeper can be viewed as the downstream retailer selling distribution services. In reality, such retailers should enjoy considerable scope in determining how to assign their “shelf space”.

$^{25}$ Different from the case where products are distributed directly to consumers and producers make pricing decisions themselves, the influence of an individual price change on the price index is no longer assumed away in the modified model. When the retailer makes the price decision for all products in a centralized fashion, a
\[ x_i + \sum_j^N (p_j - w_j) \frac{\partial x_j}{\partial p_i} = 0. \]  \hspace{1cm} (2.53)

Since the representative household has the same preferences as in the benchmark model, the demand function in this modified model remains the same as in Section 2.2. Substituting (2.8) in (2.53), we have

\[
\frac{w_i}{p_i} + \frac{\alpha (1 - \rho) \sum w_j p_j^{\alpha p - 1}}{(1 - \alpha) \sum p_j^{\alpha p - 1}} = \frac{\alpha (1 - \alpha \rho)}{1 - \alpha}, \hspace{1cm} (2.54)
\]

which defines the relationship between the retail prices \( p_i \) and wholesale prices \( w_i \). Since all producers are symmetric, the retailer offers them the same wholesale price \( w \) in equilibrium and (2.54) is simplified as

\[ p = \frac{w}{\alpha}. \hspace{1cm} (2.55) \]

This equation suggests that the retailer’s pricing decision is not affected by the number of products it carries. Substituting the retail price (2.55), the demand function for each product carried by the retailer can be written as

\[ x = \alpha^{\frac{2}{1-\alpha}} N^{\frac{1-\rho}{\rho (1-\alpha)}} w^{-\frac{1}{1-\alpha}}. \hspace{1cm} (2.56) \]

This indicates that the demand for each product decreases when the retailer carries a larger number of products and competition among these products intensifies.
Prior to the setting of the retail prices, retailer \( R \) makes take-it-or-leave-it offers \((w_i, k_i)\) to the \( N \) producers that it has chosen. It does so by maximizing the joint profits, which it fully extracts by virtue of its bargaining power. Note that with the subsidies, the total cost of production is \( C_i(x_i) = (1 - s)a_x x_i + (1 - \tau)F \). The retailer chooses \( w_i \) to maximize

\[
\pi_R = \sum_i^N \left( (p_i - (1 - s)a_x)x_i - (1 - \tau)F \right).
\]  

(2.57)

The first-order condition to this problem is

\[
\sum_j^N \frac{\partial p_j}{\partial w_i} x_j + \sum_j^N \left( p_j - (1 - s)a_x \right) \sum_m^N \frac{\partial x_j}{\partial p_m} \frac{\partial p_m}{\partial w_i} = 0.
\]  

(2.58)

Substituting (2.54) into (2.58), we obtain

\[
\sum_j^N \left( w_j - (1 - s)a_x \right) \sum_m^N \frac{\partial x_j}{\partial p_m} \frac{\partial p_m}{\partial w_i} = 0.
\]  

(2.59)

Because \( \sum_m^N \frac{\partial x_j}{\partial p_m} \frac{\partial p_m}{\partial w_i} \neq 0 \), (2.59) implies that

\[
w_j = w = (1 - s)a_x.
\]  

(2.60)

That is, the wholesale price is set equal to the marginal cost of producers. To ensure that an active producer makes non-negative profit, the retailer sets the fixed fee such that

\[
k_j = (1 - \tau)F.
\]  

(2.61)

Using (2.55) and (2.60), we are able to derive the price:

\[
p_i = \frac{w}{\alpha} = (1 - s)a_x / \alpha.
\]  

(2.62)
The demand function thus can be re-written as

\[ x_i = \alpha^{\frac{2}{1-a}} N^{\frac{1-\rho}{\rho(1-a)}((1-s)a_s)}^{\frac{1}{1-a}}. \]  

(2.63)

At the first stage, retailer $R$ chooses $N$ products (brands) to carry. The number of products is thus determined through the following maximization problem

\[ \text{Max}_N \pi_R = \sum_i \left( (p_i - w_i)x_i - k_i \right). \]  

(2.64)

Substituting (2.60) to (2.63) in the above function, we rewrite the maximization problem as

\[ \text{Max}_N \pi_R = N \left( (1-s) a_s \right)^{\frac{1}{1-a}} \left( 1 - \alpha \right)^{\frac{1+\rho}{\rho(1-a)}} N^{\frac{1-\rho}{\rho(1-a)}} - (1 - \tau) F \right). \]  

(2.65)

The first-order condition gives

\[ \left( (1-s) a_s \right)^{\frac{1}{1-a}} \left( 1 - \alpha \right)^{\frac{1+\rho}{\rho(1-a)}} N^{\frac{1-\rho}{\rho(1-a)}} \frac{1 - \alpha \rho}{\rho} = (1 - \tau) F . \]  

(2.66)

We solve (2.66) to obtain

\[ N = \left( \frac{1 - \alpha \rho}{\rho} \right)^{\frac{1+\rho}{\rho(1-a)}} \left( (1 - \tau) F \right)^{-1} \left( (1-s) a_s \right)^{\frac{1}{1-a}} \left( 1 - \alpha \right)^{\frac{\rho(1-a)}{\rho-1}}. \]  

(2.67)

Using (2.60) to (2.63) and (2.67), we can solve for the equilibrium quantities and retailer $R$’s profit function.

\[ x = \frac{\alpha \rho (1-\tau) F}{1 - \alpha \rho (1-s) a_s}, \]  

(2.68)
\[ Z = H - \left(1 - \frac{\alpha p}{\rho}\right)^{\rho^{-1}} \frac{1}{\rho^{-1}} \frac{\rho^{(1+\alpha)}}{\alpha^{(1)}} \left(1 - \tau\right)^{\rho^{-1}} \frac{\rho^{(1+\alpha)}}{\alpha^{(1)}} \left(1 - s\right)^{\rho^{-1}} \left(1 - \frac{\alpha p}{\rho} + \frac{\tau(1 - \alpha p)}{1 - \tau}\right) \], \quad (2.69)

\[ \pi_r = \frac{\rho - 1}{1 - \alpha p} \left( \frac{\rho}{1 - \alpha p} \right)^{\rho^{(1+\alpha)}} \frac{1}{\rho^{-1}} \frac{\rho^{(1+\alpha)}}{\alpha^{(1)}} \left(1 - \tau\right)^{\rho^{-1}} \frac{\rho^{(1+\alpha)}}{\alpha^{(1)}} \left(1 - s\right)^{\rho^{-1}} \left(1 - \frac{\alpha p}{\rho} + \frac{\tau(1 - \alpha p)}{1 - \tau}\right). \quad (2.70) \]

Deducting the lump sum tax from the endowment, social welfare can be written as

\[ W = \left(\sum_i x_i^{\alpha p}\right)^{1/\rho} + H - \sum_i (p_i x_i + s a x_i + \pi F) + \pi_r. \quad (2.71) \]

Substituting in (2.60) to (2.63) and (2.67) to (2.72), it becomes

\[ W = H + \left(\frac{\rho}{1 - \alpha p}\right)^{1-\rho} \alpha^{\rho-1} \left(1 - \tau\right)^{\alpha}\left(1 - \rho\right)^{\alpha}\left(1 - s\right)^{\alpha}\left(\rho - \frac{s \alpha^2}{1 - s} - \frac{\alpha(1 - \alpha p) \tau}{1 - \tau}\right). \quad (2.72) \]

### 2.5.1 Subsidy on Unit Cost of Production

As in section 2.4, we start from the case where the government subsidizes the \(N\) producers on the unit cost of production and let \(\tau = 0\) in (2.60) to (2.62) and (2.67) to (2.72). We examine first whether the presence of buyer power changes the effects of a subsidy.

**Proposition 9.** When products are sold through a retailer, an increase in the rate of government subsidy on the unit cost of production

i) reduces wholesale price and retail price;
ii) increases number of product varieties and output of each firm; 

iii) attracts resources to the differentiated goods industry; and

iv) improves social welfare if and only if 

\[ s < \frac{\rho(1 - \alpha)}{\alpha^2 \rho + \rho - \alpha} . \]

**Proof.** Differentiating (2.60), (2.62) and (2.67) to (2.69) and (2.72) with respect to subsidy \( s \), we obtain

\[ \frac{\partial w^{b1}}{\partial s} = -a_x < 0 , \quad (2.73) \]

\[ \frac{\partial p^{b1}}{\partial s} = -a_x / \alpha < 0 , \quad (2.74) \]

\[ \frac{\partial x^{b1}}{\partial s} = \frac{\alpha p}{1 - \alpha \rho} \frac{F}{(1 - \alpha \rho)^2 a_x} > 0 , \quad (2.75) \]

\[ \frac{\partial Z^{b1}}{\partial s} = -\left(1 - \frac{\alpha \rho}{\rho} \right)^{1-\alpha} \frac{\rho(1+\alpha)}{\rho^2} \frac{1-\alpha^{1-p}}{\rho^2} \left(1 - \frac{1}{s} \right) a_x \frac{\alpha}{(1 - \alpha \rho)^2 (\rho - 1)} (\rho - s + \alpha \rho) < 0 , \quad (2.76) \]

\[ \frac{\partial N^{b1}}{\partial s} = \frac{\alpha \rho}{(\rho - 1)(1 - s)} \left(1 - \frac{1 - \alpha \rho}{\rho} \right)^{1-\alpha} \frac{1+\alpha}{\rho} F^{-1} \left(1 - \frac{1}{s} \right) a_x \frac{\alpha}{(1 - \rho)(1 - s)} > 0 , \quad (2.77) \]

and

\[ \frac{\partial W^{b1}}{\partial s} = \left(1 - \frac{\alpha}{1 - \alpha \rho} \right)^{1-\alpha} \frac{1+\alpha}{\rho} F^{-1} \left(1 - \frac{1}{s} \right) a_x \frac{\alpha}{(1 - \alpha \rho)^2 (\rho - 1)} \left(\rho - \alpha \frac{\alpha^2 \rho s}{1-s} - \frac{\alpha(\rho - 1)}{1-s} \right) , \quad (2.78) \]

which is positive if and only if 

\[ s < \frac{\rho(1 - \alpha)}{\alpha^2 \rho + \rho - \alpha} < 1 . \]  

QED
Proposition 9 shows that when products are sold through the powerful retailer, a subsidy in fact has qualitatively similar effects to those when products are sold directly to consumers: The number of product varieties is increased, consumer prices are reduced, the problem of insufficient resources in the differentiated goods industry is ameliorated and social welfare is improved as long as the rate of subsidy is at a moderate level. The underlying mechanism, however, is different since now it is the retailer who makes the decisions over prices and varieties. When the government raises the rate of subsidy, retailer R reduces the wholesale prices to extract the full amount of the gains that producers could have obtained through the government subsidy. The retail prices are then lowered by the retailer and thus output expanded since the marginal costs of the retailer R \((w_i)\) is reduced. Meanwhile, retailer R is willing to carry more products as the higher rate of subsidy increases its profits and consequently the marginal gain from adding an additional product. The following two equations further confirm our analysis on the retailer’s decision process.

A variation of retailer R’s equilibrium profit function is

\[
\pi_R^{b1} = N \left( px - a_x x - F \right) + Nsa_x x . \tag{2.79}
\]

Differentiating the retailer’s profit function (2.70) with respect to \(s\), we obtain

\[
\frac{\partial \pi_R^{b1}}{\partial s} = \frac{\alpha \rho}{(1 - \alpha \rho)(1 - s)} \left( \frac{\rho}{1 - \alpha} \right)^{\frac{(1 - \alpha)}{1 - \rho}} \frac{\rho^{1 + \alpha}}{\rho^{\frac{1 - \alpha}{1 - \rho}}} F \frac{1 - \alpha \rho}{1 - \rho} \left( (1 - s) a_x \right)^{\frac{\alpha}{1 - \rho}} > 0 . \tag{2.80}
\]

That is, the subsidy originally used to motivate producers and encourage entry is fully appropriated by retailer R, who in turn finds it profitable to expand its product line. Despite the fact the retailer is the main beneficiary of the subsidy, proposition 9 suggests that part of the benefits is passed on to consumers through lower retail prices, higher output and greater
variety in the differentiated goods industry. Social welfare improves as long as the rate of subsidy is moderate where the positive effects from the increased output, variety and production efficiency in the differentiated goods industry outweigh the negative impact from reduced production in industry Z.

Given the fact that the retailer has market power both upstream and downstream, one might conjecture that there would be quantitative differences from the case where products are sold directly to consumers. We show that, as expected, retail prices are higher and the number of varieties and social welfare lower in the presence of buyer power.

Proposition 10. When products are distributed through the retailer, the retail prices are always higher than the case where products are sold directly to consumers. In addition, a marginal increase in subsidy leads to a greater reduction in retail prices in the presence of retailer buyer power.

Proof. Subtracting (2.62) by (2.23) and (2.74) by (2.28) respectively, we have

\[ p^{bl} - p^{ml} = \frac{\rho - 1 - s}{\rho} \frac{a_s}{\alpha} > 0, \quad (2.81) \]

\[ \frac{\partial p^{bl}}{\partial s} - \frac{\partial p^{ml}}{\partial s} = -\frac{a_s}{\alpha} \left( 1 - \frac{1}{\rho} \right) < 0. \quad (2.82) \]

QED
As expected retailer buyer power begets higher retail prices. What is surprising however is that the retail price falls more in response to a subsidy. This is due to the higher price level, which leaves more room for the retailer to adjust prices.

