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Maintaining a Golden Age

Demand Shock Simulations in a Fixed Coefficient Production Economy with a Government Sector and a Target Public Debt Ratio

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Abstract

In this paper I modify a model originally laid out by Thomas Michl to include endogenous government spending, saving decisions and also alter it with the inclusion of a modified Taylor rule. The tax-ratio instrument employed in the original model is tested and found to be effective at bringing stability to the economy. Demand shocks are then applied to the modified models and I conclude that, as with the original model, more than just a common fiscal response function is needed to prevent a deflation trap. The inclusion of the original form of the Taylor rule results in a slower return to a steady state debt level but results in lower long-run inflation.

Following the financial crisis of 2008 many governments found themselves in the undesirable position of having a large amount of public debt, large fiscal deficits and rising borrowing costs. As many of the countries, especially in the European Union, began to implement severe austerity measures economists and politicians began to once again talk about the sustainability of fiscal deficits. Immediately following the crisis the response of most governments was to implement massive fiscal stimulus programs that were meant to blunt the shock of a large drop in private demand in the economy. As noted in the frequent press releases from the G8 and G20 finance ministers these programs of fiscal stimulus were largely viewed as successful. Only later in 2010 did the sentiment toward fiscal stimulus start to change. The same central bank governors and finance ministers that had hailed the success of fiscal stimulus now started to believe the only way to return to prosperity was for countries that were either running large deficits or had large public debt to begin to cut their budgets significantly.

The idea behind this move toward austerity was that smaller budget deficits would lead to greater confidence in the markets and therefore lower interest rates on sovereign bonds. A recent paper (Bernoth et. al 2012) uses data from the world bond market from 1993 to 2009 and concludes that a negative fiscal position causes a rise in bond rates relative to Germany and the US, especially those of smaller countries. However, while this empirical relationship may be true, there is no consensus about the best way to achieve long-run fiscal consolidation. For instance, a study (Broda and Weinstein 2004) looking at Japan's plan to balance its budget by 2012 concluded that

"The Japanese government's target of trying to restore a primary balance of zero by 2012 is a particularly painful way of handling the transition for the current generation of workers. We have considered other approaches that smooth the transition over more generations and thus would entail lower taxes in the short run (and higher ones later). "

So while Japan did have a fiscal deficit problem, the solution was not necessarily to start cutting as much as possible as fast as possible. By contrast The OECD (Sutherland et al. 2012) concluded recently that:

"...whereas many countries require fiscal reforms to restore long-run sustainability, some may require a front-loaded budgetary package that includes revenue measures that may yield faster deficit reduction or be politically easier than spending cuts."

This conclusion seems to be supported by the most recent releases of G20 finance ministers and central bank governors. So it is clear that the solution to this problem has not yet been agreed upon by the economic community.

What is needed then is a way to examine how a country might be able to react to shocks without looking at one specific country. To this end, we move to theoretical models that describe the behaviour of an economy. The mainstream view in economics is that this type of study can be done using a dynamic stochastic general equilibrium (DSGE) model, or the so-called New Keynesian models which are in line with neo-classical thinking. These models are based on a common method of creating behavioural functions for representative agents and then aggregating them over entire populations. While these are popular models, a major issue not addressed by this school of thought is the unrealistic characteristics that are embedded in the production function. The economy is said to function on the very commonly used Cobb-Douglas production function which takes this form:

$$Y = AL^{\alpha}K^{\beta} \tag{1}$$

Where α and β are said to be the output elasticities of labour (L) and capital (K). As Godley and Lavoie (2012) point out the empirical work done by Felipe and McCombie (2006) to evaluate this type of production function simply does not support this form of production in the real world.

For an alternative to this production function, we return to the traditional ideas originally laid out by Harrod. Having been formalized by Domar (1946), this school of thinking maintains that the production function instead takes the form:

$$Y = \min\,(L, \sigma K) \tag{2}$$

And it is common to assume that output is capital constrained with no labour hoarding. This type of production function is common in the Post-Keynesian school of economic thought.

In this paper I will use the Post-Keynesian view of production along with simulation techniques similar to those outlined by Godley and Lavoie to test how a model economy reacts to demand shocks. While the model I use is not consistent with the stock flow consistent methods outlined in the Godley and Lavoie book the simulations techniques can be applied here quite well.

In this paper I use a model laid out by Michl (2012) that is based on the Classical conventional wage share model to simulate the reactions of an economy to a demand shock. I am interested in how capacity utilization adjusts and how the interest and inflation rates respond to these transitions and how quickly an economy can return to a golden age, which is defined later in section two where I outline the Michl model in more detail and outline its properties. In section three I modify the model outlined by Michl with an alternative government spending reaction function and examine how it affects the outcomes of the simulation run in the original paper. I further modify the model to include a dynamic saving decision for bond holders. A final model is generated as I modify the original interest rate response function to match what was originally suggested by Taylor. Section four shows the results from the demand shock simulations. Finally, section five outlines the conclusions that can be drawn from the simulation results.

Section 2.1- MICHL1 Model

The base model that I use to explore this topic is developed in "A Model of Fiscal and Monetary Policy" by Thomas Michl. This paper is available in the form of a working paper and will be included in a festschrift for Duncan Foley which will published by Routledge in late 2012. The model is based on his earlier work in Foley and Michl (1999, Ch. 6) and finds its roots in both Keynesian and Classical traditions. The Foley and Michl model was further expanded in another paper (Michl 2008) to include the analysis of inflation targeting, a key element in the simulations I run in this paper. The full derivation is available in the most recent working paper but I will outline it here for completeness.

We take an economy that is comprised of two classes, workers and capitalists. All taxes are paid by the capitalist class and income is distributed between the classes parametrically. That is, there is a share of profit π that is kept by the capitalist class. We assume that our economy is constrained by capital rather than labour. This model's form normalizes all variables to the level of capital. This means that our values will be given in ratio values rather than actual figures.

There are a number of different views on how capacity utilization is to be used in this type of system. Michl assumes that a monetary policy that is dedicated to the stabilization of inflation will necessarily drive capacity utilization $(u)$to a long run normal level of 1. The output-capital ratio at normal utilization is denoted by ρ so while our short run rate of profit reflects aggregate demand factors in the form $u\pi\rho$, the long run rate of profit $(\pi\rho)$ does not.

We assume that our economy has an inflation that is tied to capacity utilization. This is consistent with Corrado and Mattey (1997) who argue that capacity utilization is a useful indicator of inflationary pressures as well as business cycle fluctuations. For these reasons, we use the vertical Phillips-type curve of the form:

$$p = p_{-1} + \varphi \cdot (u_{-1} - 1) \tag{3}$$

Where p is the level of inflation at time t, u is normalized to 1. The -1 subscript indicates that we are using lagged values and φ is the sensitivity of inflation to slack in utilization. We assume that the economy can exist is a state of less than full utilization but also that factories can work overtime in order to run above normal levels.

Our central bank reaction function takes the form of a simple Taylor rule. In order for the Taylor rule to function we require the value for the natural steady state level of interest, r^*. This is a level of interest where the economy exists in an inflation neutral position, meaning that it is at full capacity $(u = 1)$. Later, we will use our expression for capacity utilization to derive a specific value for r^*. We also assume that there exists some target level of inflation $\bar{p}$. The Taylor rule takes the form:

$$r = r^* + h_m \cdot (p - \bar{p}) \tag{4}$$

A traditional Taylor would include more elements, including output gap, but Michl uses this simplified rule instead. We will assume for now that the meaning of Taylor rule implies only that the interest rate in determined by a reaction function and we make modifications later on. The standard investment function used in Kaleckian and Steindlian models is as follows:

$$\frac{I}{K} = d_0 - d_1 \cdot r + d_2 \cdot u \tag{5}$$

we know that in equilibrium this expression is equal to the growth rate of capital, g.

Government expenditures (γ) are financed in two ways, either through the issue of new bonds or through taxes. The bonds are one period indexed and represented here as b. It takes this form of the debt-capital ratio.

$$b = \frac{B}{K} \tag{6}$$

where B is total government bonds issued. The primary fiscal deficit (f) before interest payments is simply the difference between government expenditures and the tax collected.

$$f = \gamma - \tau \tag{7}$$

Since we also want to know the total fiscal deficit, we must include the interest payment on previous period bonds.

$$\frac{\Delta B}{K} = f + r \cdot b \tag{8}$$

Expanding this we have

$$\frac{B_{+1} - B}{K} = f + r \cdot \frac{B}{K} \tag{9}$$

$$\frac{B_{+1}}{K} = f + (1 + r) \cdot \frac{B}{K} \tag{10}$$

We divide both sides by growth rate of capital to get

$$b_{+1} = \frac{f + (1+r) \cdot b}{1+g} \tag{11}$$

And in a steady state we will have $b = b_{+1}$ and $r = r_{+1}$ and this solves easily to give what Michl calls the *fiscal sustainability condition* of:

$$b = \frac{f}{g-r} \tag{12}$$

I will use the fiscal sustainability condition throughout the paper to determine if the economy is in what, Michl terms, a *golden age*. In the graphs presented in the appendices we see the resulting ratio of equation 12 during each of the experiments. The golden age will appear as all points less than or equal to 1. If the values of the sustainability condition are higher than one then we are in a leaden age. In a golden age the economy is growing faster than or the same as the interest rate and can therefore support a deficit indefinitely. This is supported by Galbraith (2011, p. 5) who claims that

"...where the real interest rate is below the growth rate or even slightly negative, the fiscal balance required for stability is a primary deficit, and the sustainable deficit gets larger as the debt 'burden' grows."

If instead, the fiscal sustainability condition value is greater than one, then the economy is growing at a rate lower than the interest rate, a leaden age, and a primary fiscal surplus is required to maintain a stable public debt ratio. There will be some target public debt ratio, $\bar{b}$ that will be used with our fiscal instrument. There are two choices available for the fiscal policy instrument. Michl selects a varying tax ratio with the alternative being a government spending ratio reaction function.

Taxes are paid entirely by the capitalist class in a lump-sum payment and in the model it appears as a ratio of capital stock, τ. The goal of the tax ratio is to control the debt-capital ratio and so we will say that the lump sum taxes are paid out of the interest that is paid to the bond holders rather than out of all income. This makes for a more concentrated instrument as there is no effect on the saving out of dividends. The target public tax ratio will maintain our steady state tax ratio goal, τ^*. The fiscal instrument reaction function is therefore:

$$\tau = \tau^* + h_f \cdot (b_{-1} - \bar{b}) \tag{13}$$

Where h_f is essentially the severity of the reactions to missing the public debt ratio goal. The saving decisions are taken by the government and the capitalist class. As an alternative the model could have been created using the a lagged value of τ in place of the target value τ^*. This would add an inertial character to the tax spending ratio which would cause it to oscillate and remove the main element of stability in the equation. The main goal of our rule is to anchor the tax ratio in a steady state and so we will use equation 13 rather than the alternative based on the lagged value.

Capitalists save out of profits at a rate of s and out of net interest at a rate of s_b. The government saving is the negative of the fiscal deficit which takes the form of:

$$f + rb \tag{14}$$

The next assumption is that capitalists for the most part do not extend their planning horizons off to infinity. Since all of our values are being normalized to capital we have the following saving function.

$$\frac{S}{K} = s\pi\rho u + s_b \cdot (rb - \tau) - (\gamma - \tau + rb) \tag{15}$$

Since bonds are all held by capitalists, we might assume that the two saving rates are identical. Here we preserve them as two separate rates and say that the bond holders treat interest income differently than profit income or dividends. The only non-standard term in the equation is found in saving from interest. We see that if the tax ratio is larger than the interest payments then there is a negative effect on overall savings. As mentioned when introducing the tax response function, this results in a tax ratio instrument that is focused solely on the interest from investments rather than all income. This is a simplifying assumption made for this model as we would expect that, in the real world, this type of lump sum tax payment would come out of total income or more likely a tax rate rather than a lump sum. Combining our saving and investment functions and solving for the capacity utilization gives us an IS equation of the form:

$$u = \alpha(d_0 - d_1 r + (1 - s_b)(rb - \tau) + \gamma) \tag{16}$$

Where $\alpha = \frac{1}{c} = \frac{1}{(s\pi\rho - d_2)}$

We want r^* to be the value of r where the economy is running at full capacity utilization. We set equation 16 equal to 1 and isolate for our value of r^*.

$$r^* = \frac{d_0 - c - (1 - s_b) \cdot \tau + \gamma}{d_1 - (1 - s_b) \cdot b} \tag{17}$$

The base model uses the tax ratio as its instrument. I have included an approximate reproduction of Michl results in Appendix A1. His simulation results show a system that begins with a debt-capital ratio that is higher than the target ratio. My simulation begins in a steady state and then I apply the change to b. That is, $b = 0.5$ and $\bar{b} = 0.25$. The motion you see is the system returning to the steady state levels. This original model is labelled MICHL1. Once the natural rates of our parameters are calculated we are able to simulate the conclusions found in the Michl's paper with the following six equations.

$$p = p_{-1} + \varphi \cdot (u_{-1} - 1) \qquad \text{Vertical Phillips Curve (3)}$$

$$r = r^* + h_m \cdot (p - \bar{p}) \qquad \text{Central Bank Reaction Function (4)}$$

$$g = d_0 - d_1 \cdot r + d_2 \cdot u \qquad \text{Growth Rate of Capital (5)}$$

$$\tau = \tau^* + h_f \cdot (b_{-1} - \bar{b}) \qquad \text{Tax Ratio Reaction Function (13)}$$

$$b = \frac{f + (1 + r) \cdot b_{-1}}{1 + g} \qquad \text{Debt-Capital ratio (11)}$$

$$u = \alpha(d_0 - d_1 r + (1 - s_b)(rb - \tau) + \gamma) \qquad \text{Capacity Utilization (16)}$$

2.2 Properties of the MICHL1 Model

Michl uses equation 13, the tax ratio, as the primary fiscal instrument and in this section we explore what properties the model gains through the inclusion of this instrument. We begin by looking at the properties of the model without this instrument in place. We remove equation 13 from the MICHL1 model and set it as a constant equal to the steady state value, $\tau = \tau^*$. This model is labelled MICHL2. In the original model we have the government spending ratio constant at 0.115. For this simulation I apply a one period shock to the government spending ratio in the form of an increase of γ to 0.12. The results of this shock on MICHL1 are available in appendix A2, while results from

MICHL2 are in Appendix A3. Without the fiscal response function to hold the debt-capital ratio in place we see that the result of the shock is severe in MICHL2. The initial effect is to increase the capacity utilization which, by design, affects our inflation and therefore in the next period interest rates increase as well. The lack of a debt-ratio targeting instrument means that there is nothing to control the growth of the debt-capital ratio. So as the inflation and interest rate mechanisms try to move the capacity utilization back to the steady state level the debt-capital ratio continues to grow because taxes are unable to increase to compensate for the higher interest payments that must be made on the debt. Our vertical Phillips curve is based on a lagged value and when the capacity utilization reaches the level of 1 we still have a higher level of inflation than what we previously had in our steady state. This means that we also have a higher level of interest as it tries to compensate for the inflation rate. This also leads to a lower level of capital growth as it is inversely affected by the rate of interest. The result is that our debt-capital ratio explodes along with the interest and inflation levels and they all move away from steady state to exponential levels.

With the same experiment being run on MICHL1 we see that the inclusion of the tax ratio instrument allows the economy to absorb the shock and return to its steady state levels. When there is a one period government spending shock we see the reaction in the tax function immediately. The tax ratio increases which forces the debt-capital ratio back down to its steady state level. Even the initial spikes in inflation and interest are smaller when this instrument is in place. All variables return to their steady state values and the economy returns to a golden age. The growth rate of capital does drop below its steady state level for a few periods so there has been some adverse affects but overall since we are back on our original growth paths and the economy avoids any exponential variable states, the effect of having this tax-ratio based instrument is overwhelmingly positive.

3.1 Government Spending as the Fiscal Instrument

In MICHL1 the fiscal policy is controlled using the tax ratio. The alternative is instead to have the government react to the change in the state of the economy by adjusting its spending ratio. Modifications to the level of spending by government will better simulate the real world as it is unlikely that a government would be able to change tax ratios each period as the MICHL1 fiscal rule would suggest.

For the experiments in this section, I must make an extra assumption. Michl solves the original system for the steady state values of τ^*, g^*, and $r^*$when we have our debt-capital ratio equal to its target. Each of these expressions relies on the values of γ and $s_b$which I will be making endogenous in this section. The new forms of these variables are designed in such a way that they are affected by the short-term shock experiments I will be conducting. I assume that in the long run the natural rates for the three previously mentioned terms remain unchanged and that any changes to them in the short term will not have major impact on the overall results of the simulations.

Here I hold the tax rate constant and instead adjust government spending. The tax ratio reaction function is removed and replaced with a constant value $\bar{\tau} = \tau^*$.

There are a number of options for the form of the government spending response functions but since the original tax ratio response function related to the target debt-capital ratio, here we will use something similar. Our government spending becomes.

$$\gamma = \gamma^* + h_{f2}(b_{-1} - \bar{b}) \tag{17}$$

Here h_{f2} is a ratio less than one similar to the original h_f. The value of h_{f2} can be adjusted to increase or decrease the strength of this rule depending on how strongly the government wants to react to changes in debt ratio. This model is labelled FISCAL1. For the simulation we use $h_{f2} = 0.04$ which makes the rule rather weak but avoids any extremely fast changes in government spending. γ^* is the steady state level of spending established by Michl in his original model. Here we will use it as the basis for our fiscal reaction function. Similar to our tax-ratio based reaction function, the goal here is for stability. If we were to use a lagged value of the government spending ratio rather than the steady state value, it would result in large swings in the current government spending as it tried to move back towards the steady state. This form of the spending function leads to a constant starting point for the spending ratio selection each period, and therefore should lead to more stability.

Appendix A4 shows the results in the economy of FISCAL1 in a situation similar to the one outlined in the original paper. The debt to capital ratio is b = 0.5, while the target here is $\bar{b} = 0.25$. As we would expect there is a slow decrease in the debt ratio itself. To correct for the high debt ratio, the government instantly decreases it spending ratio. This causes a corresponding fall in the capacity utilization. By design of the response functions we then see a period of deflation in the economy which drives down interest rates. Capacity utilization rebounds and then pushes above its original level of 1 which causes inflation to return to its original levels. The growth rate of capital during this period moves much higher initially but then eventually moves lower again. In the long run there is a higher growth rate of capital. Since the interest rate and the growth rate of capital do not move in exactly the same way, the growth rate moves up while interest rates drop, our economy moves further into a golden age while the shock is being absorbed. In equilibrium the system is in a golden age and so it eventually returns to its original level without ever moving into a leaden age. The only long-term change in the economy is that the growth rate of capital is permanently higher. This would imply that if a government was able to easily change government expenditures as a response to the view that the government debt to capital ratio was higher than its target level, then they should immediately begin to takes steps to make those changes, because the reduction in the relative size of government expenditures leads to a faster rate of accumulation in the long run, while achieving the low debt to capital ratio target.

3.2 – Adjustable Saving Preferences

Tcherneva (2008) points out, based among other things, on research by Woodford (1995), that when a government finances a spending increase with the issue of new bonds, the bond holders consider this a lifetime increase in their wealth (in contrast to the Ricardian equivalence theorem put forward by Barro). The bond holders have a propensity to save out of net interest of 0.9 in the original model. Here we will include a reaction function for this rate of saving out of interest based on the current state of the debt-capital ratio. The bond holders have a rate of saving from interest that exists when the debt-capital is at its target ratio. We will call this saving rate, s_b^*. Any time the economy enters a situation where, b $> \bar{b}$ the bond holders will purchase fewer bonds to try and regain their target. The behavioural equation that we add to the model FISCAL1 is:

$$s_b = s_b^* + h_b\left[\bar{b} - b_{-1}\right] \tag{18}$$

where h_b is the responsiveness of the propensity to save out of interest income to changes in the debt-capital ratio. Here, again we wish for a stable base for the decision rule and select the steady state savings rate established my Michl rather than a lagged savings value that would cause large changes in the values between periods. This gives us the model FISCAL2. We also add the same equation to MICHL1 to get MICHL3. In the simulation, I use a value of $h_b = 0.25$ which means that for every percent the debt-capital ratio is over its target, the bond holders will save a quarter percent less of their interest revenue. This number can also be adjusted to change the strength of the reaction function. I use this value based only in an intuitive sense that people will save less as the debt ratio climbs. Further empirical analysis could be used to get a more precise value of this parameter but that work is beyond the scope of this paper. Since our assumption is that the steady state rates of τ^*, g^*, and r^* are unaffected by the endogenization of this value we expect the effect of this change to be slight but noticeable and to be acting through the value of capacity utilization rather than directly.

The results from FISCAL2 (Appendix A5) are very similar to FISCAL1 but there are noticeable differences. Rather than the smooth transition from b to $\bar{b}$, we see in FISCAL1 there is a dip below the target level. This type of trend is true of all the variables. All the movements for FISCAL2 are similar in shape and direction but are more extreme. The changes in capacity utilization, interest rates and inflation are all extreme at both the high end and the low end of the scales. The major difference we see here is that FISCAL2 moves into a leaden age before returning to its stable golden age growth rate and, as expected, we see the savings rate adjust. The growth rate of capital drops below its steady state level, which means that, even though the long-term growth rate remains unchanged, the entire system has a lower overall amount of capital than it would have had without the addition of this saving rule.

Section 3.3 Original Taylor Rule

The term Taylor Rule is used throughout the economic literature whenever referring to the reaction function of a central bank when it sets interest rate levels. In the MICHL1 model the rule is based on a steady state natural level of interest. In this section, I will instead use a reaction function that more closely resembles the original function laid out by Taylor (1993). This original version of the rule takes this form:

$$r = 0.5y + 0.5(p - \bar{p}) + r^*$$

where r* is the targeted interest rate, p is the current level of inflation, y is the output gap and $\bar{p}$ is the target level of inflation. Since we are using a fixed coefficient production function that is capital constrained we use the capacity utilization variable to substitute for the output gap term. It is not realistic to know the output gap of the current period so instead we use a one period lagged value and consider that full capacity utilization will be equal to the potential output of the economy. We use the same target level of $\bar{p}$ as in the original model. We replace the central bank reaction function in model MICHL1 with the following:

$$r = p + 0.5(u - 1) + 0.5(p - \bar{p}) + r^*$$

This modified model is TAYLOR1. In addition, we will use this original form of the Taylor rule in our FISCAL1 model to replace the original central bank reaction function to create TAYLOR2.

Section 4.1 - Demand Shock Simulations Under Fiscal Reaction Functions.

(Michl 2011, p. 13) shows that the mathematics of the investment function implies that when there is a demand shock (Δd_o) it is possible to offset it with a corresponding increase in government spending. However in the original model the government spending ratio is fixed and so the absorption of this type of shock must take place in the other variables. The models that have been defined to this point are listed in the following table.

Model	Features	Description

MICHL1	Original Model	No changes from the original model laid out by Michl.
MICHL2	$\tau = \tau^*$ (constant)	Tax ratio held constant.
MICHL3	$s_b = s_b^* + h_b\,[\bar{b} - b_{-1}]$	Saving decision added.
FISCAL1	$\tau = \tau^*$ (constant) $\gamma = \gamma^* + h_{f2}(b_{-1} - \bar{b})$	Tax ratio held constant. Government spending ratio decision rule added.
FISCAL2	$\tau = \tau^*$ (constant) $\gamma = \gamma^* + h_{f2}(b_{-1} - \bar{b})$ $s_b = s_b^* + h_b\,[\bar{b} - b_{-1}]$	Tax ratio held constant. Government spending rule added. Saving decision rule added.
TAYLOR1	$r = p + 0.5(u - 1) + 0.5(p - \bar{p}) + r^*$	Taylor rule modified.
TAYLOR2	$\tau = \tau^*$ (constant) $\gamma = \gamma^* + h_{f2}(b_{-1} - \bar{b})$ $r = p + 0.5(u - 1) + 0.5(p - \bar{p}) + r^*$	Tax ratio held constant. Government spending ratio decision rule added. Taylor rule modified.

The starting values and the values of the parameters can be found in Appendix C.

In this section we use our four models, MICHL1, MICHL3, FISCAL1 and FISCAL2 and simulate this shock. Michl states "The mathematics of the difference equations suggest that this shock will generate oscillations. A severe shock (e.g., one which involves a deflation trap) might be attacked with a discretionary fiscal stimulus…" so we expect some form of oscillation in the MICHL1 model. The goal of the modification is to see if any of the changes we make in the various models can decrease the severity of the oscillations. In all cases the shock that is applied is a change in d_o from 0.1 to 0.05, meaning that our accumulation drops by 5 percentage points.

4.2 – Demand Shock in MICHL1 and MICHL3

Results for this simulation are available in Appendix B1. In section 2 we saw that without the tax-ratio instrument that MICHL2 model becomes unstable and so I omit the demand shock simulations on it and use only MICHL1 which includes the tax mechanism. The oscillations mentioned by Michl appear in the form of the state of the economy in reference to golden and leaden ages. The economy swings between golden ages and leaden ages until the other variables return to their steady state levels. There are no long term changes in any of the variables that we observe. The shock forces the debt ratio to move slightly upward, which forces the capacity utilization down. This in turn moves our inflation and interest rates as we would expect. In fact, we see a large negative interest rate here which implies that the economy would get stuck at the zero lower bound. When we run the same shocks on MICHL3 the results are almost identical and can be found in Appendix B2. The savings decision rule only serves to amplify the movement of all variables in the system. Initial responses are all the same but when the system is trying to move back into the steady state the swings are more severe.

4.3 – Demand Shock in FISCAL1

Results from this simulation are available in Appendix B3. Initially we see that the demand shock forces the debt-capital ratio to drop for a period before the reaction functions are able to start to compensate. This also forces a drop in capacity utilization immediately after the shock, which leads to deflation period. During a single period the growth rate is pushed down but the interest rate does not react until the next period so we see that the economy is pushed into a leaden age. As designed however, the reaction functions then start to work and all of the values return to their steady state values. The oscillation that we see with the MICHL1 model is no longer present as we expected. The only long term effect that we see is that there is not a permanently higher level for the government spending ratio. Although initially we would assume that this is not a desirable state to be in, we see that the economy does return to a golden age and so the spending is sustainable even at this higher level. When compared to MICHL1 we expected that the results would be identical as the two fiscal instruments were designed to emulate a change in the fiscal stance of the government. The only difference, other than the lack of oscillation is the fact that in FISCAL1 the debt-capital ratio returns slowly to its steady state level. In MICHL we saw this ratio not only returned quickly to its original level but actually moved above it for a few periods before returning to the lower steady-state level.

