Abstract

The relations between monetary policies, asset prices, and economic growth are important and fundamental questions in macroeconomics. To address these issues, several empirical works have been conducted to investigate these relations. However, few of them have documented whether these relations differ across regimes. In this context, the general motivation of this thesis is to use dependent regime models to examine these relations for the Canadian case.

Chapter one empirically analyzes the interest rate behaviour of the Canadian monetary authorities by taking into account the asymmetry in the loss function. We employ a switching regime framework using two estimation strategies: First, we follow Caner and Hansen (2004) Threshold approach. Under this procedure we estimate the threshold values, using the Taylor empirical rules. Second, we estimate the asymmetric policy reaction function following Favero and Rovelli’s (2003) approach. The results reveal that the monetary authorities showed asymmetric preferences and that its reaction function can be better modeled with a nonlinear model. The main contribution of this chapter is to successfully interpret the parameters associated with the Bank of Canada preferences, something that Rodriguez (2008) could not do.

Chapter two tries to estimate the interest rate behaviour of the Canadian monetary authorities by expanding the arguments of the loss function for fluctuations in asset prices. Using the same methodology as in the first chapter, our findings suggest that the augmented nonlinear reaction function is a good fit for the data and gives new relevant insights into the influence of asset prices on Canadian monetary policy. These findings about the role of asset prices in the reaction function of the Bank of Canada provide relevant insights regarding the opportunities and limitations of incorporating financial indicators in monetary policy decision making. They also provide financial market participants, such as analysts, bankers and traders, with a better understanding of the impact of stock market index prices on Bank of Canada policy. Stock market stabilization plays a larger role in the interest rate decisions of the Bank of Canada than it is willing to admit.

Chapter three provides new evidence on the relation between inflation, relative price variability and economic growth to a panel of Canadian provinces over the period 1981-2008. We use the Bick and Nautz (2008) modified version of Hansen’s (1999) Panel Threshold Model. The evidence strongly supports the view that the relationship between inflation and economic growth is nonlinear. Further investigation suggests that relative price variability is one of the important channels through which inflation affects economic performance in Canadian provinces. When taking into account the cross-section dependence, we find that the critical threshold value slightly changes. It is desirable to keep the inflation rate in a moderate inflation regime because it may be helpful for the achievement of sustainable economic growth. The results seem to indicate that inflation that is too high or too low may have detrimental effects on economic growth.
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General Introduction

Modern central banks have three main tasks: (1) the pursuit of macroeconomic stability (make inflation stable, low and predictable); (2) maintaining financial stability and (3) ensuring the proper functioning of the economy, that is, achieving high and sustainable economic growth rates in order to improve the population living standards. My thesis is that both monetary theory and the practice of central banking have failed to keep up with key developments in the financial systems of advanced market economies, and that as a result of this, many central banks were to varying degrees ill prepared for the financial crisis that erupted in 2007. Since the start of the subprime crisis in August 2007, central banks all over the world have used unconventional rules to cut interest rates at a rapid pace trying to overcome the negative effects to the economy. This financial crisis shows us that central banks, including the Bank of Canada, can be reacting differently depending on the state of the economy. In this context, it becomes important to check empirically which factors were the driving forces behind their interest rate decisions. Using Taylor rules in this context might not be appropriate because during the crisis central banks have started to apply unconventional monetary policies. In this context, using the appropriate methodology, the general motivation of this thesis focuses on the empirical investigation of the different tasks of the Bank of Canada. Indeed, this dissertation consists of three chapters. The first two chapters investigate the Central Banks asymmetric preferences in the conduct of monetary policy and the third one examines the real effects of inflation on economic growth.