**Proposition 11.** At any given subsidy rate, the number of product varieties is lower in the case where products are distributed through a retailer than in the case where products are sold directly to consumers. Furthermore, a higher rate of subsidy leads to a smaller increase in the number of varieties in the presence of retailer buyer power.

**Proof.** Subtracting (2.26) from (2.67) and (2.29) from (2.77) respectively, we have

\[
N^{b1} - N^{m1} = \left(1 - \frac{\alpha \rho}{\rho} \alpha^{1+\alpha} F^{-1} \left( (1-s) \alpha_s \right) \frac{\alpha}{\rho^{1-\alpha}} \right) \left(1 - \rho^{\rho-1} \right) < 0, \quad (2.83)
\]

\[
\frac{\partial N^{b1}}{\partial s} - \frac{\partial N^{m1}}{\partial s} = \frac{\alpha \rho}{(\rho-1)(1-s)} \left(1 - \frac{\alpha \rho}{\rho} \alpha^{1+\alpha} F^{-1} \left( (1-s) \alpha_s \right) \frac{\alpha}{\rho^{1-\alpha}} \right) \left(1 - \rho^{\rho-1} \right) < 0. \quad (2.84)
\]

QED

Proposition 11 states that the retailer buyer power has both a level effect and a marginal effect. At any given subsidy rate, compared to the case where products are sold directly to consumers, the number of varieties is smaller and a marginal increase in subsidy leads to a smaller increase in variety when products are sold through a retailer. This also indicates that as the rate of subsidy increases, the number of product varieties would diverge further
between the two cases. Intuitively, in the absence of the retailer, each firm makes its entry decision independently and would enter the market as long as the revenue covers the costs. It fails to consider the fact that its entry cuts into other firms’ profits. On the other hand, when products are distributed through the retailer who makes the decision over the varieties in a centralized fashion, retailer $R$ internalizes the negative effects of more products on total profits and would always want to maintain a lower number of varieties than in the case where products are distributed to consumers directly\textsuperscript{26}. It is also because of the internalization mechanism that, although the whole amount of subsidy is taken by the retailer, the retailer increases the variety by a smaller amount. The effects of retailer buyer power on social welfare and the effectiveness of the subsidy on social welfare, on the other hand, are more complex.

**Proposition 12.** Assume that $\rho^{\frac{\rho}{\rho+1}} - \rho - 1 > 0$\textsuperscript{27}. In the range where a subsidy on the unit cost of production improves social welfare in both the case where products are sold directly to consumers and the case where products are distributed through a retailer, social welfare is always lower in the presence of retailer buyer power.

**Proof.** Subtracting (2.27) from (2.72), we have

\textsuperscript{26} The prediction that buyer power may reduce product diversity is consistent to some empirical observations. For example the UK’s Competition Commission has documented the increased concentration of UK’s grocery retail industry and the higher concentration of these retailers’ supplier base. Also it has become one of the main objectives of a few EU countries’ antitrust authorities to prevent consolidated retailers from delisting suppliers. In this respect, one related article is Inderst and Shaffer (2007) which show that a consolidated retailer may find it more profitable to reduce its supplier base and thus reducing product diversity. However the underlying rationale is different in that in their paper the motivation for the retailer to reduce the supplier base is to intensify the competition among upstream firms while in ours the motivation is to reduce the competition downstream.

\textsuperscript{27} We can show that this technical assumption holds for all relevant values of $\rho$ through numerical experiments. However, we are not able to prove it analytically.
\[
W^{b1} - W^{m1} = \frac{1}{\rho} \left( \frac{\rho}{1-\alpha \rho} \right)^{1} \frac{\alpha}{1-\rho} \frac{1}{1-\rho} (1-s) a_{s} \frac{1}{1-\rho} \left( \frac{\rho}{1-\rho} \left( 1-\alpha - \alpha^{2} \rho s \right) \right) \left( 1-\alpha - \alpha^{2} \rho s \right). \tag{2.85}
\]

Let \( D_{i} = \rho - \alpha - s \frac{\alpha^{2} \rho}{1-s} - \rho^{\rho-1} \left( 1-\alpha - \alpha^{2} \rho s \right) \). It can be shown that \( D_{i} < 0 \) if and only if

\[
s \left( \rho^{\rho-1} (\alpha^{2} \rho + 1 - \alpha) - (\rho - \alpha + \alpha^{2} \rho) \right) < \rho^{\rho-1} (1 - \alpha) - (\rho - \alpha) .
\]

Note that the right-hand side \( \rho^{\rho-1} (1 - \alpha) - (\rho - \alpha) \) can be re-written as \( \alpha < \left( \frac{\rho}{\rho^{\rho-1} - 1} \right) \). Since \( \alpha \rho < 1 \) and given the assumption \( \rho^{\rho-1} - \rho - 1 > 0 \), we can show \( \alpha < \left( \frac{\rho}{\rho^{\rho-1} - 1} \right) \) and consequently \( \rho^{\rho-1} (1 - \alpha) - (\rho - \alpha) > 0 \). Noting \( \rho^{\rho-1} (\alpha^{2} \rho + 1 - \alpha) - (\rho - \alpha + \alpha^{2} \rho) > 0 \), \( D_{i} < 0 \) (or \( W^{b1} < W^{m1} \)) if and only if \( s < s_{i} \), where

\[
s_{i} = \frac{\rho^{\rho-1} (1 - \alpha) - (\rho - \alpha)}{\rho^{\rho-1} (\alpha^{2} \rho + 1 - \alpha) - (\alpha^{2} \rho + \rho - \alpha)} .
\]

Recall from proposition 3 and 9 that a higher rate of subsidy increases social welfare in the benchmark scenario as long as \( s < \frac{1-\alpha \rho}{1-\alpha (1-\alpha \rho)} \) and in the scenario with retailer buyer power as long as \( s < \frac{\rho (1 - \alpha)}{\alpha^{2} \rho + \rho - \alpha} \). Given the assumption \( \rho^{\rho-1} - \rho - 1 > 0 \), we can show that

\[
s_{i} > \frac{1-\alpha \rho}{1-\alpha (1-\alpha \rho)} .
\]

Since \( \frac{1-\alpha \rho}{1-\alpha (1-\alpha \rho)} < \frac{\rho (1 - \alpha)}{\alpha^{2} \rho + \rho - \alpha} \), it always holds that \( W^{b1} < W^{m1} \) in the range of \( \left[ 0, \frac{1-\alpha \rho}{1-\alpha (1-\alpha \rho)} \right] \), where a subsidy improves social welfare in both scenarios.

QED
Proposition 12 is not surprising given the fact that the presence of retailer buyer power makes the representative household pay higher retail prices and consume fewer varieties, and less resources are allocated to the differentiated goods industry. Although the level of social welfare is lower in the presence of the retailer, the marginal effect of a subsidy is not necessarily smaller in this scenario.

**Proposition 13.** In the range where a subsidy on the unit cost of production improves social welfare in both the benchmark scenario and the scenario where products are distributed through a retailer,

i) the marginal effect of the subsidy on social welfare is always greater in the scenario with retailer buyer power than the benchmark scenario if \( \alpha > \alpha_1 \),

\[
\alpha_1 = \frac{\frac{1}{\rho} - \frac{\rho}{\rho - 1}}{\frac{1}{\rho} - 1};
\]

ii) if \( \alpha < \alpha_1 \), there exists a threshold level of policy \( s_2 \), where

\[
s_2 = \frac{\frac{\rho}{\rho - 1} - \frac{\rho}{\rho - 1} - \rho(1 - \alpha)}{\rho - 1} \left( \frac{1}{\rho - 1} - \frac{1}{\rho - 1} \right) \left( \rho - 1 - \rho + \rho \alpha^2 \right)
\]

such that for \( s_2 < s < \frac{1 - \alpha \rho}{1 - \alpha(1 - \alpha \rho)} \) the marginal effect of the subsidy on social welfare is greater in the scenario with buyer power while for \( s < s_2 \) the marginal effect is less in the scenario with buyer power.
Proof. Subtracting (2.80) by (2.37), we have

\[
\frac{\partial W_m}{\partial s} - \frac{\partial W_{ml}}{\partial s} = \frac{\alpha^{\rho^{-1}}}{(\rho-1)(1-s)} \left( \rho \left( 1-\alpha \right) \int_{1-s}^{1} \frac{\partial}{\partial \alpha} \left( s \alpha \left( 1+\alpha \right) - \alpha(1+\alpha) \right) - \rho^{\rho^{-1}} \left( 1-\alpha \right) - \rho^{\rho^{-1}} \left( 1-\alpha \right) \right).
\]

(2.86)

Let \( D_2 = \rho - \alpha - \frac{\alpha^2 \rho}{1-s} - \frac{\alpha(1-\rho)}{1-s} - \rho^{\rho^{-1}} \left( 1-\alpha \right) - \frac{s \alpha(1+\alpha) - \alpha(1+\alpha)}{1-s} \). It can be shown that \( D_2 > 0 \) if and only if \( s \left( \rho^{\rho^{-1}} \left( \alpha^2 \rho + 1 - \alpha \right) - \left( \rho - \alpha + \alpha^2 \rho \right) \right) > \rho^{\rho^{-1}} (1-\alpha) - \rho(1-\alpha) \). Recall that

\[
\rho^{\rho^{-1}} (\alpha^2 \rho + 1 - \alpha) - (\rho - \alpha + \alpha^2 \rho) > 0.
\]

When \( \rho^{\rho^{-1}} (1-\alpha) - \rho(1-\alpha) < 0 \) (which is equivalent to \( \alpha > \alpha_1 \)), \( D_2 > 0 \) for all \( s \). On the other hand, if \( \rho^{\rho^{-1}} (1-\alpha) - \rho(1-\alpha) > 0 \) (\( \alpha < \alpha_1 \)), \( D_2 > 0 \) if \( s > s_2 \) and \( D_2 < 0 \) if \( s < s_2 \).

It can also be easily shown that \( s_2 < \frac{1-\alpha \rho}{1-\alpha(1-\alpha \rho)} < \frac{\rho(1-\alpha)}{\alpha^2 \rho + \rho - \alpha} \). Since a higher level of government subsidy increase social welfare in the benchmark scenario as long as

\[
s < \frac{1-\alpha \rho}{1-\alpha(1-\alpha \rho)}
\]

and in the scenario with buyer power as long as \( s < \frac{\rho(1-\alpha)}{\alpha^2 \rho + \rho - \alpha} \), a subsidy increases social welfare for both scenarios in the range of \( \left[ 0, \frac{1-\alpha \rho}{1-\alpha(1-\alpha \rho)} \right] \). Therefore, when \( \alpha < \alpha_1 \), \( D_2 > 0 \) (or \( \frac{\partial W_m}{\partial s} > \frac{\partial W_{ml}}{\partial s} \)) if \( s_2 < s < \frac{1-\alpha \rho}{1-\alpha(1-\alpha \rho)} \) and \( D_2 < 0 \) if \( s < s_2 \).

QED
Proposition 13(i) establishes that if the elasticity of demand of products in the differentiated goods industry is sufficiently high, a subsidy is more effective in improving social welfare in the presence of retailer buyer power. However, Proposition 13(ii) shows that if the demand becomes less elastic, the relative effectiveness of the subsidy in the two cases depends on subsidy level.

2.5.2 Subsidy on the Fixed Cost of Production

Now consider the case where the subsidy is on the fixed cost of production and let \( s = 0 \) in (2.60) to (2.62) and (2.67) to (2.72).

**Proposition 14.** When the government subsidizes the fixed cost of production, an increase in the rate of subsidy \( \tau \)

i) reduces fixed fees that retailer \( R \) pays;

ii) has no impact on either wholesale prices or retail prices;

iii) increases the number of product varieties but reduces each firm’s output;

iv) attracts more resources to the differentiated goods industry; and

v) improves social welfare if and only if \( \tau < \frac{1}{1 + \alpha} \).
Proof. It can be clearly seen from (2.60) and (2.62) that a subsidy on the fixed cost has no impact on wholesale or retail prices. Differentiating (2.61) and (2.67) to (2.69) with respect to the subsidy \( \tau \), we have

\[
\partial k_i^{b_2} / \partial \tau = -F < 0, \quad (2.87)
\]

\[
\frac{\partial N^{b_2}}{\partial \tau} = \frac{\rho(1-\alpha)}{(\rho - 1)(1-\tau)} \left( \frac{1-\alpha \rho}{\alpha} \right)^{\frac{1}{\rho-1}} \left( \frac{1-\tau F}{\alpha F} \right)^{\frac{\rho(1-\alpha)}{\rho-1}} > 0, \quad (2.88)
\]

\[
\frac{\partial x^{b_2}}{\partial \tau} = -\frac{\alpha \rho}{1-\alpha \rho} \frac{F}{a_x} < 0, \quad (2.89)
\]

\[
\frac{\partial Z^{b_2}}{\partial \tau} = \left( \frac{1-\alpha \rho}{\rho} \right)^{\frac{1}{\rho-1}} \frac{\rho(1+\alpha)}{\rho-1} \left( (1-\tau F)^{\frac{1-\alpha}{1-\rho}} a_x^{\frac{\alpha}{1-\rho}} \right) \frac{1-\alpha \rho}{\rho(1-\tau)^2} \left( \frac{1-\tau \rho}{\rho-1} + 1 \right) < 0, \quad (2.90)
\]

and

\[
\frac{\partial W^{b_2}}{\partial \tau} = \left( \frac{\rho}{1-\alpha \rho} \right)^{\frac{1}{\rho-1}} \frac{1-\alpha \rho}{\alpha} \left( (1-\tau F)^{\frac{1-\alpha}{1-\rho}} a_x^{\frac{\alpha}{1-\rho}} \right) \frac{1-\alpha \rho}{(1-\tau)(\rho - 1)} \left( 1 - \frac{\alpha \tau}{1 - \tau} \right), \quad (2.91)
\]

which is positive if and only if \( \tau < \frac{1}{1 + \alpha} \). QED

When the government subsidizes the fixed cost of production, similar effects are observed as those in the case where products are sold directly to consumers: the number of varieties increases; output of each product is reduced; more resources are allocated to the differentiated goods industry and social welfare is improved as long as the rate of subsidy is
at a moderate level. However, the underlying mechanism is different. When the government raises the rate of subsidy, retailer $R$ negotiates a lower fixed fee by the full amount of the subsidy to appropriate the gains from producers (as can be seen from (2.61)). This increases the retailer’s profits and consequently the marginal gain from adding an additional product, which in turn motivates the retailer to carry more products. As more products are competing in the market, the demand for each variety is reduced and as a result the output of each variety in the differentiated goods industry is reduced. The following two equations further confirm what happened.