4.4 – Demand shock in FISCAL2

All results from this section can be found in Appendix B4. As in our first experiment all of the movement that we saw during the shock response in FISCAL1 are also observed in

FISCAL2. Again in this case we see that all the effects are more acute and their magnitudes slightly larger. The most interesting difference is that the inflation levels are more extreme in FISCAL2 before returning to steady state levels. It is commonly accepted that a more stable level of inflation leaves consumers better off. (Woodford (1999) and Gali and Monacelli (2005)). The inclusion of the saving reaction functions seems to have the higher level of inflation variation of all three of the models when we apply these demand shocks. We conclude therefore that compared to the form I have selected for this simulation, the tax ratio reaction function gives the least amount of variance in the inflation and therefore leaves the consumers better off. Unfortunately, as I have mentioned previously I do not believe this type of tax-ratio reaction function to be a realistic method of representing a government's possible reaction to the need for changes in fiscal policy.

4.5 – Demand Shock in TAYLOR1

All results from this section are available in Appendix B5. Since both TAYLOR1 and MICHL1 are based on Taylor type rules we expect the outcomes of the two simulations to be similar but the addition of utilization term results in some noticeable differences. The first difference is that the oscillation between golden age and leaden age is much less severe in the TAYLOR1 model. There is only a single oscillation before the system returns to the steady state golden age. This is likely the consequence of the other main difference that we observe, which is that all of the movements in the TAYLOR1 model are faster and the changes take place in a smaller number of periods. Also this could be attributed to the fact that the original Taylor rule uses larger coefficients. We see that in all cases this results in larger magnitudes of changes as well. The return to steady state takes place in fewer periods but the extreme values the variables take before they return to the steady values are higher. This even occurs in the maximum and minimum values obtained in the changes in tax ratio. The single largest factor is that with the change to the Taylor rule the steady state level of inflation ends up being lower. This means that the economy is able to return to full capacity while with a lower level of inflation. Since there are not long term adverse effects we can conclude that this new Taylor rule is a better choice. We move to the more realistic government spending reaction function in the next section.

4.6 – Demand Shock in TAYLOR2

All results from this section are available in Appendix B6. We have already seen that the results from FISCAL1 and FISCAL2 are very similar so while TAYLOR2 is FISCAL1 with the new Taylor we can forgo repeating the simulations with a version of FISCAL2 that includes the original Taylor rule and consider the results from TAYLOR2 to be adequate to explain both.

As we expect the results from TAYLOR2 are very similar to the results that we see for TAYLOR1. The shock is absorbed here by government spending of course and with the strength of the rules that we have select we see that the return of the debt-ratio to the its original steady state level is relatively slow when from compared to the MICHL1 model.

The lower long run level of inflation is also present here. Although we can see that the rest of the system does eventually return to the steady state levels that allow a sustainable deficit to be run, the number of periods that the government spending is increased may cause political problems rather than economic problems.

Section 5 – Discussion and Conclusions

In each of the demand shock simulations we see a common problem, which is that the interest rate would require a move below 0% in order for the economy to return to its steady state. None of the models or combination of reactions functions is able to fully absorb the shock without getting to the zero-interest lower bound. This is what Michl refers to as a depression level shock. There are only one-period demand shocks and yet they can cause persistent problems in the long run. Once these systems hit the lower zero percent barrier they could be stuck in a depression, which includes a deflationary trap. So while I believe that the updated models outlined in this paper have a more solid grounding in reality, they still possess some of the same short comings. We must therefore conclude that in order to effectively absorb this type of demand shock the government would need to move outside of a standard reaction function, for instance by putting in place one-time fiscal stimulus package.

If we assume that passing the lower interest rate bound is simply a result of the construction of the model and that the system was able to return to the steady-state level, then we still must concern ourselves with the altered growth path of capital. While all the models examined do return to the steady-state growth path of capital, we see that the use of a government spending ratio rule does allow the debt-capital ratio to remain lower for a much longer period of time. Although this makes no real difference in the long run of the system, we can assume that politically this is a much easier sell to the public.

The one positive change that we can see if you look only at the change in interest rates response function in TAYLOR1 is that the system returns to its steady-state growth level faster than with the rule originally laid out by Michl. It also leads to a lower level of long run inflation. We can conclude then that, although all of the models will move off the original growth path during a shock it would be best to work with a response function similar to the one originally laid out by Taylor rather than to modify it.

References

Bernoth, K., von Hagen, J., & Schuknecht, L. (2012). Sovereign risk premiums in the european government bond market. *Journal of International Money and Finance, 31*(5), 975-995. doi: 10.1016/j.jimonfin.2011.12.006

Broda, Christian and Weinstein, David E. (2012). Happy News from the dismal science: Reassessing Japanese Fiscal Policy and Sustainability. (Working Paper 10988). Retrieved from National Bureau of Economic Research. Website: http://www.nber.org/papers/w10988

Corrado, C., & Mattey, J. (1997). Capacity utilization. *Journal of Economic Perspectives, 11*(1), 151-167. Retrieved from http://search.proquest.com/docview/56723119

Domar, E. D. (1946). Capital expansion, rate of growth, and employment. *Econometrica (Pre-1986), 14*(2), 137-137.Retrieved from http://search.proquest.com/docview/214701787

Felipe, J. J. McCombie (2006). The tyranny of identity: growth accounting revisited, *International Review of Applied Economics,* 20 (3) (July) pp 283-99

Foley, Duncan K. and Michl, Thomas R. (1999). *Growth and Distribution***.** Harvard University Press, Cambridge, Mass.

Galbraigth, James K. (2011) Is the federal debt sustainable? Jerome Levy Economics Institute (Annandale). Policy Note.

Gali, Jordi, and Tommaso Monacelli. (2005) Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies* 72 (3) (July 2005): 707-34, http://www.blackwellpublishing.com/journal.asp?ref=0034-6527.

Godley Wynne Lavoie M (2012). *Monetary Economics : An Integrated Approach to Credit, Money, Income, Production and Wealth.* (New York: Palgrave Macmillan)

Michl, T. R. (2008). Tinbergen rules the taylor rule. *Eastern Economic Journal, 34*(3), 293-309. Retrieved from http://search.proquest.com/docview/56788407

Michl, T.R. , Taylor, L. And Rezai, A. (2012, Forthcoming) "A Model of Fiscal and Monetary Policy," eds. *Social Justice and Economics: Critical Economic Theory in the Spirit of Duncan Foley*, Routledge,

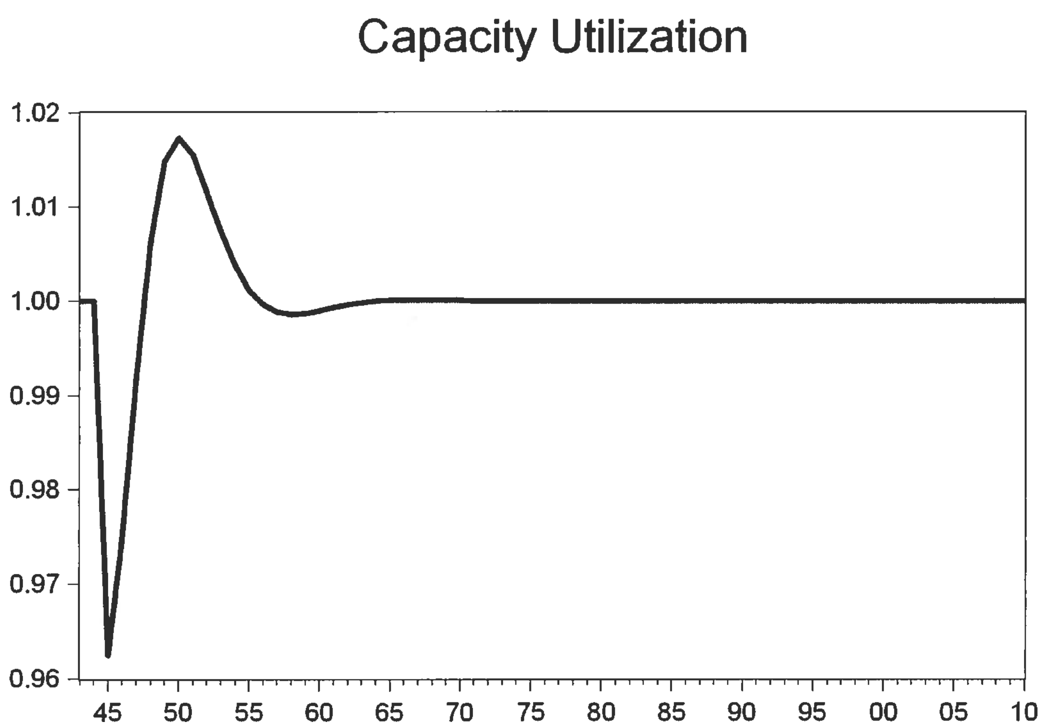
Sutherland, D., P. Hoeller and R. Merola (2012), Fiscal Consolidation: Part 1. How Much is Needed and How to Reduce Debt to a Prudent Level? OECD Economics Department Working Papers, No. 932, OECD Publishing. http://dx.doi.org/10.1787/5k9h28rhqnxt-en

Tcherneva, Pavlina R. (2008) The Return of Fiscal Policy: Can the New Developments in the New Economic Consensus Be Reconciled with the Post-Keynesian View? (Working Paper No. 539). The Levy Economics Institute.

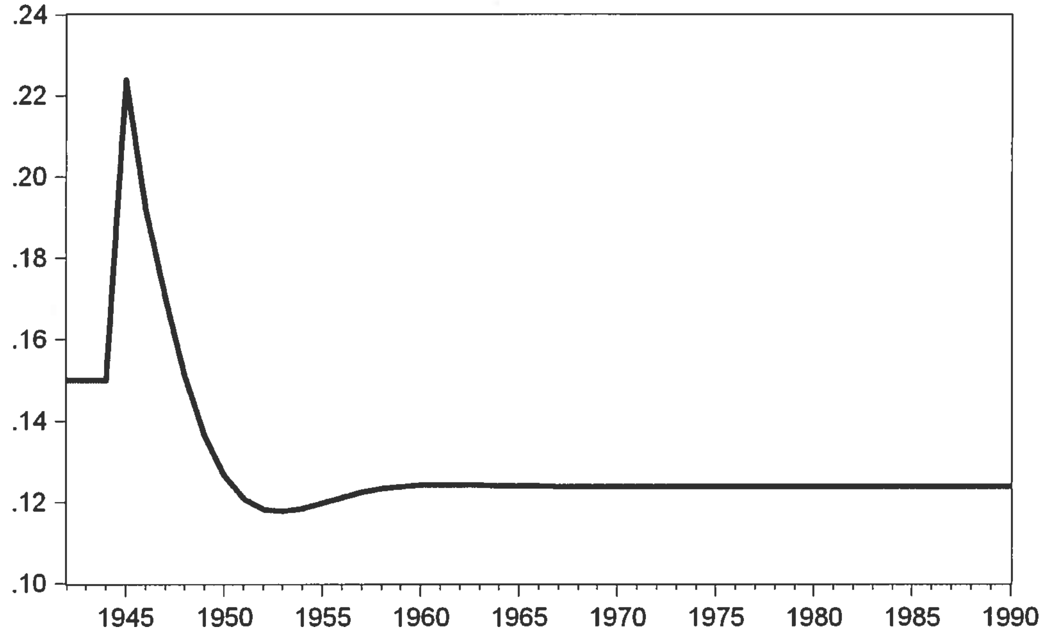
Woodford, M. (1995) "Price-level Determinacy Without Control of a Monetary Aggregate." Carnegie-Rochester Conference Series on Public Policy (43): 1-46.

Woodford, M. (1999) Inflation Stabilization and Welfare. (Working Paper 8071) Retrieved from National Bureau of Economic Research. Website: http://www.nber.org/papers/w8071

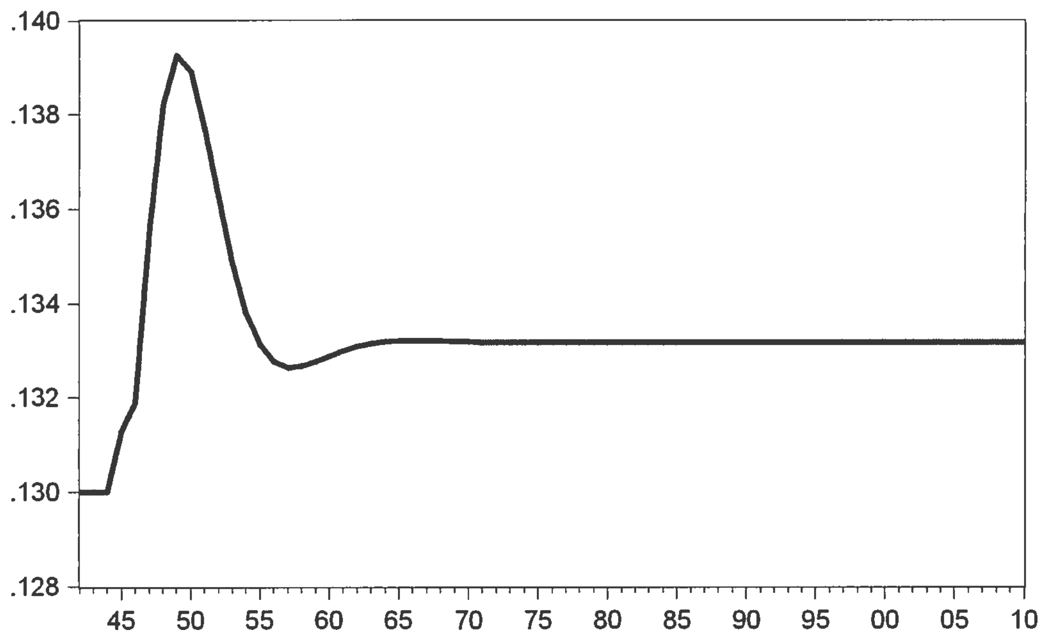
Appendix A1 – MICHL1 Change in Debt-Captial Ratio

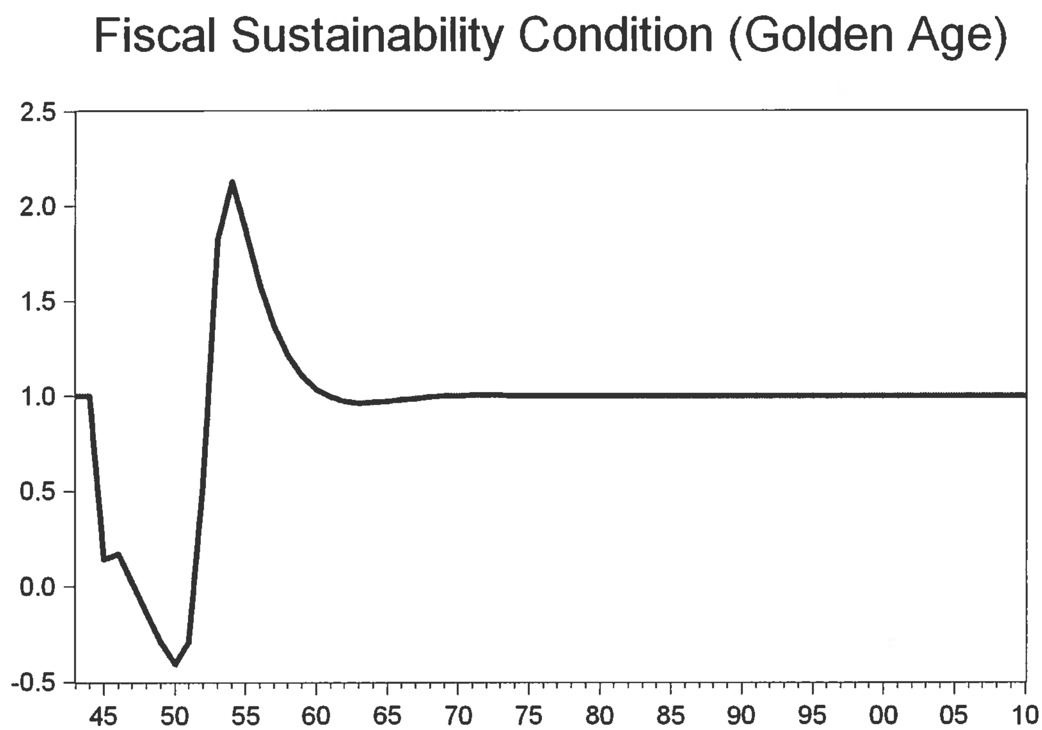


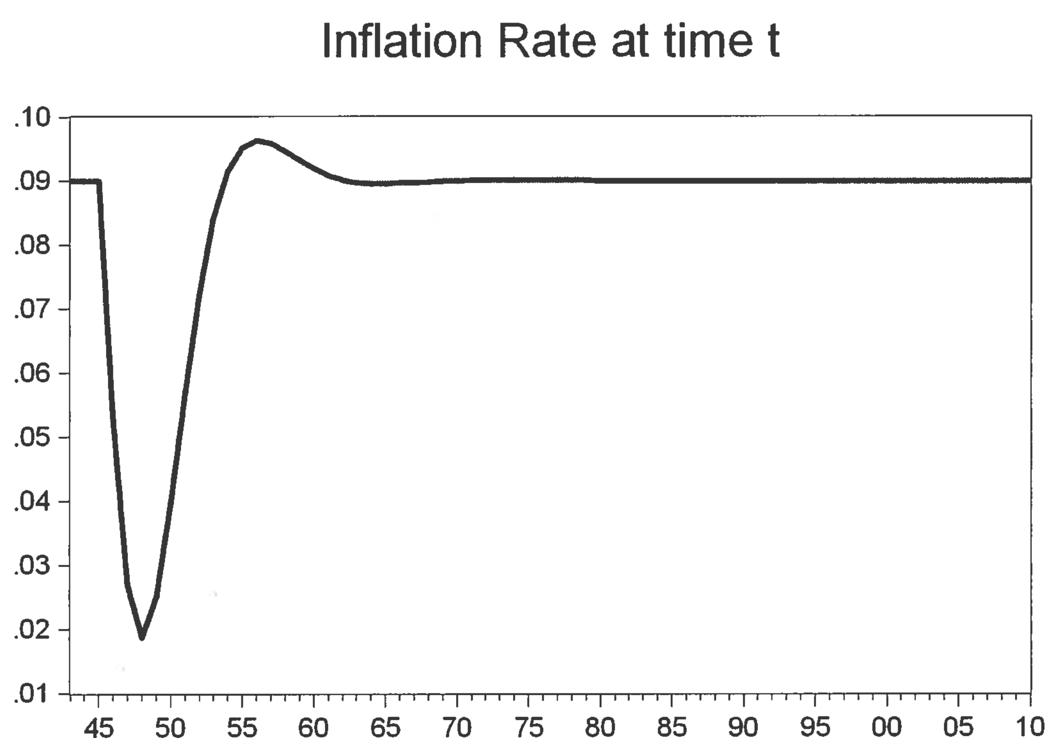
Tax (as a percentage of capital stock)

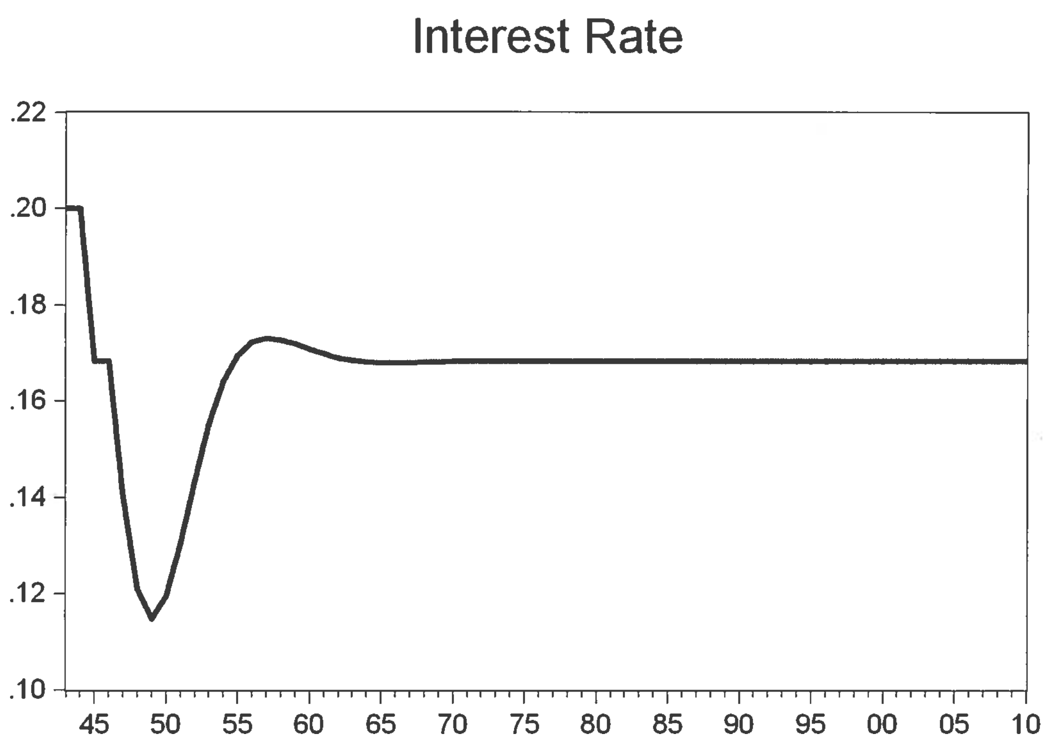


Growth Rate (I/K)

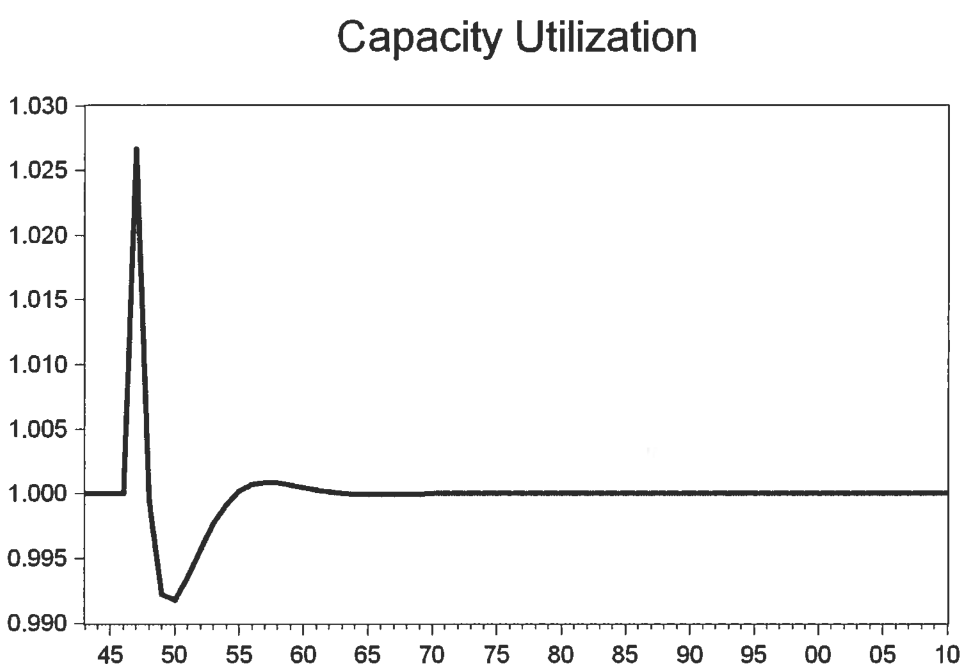



Fiscal Sustainability Condition (Golden Age)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
45
50
55
60
65
70
75
80
85
90
95
00
05
10

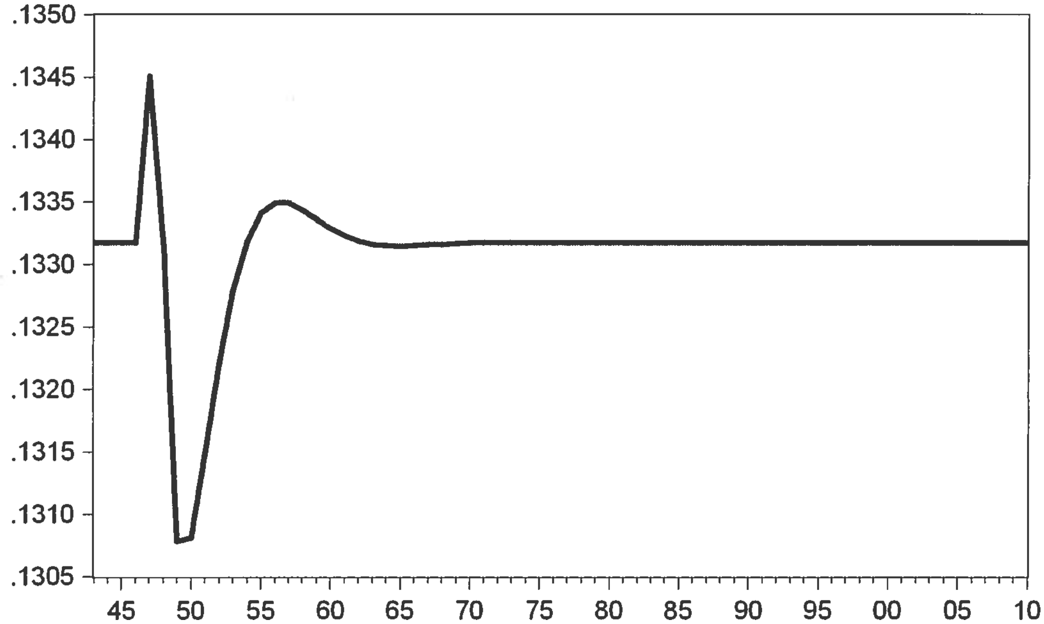

Inflation Rate at time t
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
45
50
55
60
65
70
75
80
85
90
95
00
05
10