From the methodological perspective, the model specification used in this thesis is in line with the recent literature that incorporates a nonlinear regression specification in the study of monetary policy. In this literature two nonlinear approaches prevail: The Threshold Autoregressive (TAR) methodology [Tong (1990); Terasvirta (1995); Weise (1999)] and the Markov Switching (MS) methodology (Hamilton, 1989). In both approaches, the estimated parameters are supposed to depend on the state of the system. The difference between these approaches arises from the identification of the state of the system. In the TAR methodology, the identification of the state of the system relies on the selection of a transition function depending on a transition variable through tests. Whether this transition variable is lesser or greater than a threshold value determines if an observation belongs to a regime or the another. The model that is piecewise linear is then estimated for every candidate value. The retained value is the one that provides the highest log-likelihood value. In the MS methodology, the state of the system is ruled by an unobserved process, supposed to be a one-order Markov chain. The transition probabilities of this process are incorporated in the set of parameters to be estimated. The estimation of the model is obtained through Maximum Likelihood, which provides an inference on the parameters and on the state of the system. However, the use of this approach is questionable in the present context since the parameters are recovered by estimating a first order condition. Standard regularity conditions require the relevant objective function to be smooth and twice differentiable whereas in this instance regime switching implies a discontinuity.

Unlike Bec et al. (2000) who used Smooth Transition Regression (STR) and Martin
& Milas (2004) and Petersen (2007) who applied a simple Logistic Smooth Transition Regression (LSTR) to analyze nonlinearity and structural change in monetary policy, in this paper, we retain the threshold-type regression model. Threshold models are attractive because they can allow for more flexible regression functional forms by splitting data with certain unknown threshold values. Moreover, in a threshold model the regime is determined by the level of the observable variable whereas in the Markov switching models the regime switches are exogenous and driven by an unobservable process; it is not able to account for the intuition behind the nonlinear central bank behaviour. Although there are several previous studies on the statistical inference of threshold models, only a few papers investigate statistical inference under endogenous threshold values. Tong (1983) first proposed threshold regression models for time series data. Hansen (1999) extended the threshold regression to static panel data structure and derived the corresponding asymptotic theory for threshold parameters and regression slopes. Hansen (2000) further showed the asymptotic properties of concentrated least square estimators for threshold and coefficient parameters. Caner and Hansen (2004) in turn considered endogenous regressors in threshold regression models with exogenous thresholds. They used instrument variable methods to obtain the consistent coefficient estimators.

The first chapter, The Asymmetric Reaction of Monetary Policy to Inflation and the Output gap: Evidence from Canada, investigates the existence of possible asymmetries in the Bank of Canada’s objectives. By assuming that the loss function is asymmetric with regard to positive and negative output gaps and deviations of the inflation rate from its target, we estimated a nonlinear reaction function which allows to identify and check the statistical significance of asymmetric parameters in the monetary authority’s preferences. First, we estimate a threshold model for the empirical monetary policy reaction function of the Bank of Canada that generates the existence of two policy regimes according to whether the inflation deviations or the output gap is above or below a threshold value. This estimation allows to determine threshold values and to test for the presence of asymmetry or nonlinearities. Second, to infer the monetary policy preferences and have a better interpretation of the parameters, we use these threshold values to estimate the specification rule obtained through the problem of optimization. Our findings indicate that the Bank of Canada showed asymmetric preferences and that its reaction of monetary policy can be better modeled with a nonlinear model. Over time the Bank of Canada has assigned more weight on positive deviations of inflation from the target than on negative deviations. We successfully interpret the parameters associated with the preferences of the central bank, something that Rodriguez (2008) did not achieve.

The second chapter, Efficiency of the Monetary Policy and Stability of Central Bank Preferences with respect to Asset Markets in Canada, tries to estimate monetary policy reaction functions, taking into account asset prices, using a threshold model that allows for shifts in the coefficients of the reaction function of the Bank of Canada. Our findings should give new relevant insights into the influence of stock market prices on monetary policy in Canada and should provide relevant insights regarding the opportunities and limitations of incorporating financial indicators in monetary policy decision making. They should give
financial market participants, such as analysts, bankers and traders, a better understanding of the impact of stock market index prices on the Bank of Canada policy. Our results suggest that stock market stabilization plays a larger role in the interest rate decisions of Bank of Canada than it is willing to admit. To infer the Bank of Canada’s preferences, we estimate parameters jointly with a model of the economy. The results imply that the preferences of the monetary authority have changed between the different subperiods and different regimes. The findings suggest that the introduction of inflation targeting in Canada was accompanied by a fundamental change in the objectives of monetary policy, not only with respect to the average target, but also in terms of precautions taken to keep inflation in check in the face of uncertainty about the economy.