Letting $p_i = p$, $x_i = x$, retailer $R$’s profit function can be written as

$$\pi_R = N(px - a)x + N\alpha \tau.$$  \hspace{1cm} (2.92)

Differentiating equation (2.70) with respect to $\tau$, we obtain

$$\frac{\partial \pi_R}{\partial \tau} = \frac{1}{(1 - \tau)} \left( \frac{\rho}{1 - \alpha \rho} \right)^{\frac{\rho(1-\alpha)}{1-\rho}} \alpha \frac{\rho^{(1-\rho)}}{\rho^{\rho - 1}} \left( (1 - \tau) F \right)^{1 - \alpha \rho} a^{-\alpha \rho} > 0.$$  \hspace{1cm} (2.93)

That is, retailer $R$ extracts the entire amount of subsidy given to producers and earns a higher level of profits as the rate of subsidy increases. Despite this fact, proposition 14 shows that a subsidy, similar to the case where products are sold directly to consumers, can help channel the resources to the right industry, increase product variety and improve social welfare.

Of course there are quantitative differences. The following three propositions show the effects of retailer buyer power on product variety, social welfare and the effectiveness of government subsidy.
Proposition 15. At any given subsidy rate, the number of product varieties is lower in the case where products are distributed through a retailer than when they are sold directly to consumers. Furthermore, a higher rate of subsidy leads to a smaller increase in the number of varieties in the presence of retailer buyer power.

Proof. Subtracting (2.26) from (2.67) and (2.42) from (2.88), we have

\[
N^{b_2} - N^{m_2} = \left( (1 - \tau) F \right)^{\rho(1 - \alpha)} \frac{\alpha^p}{\rho^p} \alpha^p \frac{1}{1 - \rho} \left( 1 - \frac{\rho}{1 - \alpha \rho} \right) \left( 1 - \rho^{\frac{1}{\rho - 1}} \right) < 0. \quad (2.94)
\]

\[
\frac{\partial N^{b_2}}{\partial \tau} - \frac{\partial N^{m_2}}{\partial \tau} = \left( (1 - \tau) F \right)^{\rho(1 - \alpha)} \frac{\alpha^p}{\rho^p} \alpha^p \frac{1}{1 - \rho} \left( 1 - \frac{\rho}{1 - \alpha \rho} \right) \frac{\rho(1 - \alpha)}{\rho - 1} \left( 1 - \rho^{\frac{1}{\rho - 1}} \right) < 0. \quad (2.95)
\]

QED

Proposition 15 shows that the number of varieties is smaller and a subsidy is less effective in the presence of retailer buyer power. Further, as the rate of subsidy increases, the number of product varieties between the two scenarios diverges. The underlying mechanism is similar to the case where the government subsidizes the unit cost of production. That is, in comparison with the case where products are distributed to consumers directly and producers make entry decisions independently, retailer \( R \) internalizes the negative impact of an increase in the number of varieties on its profits and therefore carries a smaller number of products. It is also due to the internalization mechanism that the retailer adds fewer new products in response to a marginal increase in subsidy.
Proposition 16. Assume that \( \rho^{\rho^{-1}} - \rho - 1 > 0 \). In the range where a government subsidy on the fixed cost of production improves social welfare for both the case where products are distributed through a retailer and the case where products are sold directly to consumers, social welfare is always lower in the case with buyer power.

Proof. Subtracting (2.27) from (2.72), we have

\[
W^{u2} - W^{u1} = \frac{\rho}{\rho^{1-\alpha} - (1-\alpha)F} \alpha^{1-\rho} \rho^{1-\rho} \left[ \rho - \frac{\alpha}{1-\tau} \alpha(1-\alpha) - \rho^{\rho^{-1}} \left( 1 - \frac{\tau}{1-\rho} \alpha(1-\alpha) \right) \right].
\]

(2.96)

Let \( D_3 = \rho - \alpha - \frac{\tau}{1-\rho} \alpha(1-\alpha) - \rho^{\rho^{-1}} \left( 1 - \frac{\tau}{1-\rho} \alpha(1-\alpha) \right) \). It can be shown that \( D_3 < 0 \) if and only if \( \frac{\tau}{1-\rho} \alpha(1-\alpha) - \rho^{\rho^{-1}} (1 - \alpha) - (\rho - \alpha) \). Under the assumption

\[
\rho^{\rho^{-1}} - \rho - 1 > 0, \quad \rho^{\rho^{-1}} (1 - \alpha) - (\rho - \alpha) > 0.
\]

Noting that \( \alpha(1-\alpha) \left( \rho^{\rho^{-1}} - 1 \right) > 0 \), \( D_3 < 0 \) if and only if \( \tau < \tau_1 \), where \( \tau_1 = \frac{\rho^{\rho^{-1}} (1 - \alpha) - (\rho - \alpha)}{\rho^{\rho^{-1}} (1 - \alpha^2 - \rho) - \rho(1 - \alpha^2)} \).

Recall from propositions 5 and 14 that a higher rate of government subsidy on the fixed cost of production increases social welfare in the benchmark scenario as long as \( \tau < \frac{1 - \alpha \rho}{1 - \alpha^2 \rho} \) and in the scenario with buyer power when \( \tau < \frac{1}{1 + \alpha} \). Given the assumption of

\[
\rho^{\rho^{-1}} - \rho - 1 > 0, \quad \text{it can be shown that } \tau_1 > \frac{1 - \alpha \rho}{1 - \alpha^2 \rho}.
\]

Since \( \frac{1 - \alpha \rho}{1 - \alpha^2 \rho} < \frac{1}{1 + \alpha} \), it always holds
that $D_3 < 0 \ (W^b < W^m)$ in the range of $\left[0, \frac{1 - \alpha \rho}{1 - \alpha^2 \rho}\right]$ where a subsidy improves social welfare for both scenarios. QED

Therefore, at any given subsidy rate that improves social welfare in both scenarios, welfare is always lower in the presence of retailer buyer power. This is not surprising given the fact that the number of varieties is lower, and fewer resources are allocated to the differentiated goods industry. Despite the lower level of social welfare, the following proposition shows that, under certain circumstances, the marginal effect of a government subsidy on social welfare is actually larger in the presence of retailer buyer power.

**Proposition 17.** In the range where a government subsidy on the fixed cost of production improves social welfare for both the case where products are distributed through a retailer and the case where products are sold directly to consumers,

i) the marginal effect of the subsidy on social welfare is always greater in the scenario with buyer power than the benchmark scenario when $\alpha > \alpha_t$, where

$$\alpha_t = \frac{\rho^{\rho^{-1}-1}}{\rho^{\rho^{-1}}-1}$$; and

ii) if $\alpha < \alpha_t$, there exists a threshold level of policy $\tau_z$, where
\[
\tau_2 = \frac{\frac{1}{\rho^{\rho-1}}(1-\alpha\rho)-(1-\alpha)}{\rho^{\rho-1}(1-\alpha^2\rho)-(1-\alpha^2)}
\]

such that for \( \tau_2 < \tau < \frac{1-\alpha\rho}{1-\alpha^2\rho} \) the marginal effect of the subsidy on social welfare is greater in the scenario with buyer power while for \( \tau < \tau_2 \) the marginal effect is less in the scenario with buyer power.

**Proof.** Subtracting (2.43) from (2.91), we have

\[
\frac{\partial W^{b2}}{\partial \tau} - \frac{\partial W^{m2}}{\partial \tau} = \left(1-\alpha\rho(\alpha^{\frac{\rho}{\rho-1}})\right) \left(\frac{\rho}{1-\alpha}\right)^{\frac{1}{\rho-1}} (1-\tau)^{\frac{\rho}{\rho-1}} \left(1-\alpha\right) - \rho^{\frac{\rho}{\rho-1}} \left(1-\alpha\rho(1-\alpha)\right). \tag{2.97}
\]

Let \( D_4 = (1-\alpha)\left(1-\frac{\alpha\tau}{1-\tau}\right) - \rho^{\frac{\rho}{\rho-1}} \left(1-\alpha\rho - \frac{\tau\alpha(1-\alpha)}{1-\tau}\right) \). It can be shown that \( D_4 > 0 \) if and only if

if \( \frac{\tau}{1-\tau}(1-\alpha) > \rho^{\frac{\rho}{\rho-1}} - (1-\alpha) \). Since \( \rho^{\frac{\rho}{\rho-1}} - 1 > 0 \), \( D_4 > 0 \) when

\( \rho^{\frac{\rho}{\rho-1}}(1-\alpha\rho)-(1-\alpha) < 0 \) (which is equivalent to \( \alpha > \alpha_4 \)). On the other hand, if

\( \rho^{\frac{\rho}{\rho-1}}(1-\alpha\rho)-(1-\alpha) > 0 \) (or \( \alpha > \alpha_4 \)), \( D_4 > 0 \) if and only if \( \tau > \tau_2 \).

Furthermore, it can be shown that \( \tau_2 < \frac{1-\alpha\rho}{1-\alpha^2\rho} < \frac{1}{1+\alpha} \). Since a higher level of subsidy increases social welfare in the benchmark scenario as long as \( \tau < \frac{1-\alpha\rho}{1-\alpha^2\rho} \) and in the scenario with buyer power as long as \( \tau < \frac{1}{1+\alpha} \), a subsidy increases social welfare for both.
scenarios in the range of $\left[0, \frac{1-\alpha\rho}{1-\alpha^2\rho}\right]$. Therefore, when $\alpha < \alpha_1$, \( \frac{\partial W^{b^2}}{\partial s} > \frac{\partial W^{m^2}}{\partial s} \) if

$$\tau_2 < \tau < \frac{1-\alpha\rho}{1-\alpha^2\rho} \quad \text{while} \quad \frac{\partial W^{b^2}}{\partial s} < \frac{\partial W^{m^2}}{\partial s} \quad \text{if} \quad \tau < \tau_2.$$  

\[QED\]

Similar to the case where the government subsidizes the unit cost of production, Proposition 17(i) establishes that an increase in the rate of subsidy on the fixed cost of production is always more effective in improving social welfare in the presence of retailer buyer power when the elasticity of demand of products in the differentiated goods industry is sufficiently high. In addition, Proposition 17(ii) shows that when the demand is less elastic, the relative effectiveness of the subsidy in the two scenarios depends on the level of the subsidy.

2.5.3 Combination of the two types of subsidies

Recall from section 2.4.3 that a combination of optimal subsidies on the unit cost and fixed cost is able to achieve the social optimum. In this section we examine whether this holds true in the presence of retailer buyer power.

\textit{Proposition 18.} When products are sold through a retailer with all the power, a combination of a subsidy on the unit cost of production and a subsidy on the fixed costs achieves the social optimum. The optimal subsidizing rates are $s^{b^v} = \tau^{b^v} = 1 - \alpha$. 

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Proof. The government maximizes social welfare by choosing \( s \) and \( \tau \)

\[
\max_{s,\tau} W = H + \frac{1}{\rho (1 - \alpha \rho)} \left( \frac{\rho}{1-\rho} \right)^{1-\alpha \rho} \frac{1+\alpha \rho}{\alpha^{\rho^{-1}}} \left( (1 - \tau) F \right)^{1-\alpha \rho} \left( (1 - s) a_i \right)^{\alpha \rho} \left( \rho - \alpha - \frac{s \alpha^2 \rho}{1-s} - \frac{\alpha (1 - \alpha \rho) \tau}{1-\tau} \right)
\]

(2.98)

The first-order conditions are

\[
\left( \frac{\rho}{1-\alpha \rho} \right)^{1-\alpha \rho} \frac{1+\alpha \rho}{\alpha^{\rho^{-1}}} \frac{(1 - \tau) F}{(1-s)(\rho-1)} \left( (1 - s) a_i \right)^{\alpha \rho} \left( \rho - \alpha - \frac{s \alpha^2 \rho}{1-s} - \frac{\alpha (1 - \alpha \rho) \tau}{1-\tau} - \frac{\alpha (\rho - 1)}{1-s} \right) = 0 \tag{2.99}
\]

\[
\left( \frac{\rho}{1-\alpha \rho} \right)^{1-\alpha \rho} \frac{1+\alpha \rho}{\alpha^{\rho^{-1}}} \frac{(1 - \tau) F}{\rho (1-\tau)(\rho-1)} \left( (1 - s) a_i \right)^{\alpha \rho} \left( \rho - \alpha - \frac{s \alpha^2 \rho}{1-s} - \frac{\alpha (1 - \alpha \rho) \tau}{1-\tau} - \frac{\alpha (\rho - 1)}{1-\tau} \right) = 0 \tag{2.100}
\]

Solving (2.99) and (2.100), we obtain

\[
s^{b*} = \tau^{b*} = 1 - \alpha \tag{2.101}
\]

Substituting (2.101) in (2.62) and (2.67) in (2.70), it can be shown that with the subsidies, the prices, quantities and varieties and consequently social welfare are the same as those under the unconstrained social optimum. QED

Therefore, in the presence of retailer buyer power, although a subsidizing policy that combines the two types of subsidies increases the retailer’s profits, it does help to reach the social optimum. Further, a comparison with the results from proposition 8 demonstrates that the rate of subsidy to reach the social optimum in the presence of retailer buyer power is larger than the case where products are directly sold to consumers. This is consistent with the
results from section 2.5.1 and 2.5.2 that at any given subsidy rate, the number of product varieties and social welfare are always lower when products are sold through a retailer.