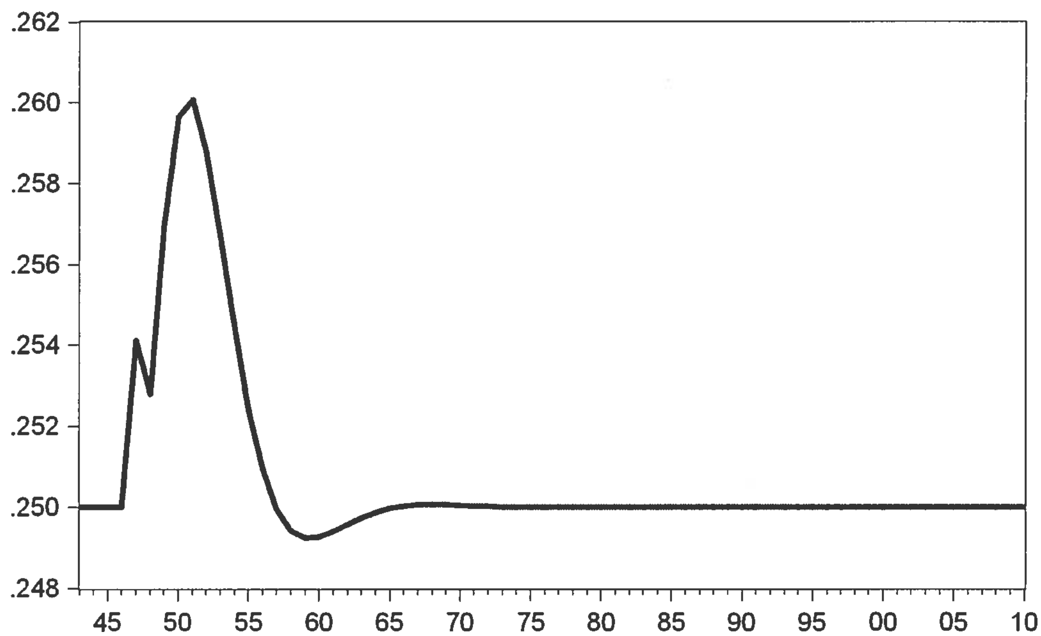
Appendix A2 – MICHL1 Government Spending Shock



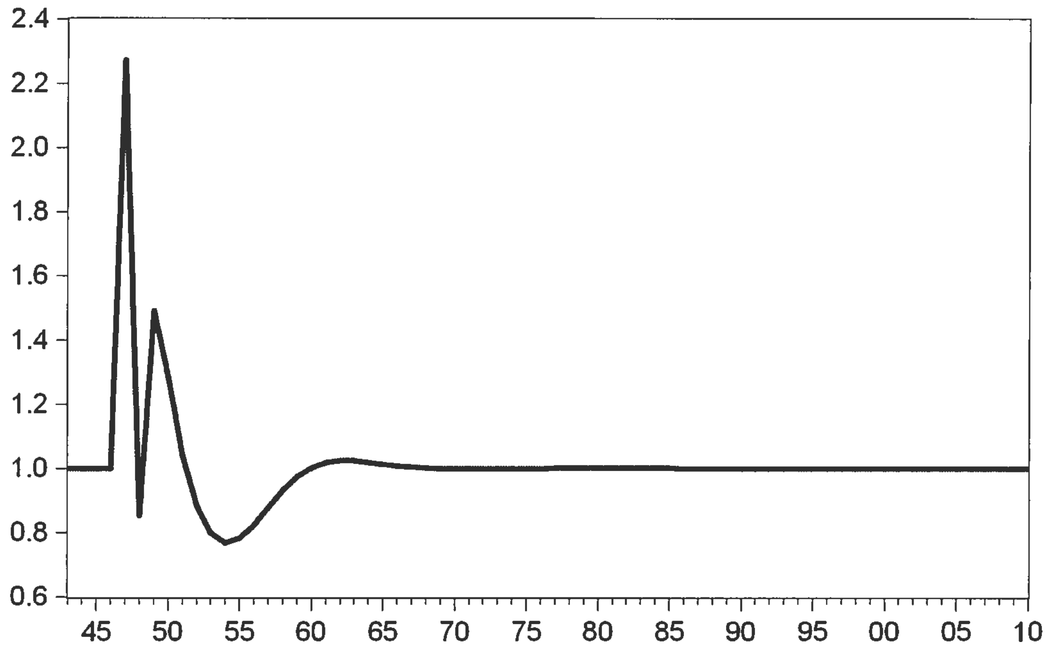
Growth Rate (I/K)



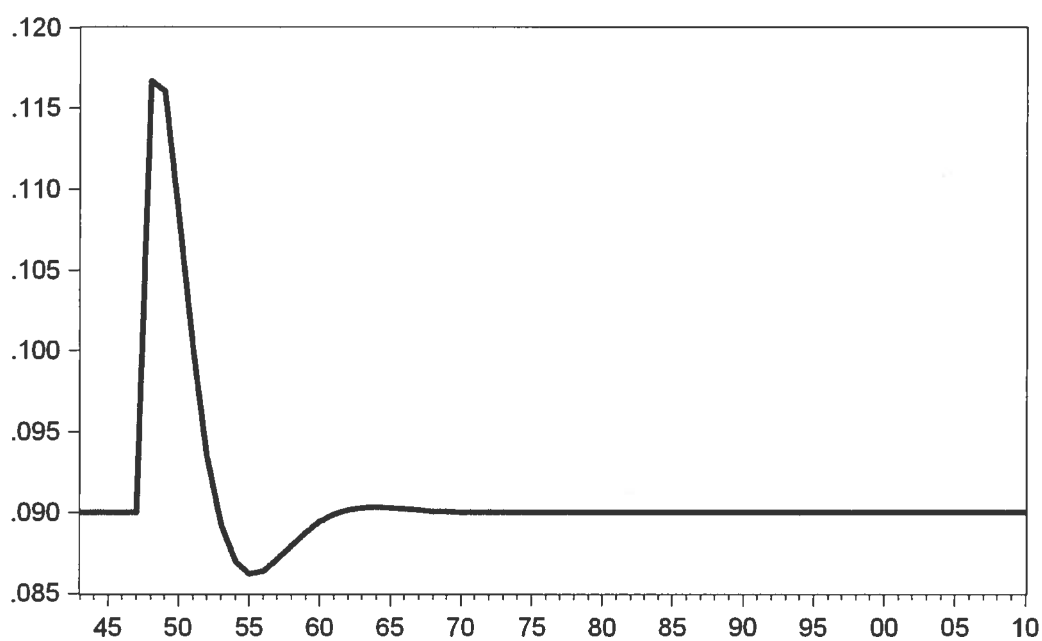
Debt-Capital Ratio



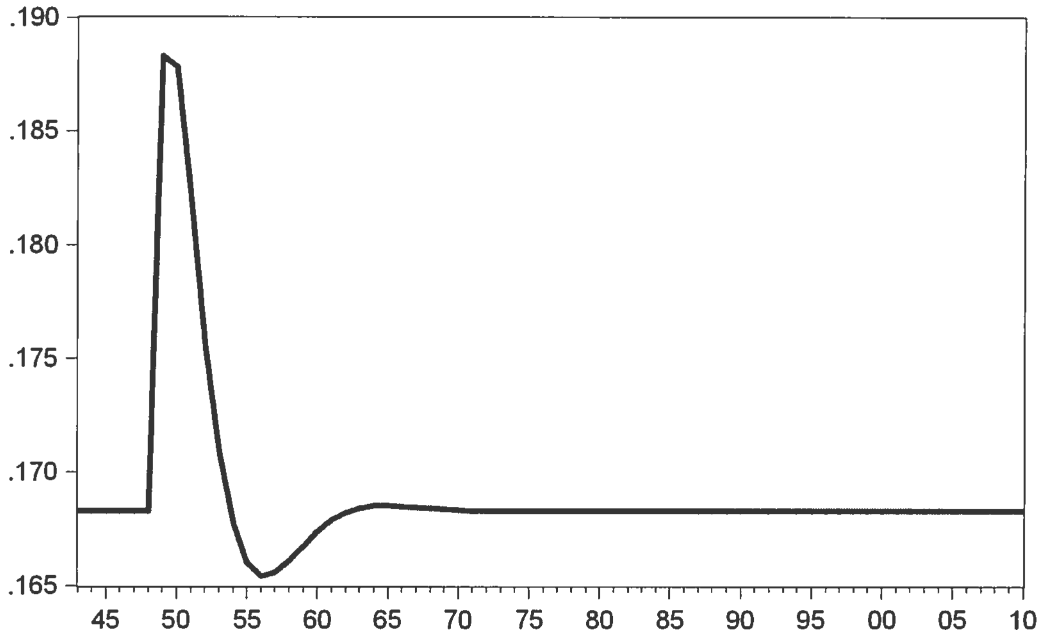
Fiscal Sustainability Condition (Golden Age)



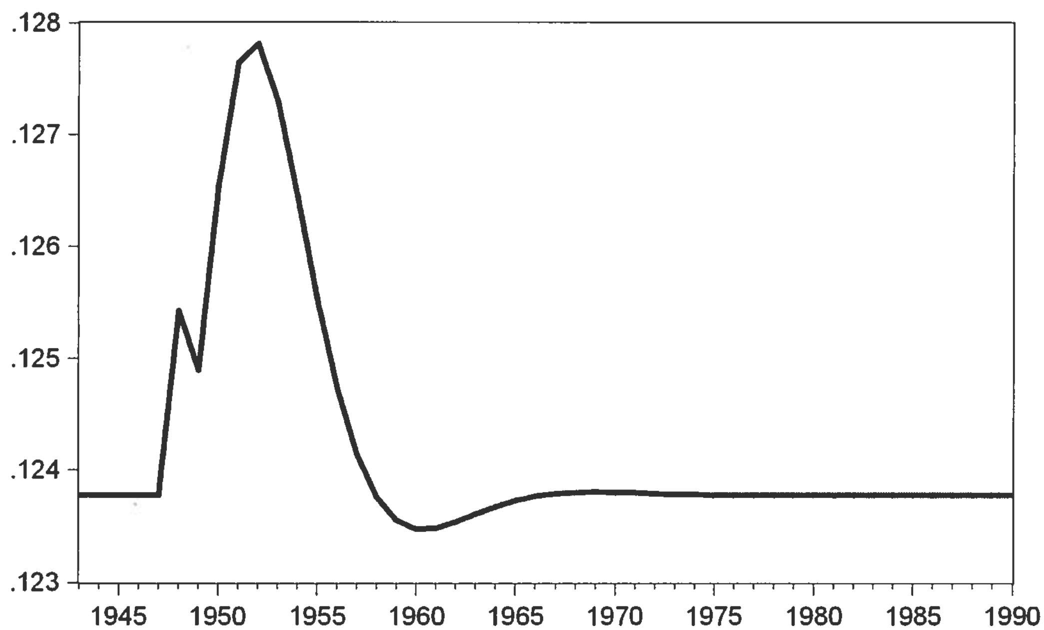
Inflation Rate at time t



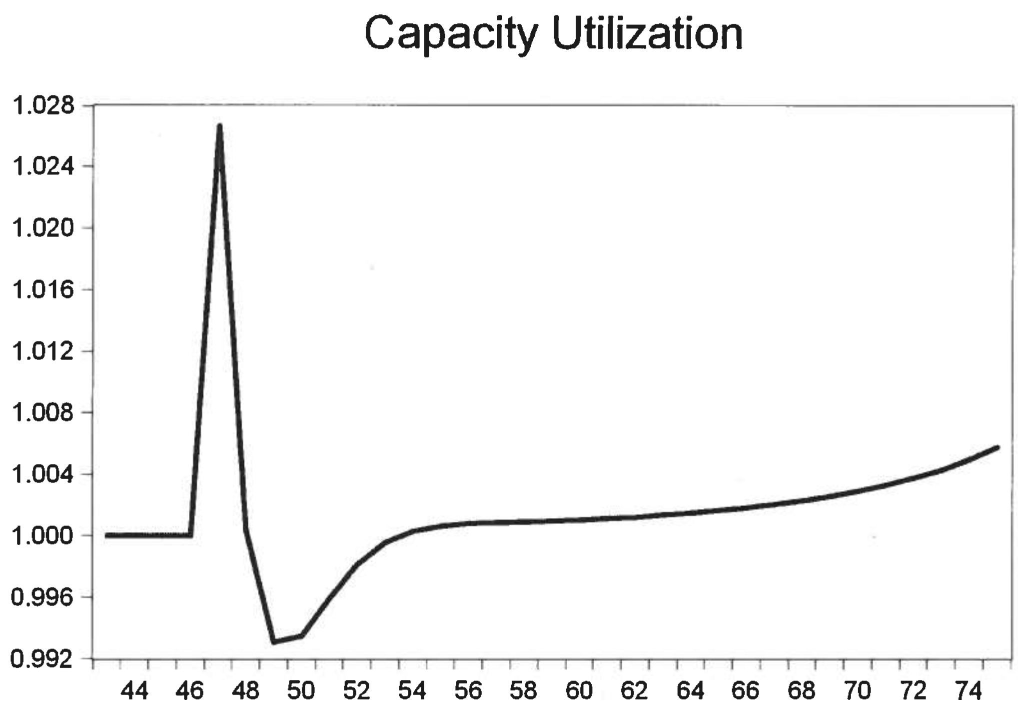
Interest Rate

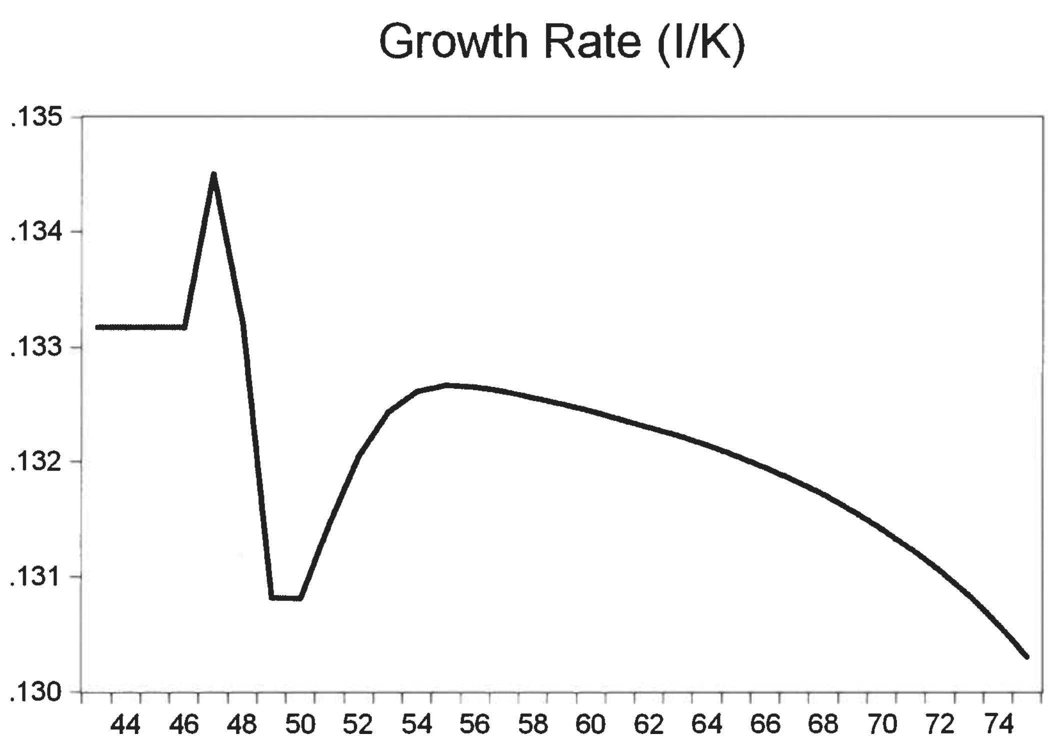


Tax (as a percentage of capital stock)

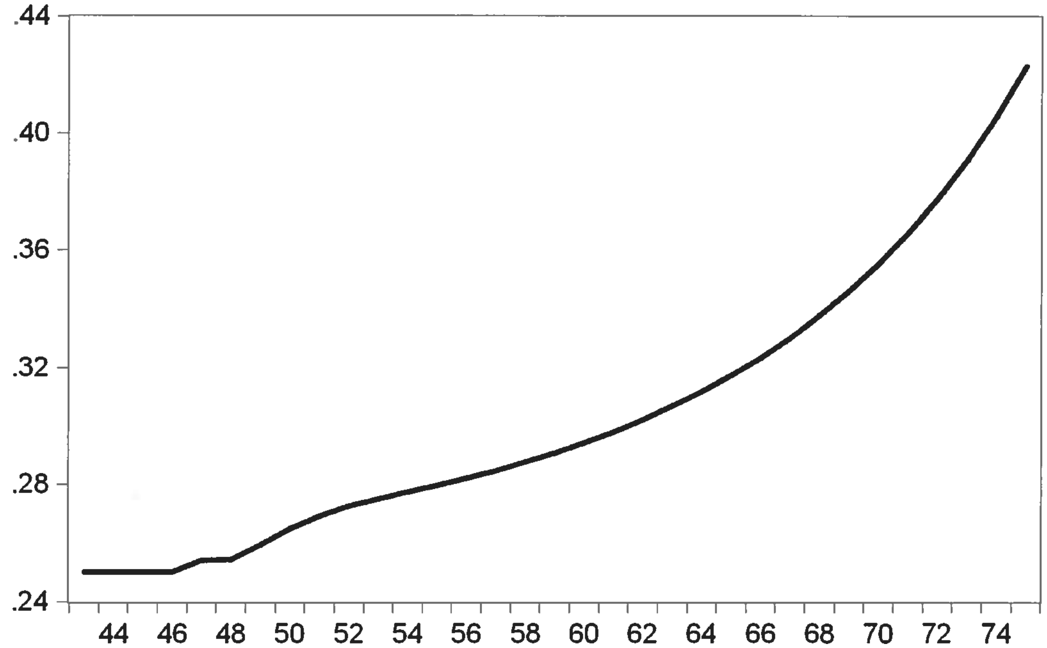


Appendix A3 – MICHL2 Government Spending Shock

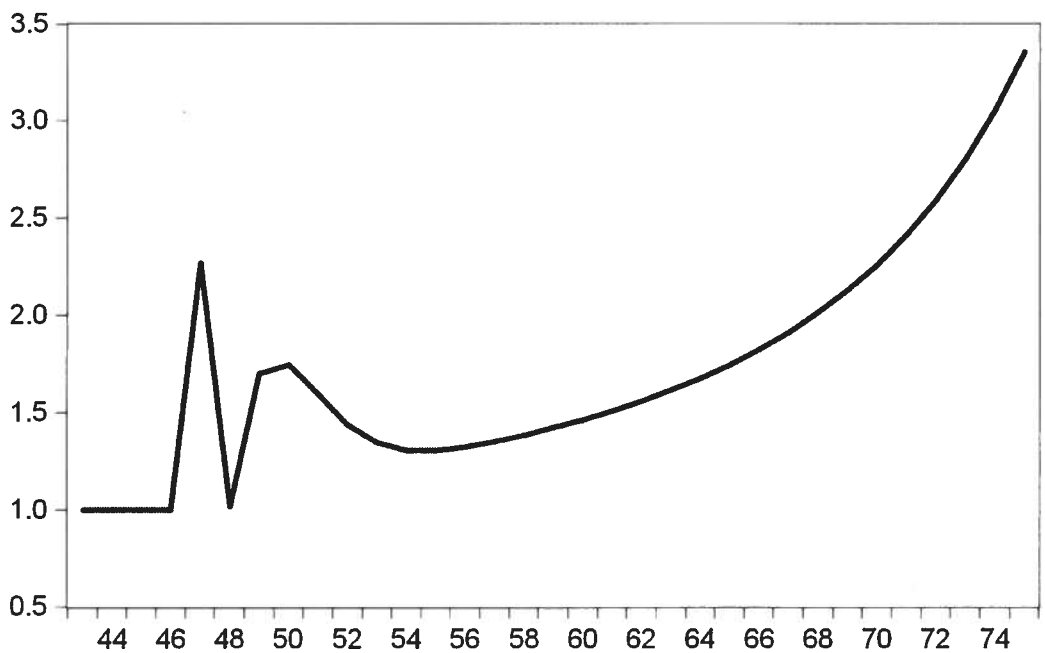


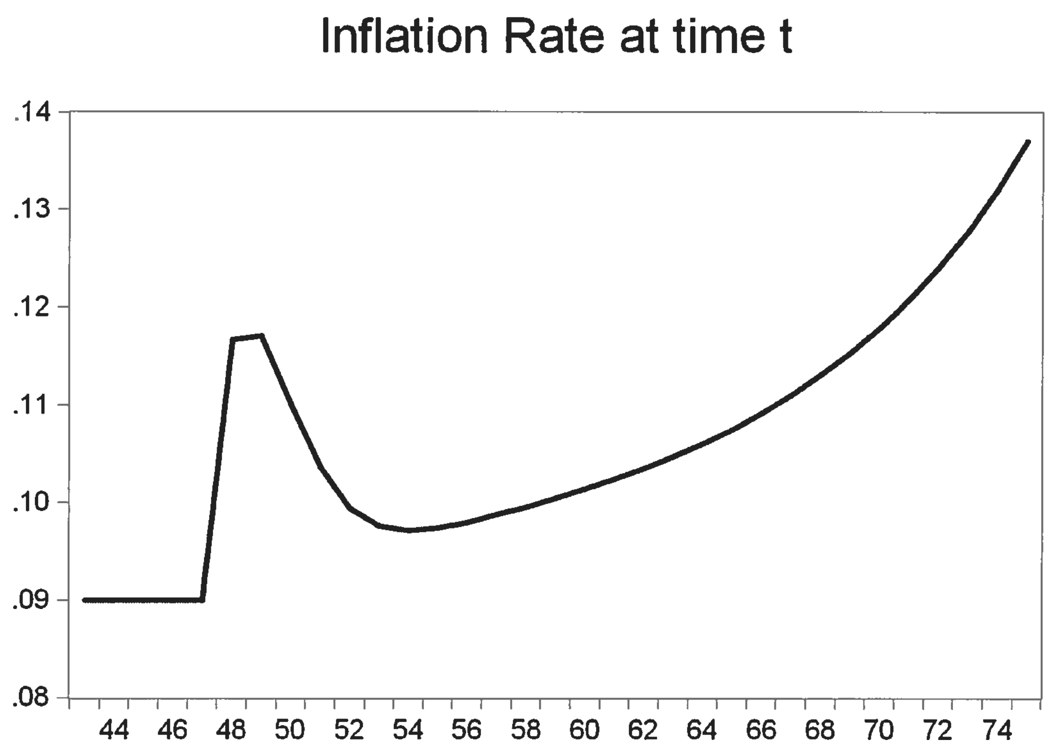


Debt-Capital Ratio

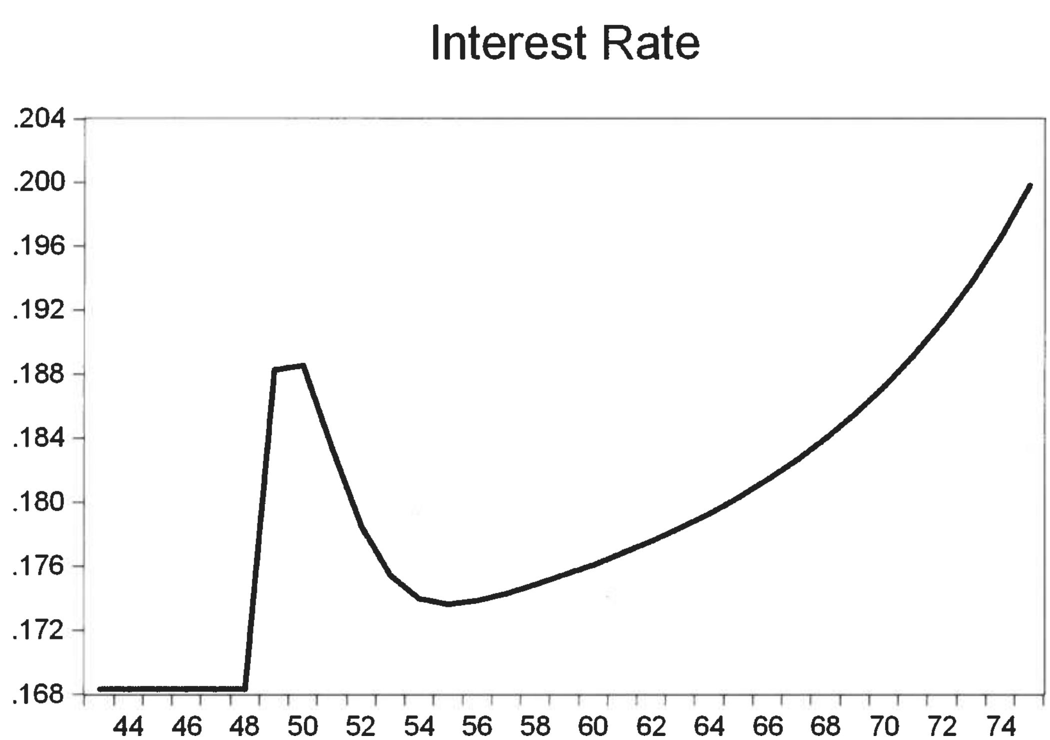


Fiscal Sustainability Condition (Golden Age)