The third chapter, Inflation and Economic Growth with Cross-section Dependency: An Empirical Analysis in terms of Relative Price Variability across Canadian Provinces, examines the empirical relationship between inflation and economic performance or living standards through relative price variability by using data for Canadian provinces over the period 1981-2008. The results, based on the relatively novel panel threshold model, suggest the presence of a statistically significant double threshold. Further investigation suggests that relative price variability is an important channel through which inflation affects economic performance in Canadian provinces. Our findings indicate that in a low and moderate inflation regime the marginal impact of inflation is insignificant and even positive; in extremely high inflation regimes, the marginal impact of inflation on economic growth is significantly negative. Using relative price variability as an inflation channel improves our results in terms of magnitude and significant level. Controlling for cross-section dependency, we find more consistent results. On the basis of this study, we conclude that it is desirable to keep the inflation rate in the moderate inflation regime and therefore the Bank of Canada should concentrate on those policies which keep the inflation rate between 1.8 percent and 4.6 percent because it may be helpful for the achievement of sustainable economic growth that improve the living standards of Canadian provinces. This information gives an important signal for Canadian policymakers.
Essay 1

The Asymmetric Reaction of Monetary Policy to Inflation and the Output gap: Evidence from Canada
1. Introduction

The positive theory of monetary policy in industrial countries has reached a broad consensus since the influential paper of Taylor (1993). Monetary policy is conceptualized as follows: policy authorities minimize a linear combination of the quadratic central bank loss function of inflation and output from their respective targets (the inflation and the output gaps in what follows), and the main policy instrument is the short term rate of interest. The majority of the literature of the last decade has implemented this perspective by estimating Taylor rules or reaction functions in which the short term rate is a linear function of the, currently expected, future values of the inflation and of the output gaps (Clarida et al., 1998, 2000). It is well known that, as a theoretical matter, such linear reaction functions are obtained when the expected value of a loss function that is quadratic in the inflation and output gaps is minimized subject to a linear dynamic structure of the economy.

More recently some of the literature has considered the possibility that, as a positive matter, the loss function of monetary policymakers may not be quadratic and consequently the Taylor rules derived from such functions are not necessarily linear. For example, monetary authorities may dislike positive inflation deviations more than negative ones, or make more efforts to reduce the output gap when the inflation goal has been achieved. The public dislikes unemployment more than inflation, especially when inflation rates are low; then, during recessions voters may prefer an increase in inflation (to reduce unemployment) that is larger than the decrease in inflation they would want (to increase unemployment) during booms. Since central bankers respond in part to the political power of policymakers, they may reflect some of these preferences. Blinder (1998) for instance suggests that political demands may lead to asymmetric central bank behaviour.\(^1\) This suggests that during nor-

\(^1\)In most situations, the central bank will take far more political heat when it tightens preemptively to avoid higher inflation than when it eases pre-emptively to avoid higher unemployment.
mal times the central banks may be more averse to negative than to positive output gaps. In fact, despite its analytical convenience, a quadratic loss function that penalizes positive and negative output gaps to the same extent does not appear to be realistic.

Evidently, policymakers are averse to negative output gaps but it is less evident that they are, given inflation, equally averse to positive output gaps. They may even, for a given inflation rate, be indifferent between different magnitudes of positive output gaps. Cukierman (2000, 2002) shows that in the last case, and in the presence of uncertainty and rational expectations, there is an inflation bias even if the output target of the central banks is equal to the potential level. Cukierman and Muscatelli (2008) refer to central bank loss functions that display this type of asymmetry as recession avoidance preferences (RAP). In the presence of uncertainty about future shocks such an asymmetry leads the central bank to take more precautions against negative than against positive output gaps. In their papers, Cukierman and Gerlach (2003) and Ruge-Murcia (2003) test this hypothesis empirically. Using a natural rate framework and cross sectional data for the OECD countries, Cukierman and Gerlach (2003) find evidence supporting a half quadratic specification of output gap losses. In a similar study, Doyle and Falk (2006) arrive at the same conclusions. Ruge-Murcia (2003), using a natural rate framework and time series data for the US, find that it provides a better explanation for the behaviour of US inflation than does the Barro-Gordon (1983) inflation bias model.