2.6 Discussion

In this section, we discuss a few assumptions made in this chapter and how these assumptions would affect the results.

2.6.1 Income Effects on Consumption Behaviors

In this model the income effects are assumed away. As long as the effects are not significantly large, it would not qualitatively impact our results. In the presence of buyer power, the entire amount of subsidy is appropriated by the retailer (as seen from (2.79) and (2.91)). Since the profits of the retailer are given back to the representative household, a lump sum tax would not affect the household’s disposable income and consequently the consumption decision. Thus a subsidy still works to increase product variety as in the case where income effects are assumed away. On the other hand when the products are sold directly to consumers, the tax reduces the household’s disposable income and consequently the demand for the differentiated goods. Given the fact that the effects of a subsidy on producers’ profits are of the first order, it will increase producers’ profits and promote product diversity. The difference is that, when income effects are taken into account, the effects of subsidies on product variety become smaller. In comparison with the case where products are sold through the retailer, if the income effects are not significantly large, the result that retailer buyer power reduces the effectiveness of subsidies still holds.
2.6.2 Income Distributional Effects

Related to the above is the assumption of one representative household, which implies that the income distribution problems are neglected. Alternatively suppose there are two representative households, one is rich and the other is poor. The differentiated goods are regarded normal by the rich but luxury by the poor. The demand faced by producers thus is the combined demand of the two households. Taxes are levied and also firms’ profits distributed to households in proportion to their endowment. In the presence of buyer power, a subsidy does not affect either household’ disposable income and consequently would not have distributional effects. On the other hand, when products are distributed directly to consumers, a subsidy leads to a further divergence of the two households’ consumption behaviours since in this case a lump-sum tax decreases disposable income and the poor reduces its demand for the differentiated goods more than the rich. That is, as the rate of the subsidy increases, the poor is increasingly taxed to support an industry where it has less consumption and consequently benefit less from the subsidizing policy. When the differences in consumption preferences of the two households are large or the subsidizing rate is high, the poor is likely to be hurt as the subsidy and the demand from the rich support a high level of product diversity in the differentiated goods industry while at the same time the poor is paying for the expansion of this industry but can only afford a small amount of these goods. As a result, a subsidy could reduce social welfare if the decrease of the poor’s utility outweighs the increase of the rich. Also, welfare is not necessarily always higher at any subsidizing rate than the case where products are distributed through the retailer. Despite the differences between the two households’ consumption behaviours and welfare, the effects of a subsidy on producers’ profits depend on how the combined demand responds to
the lump sum tax. As long as the income effects on the combined demand is not significantly large, a subsidy increases product diversity and is more effective in this respect compared to the case where products have to sell through a downstream retailer.

2.6.3 Trading Interface between the Retailer and Producers

Another important assumption of this model is that the retailer uses the all-or-nothing-pricing and has the power to decide the number of upstream products. Suppose first we modify the model so that the two-part tariff is negotiated between the retailer and its supplier. In this case, the wholesale price will still be set equal to the marginal cost to maximize the joint profits but the fixed fee will be used to split the gain from the trade. Greater buyer power is manifested through less fixed fees paid by the retailer. A subsidy increases the joint profits of the retailer and its supplier and consequently the share of both parties’ profits. The retailer would still want to increase the number of products it carries but to a less extent compared to the case of “all-or-nothing-pricing” since it can only capture part of the benefits from the subsidy. Thus compared to the case where products are distributed to consumers directly, the increase in the number of product varieties in response to a subsidy is even less. We could further modify the model so that at the first stage it is the producers instead of the retailer that decide whether a product should be introduced. In this case the number of product varieties hinges on producers’ profit level. Since a subsidy increases producers’ profits, the number of product varieties will increase. Compared to the case where products are distributed directly to consumers, a subsidy increases the joint profits of the retailer and its supplier to a greater extent since the retailer is able to internalize the downstream competition among the products it carries. As a result the supplier tends to obtain greater
profits. On the other hand, part of the increased profits is appropriated by the retailer. When the retailer buyer power is relatively large, the increase in the producers’ profits is less and thus marginal increase in the number of product varieties as a result of a subsidy remains smaller in the presence of buyer power as predicted by our model.

2.6.4 Strategic interaction among producers

We maintain the usual assumption of a Dixit-Stiglitz model of monopolistic competition that the influence of an individual price change on the price index is negligible for the case where products are distributed directly to consumers. That is, the strategic interactions among producers are assumed away. A relaxation of this assumption would not change our results qualitatively. When a producer sets its own price in this case, it not only accounts for the direct but also the indirect effects of its own price change on the demand for its products. The latter effect is positive and thus the own elasticity of demand is smaller compared to the case where the strategic interaction among producers are assumed away\(^{28}\). As a result although a subsidy on the unit cost of production increases profits, the magnitude is smaller leading to a smaller increase in the number of product varieties. Compared to the case where products are distributed through a retailer, the effectiveness of a subsidy on product varieties remains greater. In both cases the direct and indirect effects of an individual price change on demand is accounted for. However in the presence of buyer power, the retailer internalizes any negative effects of “excess competition” from too many products. A subsidy thus is

\[^{28}\text{After relaxing the assumption, the own elasticity of demand changes from } -1/(1-\alpha p) \text{ to}\]

\[-\frac{1}{1-\alpha p} + \frac{\alpha(p-1)}{(1-\alpha p)(1-\alpha)} \left( \sum_j p_{,\alpha p^{-1}} \right)^{-1} p_{,\alpha p^{-1}}. \]

The second part is positive and reflects how an individual price change would increases the general price index and in turn increases its own demand.
always less effective in the presence of buyer power. On the other hand, the effect of a subsidy on the fixed costs should not change as the price decisions of producers remain unchanged. The results in section 2.4 and 2.5 still hold.

2.7 Conclusion

The wide use of government subsidies to promote cultural diversity and the increasingly concentrated retail market for the cultural products motivated our study. In this paper, we examine the implication of retailer buyer power on product diversity and the effectiveness of government subsidies in the presence of such power. Using an extension of the Dixit-Stiglitz model of monopolistic competition, we compare the equilibrium number of product varieties, social welfare and the effects of subsidies in the case where products are sold directly to consumers and the case where they are sold through a monopoly retailer with buyer power.

We demonstrate that in the framework of monopolistic competition retailer buyer power exacerbates the problem of insufficient product diversity and reduces social welfare. However, the mechanism is different from what has been commonly argued as the hold-up problem of suppliers. Our paper shows that when the retailer is faced with the choice over a group of substitutable products, it may find it more profitable to carry a smaller number of products in order to reduce the level of competition in the downstream market. This is of particular relevance to the case where producers cannot access end consumers efficiently without using the retailer. The retailer acts as the gatekeeper to the downstream market and determines the number of products that are available to consumers. One example is the cable or satellite television company which controls subscriber access to the many potentially
available pay- and specialty-channels. Such a mechanism reflects the important role of the retailer in determining what is accessible to consumers, and does not necessarily reflect consumer preferences.

Product diversity is an important dimension of consumer and total welfare. Our paper brings forth a mechanism by which the buyer power may result in a loss in diversity and harm end consumers. Empirically, the ability of the antitrust authority to restrain the exercise of buyer power is rather limited (OECD 2008). The task of sustaining a desirable level of product diversity in the presence of buyer power, if required, therefore may be better assigned to a regulatory agency.

Common measures used by regulatory agencies to sustain or develop a level of pluralism in cultural expression include the imposition of reasonable scheduling, expenditure requirements on cultural gatekeepers and subsidies and tax incentives. Subsidies are more likely than other measures to be the subject of continuing evaluation and are widely used by governments across countries. Our paper shows that both a subsidy on the unit costs and a subsidy on the fixed costs that are given to producers raise the number of product varieties and enhance social welfare. Moreover, a proper combination of the two types of subsidies is able to achieve the social optimum. These results are true even when products are distributed through a downstream gatekeeper who has all the bargaining power and is able to appropriate the entire amount of subsidy given to producers. Further, in comparison with the case where products are distributed directly to consumers, retailer buyer power reduces the effectiveness of the subsidies in promoting product diversity as the retailer internalizes any “excess” competition it brings by introducing additional products.
Chapter 3

Subsidy on Complementary Products
in a Model of Monopolistic Competition
3.1 Introduction

The market under monopolistic competition yields suboptimal product diversity and a government subsidy could be welfare enhancing according to a number of analyses. Dixit and Stiglitz (1977) and Spence (1976) demonstrate that the equilibrium number of differentiated products can be higher or lower than that of the (constrained) social optimum using models based on a representative consumer formulation. Furthermore, Spence shows that in the case of insufficient product diversity, a subsidy to induce additional varieties is welfare enhancing. Costrell (1990) extends their models and allows for a variable elasticity of substitution among differentiated products. He concludes that no matter whether the market yields too many or too few product varieties, an infinitesimal subsidy always improves social welfare. Broer and Heijdra (2001) and Heijdra (1998, p. 671) show in a dynamic model of a closed economy that a subsidy is a useful instrument to improve social welfare under standard Dixit-Stiglitz (1977) assumptions because it corrects the monopoly distortion from factor demands and yields the correct number of firms.

With the exception of Spence (1976), these papers assume demand linkages where goods are substitutes with no complements in existence. But in reality some markets are composed of complementary products\textsuperscript{29}. One typical example is the adoption of hydrogen cars\textsuperscript{30} and billions of dollars spent by governments worldwide in subsidies in an effort to make this a reality\textsuperscript{31}. The market of hydrogen vehicles consists of not only the specially

\footnote{29 For empirical support for the importance of complementarities, see e.g. Bartelsman et al. (1994), Cooper and Haltiwanger (1996), Cooper and Johri (1997), and Miyagawa (1996).}

\footnote{30 The adoption of hydrogen vehicle is supported by governments mainly for the following two benefits. First, there are a number of different ways to produce hydrogen which allow the nation to be less susceptible to price and supply shocks. Second, it is a clean energy and can achieve significant emission reduction.}

\footnote{31 The Economist, Technology Quarterly Q3 2008}
designed parts and vehicles, but also hydrogen infrastructure, hydrogen production, storage and distribution devices and vehicle repair services etc. These products are complementary as the use of one of these goods becomes highly inconvenient or sometimes impossible without penetration of some or most of them.

Complementary goods are different from substitutable goods in that the marginal benefit from consuming each of the complementary goods increases with the consumption of other complementary goods. Thus the greater is the variety or consumption of other goods, the greater is the profits of the firm producing one of these goods. Two properties then follow given demand complementarity: First, since the entering firm does not take into account its positive effects on other firms’ profits, the externalities in the payoff of the firms will result in lower product diversity; Second, there could be coordination failure in entry decisions where although mutual gain could be realized, no firm has an incentive to enter the market when it believes that no other firms would enter the market. This leaves open the question of whether or not there exist mechanisms that can overcome these coordination failures.

Most of theoretical work related to product complementarity (for example, Economides and Salop 1992, Church and Gandal 1992 and 1996, Carter and Wright 1994; Brueckner 2004) assume an exogenous number of products to be consumed or to be combined into a composite good. Very few study optimal product diversity in the framework of monopolistic competition and none has studied the effects of a subsidy on social welfare. One related article is Spence (1976) who argues that when products are complementary, both the fixed cost of production and the externality in payoff tends towards too few products under monopolistic competition. He uses a partial equilibrium model and shows that when the
income effects are assumed away, there is a unique equilibrium where both the output and the number of product varieties are lower than the social optimum. Although sufficient for his purpose, Spence does not consider situations where none of the complementary products is produced or all resources are used to produce these products. Neither does he consider the effects of a government subsidy.

Also related is the paper by Lin (1996) who examines the welfare effects of tariff protection when import and domestic goods are complements in a framework of monopolistic competition. He demonstrates that with complementary goods, tariff protection which is shown to benefit the importing country in the case of substitutable products may become a welfare-reducing policy if it harms the import complementing sector more than it assists the import substituting sector. An explicit analysis of optimal product diversity and the effects of tariff on product diversity is not included, though.