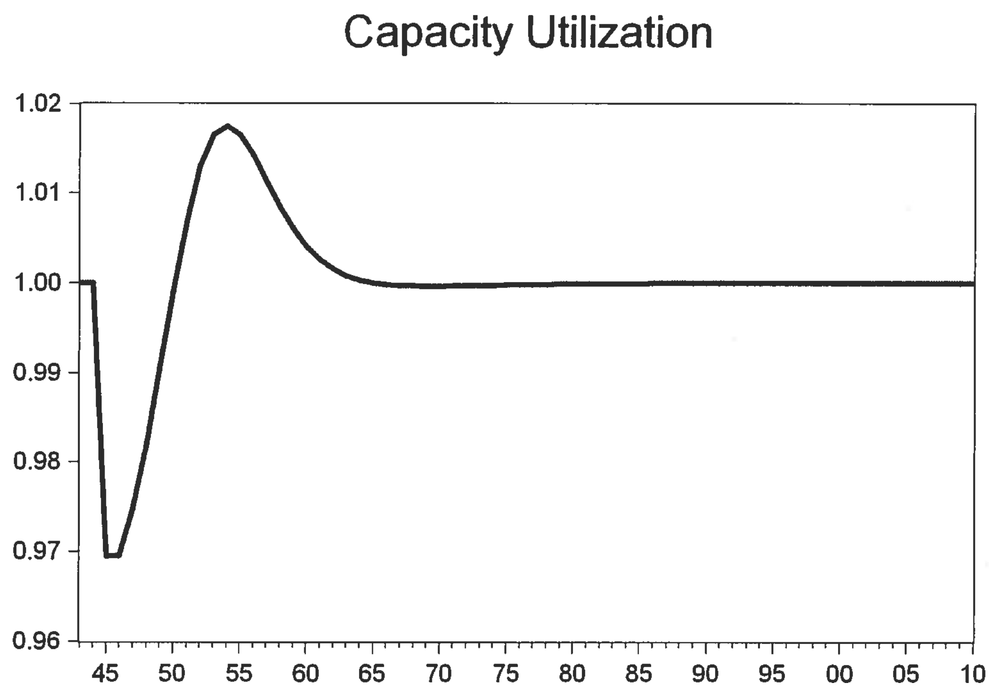


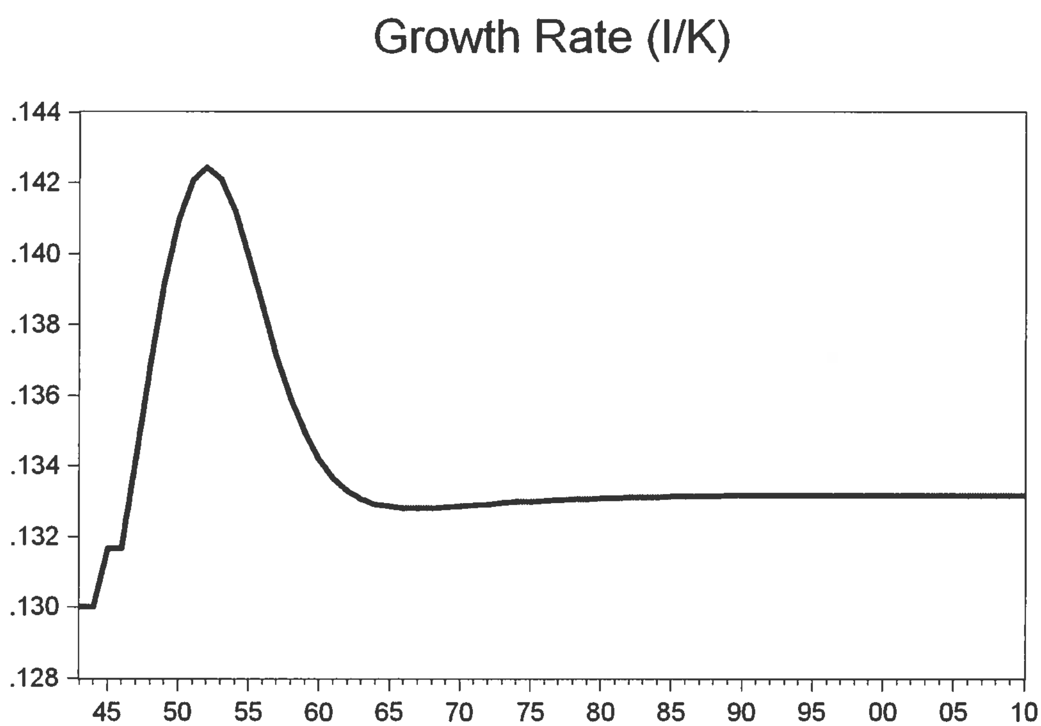
Inflation Rate at time t
.14
.13
.12
.11
.10
.09
.08
44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74



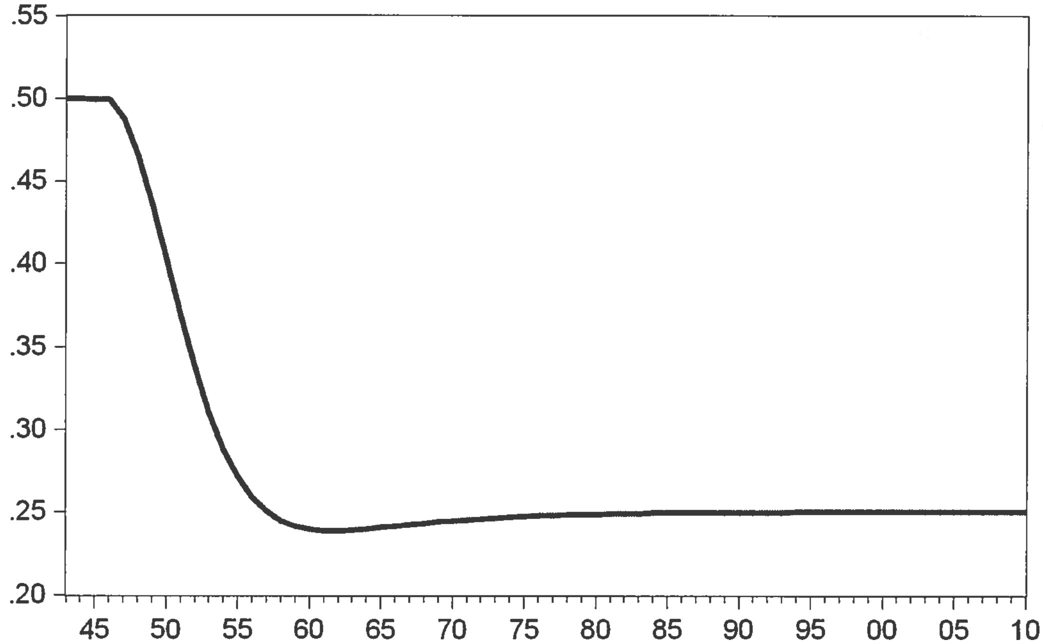
Interest Rate
.204
.200
.196
.192
.188
.184
.180
.176
.172
.168
44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74

Appendix A4 – FISCAL1 change in Debt-Capital Ratio

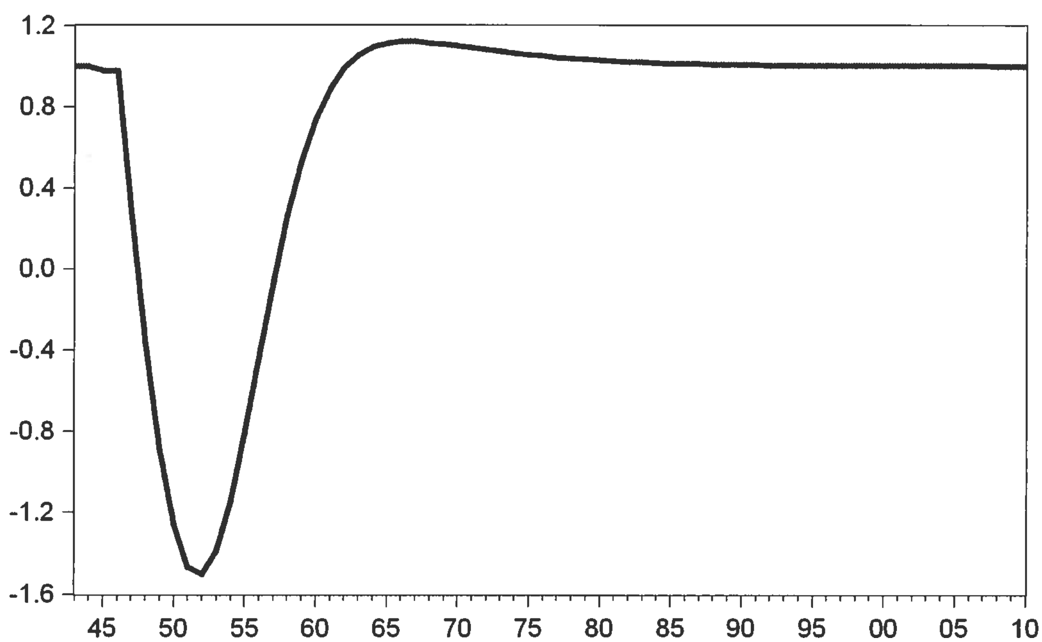




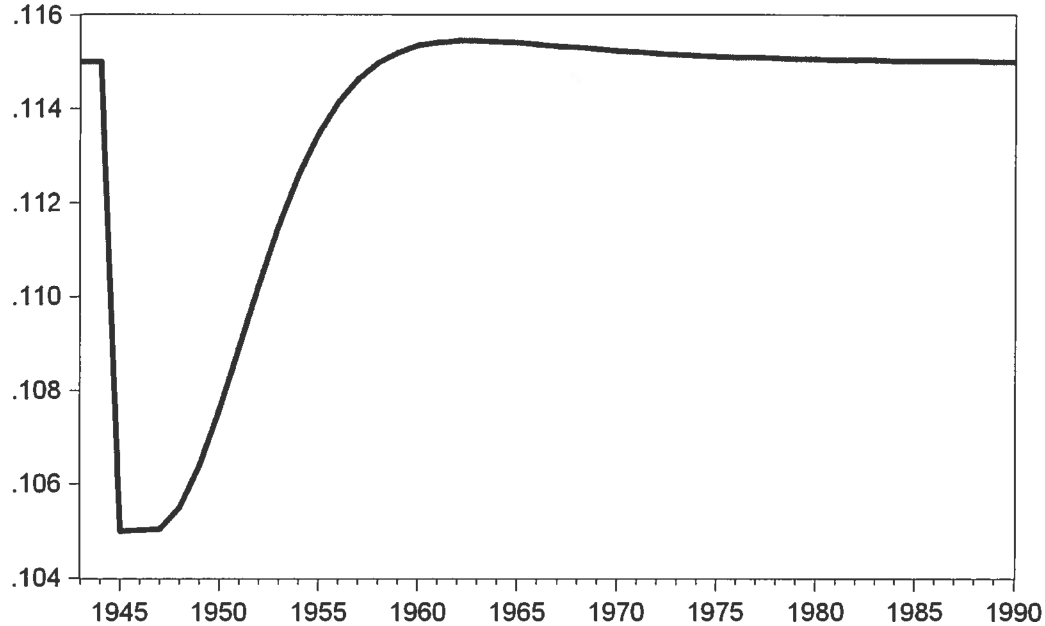
Debt-Capital Ratio



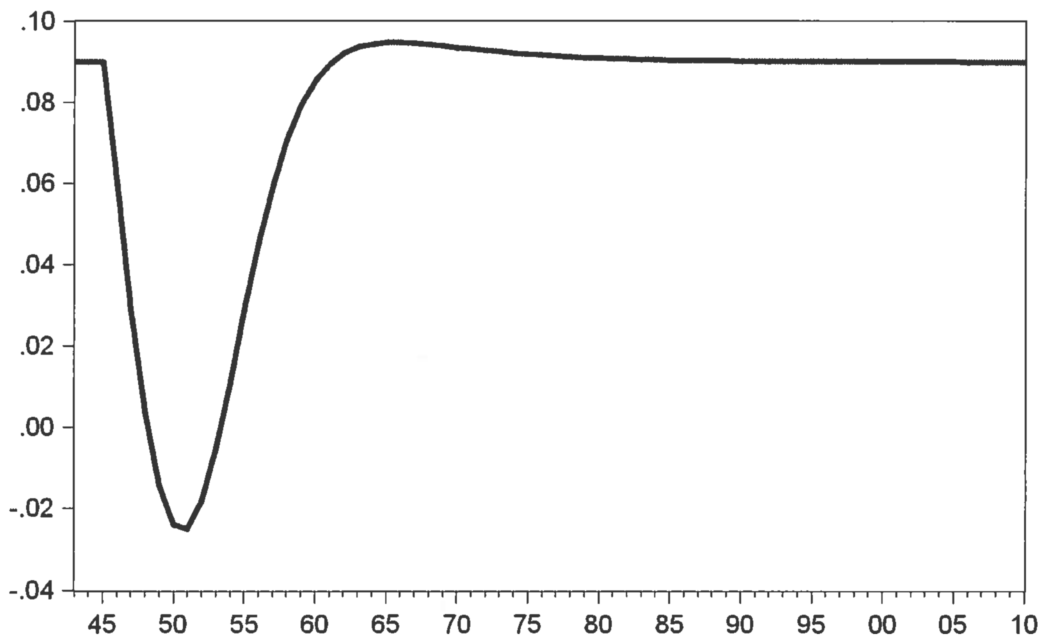
Fiscal Sustainability Condition (Golden Age)

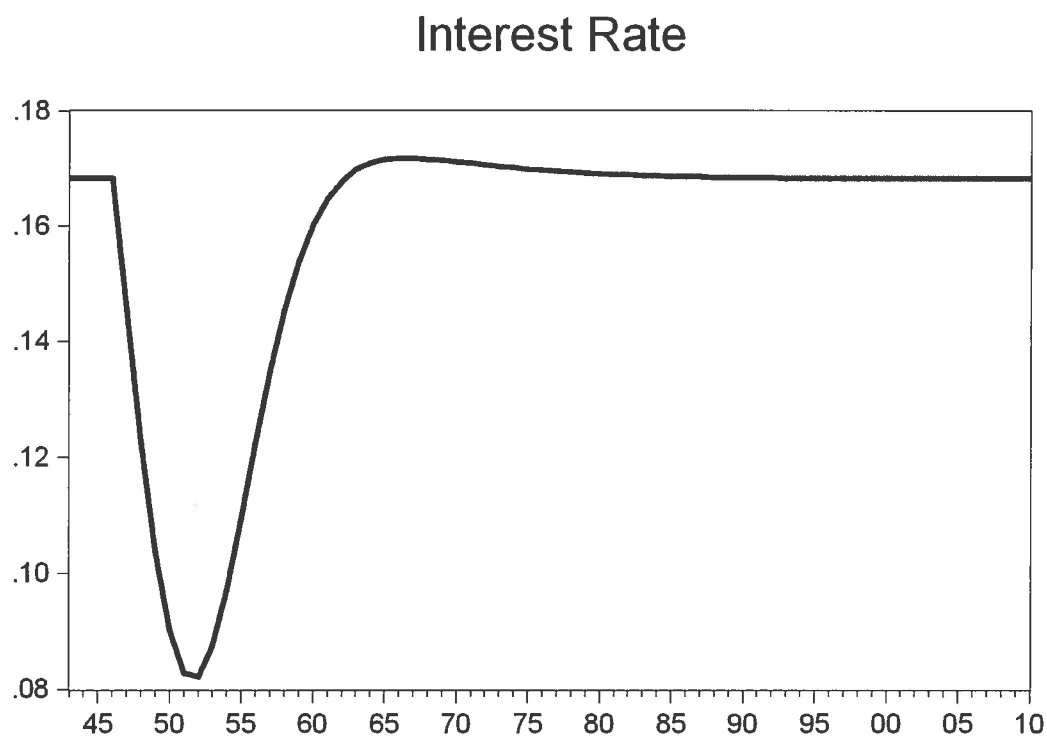


Government Spending Ratio

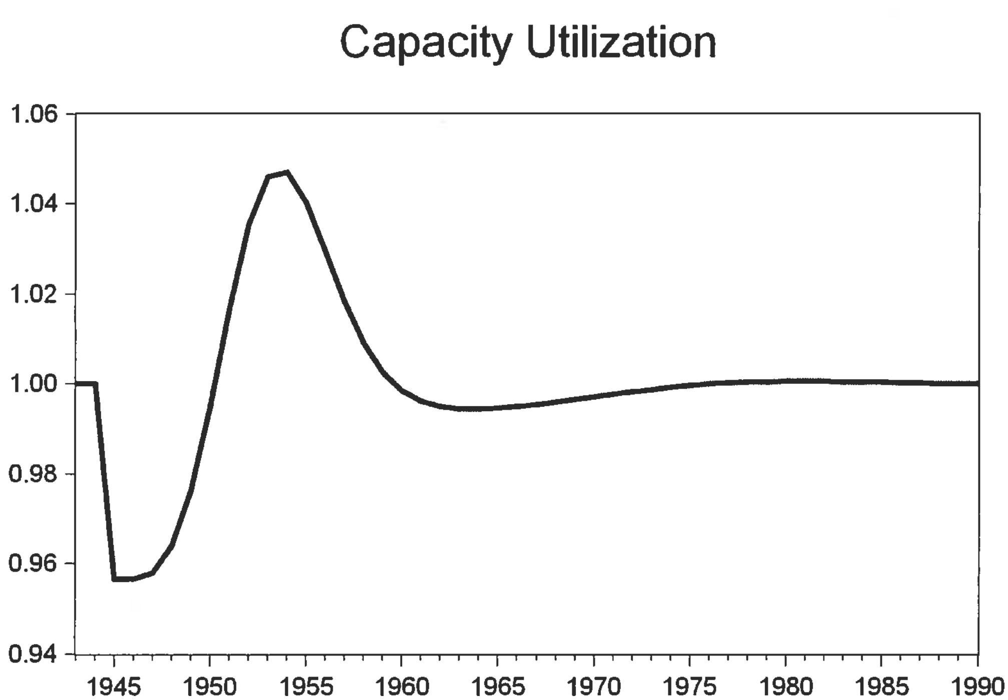


Inflation Rate at time t

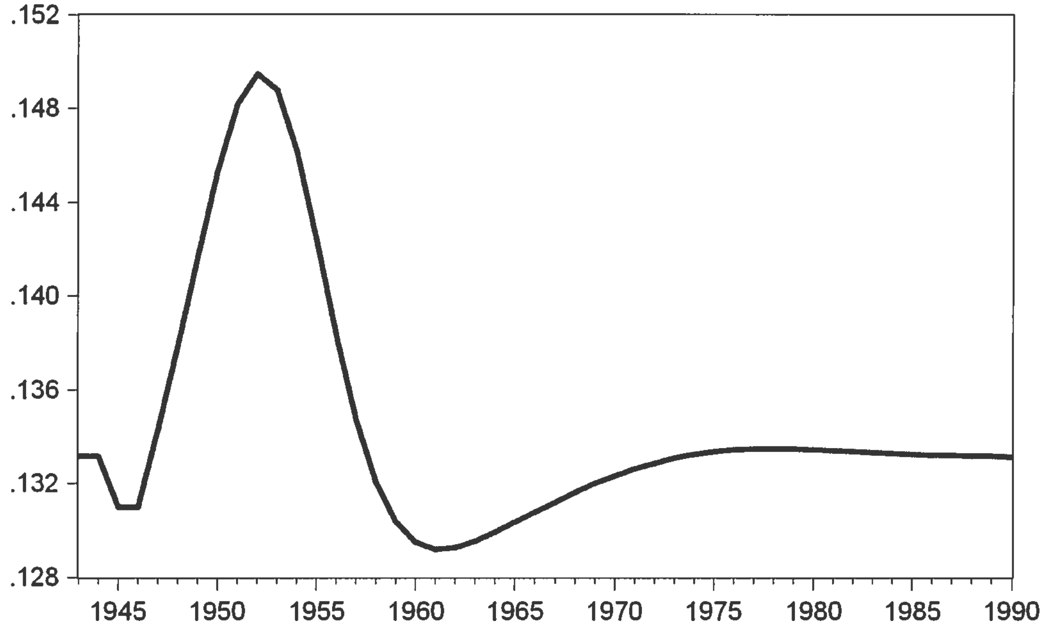




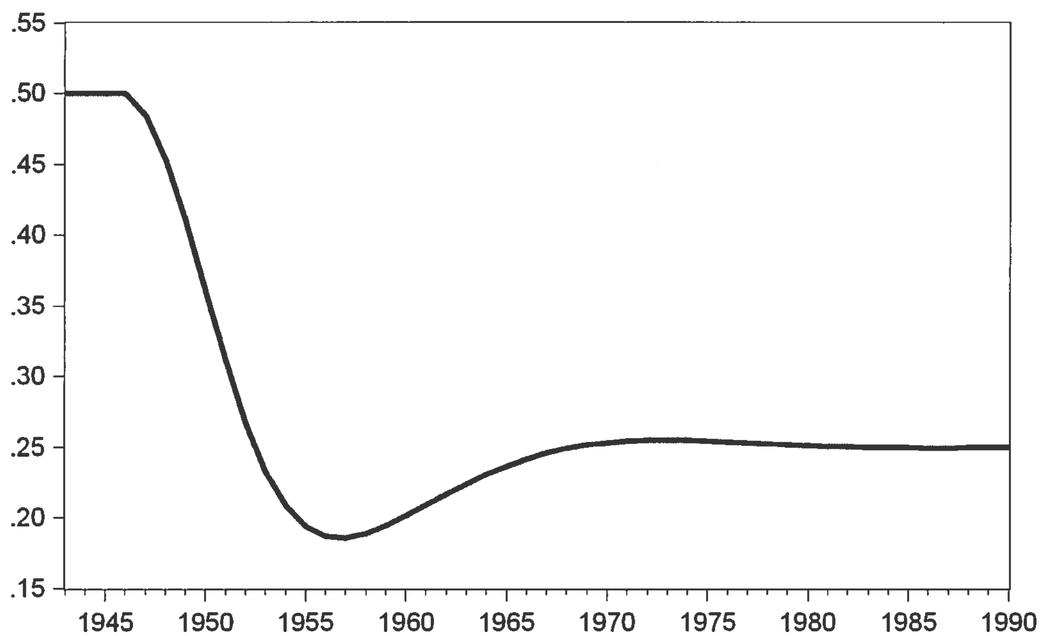
Appendix A5 – FISCAL2 change in Debt-Capital Ratio



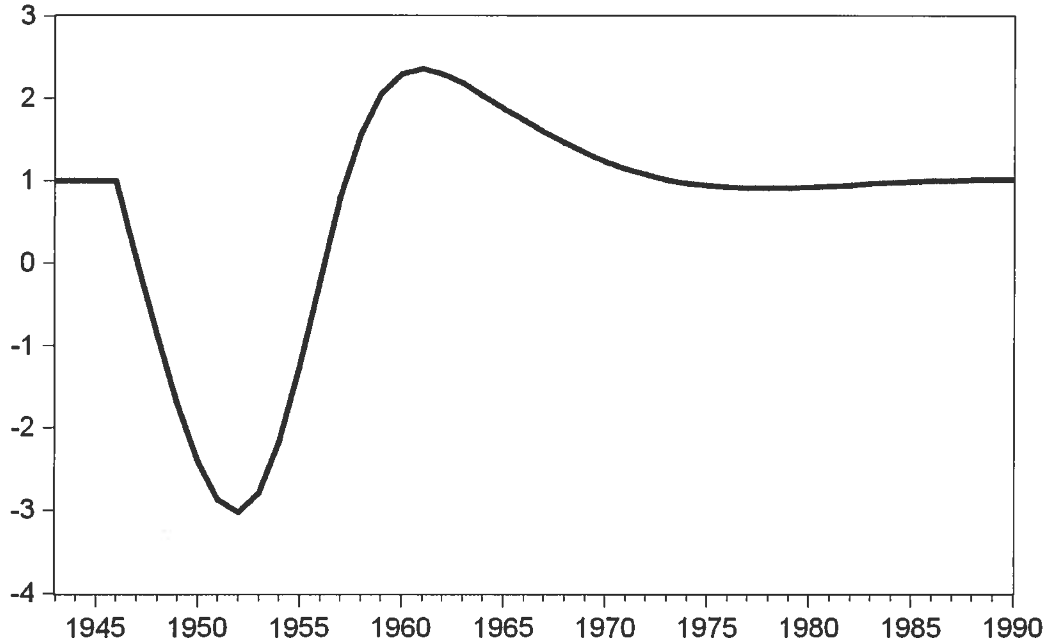
Growth Rate (I/K)



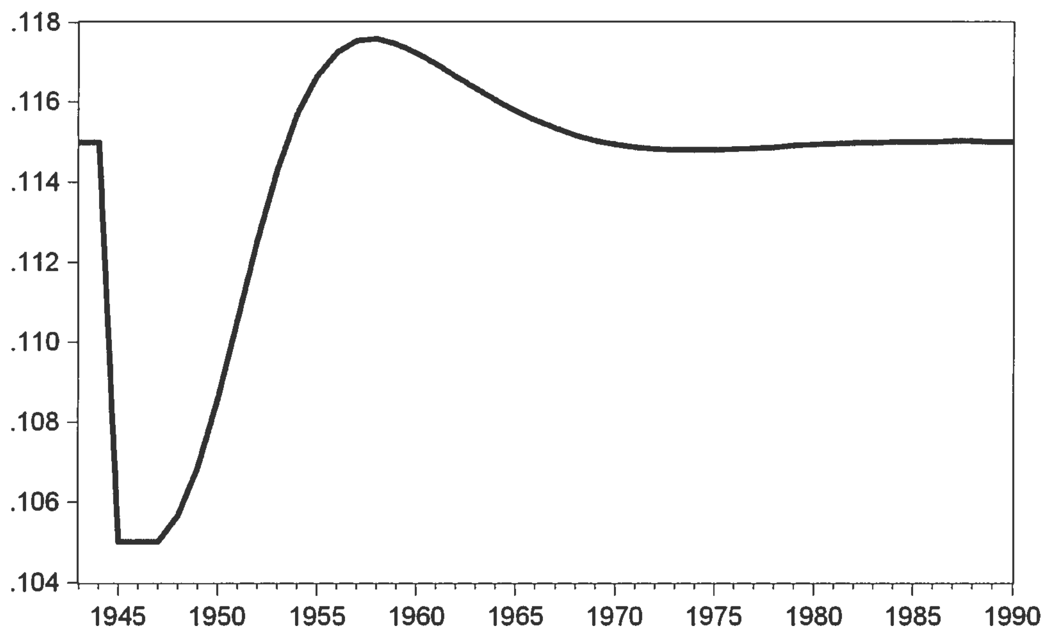
Debt-Capital Ratio



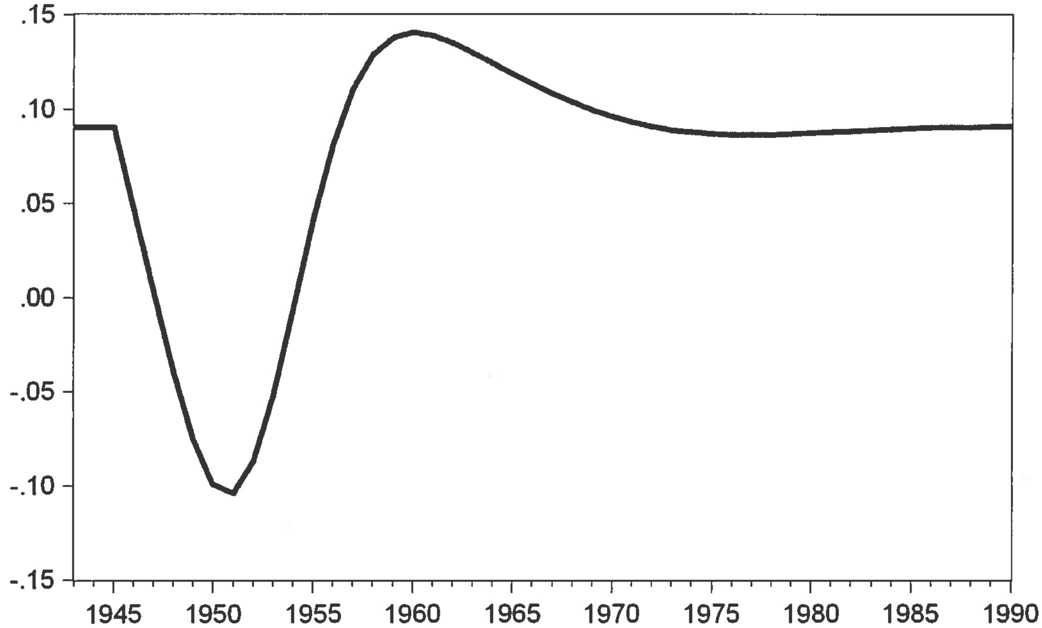
Fiscal Sustainability Condition (Golden Age)