This paper seeks to re-examine product diversity and the effects of a government subsidy in the case of complementary products under monopolistic competition. This examination builds upon the standard Dixit-Stiglitz model of monopolistic competition which posits a representative consumer utility function defined over the quantities of a number of differentiated products and an outside numeraire good. What is different is that we assume the differentiated products are complementary instead of being substitutable. Our results show that depending on the magnitude of the fixed cost of production there are three possible market equilibriums. When the fixed costs are sufficiently low, the complementary goods industry absorbs all resources, leaving no products being produced in the other industry (Equilibrium I). On the other hand if the fixed costs are sufficiently high, the complementary goods industry becomes non-existent because revenues fail to cover the fixed
cost of production (Equilibrium III). Only when the fixed costs are intermediate, products from both the complementary goods industry and the rest of economy will be produced (Equilibrium II). It is also within this range is there an overlap of two or all of these three equilibriums. When this occurs, firms’ beliefs play a critical role in determining which equilibrium prevails. If no firm expects others to enter this market, the limited demand for a single product will not generate enough revenue and no firm is willing to enter the market. Hence the complementary goods industry would not exist (Equilibrium III). On the other hand, if each firm expects others to enter the market, the greater variety increases demand for all complementary goods and production becomes profitable. As a result, firms would indeed enter and the economy ends up with greater product diversity (Equilibrium I or II).

When comparing the three market equilibriums with the optimum constrained by monopolistically competitive pricing, we find that the number of product varieties could be higher or lower than that of the constrained optimum, which implies that there is distortion in product diversity and it is not merely a by-product of monopolistic competitive pricing. An infinitesimal subsidy on the fixed cost of production may induce a switch of the market equilibriums and always improves social welfare if it does induce the switch. In the case where the subsidy does not induce a switch the policy is proved futile in improving social welfare.

Our paper is organized as follows. Section 3.2 outlines the model; Section 3.3 solves for the multiple market equilibriums. Section 3.4 studies the constrained social optimums and identifies possible market failures. Section 3.5 studies the effects of government subsidy. Section 3.6 concludes.
3.2 The Model

Consider a monopolistically competitive industry with \( N \) producers. The \( N \) producers produce products (brands) that are viewed by consumers as complementary products, indexed by \( i \). The rest of the economy is aggregated into one good \( Z \), which is chosen to be the numeraire good. The market for \( Z \) is perfectly competitive.

Following Dixit and Stiglitz (1977) and Spence (1976), we model the benefits of variety through strictly convex preferences of a representative consumer over \( \tilde{N} \) potential products, among which only \( N \) is produced. Assume the preferences take the functional form

\[
U = Z + \left( \sum_{i} x_i^{\alpha} \right)^{\frac{1}{\rho}}.
\]

For products \( x_i \) and \( x_j \) to be complementary, we need \( \partial^2 U / \partial x_i \partial x_j > 0 \) which requires \( \rho < 1 \). Meanwhile positive marginal utility of consumption implies \( \alpha > 0 \). In a symmetric situation where the representative consumer consumes the same quantity of \( N \) complementary products, the utility function is re-written as

\[
U = Z + N^{\frac{1}{\rho}} x^\alpha.
\]

It is assumed that \( \alpha < 1 \) to ensure that utility is concave in \( x \). Also we let \( \rho > 0 \) so that utility increases with the number of products, indicating consumer’s preference for diversity.

The representative consumer is endowed with \( H \) units of labor, which is supplied inelastically. Labor is the only factor of production and is perfectly mobile across the two industries. For simplicity, we normalize the wage rate to 1. Thus the budget constraint of the consumer is

\[
I = Z + \sum_{i} p_i x_i = I,
\]

where \( I \) is the consumer’s disposable income and equals the wage the consumer receives \( (H) \) plus the profits of the firms \( (\sum \pi_i) \) and minus lump-sum tax by the government, as the case may be.
The consumer maximizes his utility subject to the budget constraint, which is formulated as follows

$$\max_{x_i, Z} U = Z + \left( \sum_{i}^{N} x_i^{\alpha} \right)^{\frac{1}{\rho}}$$

$$s.t. \ Z + \sum_{i}^{N} p_i x_i \leq I ,$$

$$Z \geq 0.$$ (3.3)

Note that the quasi-linear utility function permits the possibility that the consumer purchases zero units of Z.\footnote{The consumer will not set $x_i = 0$ because the marginal utility of consuming $x_i$ approaches infinity as $x_i$ is close to zero. In other words, the consumer will consume a positive quantity of each complementary goods as long as it is available in the market.}

To solve this problem, we set up the Lagrangean $L$

$$Z + \left( \sum_{i}^{N} x_i^{\alpha} \right)^{\frac{1}{\rho}} + \lambda_0 \left( I - Z - \sum_{i}^{N} p_i x_i \right) + \lambda_Z Z .$$ (3.4)

The Kuhn-Tucker conditions are:\footnote{Since the utility strictly increases with $Z$ and $x_i$, the budget constraint is binding and consequently $\lambda_0 \geq 0$.}

$$\frac{\partial L}{\partial Z} = 1 - \lambda_0 + \lambda_Z = 0 ,$$ (3.5)

$$\frac{\partial L}{\partial x_i} = \alpha \left( \sum_{i}^{N} x_i^{\alpha} \right)^{\frac{1}{\rho}-1} x_i^{\alpha-1} - \lambda_0 p_i = 0 ,$$ (3.6)

$$\lambda_0, \lambda_Z \geq 0 ,$$ (3.7)
Consider first the situation where $Z=0$. In this case $\lambda_0 \geq 0$. Following the Dixit-Stiglitz model of monopolistic competition, define $q = \left( \sum \frac{p_i \alpha_p}{\alpha_p} \right)^{-1}$ as the price index. Solving the above set of equations we have a corner solution where

\[
x_i^1 = IqP_i \frac{1}{1-\alpha_p},
\]

(3.10)

\[
Z^1 = 0,
\]

(3.11)

\[
\lambda_0^1 = c \alpha p \frac{1-\alpha_p}{\alpha_p},
\]

(3.12)

\[
\lambda_z^1 = c \alpha p \frac{1-\alpha_p}{\alpha_p} - 1 \geq 0.
\]

(3.13)

Since $\lambda_z \geq 0$, equation (3.13) implies that when $Z=0$, it must be that $I^1 q \frac{1-\alpha_p}{\alpha_p} \leq \alpha \frac{1}{1-\alpha}$. Let $p_i = p$, the condition can be re-written as $N \frac{1-\alpha_p}{\alpha_p} \geq I^1 \alpha \frac{1}{1-\alpha} \frac{1}{p \alpha \frac{1-\alpha_p}{\alpha_p}}$, which implies that a corner solution (with $Z=0$) emerges if the number of product varieties is sufficiently large.

When $Z > 0$, $\lambda_z = 0$ and we have

\[
x_i^2 = \alpha^{1-a} q \frac{1-\rho}{\rho(1-\alpha)} p_i \frac{1}{1-\alpha_p},
\]

(3.14)

\[
Z^2 = I - \alpha^{1-a} q \frac{1-\alpha_p}{\rho(1-\alpha)},
\]

(3.15)
\( \hat{\lambda}_0^2 = 1 \). \hspace{5cm} (3.16)

Let \( p_i = p, \ Z^2 = I - \alpha^{\frac{1}{1-a}} N(1-a^i) p^{\frac{\alpha}{(1-a^i)}} \). \ Z^2 > 0 \) implies that demands for \( x_i \) and \( Z \) are given by (3.14) and (3.15) if the number of product varieties is not too large.

On the production side, suppose that one unit of labor produces one unit of \( Z \). Since the market of \( Z \) is perfectly competitive, the price of good \( Z \) equals its marginal cost, which is the wage rate \( 1 \). On the other hand, production of the complementary products involves a fixed cost \( F \) in units of labor and a marginal cost of \( a_x \) units of labor. Hence the level of output produced using \( L_i \) units of labor is \( x_i = \max \{ 0, (L_i - F)/a_x \} \). This implies that the total cost of production is \( C_i(x_i) = L_i = a_x x_i + F \).

Firms play a two-stage game. At stage one, each of the \( \tilde{N} \) potential firms decides whether to enter the market. At stage two, firms that have entered the market choose \( p_i \) to maximize their own profits.

### 3.3 Market Equilibriums

Generally speaking, there are three possible equilibriums, which will be numbered as equilibrium \( I, II \) and \( III \). Equilibrium \( I \) is derived when the demand of the consumer is represented by the corner solution where \( Z = 0 \) and the demand for \( x_i \) is given by (3.10). In equilibrium \( II \), the demands for \( Z \) and \( x_i \) are positive, given by (3.14) and (3.15). Equilibrium \( III \) explores the possibility of coordination failure through examining the entry decision of a potential firm when it expects no other firms to enter the market. In this last equilibrium, a
monopolist makes negative profits and thus the complementary goods industry is nonexistent even though a higher number of firms may generate greater demand and make this industry viable. We will show that each of these equilibriums can arise under certain conditions. In Table 3.1, an overview of the prices, the number of product varieties and the quantity of each product in these equilibriums is presented.

### Table 3.1 Overview of Results

<table>
<thead>
<tr>
<th>Equilibrium I</th>
<th>Equilibrium II</th>
<th>Equilibrium III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^1 = \frac{1}{\alpha \rho} a_x$</td>
<td>$p^2 = \frac{1}{\alpha \rho} a_x$</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>$N^1 = \frac{(1-\alpha \rho)H}{F}$</td>
<td>$N^2 = \left( (1-\alpha \rho)(\alpha \rho)\frac{\alpha}{1-\alpha} a_x \frac{1}{\alpha \rho} a_x \frac{1}{\alpha-1} F^{-1} \right)^{\frac{\rho(1-\alpha)}{1-\rho}}$</td>
<td>$N^3 = 0$</td>
</tr>
<tr>
<td>$x^1 = \frac{\alpha \rho}{(1-\alpha \rho)} \frac{F}{a_x}$</td>
<td>$x^2 = \frac{\alpha \rho}{(1-\alpha \rho)} \frac{F}{a_x}$</td>
<td>$x^3 = 0$</td>
</tr>
<tr>
<td>$Z^1 = 0$</td>
<td>$Z^2 = H - (1-\alpha \rho) \frac{1-\alpha \rho}{1-\rho} (\alpha \rho) \frac{\alpha}{1-\rho} a_x \frac{1-\alpha \rho}{1-\rho} a_x \frac{\alpha}{1-\rho}$</td>
<td>$Z^3 = 1$</td>
</tr>
</tbody>
</table>

In the following analysis, we assume the endowment $H$ is sufficiently large so that

$$H \geq \alpha^{\frac{1}{1-\alpha}} \left( \frac{\alpha \rho}{a_x} \right)^{\frac{\alpha}{1-\alpha}}. \quad (3.17)$$

This assumption ensures that the endowment is large enough for the consumer to afford at least one product variety in the complementary goods industry.
3.3.1 Equilibrium I

We consider first the scenario of a corner solution where $Z = 0$. Firm $i$’s profit-maximization problem at stage two is

$$
\max_{p_i} \pi_i = (p_i - a_s)x_i - F.
$$

(3.18)

So long as the prices of the products in the group are not of different orders of magnitude, the elasticity of $\sum p_j^{\frac{\alpha p}{\alpha p - 1}}$ with respect to $p_i$ is of the order $(1/N)$. We shall assume that $N$ is reasonably large, and accordingly neglect the effect of $p_i$ on $\sum p_j^{\frac{\alpha p}{\alpha p - 1}}$. Substituting demand functions (3.10) and (3.11) in the profit-maximization problem, it is straightforward to derive the equilibrium price as shown in the first column of Table 1.

Under monopolistic competition, each firm attempts to maximize profits and entry occurs until the marginal firm can just break even. That is, $\pi_i = 0$, $i=1...N$. The representative consumer’s disposable income is $I = H + \sum \pi_i = H$. Consider a symmetric situation where $x_i = x$ and $p_i = p$ for all firms, we have the equilibrium variety and quantities, which are also summarized in Table 3.1.

Recall from (3.13) that the corner solution emerges only if $Hq^{\frac{1-\alpha p}{\rho(1-\alpha)}} \leq \alpha^{1-\alpha p}$. Substituting the equilibrium price ($p^*$) and variety ($N^*$) into this condition, we obtain

$$
F \leq F_1, \text{ where } F_1 = \alpha^{1-\alpha p}(1-\alpha p)(\alpha p)^{\frac{\alpha p}{1-\alpha p}} H^{\frac{1-\rho}{\rho(1-\alpha p)}} a_s^{\frac{\alpha p}{1-\alpha p}}.
$$

(3.19)

Meanwhile $N^* \geq 1$ implies that

$$
F \leq (1-\alpha p)H.
$$

(3.20)
Given assumption (3.17), we can show that

$$\frac{\rho}{1-\rho} (1-\alpha \rho) H \geq \alpha \frac{\rho}{1-\rho} (1-\alpha \rho) (\alpha \rho) \frac{1-\rho}{1-\alpha \rho} A \frac{1-\rho}{1-\alpha \rho} \frac{1}{1-\alpha \rho}.$$ 

The inequality in (3.20) is always satisfied as long as (19) is satisfied.

Therefore the necessary condition for equilibrium I to exist is that the fixed costs are sufficiently small so that more firms are able to survive. Since the consumer will always want to consume some \(x_i\), as the marginal utility of consuming \(x_i\) when \(x_i\) approaches zero is infinite while the marginal utility of consuming \(Z\) is constant at 1, they tend to consume none of \(Z\) when there is a large number of product varieties in the complementary goods industry.

3.3.2. Equilibrium II

To derive the results for the scenario of an interior solution where \(Z \neq 0\), we substitute demand functions (3.14) and (3.15) in the profit maximization problem (3.18) and the zero profit function \(\pi_i = 0\) and assume a symmetric situation to obtain the equilibrium price, variety and quantities as shown in the second column of Table 3.1. It is not surprising to see that both the price and the quantity of each product under equilibrium II are the same as those under equilibrium I. In both equilibriums, each firm is the sole producer of its own brand and is aware of its monopoly power. It sets the price as a markup over its marginal cost and the markup depends on the elasticity of demand faced by each firm, which is constant at \(1/(1-\alpha \rho)\) for both scenarios. As a result, the equilibrium price charged by firms is the same across the two equilibriums. Further, since firms earn zero profit at equilibrium and the markup is the same, the output per firm has to be the same under the two equilibriums.

Since \(N^2 \geq 1\), equilibrium II can emerge only if

$$F \geq (1-\alpha \rho) (\alpha \rho) \frac{1}{1-a} \alpha \frac{1}{1-a} \frac{a}{1-a}.$$ 

Meanwhile \(Z^2 > 0\) implies

$$F < \alpha \frac{\rho}{1-\rho} (1-\alpha \rho) (\alpha \rho) \frac{1-\rho}{1-\alpha \rho} A \frac{1-\rho}{1-\alpha \rho} \frac{1}{1-\alpha \rho}.$$ 

Under assumption (3.17),
it can be shown that \( (1 - \alpha \rho) (\alpha \rho)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} a_x^{\frac{1}{1-\alpha}} \leq \alpha^{\frac{1}{1-\alpha}} (1 - \alpha \rho) (\alpha \rho)^{\frac{1}{1-\alpha}} H^{\frac{1}{1-\alpha}} a_x^{\frac{1}{1-\alpha}} \).

Therefore we can conclude that the necessary condition for equilibrium II to emerge is

\[
F_2 \leq F < F_1, \text{ where } F_2 = (1 - \alpha \rho) (\alpha \rho)^{\frac{1}{1-\alpha}} c^{\frac{1}{1-\alpha}}. \tag{3.21}
\]

Equation (3.21) implies that the fixed costs have to be within an intermediate range in order for equilibrium II to exist. When the fixed costs are small and entry barriers low, there tend to be many firms providing lots of product choices. The consumer will simply forgo \( Z \) to purchase products in the complementary goods industry and the market yields equilibrium I. On the other hand, when the fixed costs are too high, the revenue of a monopolistic competitive firm simply cannot cover the fixed costs of production and no firm is willing to enter this market. In this case, no products are produced in the complementary goods industry. This is explored in the following section.

3.3.3 Market Equilibrium III

We wonder whether coordination failure would occur so that without coordination no single firm is willing to enter the market even though more firms are likely to generate a demand that would make this industry viable. Consider the problem of a potential entrant who expects no other firms to enter the market. This entrant faces the demand function \( x = \alpha^{\frac{1}{1-\alpha}} p^{\frac{1}{1-\alpha}} \), which is obtained from (3.14) by setting \( N=1 \) in \( q \). The entrant's profit maximization problem is

\[
\text{Max}_p \pi = (p - a_x) \alpha^{\frac{1}{1-\alpha}} p^{\frac{1}{1-\alpha}} - F. \tag{3.22}
\]

The first-order condition of this optimization problem is
\[
\frac{1}{\alpha} \left( \frac{1}{1-a} \right)^{-\alpha} \left( 1 - \frac{p-a_x}{(1-\alpha)p} \right) = 0.
\]

(3.23)

From (3.23), we can solve for the price, quantities and profits if the firm does enter as a monopolist.

\[
p^m = \frac{a_x}{\alpha},
\]

(3.24)

\[
x^m = \alpha^{1-a} a_x \frac{1}{1-a},
\]

(3.25)

\[
Z^m = H - \alpha^{1-a} a_x \frac{\alpha}{1-a},
\]

(3.26)

\[
\pi^m = (1-\alpha) \alpha^{1-a} a_x \frac{\alpha}{1-a} - F.
\]

(3.27)

\(Z^m > 0\) implies \(H > \alpha^{1-a} a_x \frac{\alpha}{1-a}\), which is satisfied given assumption (3.17). The potential entrant will not enter the market if \(\pi^m\) given by (3.27), is negative. Thus the necessary conditions for this case is

\[
F > F_3, \text{ where } F_3 = (1-\alpha) \alpha^{1-a} a_x \frac{\alpha}{1-a}.
\]

(3.28)

That is, if firms hold the belief that no others will enter the market and if the fixed costs are sufficiently large, even a monopolist earns negative profits and no firm would want to enter the complementary goods industry. Given the fact that the entry of a complementary product increases demand for other firms’ products, one might wonder whether the coordinated entry of a sufficiently large number of firms could make the market viable. Section 3.4 shows that this could be true, which leaves open the possibility of mechanisms to address coordination failure and increase product diversity.
3.3.4 Comparison of the Three Market Equilibriums

Now it is easy to compare the number of product varieties under the three market equilibriums. Since \( p^1 = p^2, \ x^1 = x^2 \) and \( Z^1 < Z^2, \ N^1 > N^2 \) given the budget constraint \( Z + \sum_{i}^{N} p_i x_i - H = 0 \). In addition since \( N^3 = 0 \), we conclude that

\[
N^1 > N^2 > N^3 = 0.
\]  

To determine the circumstances where the three market equilibriums emerge, we need to compare the relative magnitude of \( F_1, F_2 \) and \( F_3 \). Divide \( F_2 \) by \( F_3 \), we have

\[
\frac{F_2}{F_3} = \frac{(1 - \alpha \rho)}{1 - \alpha} \rho^{\frac{\alpha}{1 - \alpha}} < 1.
\]  

We then compare \( F_1 \) and \( F_3 \) by dividing the two to obtain

\[
\frac{F_1}{F_3} = \frac{1 - \alpha \rho}{1 - \alpha} \left[ (1 - \rho)^{\frac{\alpha}{1 - \alpha}} \right]^{\frac{\alpha}{1 - \alpha}} \left[ (\alpha \rho)^{\frac{\alpha}{1 - \alpha}} \right] \left( Ha \right)^{\frac{\alpha}{1 - \alpha}}.
\]  

It is not difficult to see that \( F_1/F_3 \) increases with endowment \( H \) and it goes to infinity when \( H \) approaches infinity. Meanwhile given assumption (17), \( F_1/F_3 \) equals \( (1 - \alpha \rho)^{\frac{\alpha}{1 - \alpha}}\left(1 - \alpha\right) < 1 \) when \( H \) is at its minimum. Hence there must exist \( \tilde{H} \) where

\[
\tilde{H} = \left(1 - \alpha \rho\right)^{\frac{1 - \alpha}{1 - \alpha}} \left(1 - \rho\right)^{\frac{\alpha}{1 - \alpha}} \left(\alpha \rho\right)^{\frac{\alpha}{1 - \alpha}} \left( Ha \right)^{\frac{\alpha}{1 - \alpha}} \text{ so that if } H > \tilde{H}, \ F_1 > F_3 \text{ and vice versa. Further it has been shown that } F_2 \leq F_1. \text{ Combining the results on the relative magnitude of } F_1, F_2 \text{ and } F_3 \text{ with the necessary conditions for the three equilibriums (3.19) (3.21) and (3.28), we have the following proposition.}

\[\text{34 Letting } F(\rho) = (1 - \alpha \rho)^{\frac{\alpha}{1 - \alpha}}\left(1 - \alpha\right) \text{ it can be shown that } \partial F(\rho)/\partial \rho > 0. \text{ Since } F(1) = 1 \text{ and } 0 < \rho < 1, \ F(\rho) < 1.\]
Proposition 1. The equilibrium of the complementary goods market depends on the level of endowment $H$ and the magnitude of the fixed costs of production $F$. In the case of $H > \tilde{H}$, only equilibrium I emerges when $F \leq F_2$; either equilibrium I or II occurs when $F_2 \leq F \leq F_3$; all three equilibriums are possible when $F_3 < F < F_1$ and only equilibrium III exists when $F > F_1$. On the other hand if $H < \tilde{H}$, only equilibrium I exists when $F \leq F_2$; both equilibriums I and II are possible when $F_2 \leq F < F_1$; a market structure of monopoly or oligopoly could occur when $F_1 < F < F_3$ and only equilibrium III exists when $F > F_3$.

Figure 3.1: Circumstances for Multiple Equilibriums

Scenario 1: $H > \tilde{H}$

<table>
<thead>
<tr>
<th>Equilibrium I</th>
<th>Equilibrium I or II</th>
<th>Equilibrium I, II or III</th>
<th>Equilibrium III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2$</td>
<td>$F_3$</td>
<td>$F_1$</td>
<td>Fixed Costs of Production $F$</td>
</tr>
</tbody>
</table>

Scenario 2: $H < \tilde{H}$

<table>
<thead>
<tr>
<th>Equilibrium I</th>
<th>Equilibrium I or II</th>
<th>Monopoly or Oligopoly</th>
<th>Equilibrium III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2$</td>
<td>$F_1$</td>
<td>$F_3$</td>
<td>Fixed Costs of Production $F$</td>
</tr>
</tbody>
</table>

Proposition 1 is illustrated in Figure 3.1. The first diagram in Figure 3.1 is about the case where $H > \tilde{H}$. When the fixed costs are sufficiently small ($F \leq F_2$), equilibrium I
prevails and there tends to be a large number of product varieties being produced. Since the representative consumer will always want to consume a positive quantity of every complementary good, the consumer, given the limited endowment, will choose to forego $Z$ and purchase only products in the complementary goods industry. On the other hand, when the fixed costs are sufficiently large ($F \geq F_1$), no firm is willing to enter the market since the revenue would never cover the fixed costs. As a result the complementary goods industry will not exist and only equilibrium $III$ would emerge. Only for fixed costs in the intermediate range are there multiple equilibriums. When $F_2 \leq F \leq F_3$, either equilibrium $I$ or $II$ could occur. In this case a monopoly firm would earn positive profits and thus would enter the market even if no one else does so. As a result, there are always firms operating in the complementary goods industry. In contrast, if the fixed costs are larger so that $F_3 < F < F_1$, not only equilibriums $I$ and $II$ but also equilibrium $III$ are possible. Which of these equilibriums prevails depends on firms’ beliefs. If no firm expects others to enter, the complementary goods industry will not exist because the limited demand will not generate enough revenue to cover the fixed costs. However, if firms simultaneously expect entry by other firms, given demand complementarity, demand increases and the market yields a positive number of product varieties (equilibrium $I$ or $II$). Note, however, that equilibrium $II$ is unstable. If it happens that the “right” number of firms enters the market simultaneously, the market would yield equilibrium $II$. Otherwise, the market would always end up at equilibrium $I$ since as each firm enters the market the demand and consequently firms’ profits increase. This induces further entry and the cycle reinforces itself until all resources are drawn to the complementary goods industry. Furthermore, the instability of equilibrium $II$ also implies that a small shock could start a cumulative process and switch equilibrium $II$
to equilibrium I or III. In fact in Section 5, we show that an infinitesimal subsidy on the fixed cost of production induces such a switch.

The second diagram in Figure 3.1 illustrates the case $H < \tilde{H}$. In this scenario, the incentives for a firm to enter (or stay out of) the market are quite similar to that of scenario 1 except that the representative consumer in this case has less income to spend and consequently the conditions for equilibriums I and II to exist become more restrictive ($F_1 < F_3$). As a result in the full range of $F_2 \leq F < F_1$ where both equilibriums I and II are possible, equilibrium III can no longer exist. Furthermore, in the range of $F_1 < F < F_3$, none of the three aforementioned equilibriums will emerge. In this area, a monopolist earns positive profits. However, because of the large fixed costs relative to the endowment, there may not be enough resources to support a second firm in this market. If this is the case, monopoly equilibrium prevails. Otherwise, the market will be served by a small number of firms (i.e. oligopoly). Since the focus of this paper is monopolistic competition, this situation is not explored in details.

### 3.4 Constrained Optimum and Market Equilibriums

Spence demonstrates that complementary products are undersupplied in a monopolistically competitive equilibrium. In particular, he argues that two forces are at work tending to eliminate products that should have been produced. First, because of setup costs, revenues may fail to cover the costs of a socially desirable product. Second, the introduction of a new product increases other firms’ profits as well as consumer surplus. An entering firm does not
take this into account and may not enter when it is generating a net social benefit. In this section we explore whether this is still the case when there are multiple equilibriums.

3.4.1 Constrained optimum

In the case of an optimum constrained by monopolistically competitive pricing, the social planner’s aim is to choose the number of products to maximize consumer utility subject to the economy’s resource constraint and the monopolistically competitive pricing rule. Given the consumer’s demand function (3.14) and letting \( x_i = x \) and \( p_i = p \) for all \( i \), we are able to derive the following relationship between the price and the output of each complementary product (3.35). The maximization problem can be written as follows:

\[
\begin{align*}
\text{Max}_{N} \quad & U = Z + N^{1/a} x^a \\
\text{Subject to:} \quad & Z = H - N(a \cdot x + F), \\
& p = \frac{a \cdot x}{\alpha p}, \\
& p = \alpha N^{a^{-1}} x^{a^{-1}}. 
\end{align*}
\]

Further simplifying the maximization problem, we have

\[
\begin{align*}
\text{Max}_{N} U = H + \alpha^{1-a} \left( \frac{a \cdot x}{\alpha \rho} \right)^{1-a^{-1}} \left( \frac{1}{\alpha \cdot \rho} - 1 \right) a \cdot N^{\frac{1-a}{a^{-1}}} - NF.
\end{align*}
\]
\[
\frac{\partial^2 U}{\partial N^2} = \alpha^{1-a} \left( \frac{a_s}{\alpha \rho} \right)^{1-a} \left( \frac{1}{\alpha^2 \rho} - 1 \right) \frac{(1-\alpha \rho)(1-\rho)}{\rho^2 (1-\alpha)^2} a_s N^{\frac{1-\rho}{\rho^2(1-\alpha)}} > 0. \tag{3.37}
\]

Since the second order derivative of \( U \) is positive, utility reaches the maximum when \( N \) is either at its maximum (\( N^c \)) or minimum (\( N^2 = 0 \)).