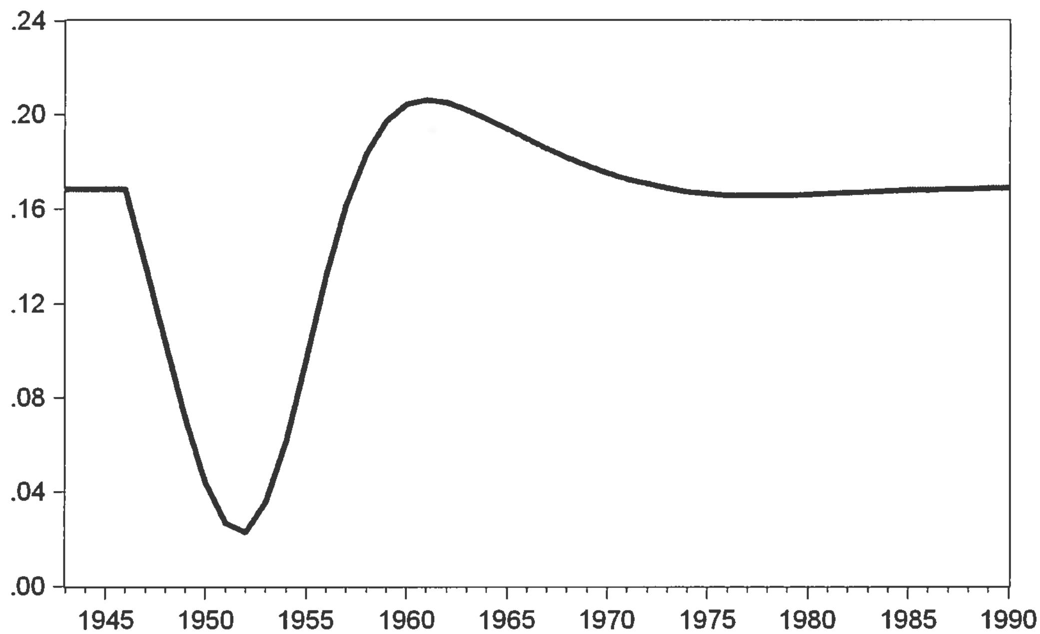
Government Spending Ratio

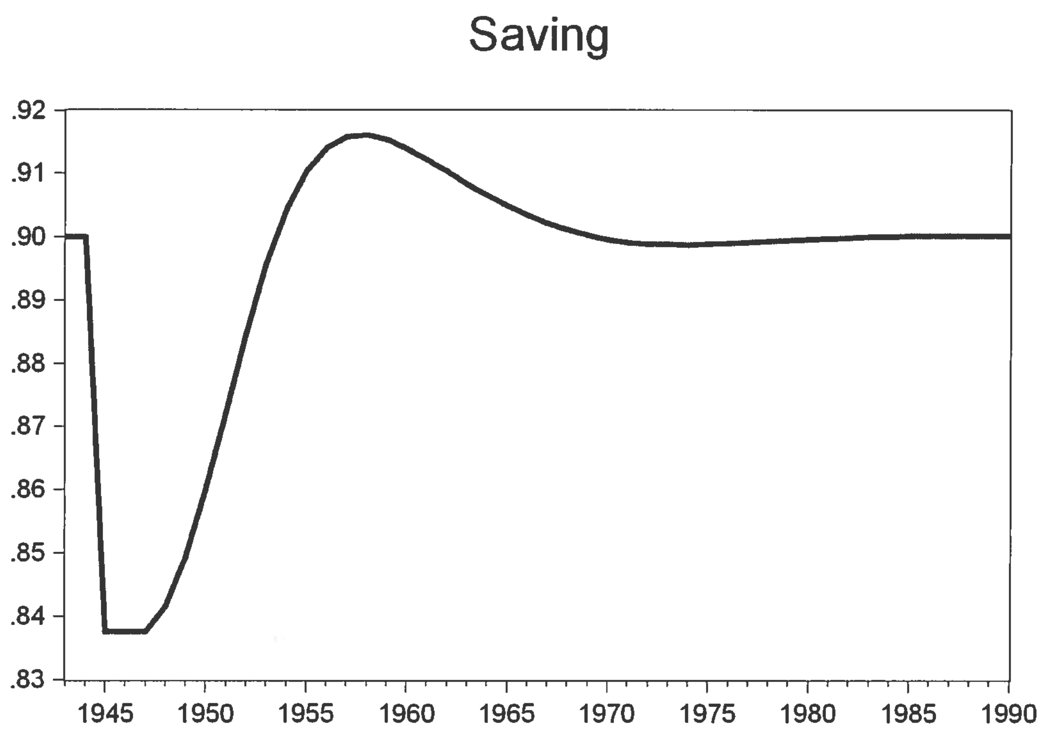


Inflation Rate at time t

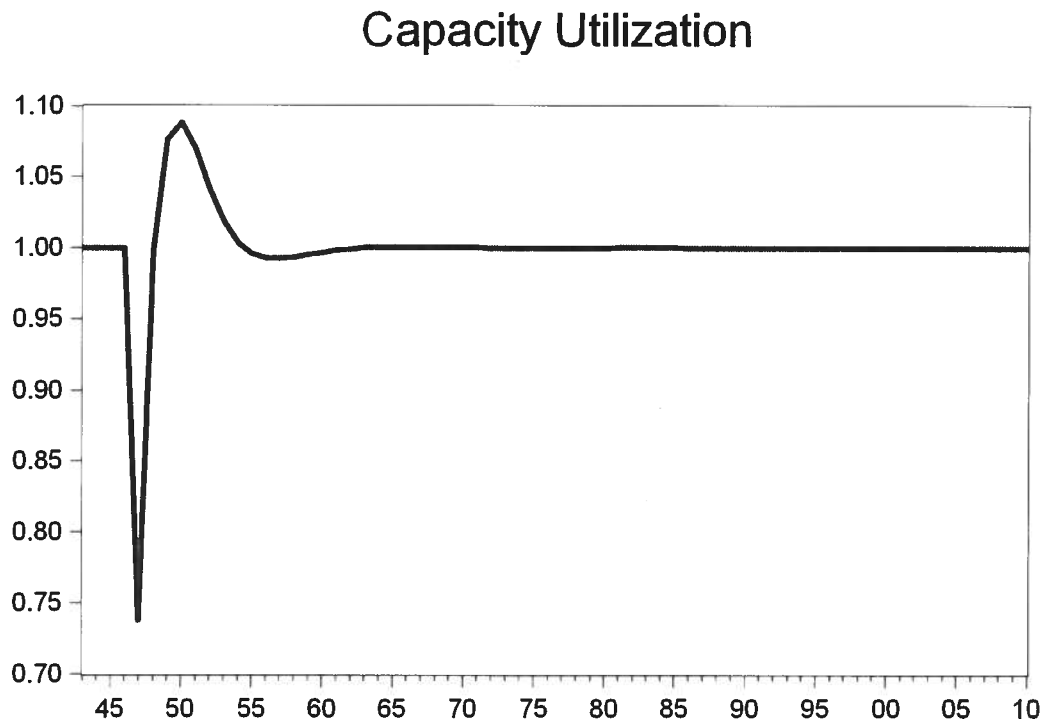


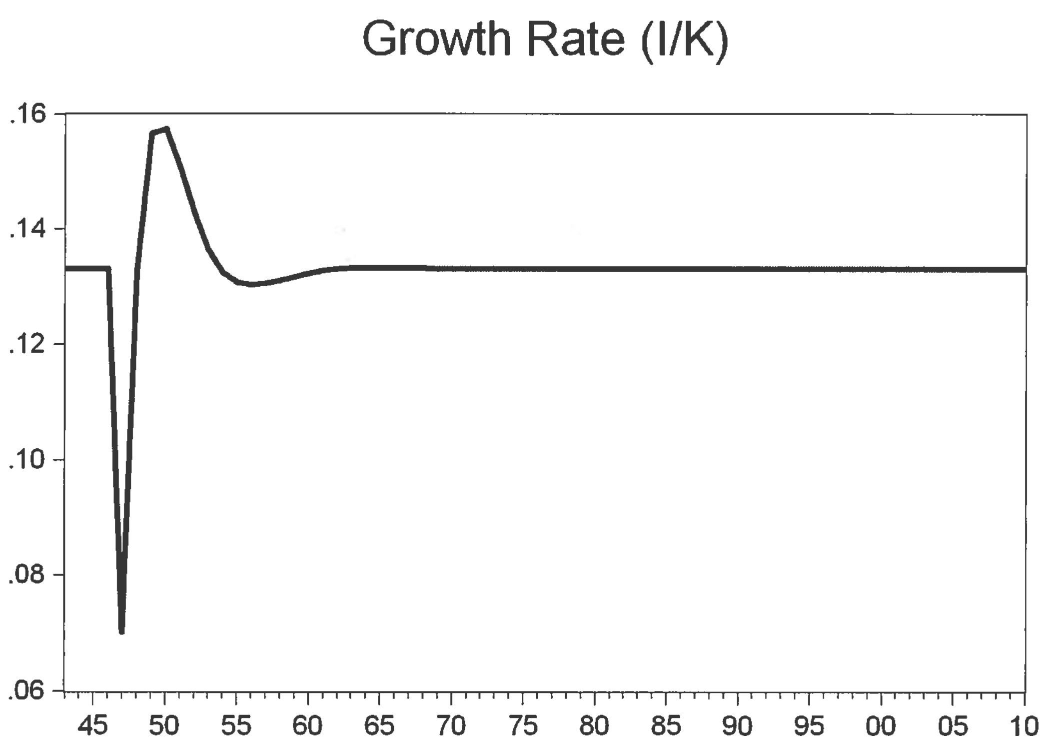
Interest Rate



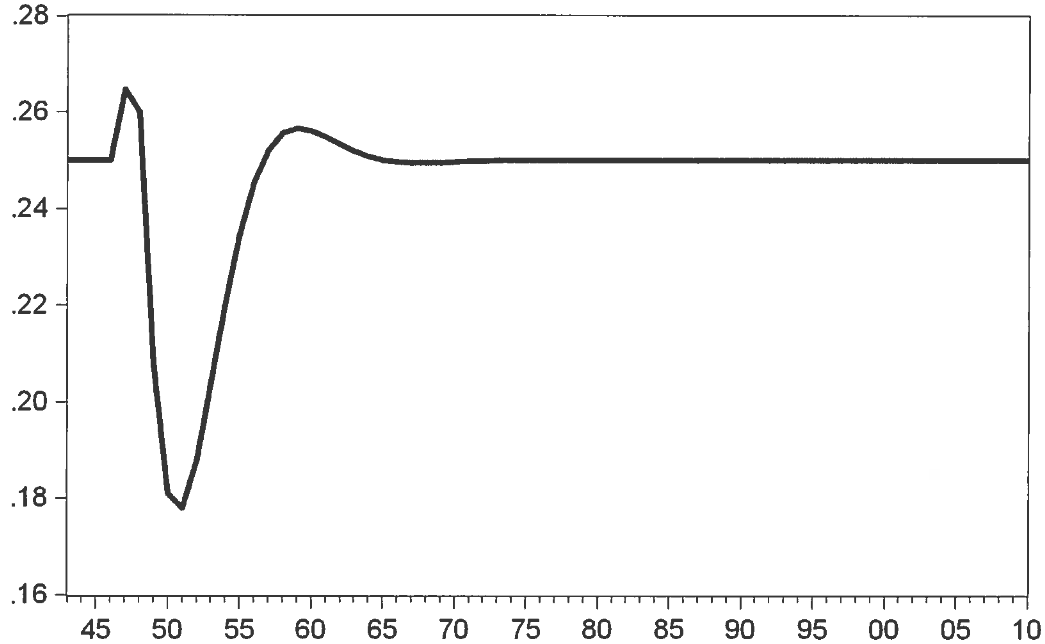

Saving
.92
.91
.90
.89
.88
.87
.86
.85
.84
.83
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990

Appendix B1 - MICHL1 Demand Shock

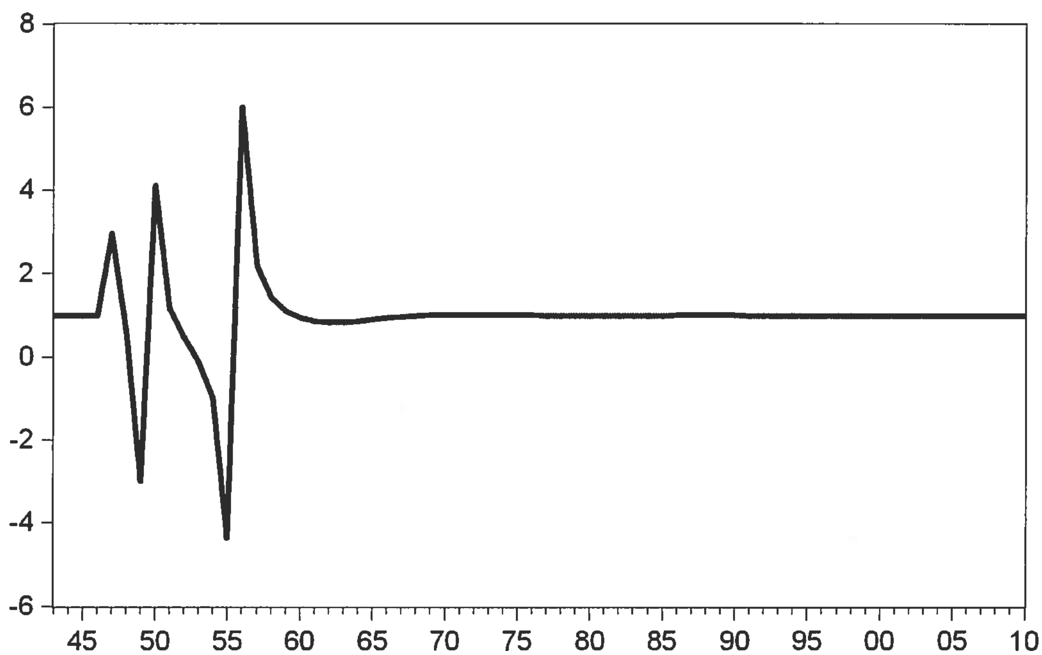


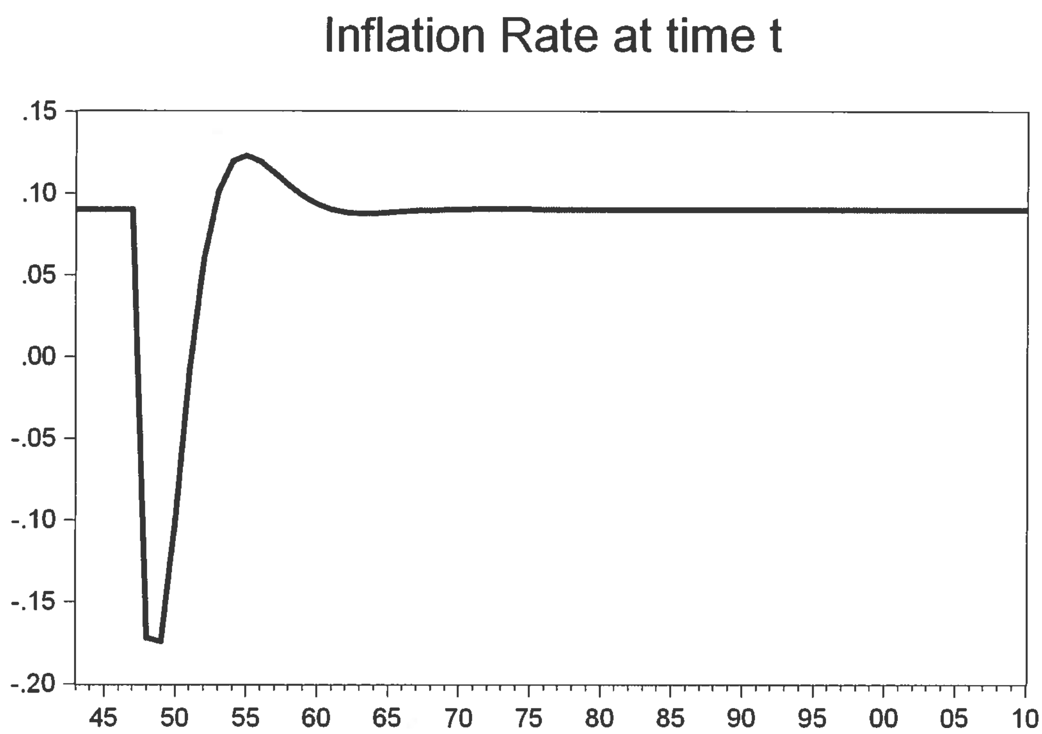


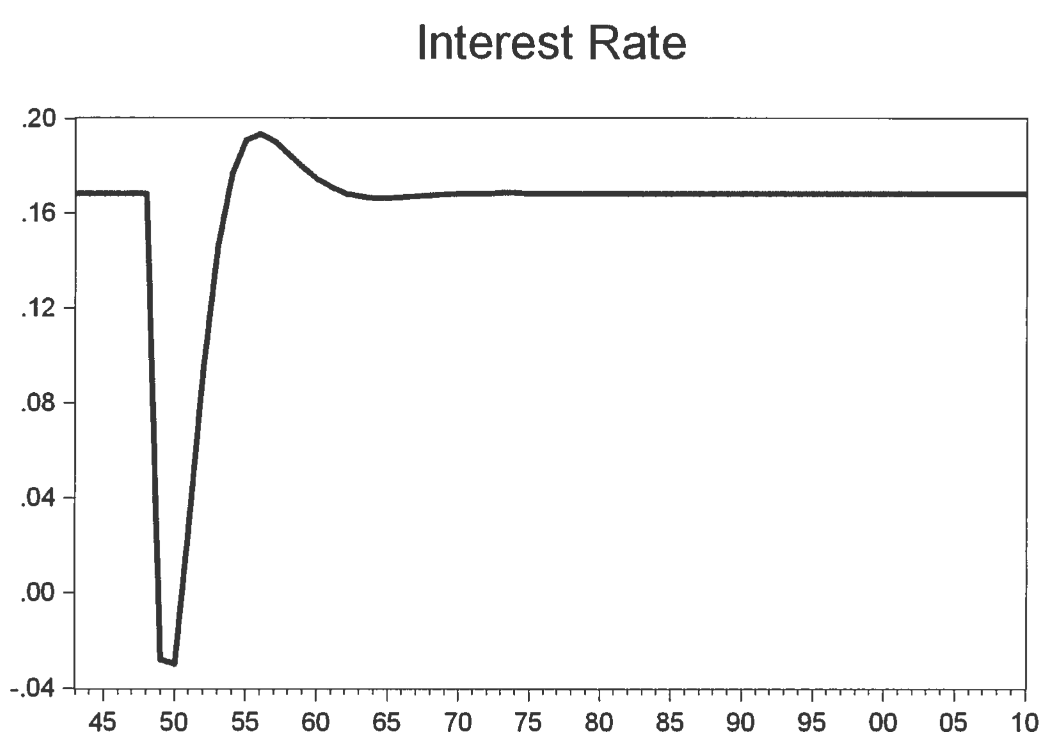
Debt-Capital Ratio



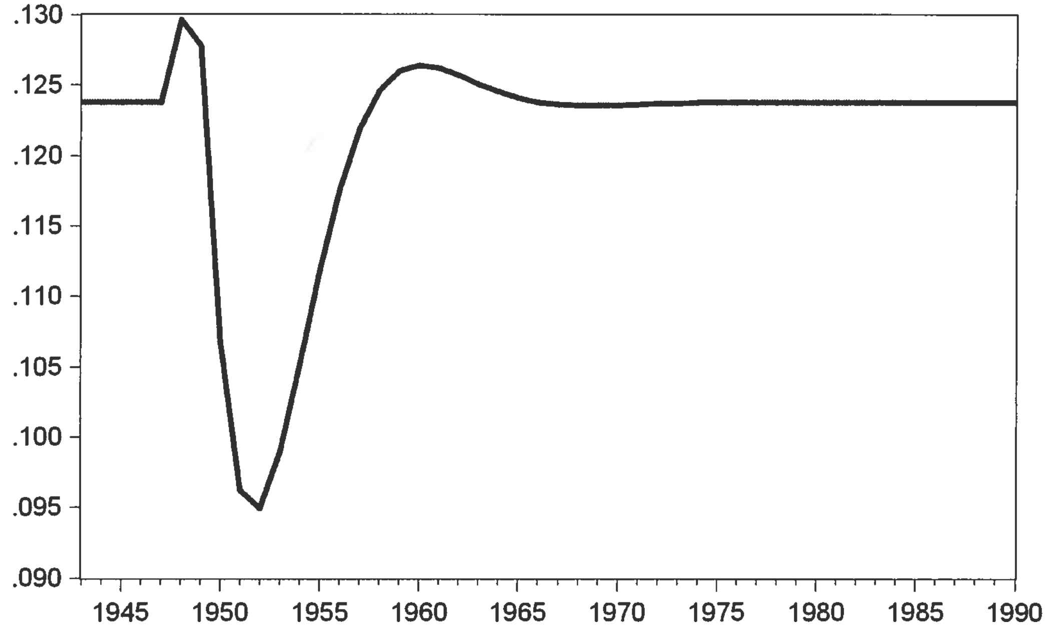
Fiscal Sustainability Condition (Golden Age)




Inflation Rate at time t
.15
.10
.05
.00
-.05
-.10
-.15
-.20
45
50
55
60
65
70
75
80
85
90
95
00
05
10

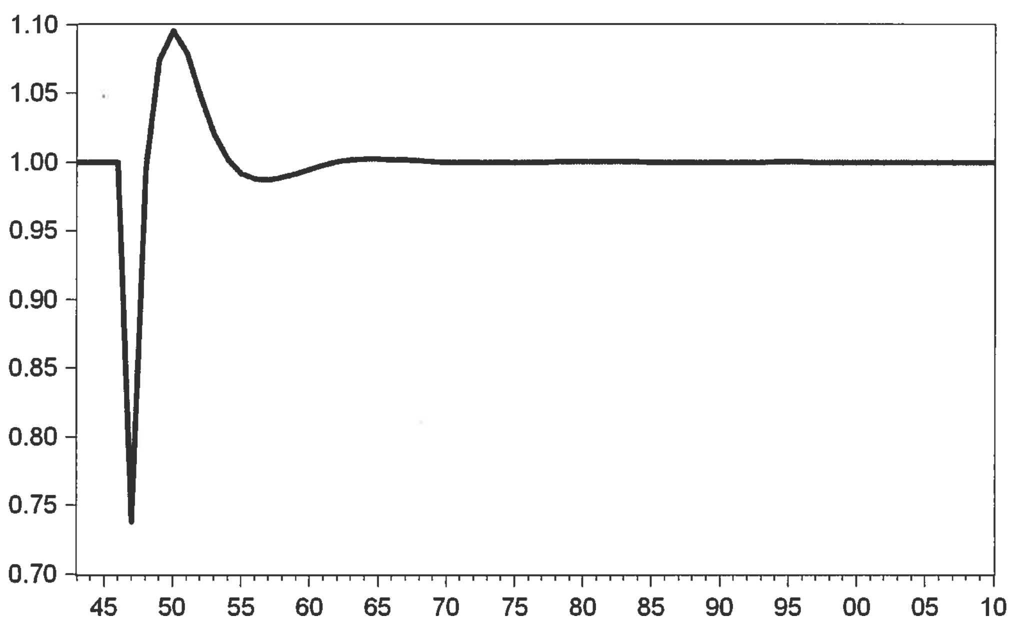

Interest Rate
.20
.16
.12
.08
.04
.00
-.04
45
50
55
60
65
70
75
80
85
90
95
00
05
10

Tax (as a percentage of capital stock)