Hence, to determine the coordinated optimum, we need to compare the social welfare at \( N = N^c_1 \) and \( N = 0 \). To do so, we first derive a condition that determines \( N^c_1 \). Recall the economy’s resource constraint \( Z = H - N(a_s x + F) \). Substituting the monopolistically competitive price (3.34) and demand function (3.35), we obtain \( Z \) as a function of \( N \):

\[
Z = H - a_s \alpha^{1-a} \left( \frac{a_s}{\alpha \rho} \right)^{1-a} N^{\frac{1-\alpha \rho}{\rho^2(1-\alpha)}} - NF. \tag{3.38}
\]

Differentiate both sides with respect to \( Z \), we have

\[
\frac{\partial N}{\partial Z} \left( a_s \alpha^{1-a} \left( \frac{a_s}{\alpha \rho} \right)^{1-a} \frac{1-\alpha \rho}{\rho (1-\alpha)} N^{\frac{1-\alpha \rho}{\rho^2(1-\alpha)}} + F \right) = -1. \tag{3.39}
\]

This implies that \( \partial N / \partial Z < 0 \) and \( N \) reaches a maximum when \( Z = 0 \). Using (3.38) and letting \( Z = 0 \), we obtain the function that determines the maximum number of varieties \( N^c_1 \)

\[
a_s \alpha^{1-a} \alpha^{1-a} \left( \alpha \rho \right)^{1-a} \left( N^c_1 \right)^{\frac{1-\alpha \rho}{\rho^2(1-\alpha)}} + FN^c_1 = H. \tag{3.40}
\]

Accordingly, the utility when \( N \) is at its maximum is

\[
U^c_1 = \frac{H - FN^c_1}{\alpha^2 \rho}. \tag{3.41}
\]
On the other hand, the representative consumer spends all of his income on Z if \( N=0 \), in which case

\[ U^{c2} = H. \]  

(3.42)

Since \( H < (H - N^{c1}F)/\alpha^2 \rho \) when \( F < (1 - \alpha^2 \rho)H/N^{c1} \). The maximized utility is \( U^{c1} \) when \( F < F_4 \) and \( U^{c2} \) when \( F > F_4 \), where

\[ F_4 = \frac{(1 - \alpha^2 \rho)H}{N^{c1}}. \]  

(3.43)

**Proposition 2**: The constrained optimum depends on the magnitude of the fixed costs of production. If \( F < F_4 \), the constrained optimal number of product varieties in the complementary goods industry is \( N^{c1} \) given by the equation (3.40) and social welfare is given by (3.41). If \( F > F_4 \), the constrained optimal number of product varieties in the complementary goods industry is 0 and the level of social welfare is at \( H \).

For ease of discussion, we will refer to the constrained optimum associated with \( F < F_4 \) as optimum I, and the other as optimum II.

3.4.2 *Comparison between constrained optima and market equilibriums*

To compare the number of product varieties under the market equilibriums with that under the constrained optima, it is necessary to first examine, when the constrained optimum is at optimum I (or II), which of the three market equilibriums would emerge. To achieve this,
we compare the relative magnitudes of $F_1$, $F_3$, and $F_4$. A simple manipulation of (3.43) gives

$$N^{c1} = \left(1 - \alpha^2 \rho \right) H / F_4.$$ Substitute $N^{c1}$ in (3.40) to solve for $F_4 = (1 - \alpha^2 \rho) \left( \alpha^2 \rho \right)^{\frac{1-p}{1-\alpha p}} H^{\frac{1-p}{1-\alpha p}} a_\rho^{\frac{1-p}{1-\alpha p}}$.

Divide $F_4$ by $F_1$ and $F_4$ by $F_3$ respectively to obtain

$$\frac{F_4}{F_1} = \alpha^{\frac{\rho(1-\alpha)}{1-\alpha p}} \frac{1 - \alpha^2 \rho}{1 - \alpha \rho} > 1,$$  \hspace{1cm} (3.44)

and

$$\frac{F_4}{F_3} = \frac{1 - \alpha^2 \rho}{1 - \alpha} \alpha^{\frac{\rho(1-\alpha)}{1-\alpha p}} \frac{a_\rho}{\rho^{1-\alpha}}.$$ \hspace{1cm} (3.45)

From (3.45), it can be shown that $F_4$ could be higher or lower than $F_3$. Recall that the optimal number of product varieties is $N^{c1}$ when $F < F_4$ while the number is 0 when $F > F_4$. This combined with (3.44), (3.45) and the results from proposition 1 implies that, when comparing the number of product varieties to that under the constrained optimum, there are three scenarios depending on the level of endowment and the fixed cost of production. When $H > \tilde{H}$ and $F < F_4$, all three market equilibriums are possible and thus the equilibrium number of product varieties could be $N_1$, $N_2$ or $N_3$ while the corresponding optimal number of product varieties is $N^{c1}$. The second scenario is when $H < \tilde{H}$ and $F < F_4$. One of equilibriums I and II could emerge if $F_4 < F_3$ while all three equilibriums are possible if $F_4 > F_3$. For this scenario the optimal number of product varieties remains as $N^{c1}$. The last
is when \( F > F_\downarrow \). Only equilibrium \( III \) would occur. The equilibrium number of product varieties is the same as that under the constrained optimum as \( N^3 = N^{c_2} = 0 \).

We now examine the relative magnitudes of market product varieties as compared to those under the constrained optimums. It is straightforward to show that \( N^{c_1} > N^3 = N^{c_2} = 0 \). To compare \( N_I \) with \( N^{c_1} \), we re-write the left-hand side of (3.40) as a function of \( N \)

\[
\begin{align*}
  f(N) &= a_x \alpha^{\frac{1}{1-\alpha}} (\alpha \rho)^{\frac{1}{1-\alpha}} N^{\rho(1-\alpha)} + NF .
\end{align*}
\]

Differentiating (3.46) with respect to \( N \), we have \( \frac{\partial f(N)}{\partial N} > 0 \), which indicates that \( f(N) \) is an increasing function of \( N \). Substituting the equilibrium number of products under market equilibrium \( I \), \( N^1 \), in (3.46), we obtain

\[
\begin{align*}
  f(N^1) &= a_x \alpha^{\frac{1}{1-\alpha}} (\alpha \rho)^{\frac{1}{1-\alpha}} (1-\alpha \rho)^{\rho(1-\alpha)} N^{\rho(1-\alpha)} F^{\frac{1-\alpha}{\rho(1-\alpha)}} H^{\rho(1-\alpha)} + (1-\alpha \rho) H .
\end{align*}
\]

Using the necessary condition for market equilibrium \( I \) (3.19), we can show that \( f(N^1) \geq H \).

Since \( f(N^{c_2}) = H \) and \( \frac{\partial f(N)}{\partial N} > 0 \),

\[
N^1 \geq N^{c_1} .
\]

The comparison of \( N^2 \) and \( N^{c_1} \) can be easily achieved by resorting to the demand function for the complementary goods \( x = \alpha^{\frac{1}{\rho(1-\alpha)}} N^{\rho(1-\alpha)} \rho(1-\alpha) \) and the societal resource constraint \( Z = H - Na_x x - NF \). From \( Z^2 > Z^I = 0 \), we know that \( N^2 (a_x x^2 + F) < N^{c_1} (a_x x^{c_1} + F) \),

---

35 The situation of monopoly or oligopoly is not explored because the focus of this paper is monopolistic competition.
which implies that $x^2 < x^c$ if $N^2 \geq N^c$. This conflicts with the demand function which indicates $x^2 \geq x^c$ when $N^2 \geq N^c$. Thus we can conclude that $N^2 < N^c$. Combining this with (3.29), (3.48) and keeping in mind that $N^3 = 0$, we have

$$N^1 \geq N^c > N^2 > N^3 = 0 .$$

(3.50)

Proposition 3: There are 3 scenarios when comparing the equilibrium number of product varieties with that under the constrained optimums.

a) All three market equilibriums are possible if either of the following two conditions is satisfied: i) $H > \tilde{H}$ and $F < F_4$ or ii) $H < \tilde{H}$, $F < F_4$ and $F_4 > F_3$. In this scenario, the optimal number of product varieties is $N^c$ and $N^1 \geq N^c > N^2 > N^3 = 0$.

b) Either equilibrium I or II would emerge if $H < \tilde{H}$ and $F < F_4 < F_3$. In this scenario, the optimal number of product varieties is $N^c$ and $N^1 \geq N^c > N^2$.

c) Only equilibrium III would exist if $F > F_4$. In this case, the optimal number of product varieties is $N^c$ and $N^3 = N^c = 0$.

Therefore, although there are situations where the equilibrium number of product varieties is lower than the optimum as predicted by Spence (1976), our model shows that in some cases the variety in fact could be greater. For example, when the fixed costs are low ($F \leq F_2$) or when the fixed costs are slightly higher ($F_2 \leq F < F_4$) and firms expect all others to enter, the market yields an excess number of product varieties. Compared with the constrained optimum, although firms charge the same price, the higher number of product
varieties increases demand for each product and thus more firms are able to survive. Since in both cases all resources are used in the complementary goods industry, higher product variety under market equilibrium means a production scale smaller than the optimum, resulting in efficiency loss.

Furthermore, propositions 1 and 3 combined imply that there are situations of coordination failure. For example, when $H > \tilde{H}$ and $F_3 \leq F \leq F_1$, if no firm expects other firms to enter the market, no complementary products would be produced since a single monopolist makes negative profits and equilibrium $III$ prevails. However if firms hold a different belief and think that other firms will enter the market, a greater number of product varieties will generate increased demand and the industry becomes viable (Equilibriums $I$ or $II$ emerges). A mechanism/policy that works to change firms’ belief and promote better coordination among firms could enhance product diversity and improve social welfare. In the following section we explore one such policy.

### 3.5 Effects of Government Subsidy

Production subsidy is among the set of commonly used industry policies in many countries. In the literature on monopolistic competition, a moderate level of subsidy is shown to be welfare enhancing under standard Dixit-Stiglitz assumptions in the case of substitutable products (Costrell 1990 and Spence 1976). Thus it would be interesting to examine whether it holds true when products are complementary. This section focuses on the scenario of $H > \tilde{H}$ as in this case all three market equilibriums are possible and a small shock could start a cumulative process that leads to a switch of equilibriums.
Suppose that the government offers a subsidy on the fixed cost of production at the rate of $s$ so that the producers’ total costs become $C_i(x_i) = a_x x_i + (1-s) F$. The subsidy is financed through a lump sum tax on the representative consumer’s income. We study the case of an infinitesimal subsidy on the fixed cost of production when the initial rate of subsidy is zero.

Consider first the situation where the market is at equilibrium $I$. Using the demand function (3.10) derived in section 3.2 and multiplying $F$ by $(1-s)$ in the producers’ profit maximization problem (3.18) and zero-profit condition, we are able to solve the equilibrium price, quantities and variety:

$$ p^i = \frac{1}{\alpha \rho} c, \quad (3.51) $$

$$ N^i = \frac{(1-\alpha \rho) H}{F(1-s)}, \quad (3.52) $$

$$ x^i = \frac{\alpha \rho}{(1-\alpha \rho)} \frac{F(1-s)}{a_x}, \quad (3.53) $$

$$ Z^i = 0. \quad (3.54) $$

Note that disposable income is a function of the number of product varieties $I = H - NsF$. Deducting the amount of lump sum tax from the consumer’s endowment, the number of product varieties is re-written as

$$ N^i = \frac{(1-\alpha \rho)}{F(1-s \alpha \rho)} H. \quad (3.55) $$
Social welfare thus is

\[ W^1 = \left( \sum_{i} x_i^\alpha \right)^{\frac{1}{\rho}} = \left( \frac{1 - \alpha \rho}{F} \right)^{\frac{1 - \alpha}{\rho}} \left( \frac{\alpha \rho}{a_s} \right)^{\frac{1}{\rho}} H^{\frac{1}{\rho}} (1 - s \alpha \rho)^{\frac{1}{\rho}} (1 - s)^{\alpha}. \tag{3.56} \]

**Proposition 4.** If the market is at equilibrium I, an infinitesimal subsidy on the fixed cost of production increases the number of product varieties, but reduces output per product. Social welfare remains unchanged.

**Proof.** When the market is already in equilibrium I, an infinitesimal subsidy increases firms’ profits and thus will not cause a switch to either equilibrium II or III. Thus we can study the effects of an infinitesimal subsidy using comparative statics. Differentiating the number of product varieties, output per product and social welfare with respect to \( s \) and letting \( s=0 \), we have

\[ \frac{\partial N^1}{\partial s} = \frac{\alpha \rho (1 - \alpha \rho) H}{F} > 0, \tag{3.57} \]

\[ \frac{\partial x^1}{\partial s} = -\frac{\alpha \rho F}{(1 - \alpha \rho) a_s} < 0, \tag{3.58} \]

\[ \frac{\partial W^1}{\partial s} = 0. \tag{3.59} \]

QED
Intuitively when the government subsidizes firms, an infinitesimal subsidy reduces firms’ fixed cost of production and enhances their profits. As a result more firms enter the market and the number of product varieties increases. Although products are complementary in nature, the representative consumer has to assign the limited income among a higher number of products. As a result, demand for each product decreases and consequently the output of each firm is reduced. Social welfare tends to decrease because firms operate at a smaller scale which leads to efficiency loss. However the decrease in social welfare is compensated by the increase in welfare as a result of more variety in the case of an infinitesimal subsidy\textsuperscript{36}.

Now consider the situation where the initial market equilibrium is at $II$. Using the demand function (3.14) and multiplying $F$ by $(1-s)$ in the producers’ profit maximization problem (3.18) and zero-profit condition, we are able to solve for the equilibrium price, quantities and varieties.

\begin{equation}
 p^1 = \frac{1}{\alpha \rho} c, \tag{3.60}
\end{equation}

\begin{equation}
 x^1 = \frac{\alpha \rho}{(1-\alpha \rho)} \frac{F(1-s)}{a_s}, \tag{3.61}
\end{equation}

\begin{equation}
 Z^2 = H - (1-\alpha \rho) \frac{\rho(1-\alpha)}{\rho-1} (\alpha \rho) \frac{\alpha}{\rho} \frac{\rho}{\rho-1} ((1-s)F)^{\frac{\rho(1-\alpha)}{\rho-1}} a_s \frac{\alpha}{\rho-1} \frac{F}{(1-\alpha \rho)} (1-s \alpha \rho), \tag{3.62}
\end{equation}

\textsuperscript{36} We also explore the situation of a discrete subsidy. An increase in the rate of subsidy increases the number of product varieties and reduces the output per firm, which is the same as the case of an infinitesimal subsidy. What is different is that social welfare decreases with the subsidy. Recall from (3.53) that the equilibrium number of product varieties under equilibrium $I$ is higher than that under the constrained optimum. A positive subsidy pushes the number of product varieties even higher and the equilibrium deviates further from the optimum.
\[ N^2 = \left( \frac{1 - \alpha \rho \alpha^\frac{1}{1-\alpha}}{1 - \alpha \rho} \right)^{\frac{1}{1-\alpha}} \left( (1 - s) F \right)^{-1} a_s \alpha^\frac{1}{1-\alpha} \left( 1 - \alpha \right)^{\frac{\rho(1-\alpha)}{\rho - 1}}, \quad (3.63) \]

\[ W^2 = H + (1 - \alpha \rho) \left( \frac{1}{1-\rho} \alpha \right) \left( \frac{1}{1-\rho} F \right)^{\rho(1-\alpha)} \left( 1 - \alpha \right)^{1-\rho} F \left( 1 - \alpha - s(1 - \alpha^2 \rho) \right). \quad (3.64) \]

**Proposition 5.** If the market is initially at equilibrium II, an infinitesimal subsidy on the fixed cost of production induces a switch from equilibrium II to equilibrium I. It increases the number of product varieties and improves social welfare. The output per firm remains unchanged.

**Proof.** Since the market is initially at equilibrium II, it must hold that \( F_2 \leq F < F_1 \). Recall from proposition 1 that equilibrium I could also emerge within this range and that equilibrium II is unstable as a small shock would trigger a cumulative process. An infinitesimal subsidy increases firms’ profits and encourages entry. This increases demand and induces further entry. The cycle reinforces itself until all resources are drawn to the complementary goods industry. Therefore, an infinitesimal subsidy would always induce a switch from equilibrium II to equilibrium I.

We now examine the changes in varieties, output and welfare when the equilibrium switches to equilibrium I. We have shown that \( N^1 > N^2 \) and \( x^1 = x^2 \). Let \( s=0 \) and subtract (3.56) by (3.64) to obtain

\[ W^1 - W^2 = \left( \frac{1 - \alpha \rho}{F} \right) \left( \frac{1}{1-\rho} \alpha \right) \left( \frac{1}{1-\rho} \right)^{\alpha} \left( \frac{1}{1-\rho} \alpha \right) \left( \frac{1}{1-\rho} F \right)^{\alpha} \left( 1 - \alpha - s(1 - \alpha^2 \rho) \right). \quad (3.65) \]
Differentiate (3.65) with respect to $F$, we have

$$\frac{\partial (W^1 - W^2)}{\partial F} = -\frac{1-\alpha \rho}{F \rho} \left( \frac{1-\alpha \rho}{F} \right)^{1-\alpha \rho} \left( \frac{\alpha \rho}{a_i} \right)^{\frac{1}{\rho}} - \frac{1-\alpha \rho}{1-\rho} \left(1-\alpha \rho\right)^{\frac{1-\alpha \rho}{\rho}} \left( \frac{\alpha \rho}{a_i} \right)^{\frac{1}{\rho}} \left( \frac{1-\alpha \rho}{1-\beta} \right) \left( \frac{a_i}{F} \right)^{\frac{1}{\rho}} F^{\frac{1}{\rho}} (1-\alpha) < 0. \tag{3.66}$$

That is, the smaller are the fixed costs of production, the more likely does an infinitesimal subsidy improve social welfare. This is consistent with the results from Section 3.4 that when the fixed costs are relatively small, the constrained optimum is achieved when $Z=0$ and all resources are used in the complementary goods industry.

Since the initial equilibrium is at $II$, the fixed costs of production must be within the range of $F_2 \leq F < F_1$. Substitute $F_1$ in (3.65) to obtain $W^1 - W^2 = 0$. Using (3.66), we can conclude that $W^1 > W^2$ in this range. \textit{QED}

Therefore if the initial market equilibrium is at $II$, an infinitesimal subsidy always leads to a switch from equilibrium $II$ to equilibrium $I$. More firms enter the market and all consumer income is spent in the complementary goods industry. It has been shown that although the number of product varieties increases, output per firm remains unchanged. This implies there is no trade-off between product variety and economies of scale. The extra product varieties are generated by drawing on the resources from another industry. Although now there is excess product variety, social welfare improves compared to the situation where the equilibrium is at $II$.\textsuperscript{37}

\textsuperscript{37} A discrete subsidy is similar in that it induces a switch from equilibrium $II$ to equilibrium $I$ and consequently increases the number of product varieties. What is different is that the output of each firm is reduced. After all resources are already being utilized in the complementary goods industry, as the number of product varieties increases, the demand for each variety has to decrease given the fixed amount of endowment. In this case, the
Finally consider the situation where the market is at equilibrium \( III \) and no products in the complementary goods industry are produced. Since the initial market is at equilibrium \( III \), \( F > F_3 \). Proposition 1 states that in the range of \( F_3 < F < F_1 \), all three market equilibriums are possible. Thus if the fixed cost of production is within this range, the market may remain at equilibrium \( III \) or move to equilibrium \( I \) or \( II \) depending on firms’ beliefs. If firms still expect no others to enter the market, the market will simply remain at its initial equilibrium. However, if an infinitesimal subsidy sends out positive signals and changes firms’ beliefs, as more firms enter the market, demand and consequently profits increase due to product complementarity, and the equilibrium moves to equilibrium \( I \) or \( II \). Again Equilibrium \( II \) is unstable in a sense that if the “right” number of firms enter the market simultaneously, it is likely to emerge. Otherwise, the market moves to equilibrium \( I \). Another situation is when the fixed costs are so high (\( F > F_1 \)) that revenues will not cover the fixed cost of production. In this case the market will always remain at equilibrium \( III \).

**Proposition 6.** If the market is initially at equilibrium \( III \) and \( F_3 < F < F_1 \), an infinitesimal subsidy induces a switch to equilibrium \( I \) or \( II \) as long as the subsidy changes firms beliefs. On the other hand, if the subsidy does not change beliefs, the market stays unchanged. A subsidy has no impact on equilibrium if \( F > F_1 \).

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economies of scale are traded for a higher number of product varieties. Social welfare no longer always improves with the subsidy. In fact when the fixed costs are too large (close to \( F_2 \)) or the subsidy is too large, a subsidy reduces social welfare. Only when the subsidy is at a moderate level and the fixed costs are relatively small does social welfare improve.
Proposition 6 indicates that when the fixed cost of production is within the range \( F_3 < F < F_1 \), whether an infinitesimal subsidy addresses the coordination failure relies heavily on how it influences firms’ beliefs. If the subsidy sends out positive signals that change firms’ beliefs, a self-enforcing process would start and more product varieties would be introduced. Otherwise, a subsidy is futile and none of the aspects of equilibrium III would be changed. Furthermore, a combination of the above proposition with proposition 3 shows that when the fixed cost of production is relatively high in the sense that \( F_i < F < F_4 \), the number of product varieties is under-supplied and an infinitesimal subsidy can no longer work to promote product diversity even if it does change firms’ beliefs. In this case, a higher rate of subsidy that reduces costs and increases firms’ profits is required. In the following proposition, we examine whether an infinitesimal subsidy improves social welfare if it does induce a switch of equilibriums.

**Proposition 7.** If the market is initially at equilibrium III and an infinitesimal subsidy induces a switch of equilibriums, social welfare improves.

**Proof.** Social welfare under equilibrium III is

\[
W^3 = Z = H.
\] (3.67)

Dividing (3.56) by (3.67) and letting \( s=0 \), we obtain

\[
\frac{W^1}{W^3} = \left( \frac{1 - \alpha \rho}{F} \right)^{1-\alpha \rho} H^{1-\rho} \left( \frac{\alpha \rho}{a_x} \right)^{a}.
\] (3.68)
which is greater than 1 if and only if $F < (1 - \alpha \rho) \left( \frac{\alpha \rho}{a_x} \right)^{\frac{\alpha}{1-\alpha \rho}} H^{\frac{1-\rho}{1-\alpha \rho}}$. Recall that an infinitesimal subsidy can induce a switch from equilibrium III to I when $F_3 < F < F_1$. It can be shown that $(1 - \alpha \rho) \left( \frac{\alpha \rho}{a_x} \right)^{\frac{\alpha}{1-\alpha \rho}} H^{\frac{1-\rho}{1-\alpha \rho}} F_1 = \alpha^{\frac{\rho}{1-\alpha \rho}} > 1$. Using (3.68), we conclude that $W^1 > W^3$ in the range of $F_3 < F < F_1$.

In comparing the relative magnitudes of social welfare between equilibriums II and III, we subtract (3.67) from (3.64) and let $s=0$ to obtain

$$W^2 - W^3 = (1 - \alpha \rho) \left( \frac{1-\alpha \rho}{1-\rho} \right) \frac{\alpha \rho}{a_x} \left( \frac{a_x}{a_x - \rho} \right)^{\frac{1-\alpha \rho}{1-\rho}} F^\frac{1-\rho}{1-\alpha \rho} (1 - \alpha) > 0. \quad (3.69)$$

QED

Therefore when an infinitesimal subsidy changes the market expectations and induces a switch of equilibriums, it always enhances social welfare. This is consistent with the results from section 3.4 that within the range of $F_3 < F < F_1$, the constrained optimum is at optimum I and the optimal number of product varieties is higher than zero ($N^3$). A subsidy that increases the number of product varieties tends to improve social welfare$^{38}$.

$^{38}$The results under a discrete subsidy are qualitatively similar to those in the case of an infinitesimal subsidy. It can be shown that equilibrium III will always move to equilibrium I or II when $F_3 < F < F_1/(1-s)$, remain at equilibrium III or move to equilibrium I or II when $F_3/(1-s) < F < (1-s) a_x$ and always remain at equilibrium III when $F > (1-s) a_x$ and always remain at equilibrium III when $F > (1-s) a_x (1-s a_x)$ and always remain at equilibrium III when $F > (1-s) a_x (1-s a_x)$ and always remain at equilibrium III when $F > (1-s) a_x (1-s a_x)$. In the case where a subsidy induces a switch of equilibriums, the number of product varieties increases and social welfare improves as long as the rate of subsidy is at a moderate level.
3.6 Concluding Remarks

Using an alternative assumption that products are complementary, we have studied the market provision of product diversity under monopolistic competition and the effects of an infinitesimal subsidy on product variety and social welfare. We show that demand complementarity leads to multiple equilibriums and the number of product varieties could be higher or lower than the optimum constrained by monopolistic competitive pricing. When the fixed costs are sufficiently small, monopolistic competition yields an excess number of product varieties. An infinitesimal subsidy pushes the number of product varieties even higher and economies of scale are sacrificed, resulting in efficiency loss. Social welfare remains unchanged in the case of an infinitesimal subsidy but decreases when the subsidy is discrete. On the other hand, when fixed costs are large, the market could have too few products. In some cases, the complementary goods industry becomes non-existent as revenues do not cover the large amount of set up costs. In this case a subsidy could be welfare enhancing. Thus to ensure welfare enhancement, our paper suggests the necessity of investigating the pattern of demand interdependence and assessing the magnitude of fixed costs of production before choosing a policy.

Our results also show that when fixed costs are intermediate, there is an overlap of two or all three market equilibriums and the equilibrium outcomes depends crucially on firms’ beliefs. Thus coordination failure in entry decision could occur where mutual gains can be realized but no firm has an incentive to deviate. We demonstrate that an infinitesimal subsidy on the fixed cost of production induces a switch of equilibriums if it changes firms’ beliefs. Moreover, it enhances product diversity and improves social welfare.
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