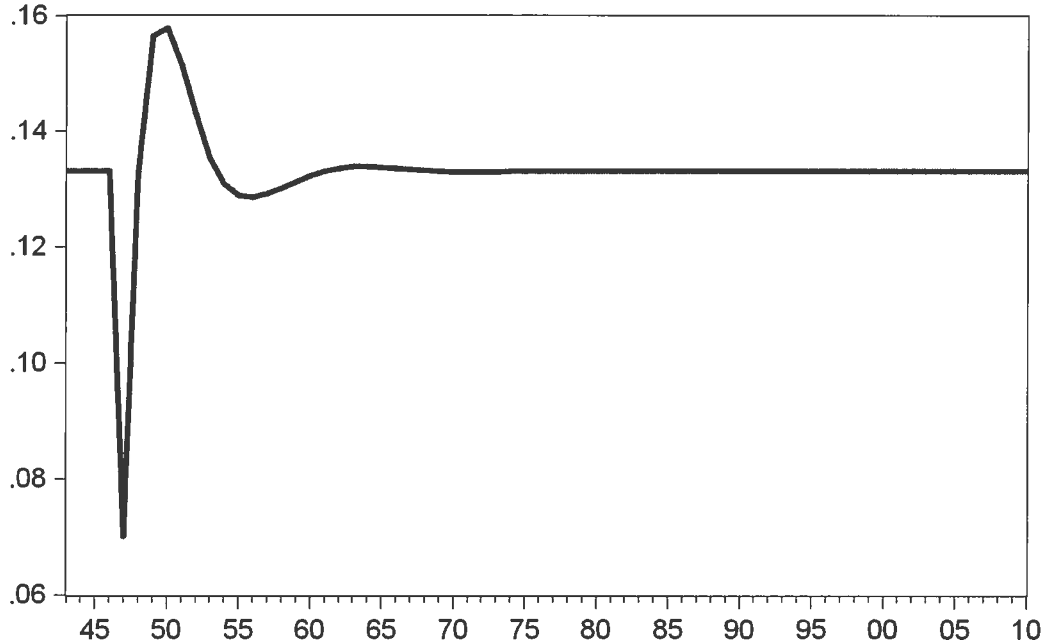


Appendix B2 – MICHL3 Demand Shock

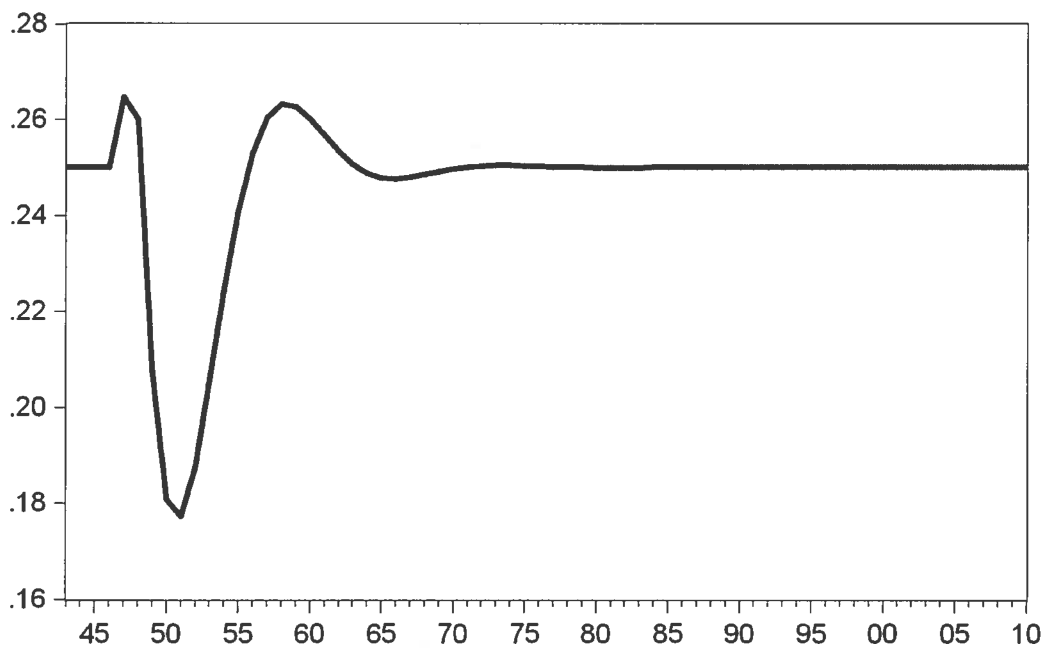
Capacity Utilization



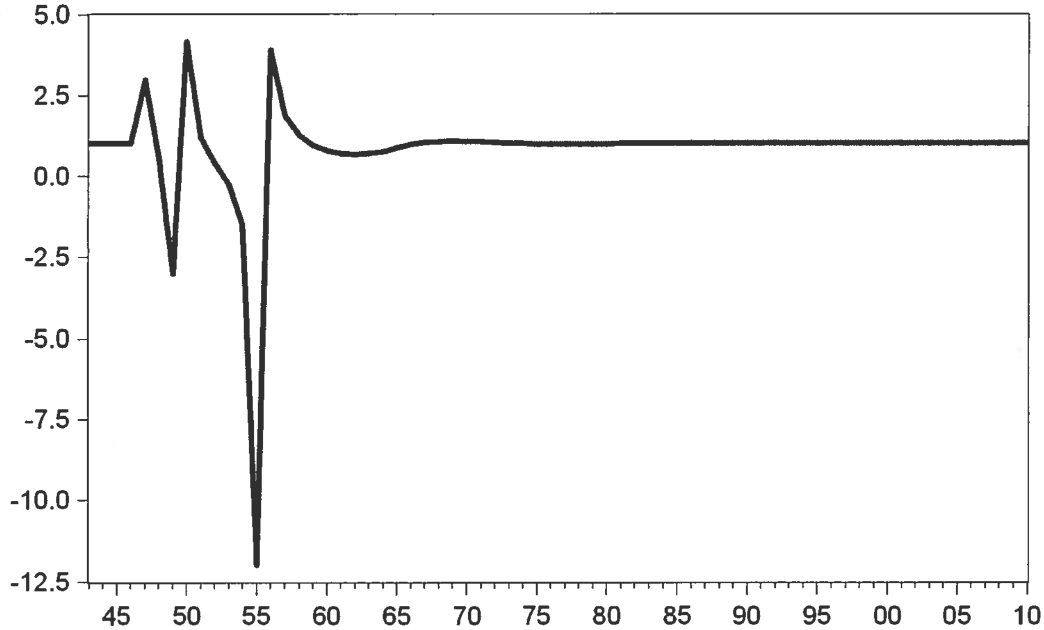
Growth Rate (I/K)



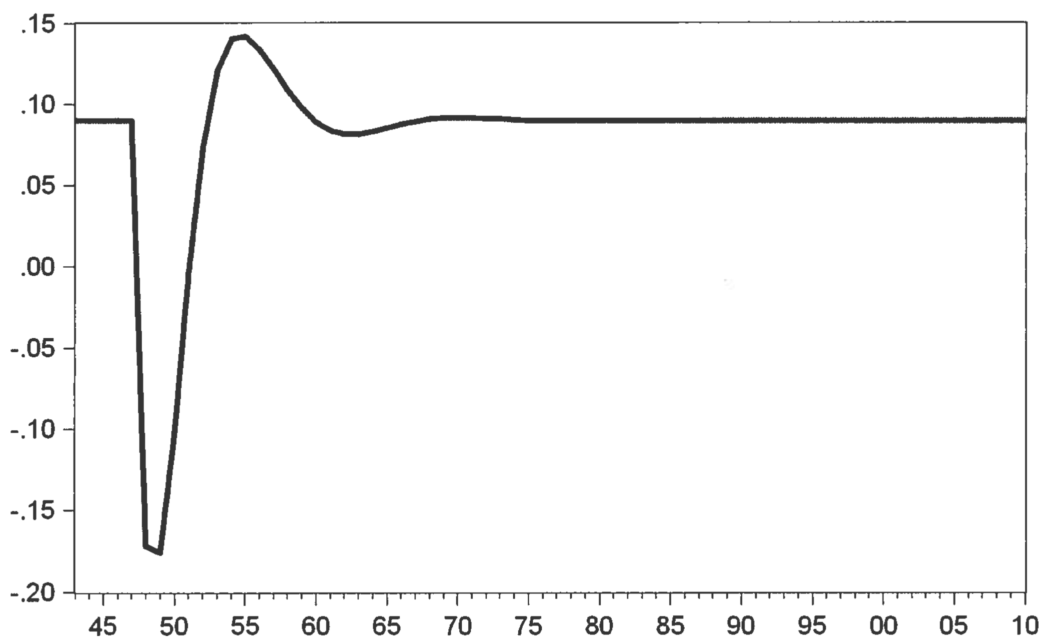
Debt-Capital Ratio



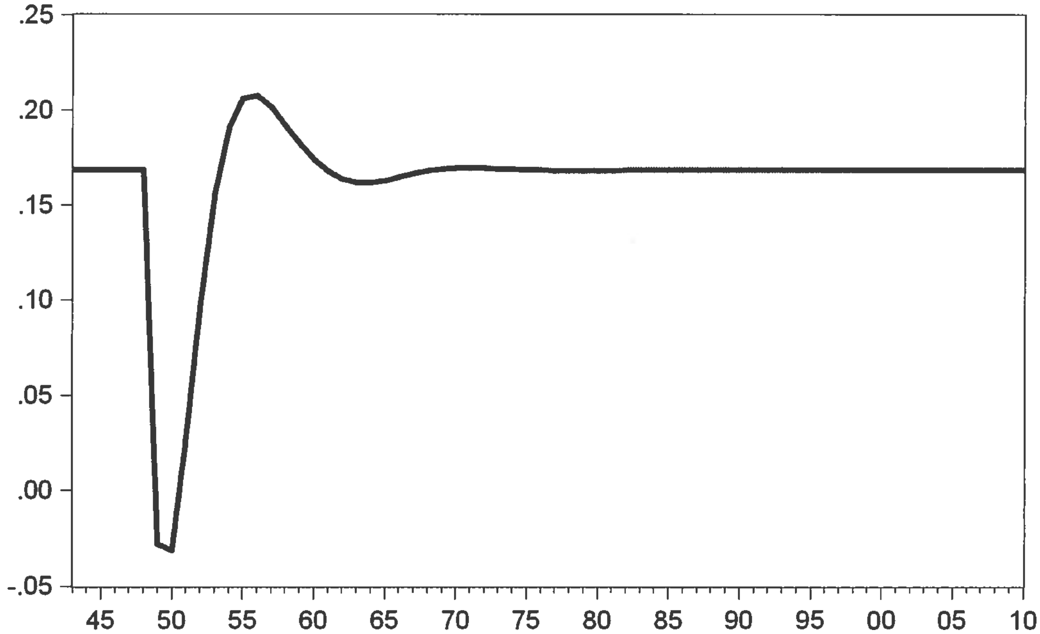
Fiscal Sustainability Condition (Golden Age)



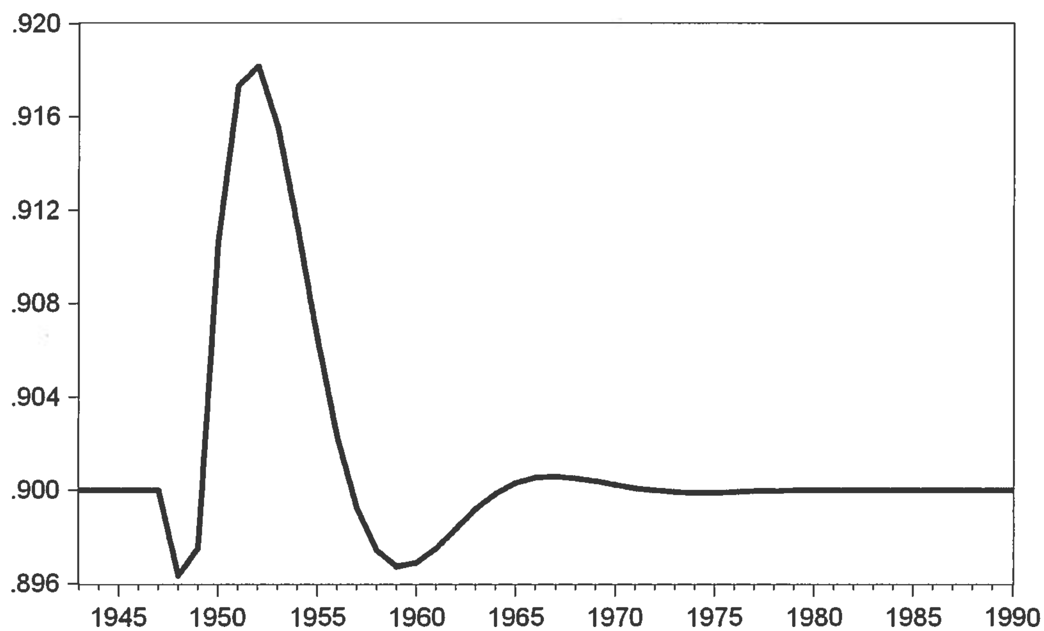
Inflation Rate at time t

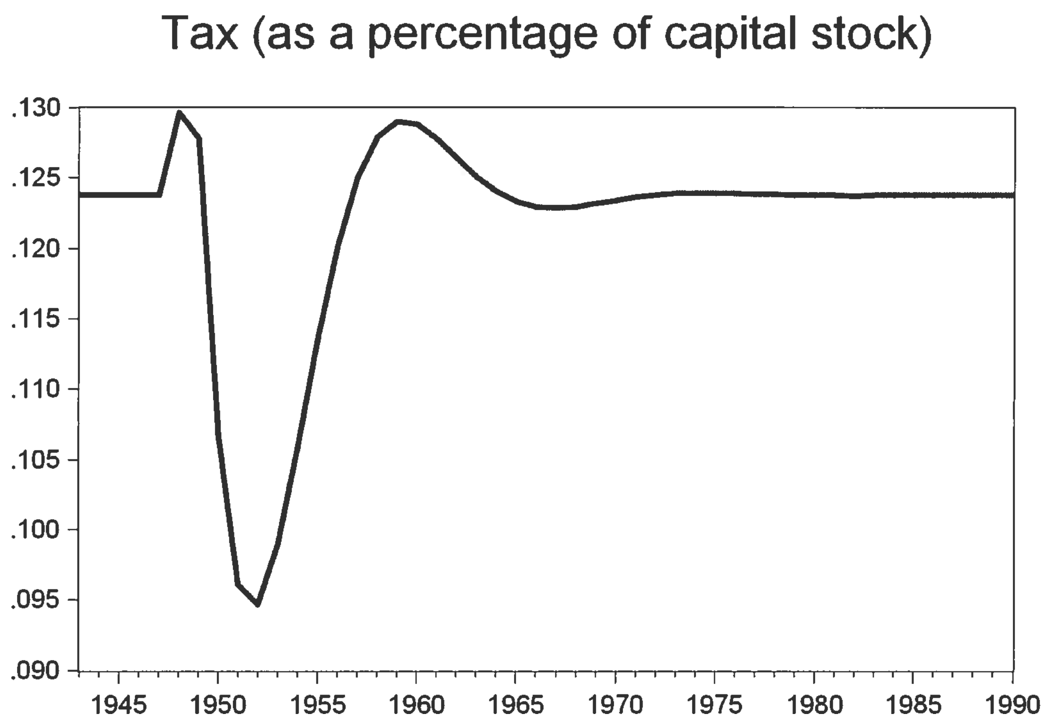


Interest Rate

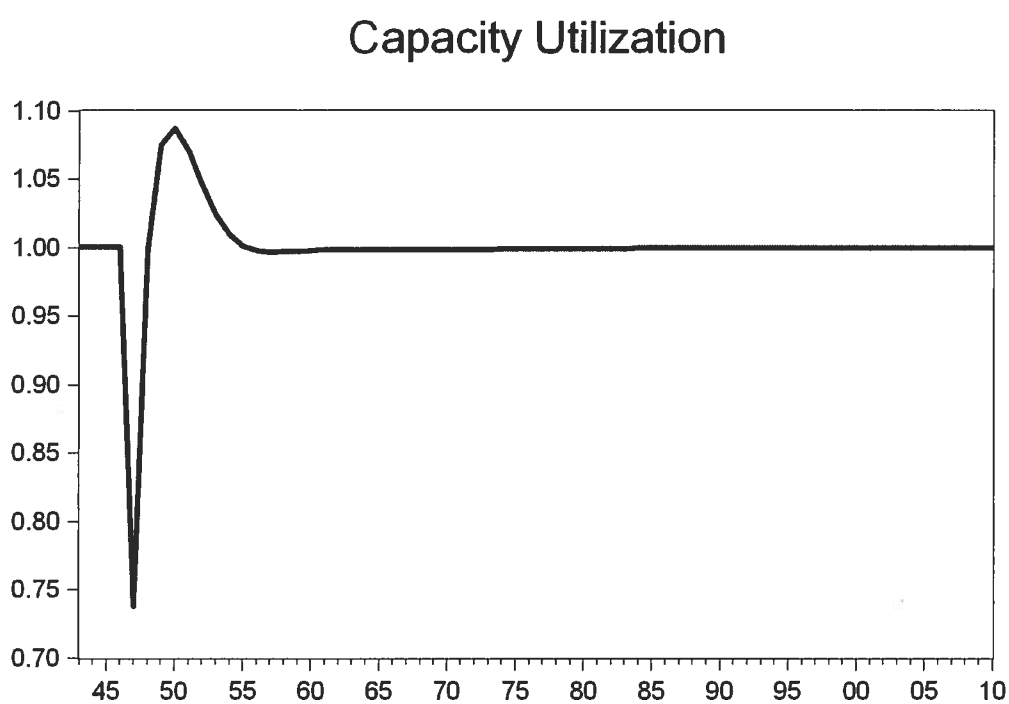


Saving

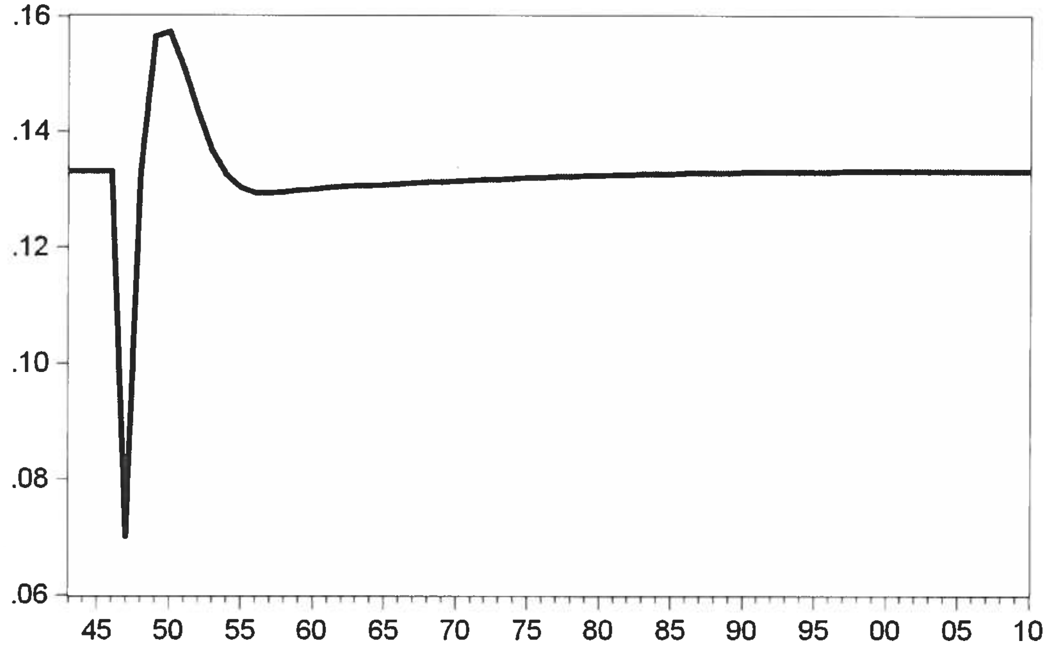




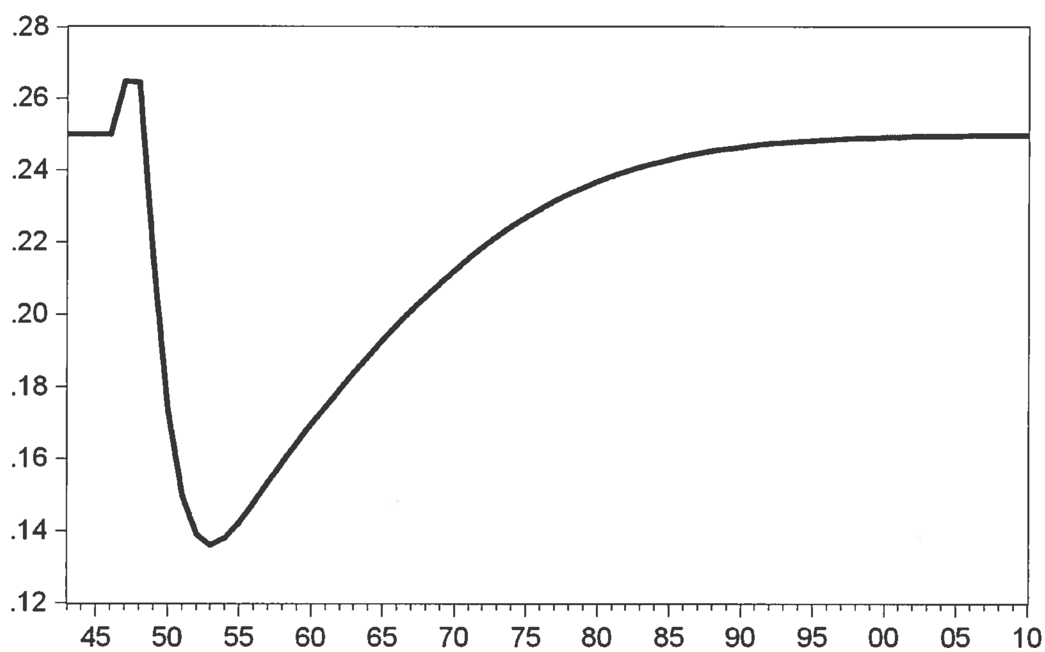
Appendix B3 – FISCAL1 Demand Shock



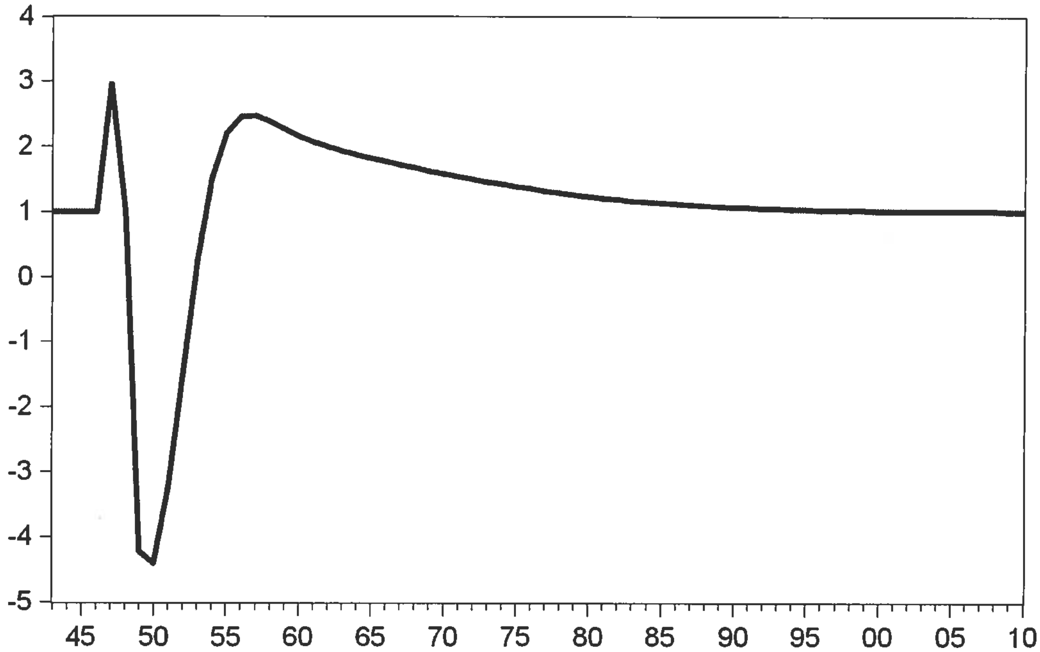
Growth Rate (I/K)



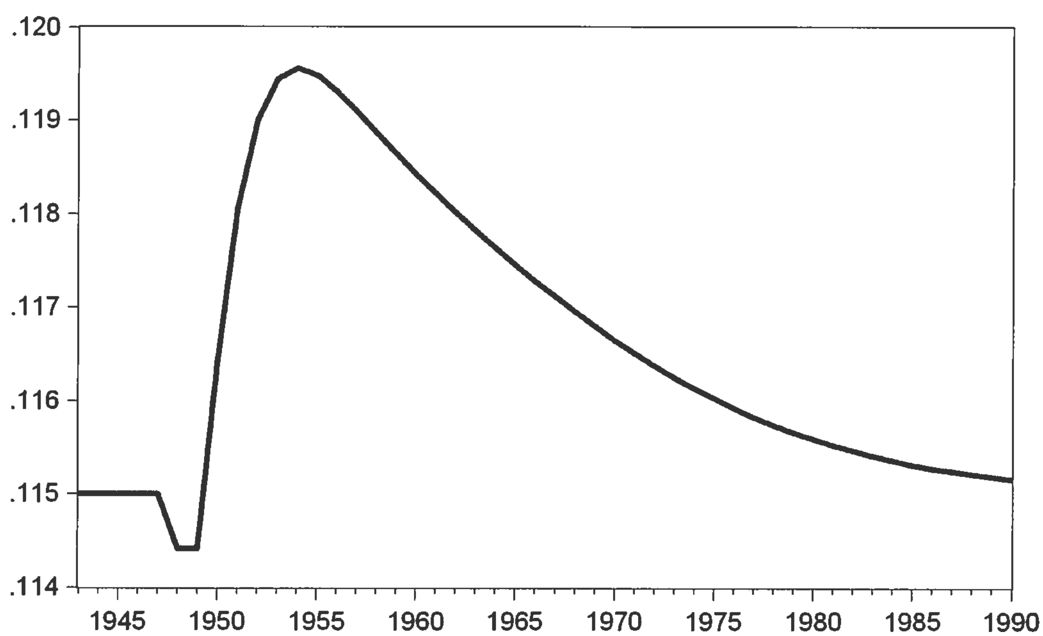
Debt-Capital Ratio



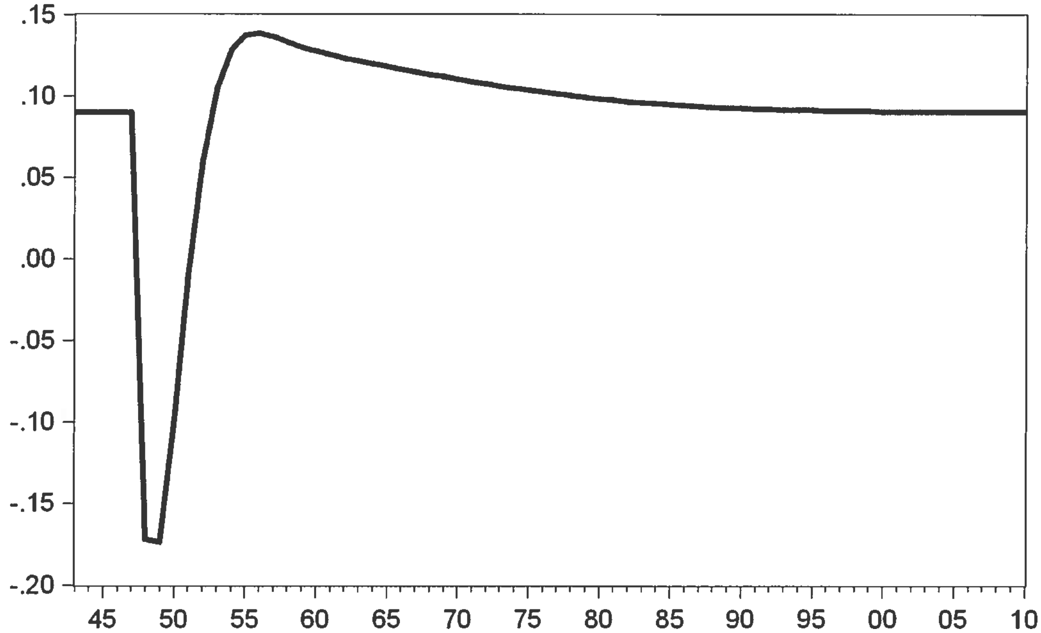
Fiscal Sustainability Condition (Golden Age)



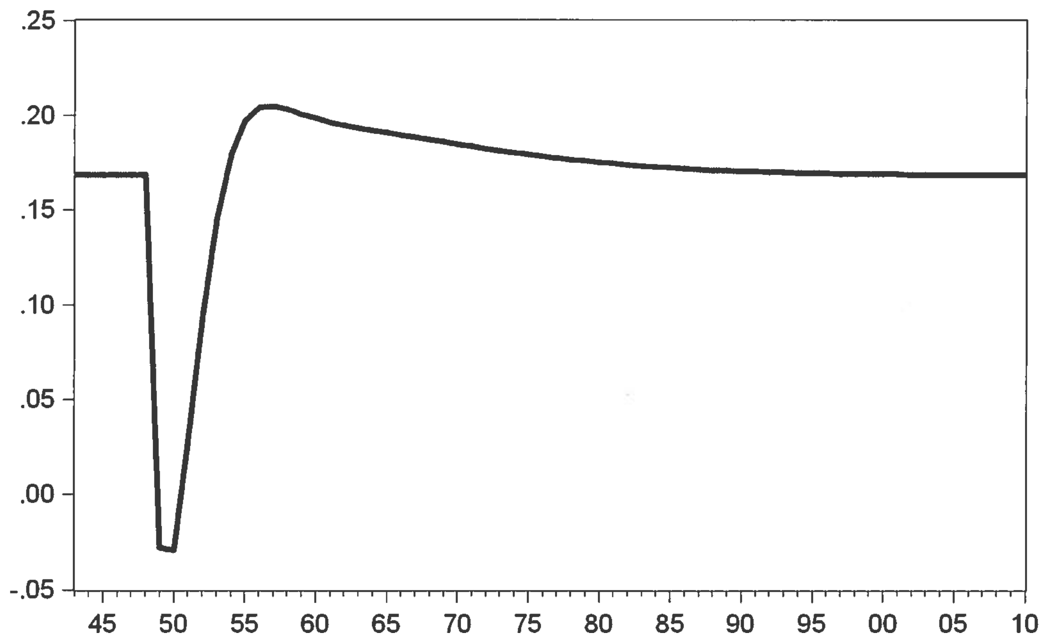
Government Spending Ratio



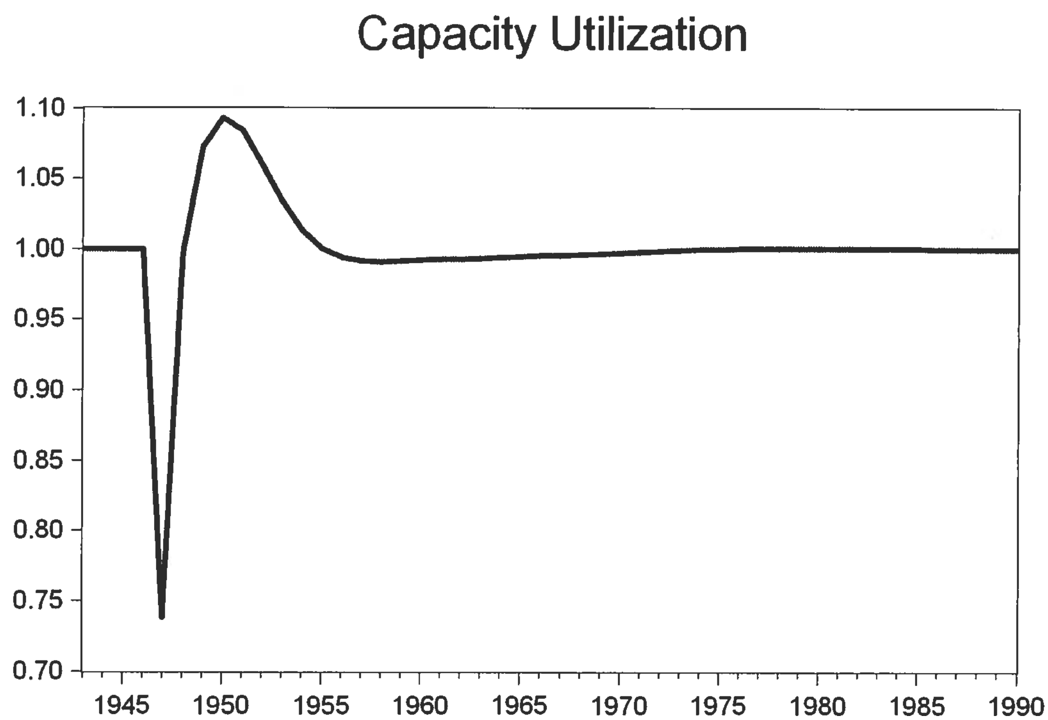
Inflation Rate at time t

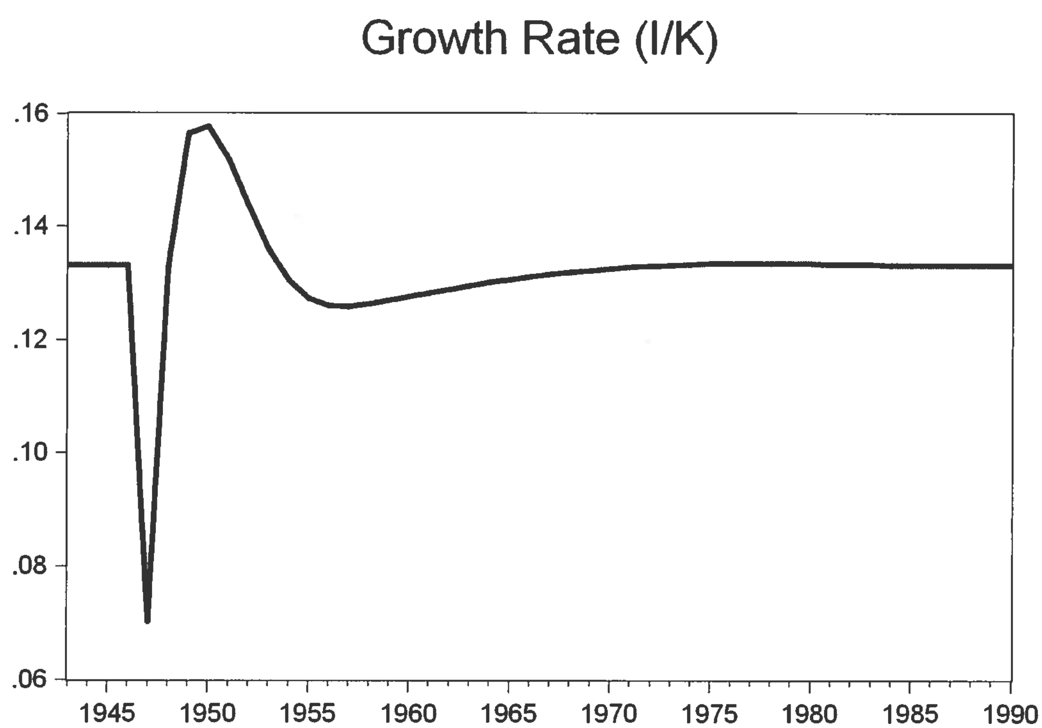


Interest Rate

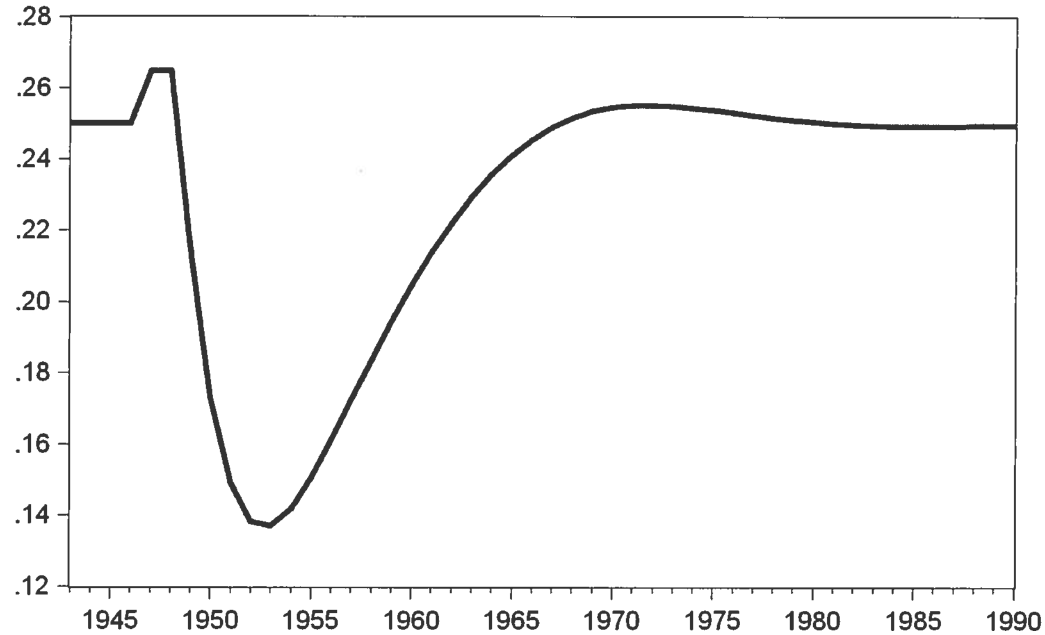


Appendix B4 – FISCAL2 Demand Shock

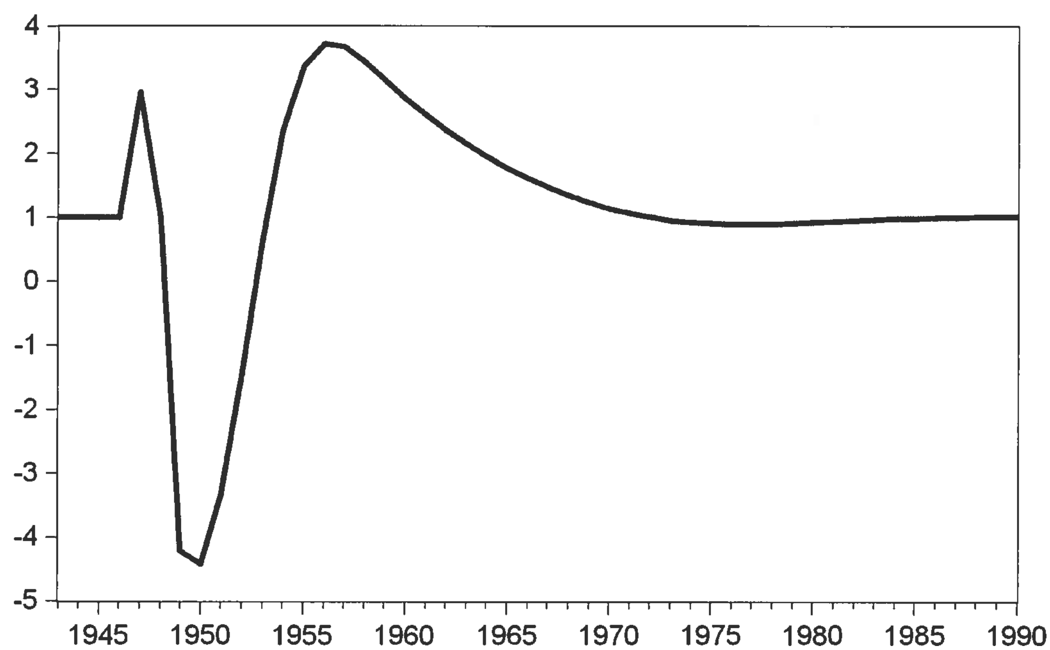




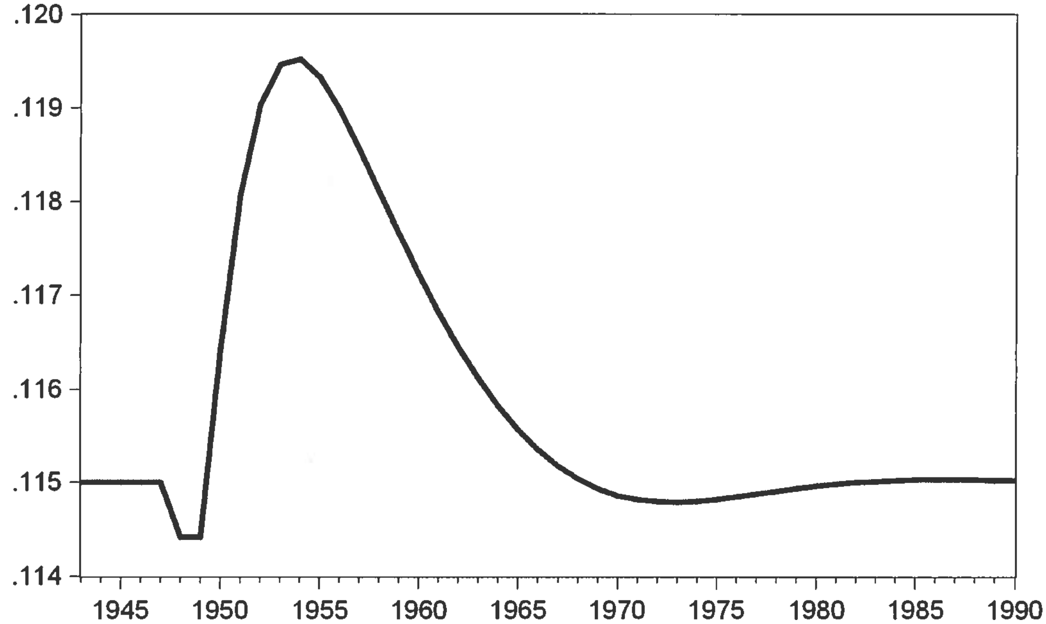
Debt-Capital Ratio



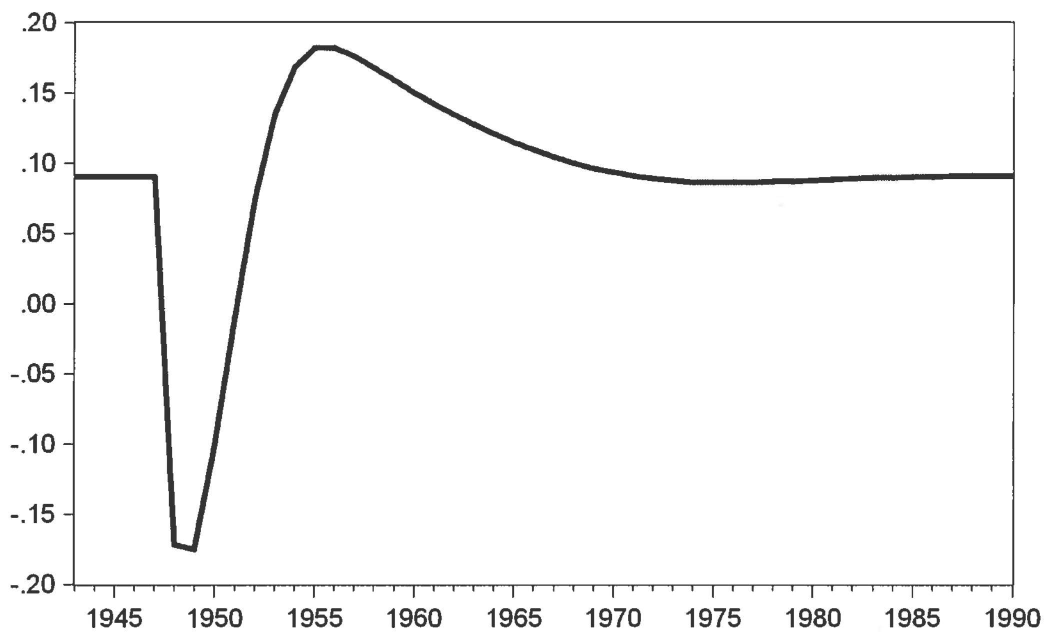
Fiscal Sustainability Condition (Golden Age)



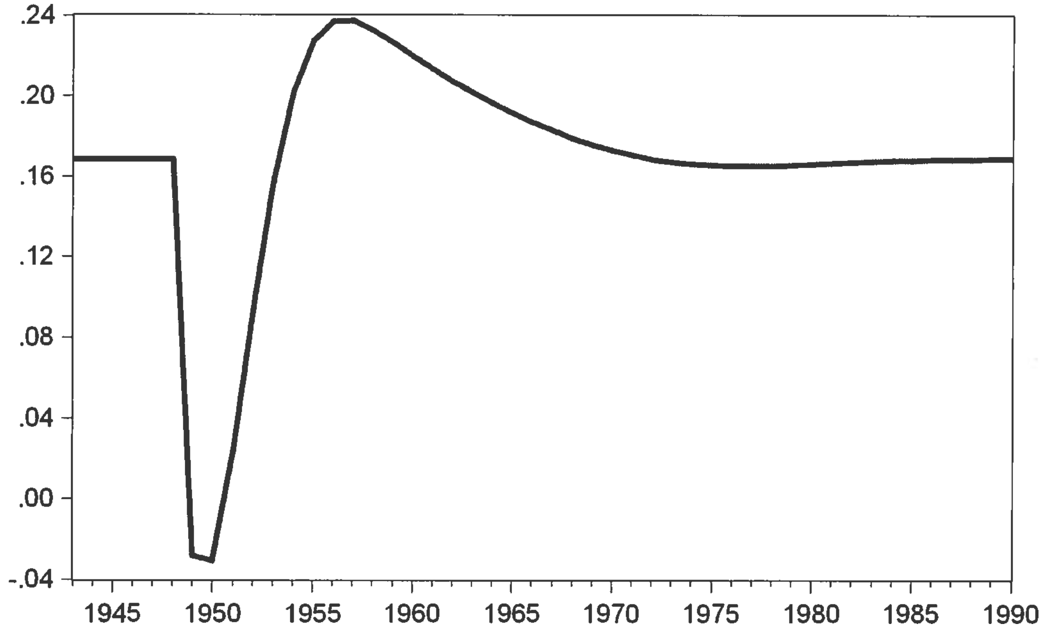
Government Spending Ratio



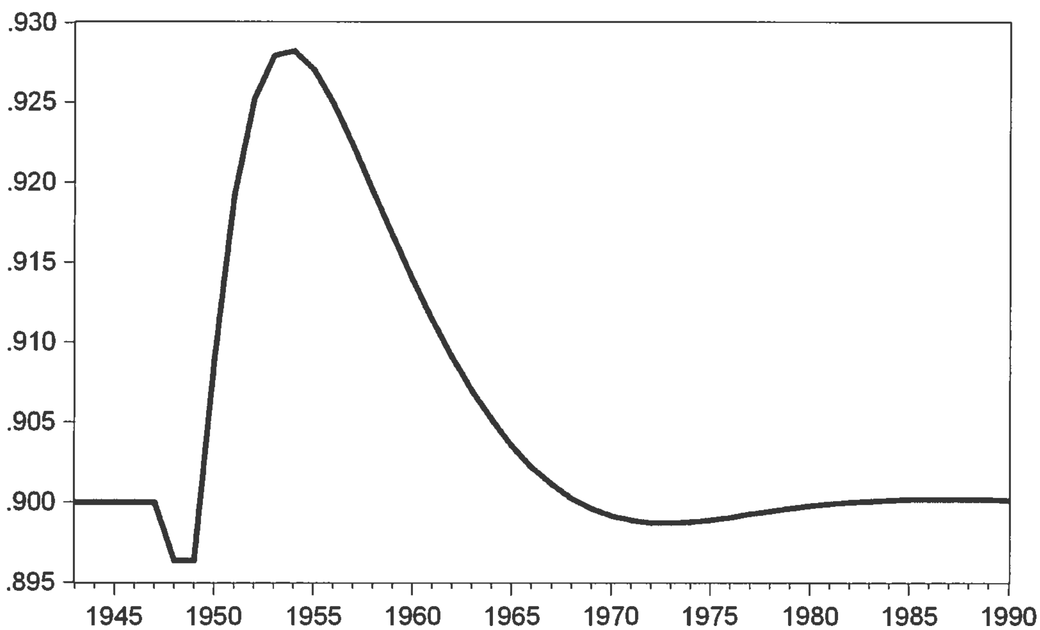
Inflation Rate at time t



Interest Rate

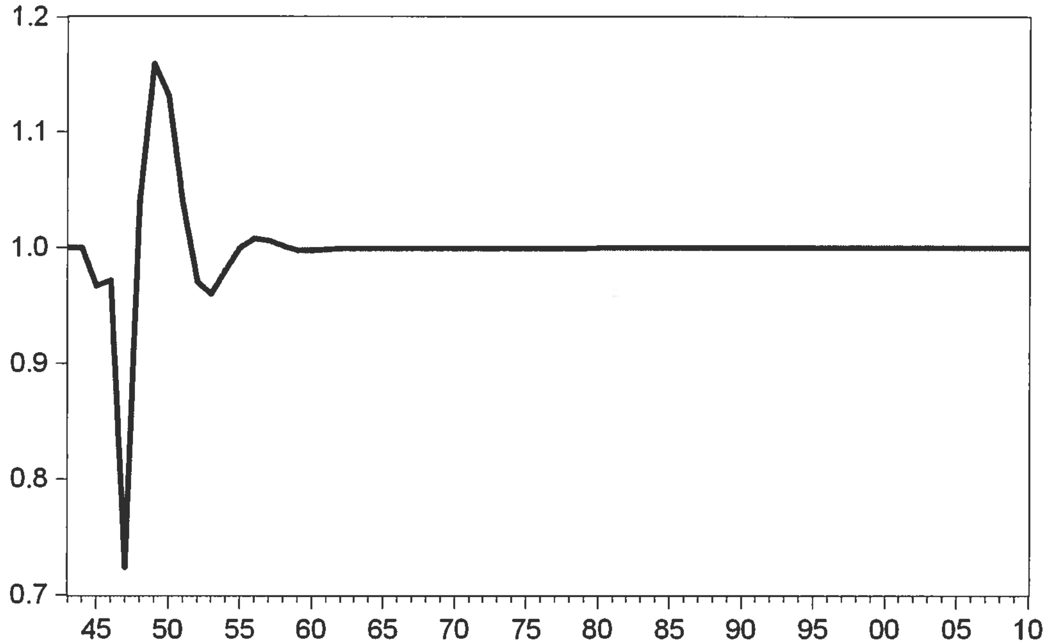


Saving

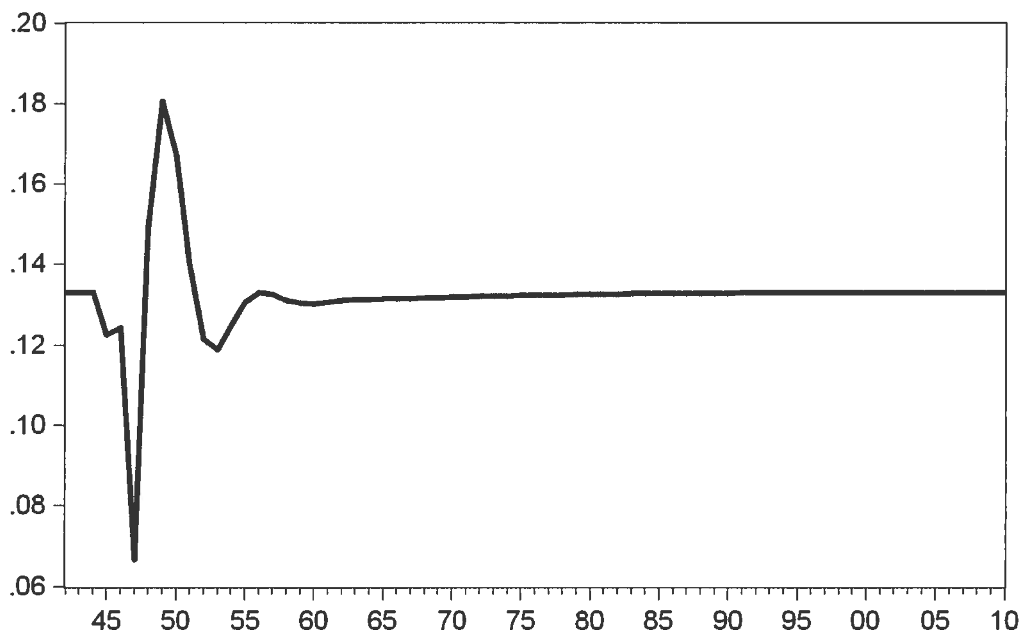


Appendix B5 – TAYLOR1 Demand Shock

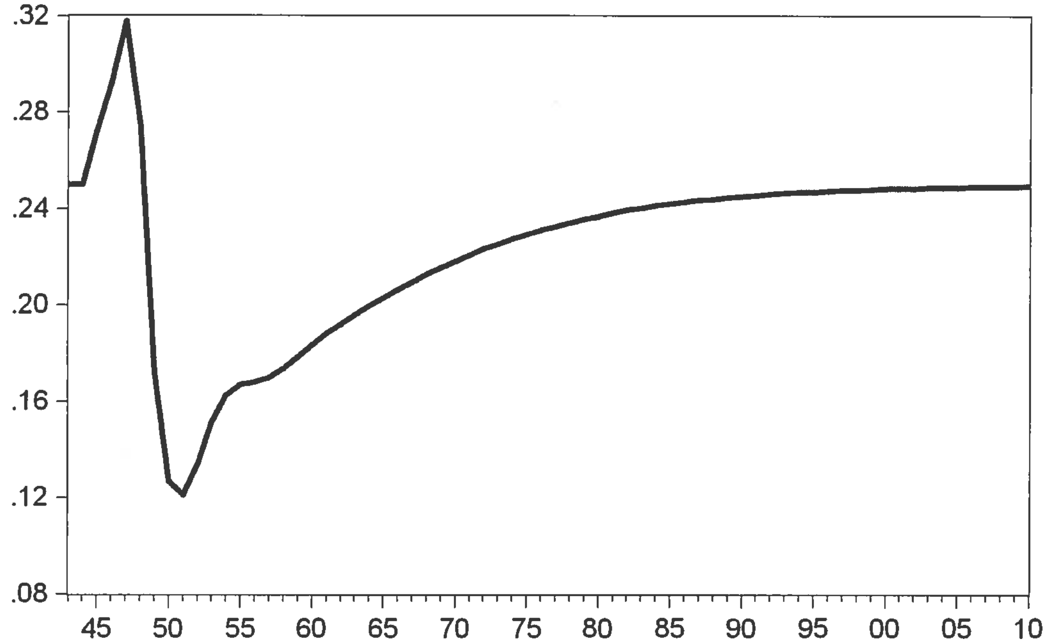
Capacity Utilization



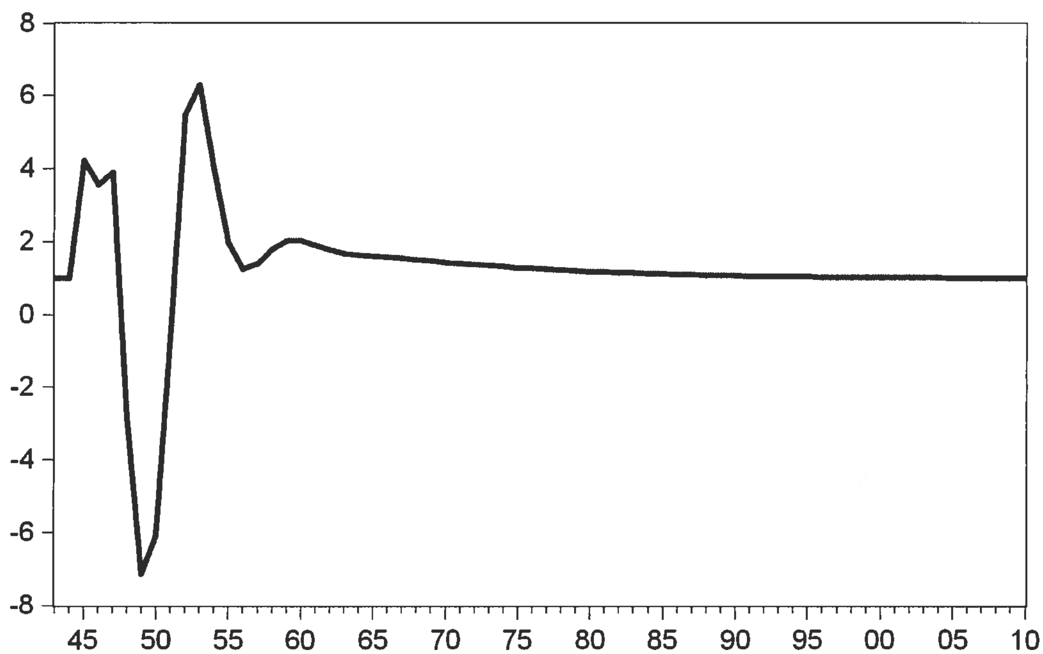
Growth Rate (I/K)



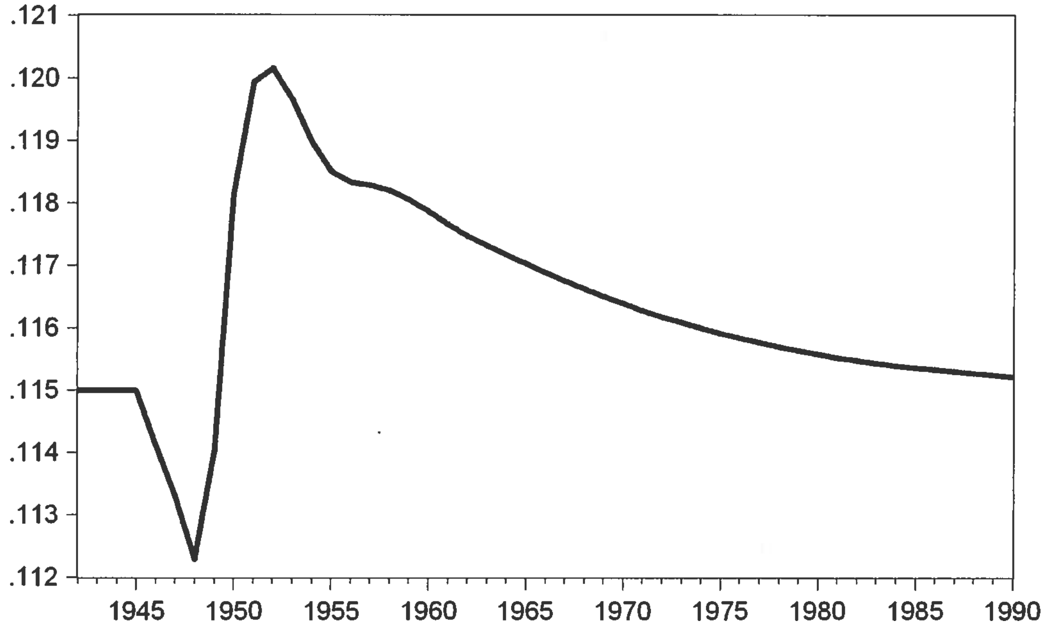
Debt-Capital Ratio



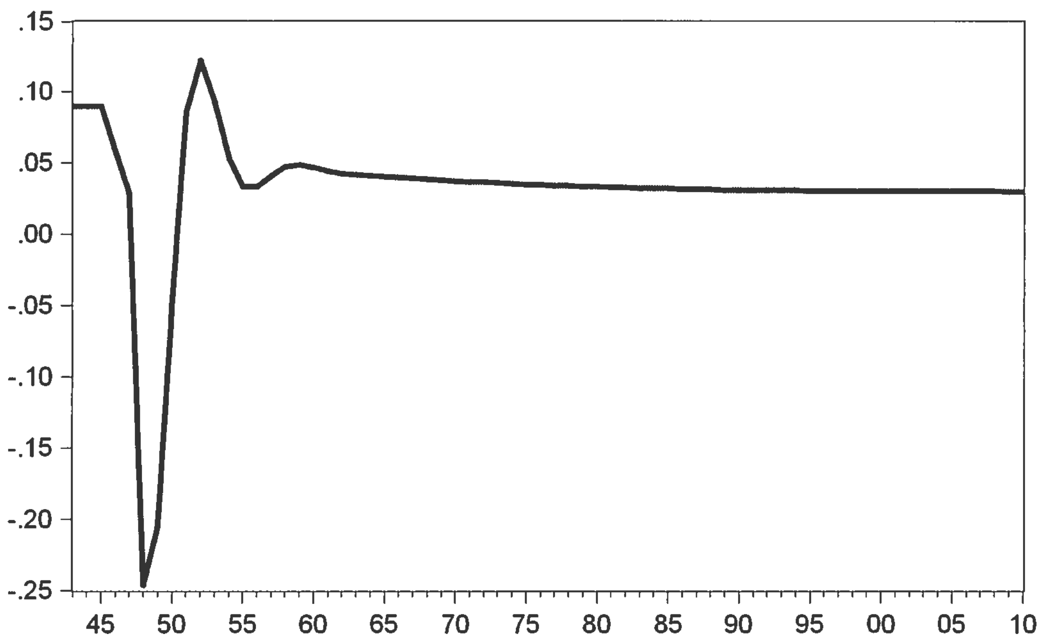
Fiscal Sustainability Condition (Golden Age)

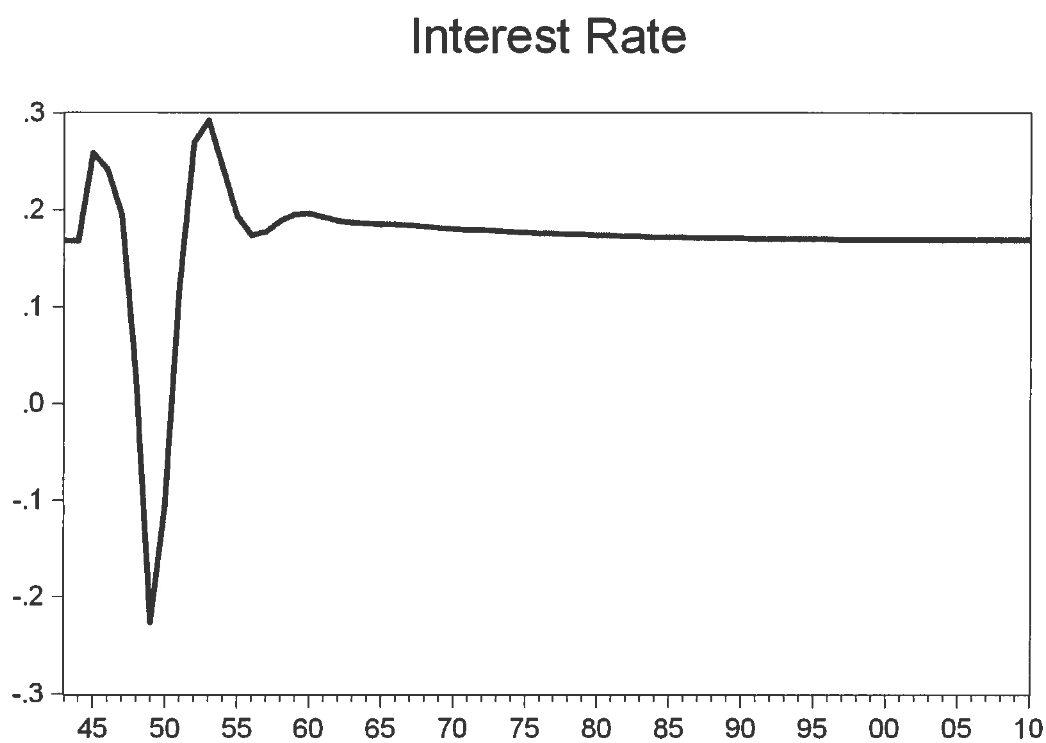


Government Spending Ratio

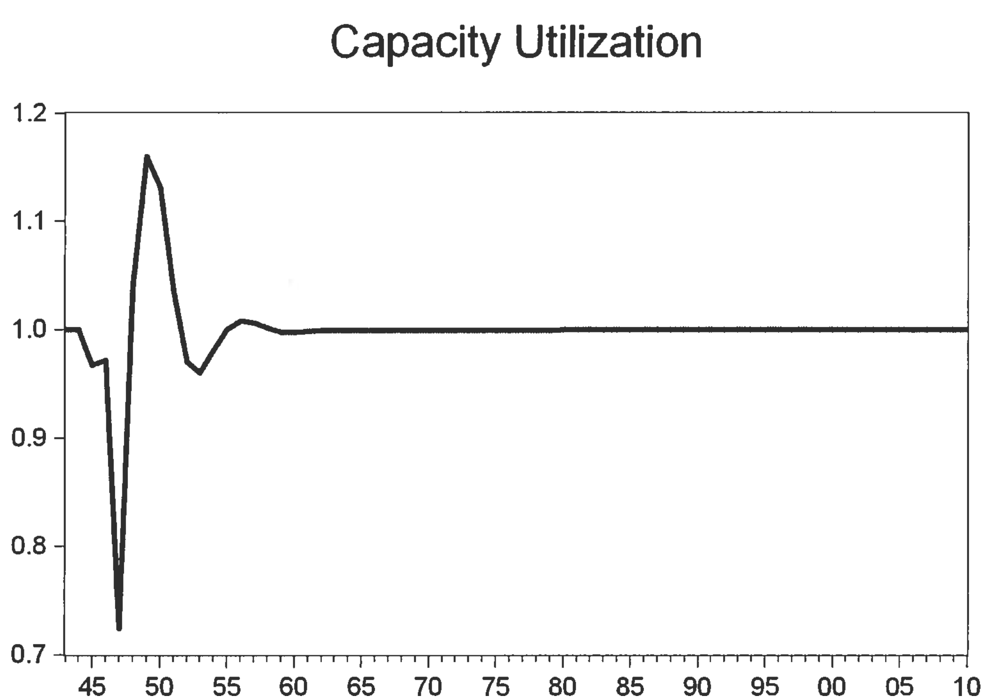


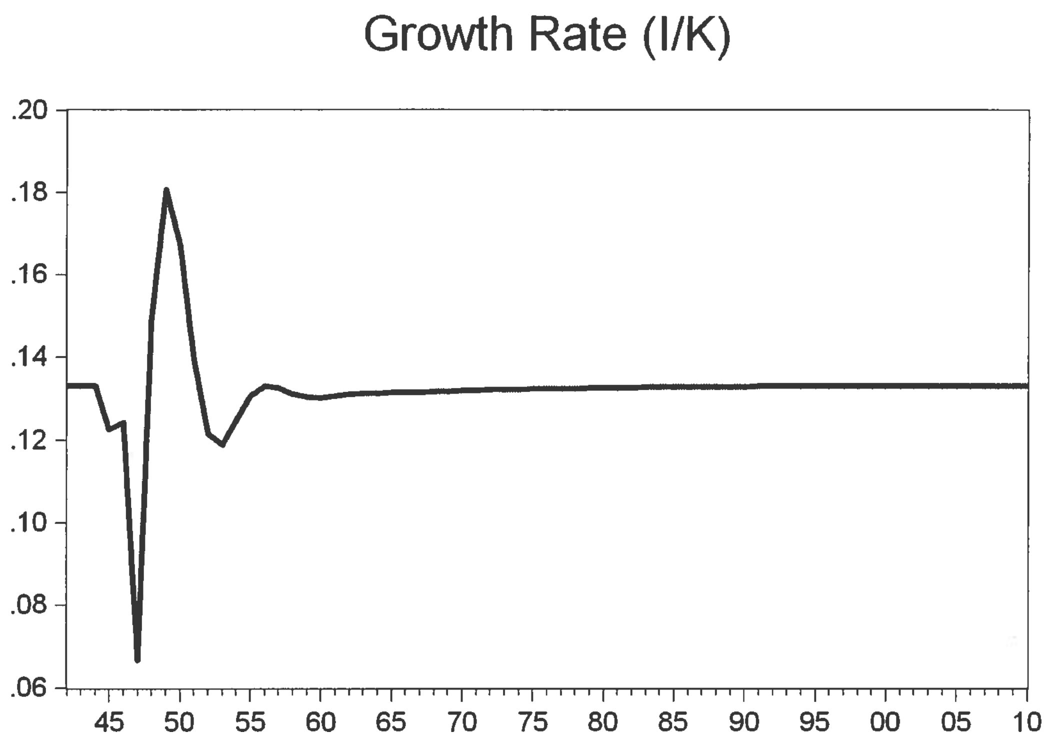
Inflation Rate at time t

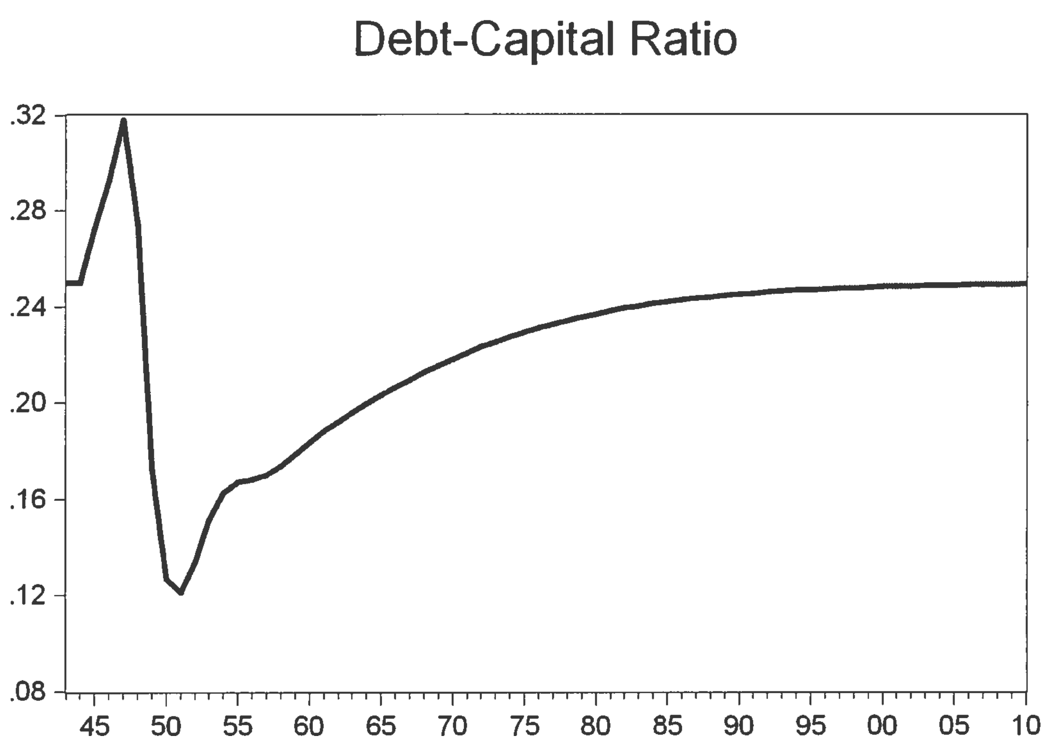


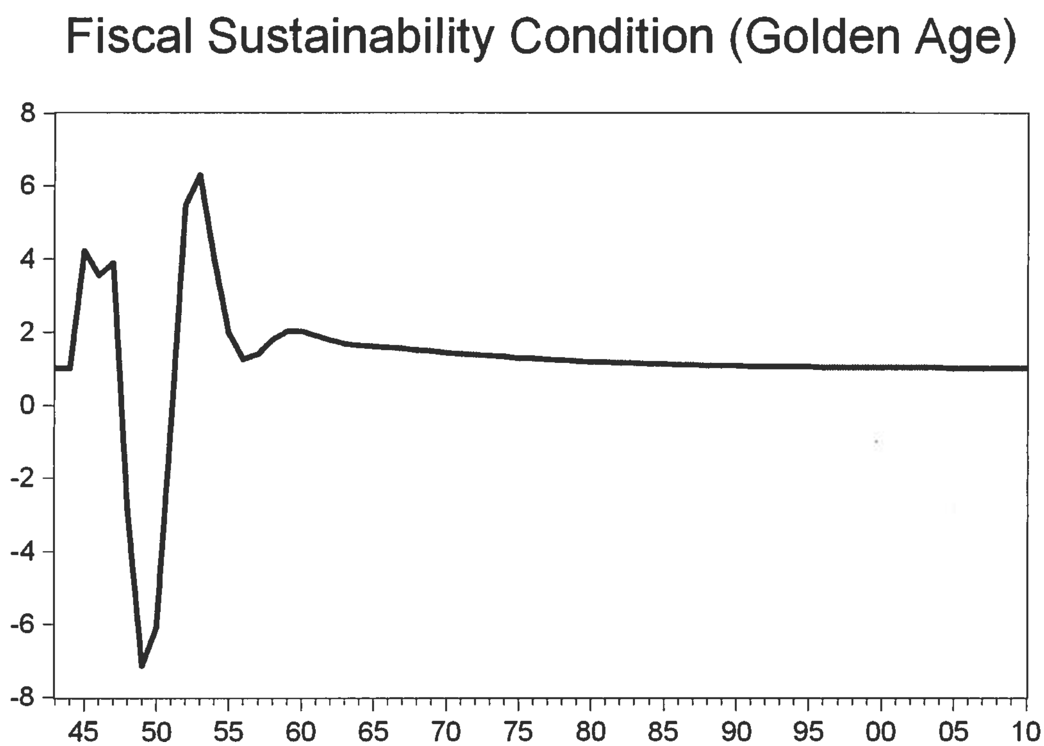


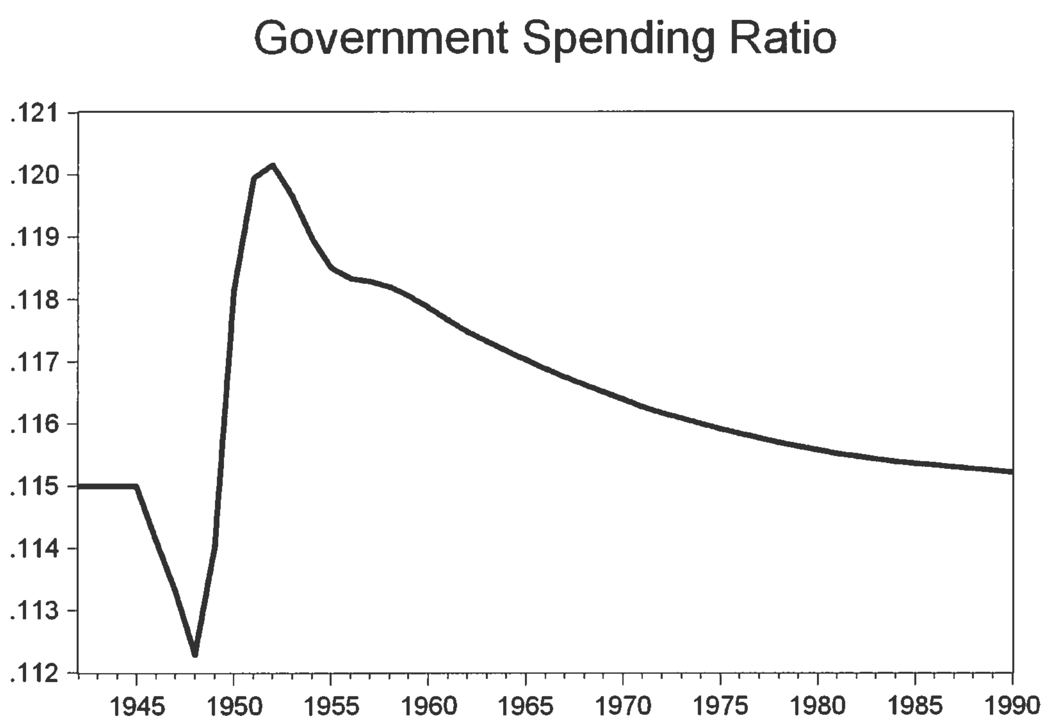
Appendix B6 – TAYLOR2 Demand Shock



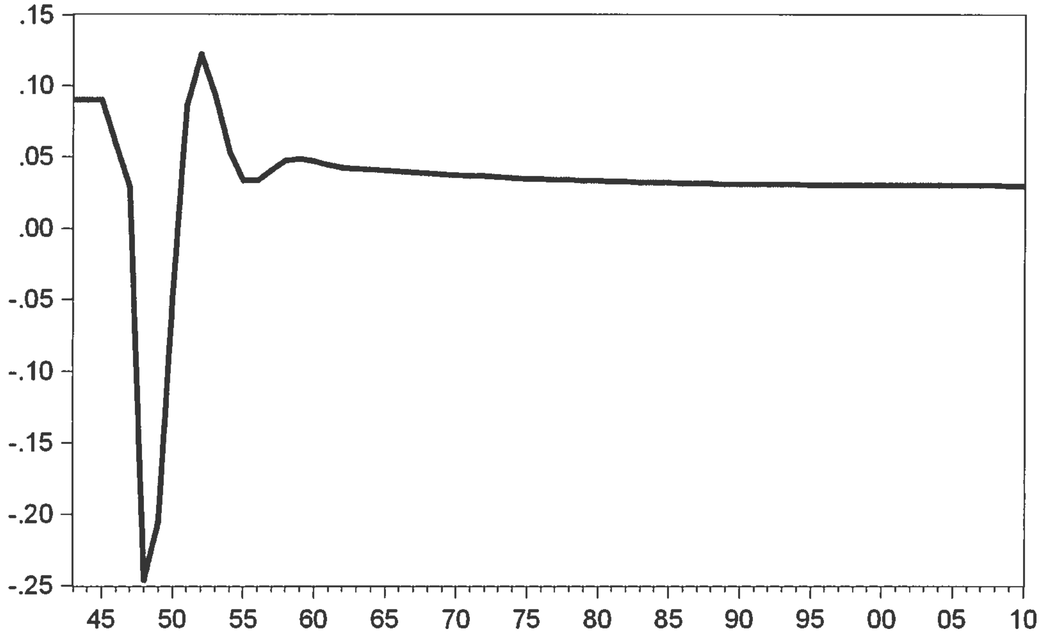

Growth Rate (I/K)
.20
.18
.16
.14
.12
.10
.08
.06
45
50
55
60
65
70
75
80
85
90
95
00
05
10


Debt-Capital Ratio
.32
.28
.24
.20
.16
.12
.08
45
50
55
60
65
70
75
80
85
90
95
00
05
10

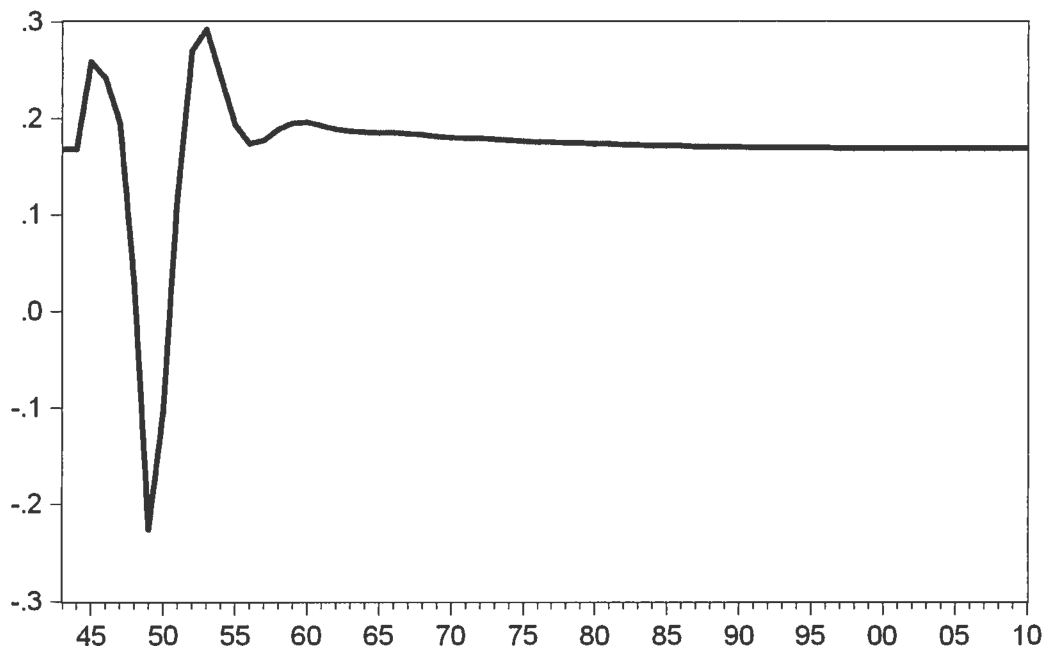

Fiscal Sustainability Condition (Golden Age)
8
6
4
2
0
-2
-4
-6
-8
45 50 55 60 65 70 75 80 85 90 95 00 05 10


Government Spending Ratio
.121
.120
.119
.118
.117
.116
.115
.114
.113
.112
1945 1950 1955 1960 1965 1970 1975 1980 1985 1990

Inflation Rate at time t



Interest Rate



Appendix C – Parameters and Starting Values

Parameter/Variable	Value
$\bar{b}$	0.25
$\bar{p}$	0.09
φ	1
h_m	0.75
r^*	0.168292683
d_0	0.1
d_1	0.1
d_2	0.05
h_f	0.4
h_{f2}	0.04
s_b	0.25
α	5.263157895
s_b	0.9
γ	0.115
τ^*	0.123780488