On the other hand, during periods of inflation stabilization in which monetary policymakers are trying to build up credibility, they may be more averse to positive than to negative inflation gaps of equal size. Following Cukierman and Muscatelli (2008), we refer to objective functions that display this type of asymmetry as inflation avoidance preferences (IAP). In the presence of uncertainty about future shocks this leads policymakers to react

\[^2\text{In this specification losses from negative output gaps are quadratic and, given inflation, there are no losses from positive output gaps.}\]
more vigorously to positive than to negative inflation gaps.

Most studies about the so-called asymmetric preferences are concentrated on the U.S. and European cases. To the best of our knowledge, no similar work has been done for Canada. We allow asymmetric preferences for the central bank loss function. Also, this paper employs an intertemporal specification of the reaction function, which is derived from the central bank’s objective function.

Asymmetric objectives generally lead to nonlinear reaction functions. To understand the meaning of such theoretical nonlinearities for the loss functions of monetary policymakers, this research provides an empirical assessment of the monetary policy rules in Canada for the period 1961:1 to 2008:4. We are trying to answer the question of whether the preferences for inflation and output gap of the Bank of Canada are asymmetric. In other words, this study investigates whether there is significant evidence of asymmetries in the revealed preferences of the Canadian monetary policymaker. Our analysis introduces a threshold effect in a central bank loss function and then we test the relevance of the nonlinearity hypothesis. The contribution of this paper is to provide some evidence supporting the idea that the preferences of the Bank of Canada may not be symmetric, and therefore it adds another empirical result to the literature of asymmetric preferences of central banks.

There is a growing literature that explores both the existence and the effects of asymmetries or nonlinearities in monetary policy rules. Most of this research has focused on the estimation of nonlinear policy reaction functions exploiting the well-known result that if an asymmetry in central bank preferences exists, then the optimal policy rule is nonlinear (see Bec et al., 2002; Kim et al., 2002; Martin and Milas, 2004). However, evidence of nonlinearity in policy reaction functions may be ultimately uninformative about the asymmetry of the policymaker’s loss function. The reason is that policy reaction coefficients, as complex convolutions of the structural parameters, do not reveal the policymaker’s preferences.
Dealing with this issue requires the specification of a structural model of the economy so as to uncover the coefficients of the policymaker’s loss function. Empirical studies using this structural approach are much more scarce, and seem limited to Dolado et al. (2004) and Surco (2003, 2007). Assuming a linex function for the central bank’s loss function and allowing for nonlinearities in the AS curve, Dolado et al. (2004) find that the optimal monetary policy rule must include the conditional variance of inflation as an argument. However, their model induces asymmetric responses only when the AS curve is nonlinear. Their empirical estimations for the U.S. suggest the existence of a nonlinear Fed’s monetary policy reaction during the Volcker-Greenspan period (post-1982) driven by asymmetric preferences regarding inflation deviations instead of convexities in the AS curve. According to this result, over this period the Fed would have weighted more severely positive inflation deviations than negative ones. For the previous Burns-Miller period (pre-1979), their estimations cannot reject the existence of quadratic preferences. Surico (2007) shows that if the central bank has a cubic specification for the loss function and takes discretionary actions in a standard New Keynesian Model, then the optimal monetary policy reaction must add squared terms of inflation deviations and output gap. Using U.S data, he finds asymmetric preferences of the Fed with respect to the output gap in the pre-Volcker era. During this period, this kind of preferences induced stronger reactions to output contractions than expansions of the same magnitude.

Both these papers, however, seem to have some drawbacks. Dolado et al. (2004) have to restrict their policymaker loss function to a regime of strict inflation targeting and thus are not able to test for Cukierman’s asymmetry. Furthermore, their econometric strategy is not a truly simultaneous estimation of macro systems, as the conditional variance of inflation included in the optimal policy reaction function is generated in a first step prior to

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the rule estimation. In turn, Surico (2003, 2007) models the structure of the economy with purely forward-looking equations and with the instantaneous transmission of interest rate changes to output and inflation. The resulting lack of persistence and of policy lags implies that his model is not data-consistent. Outstanding to these problems, it is hard to assess whether Dolado et al.’s and Surico’s empirical results on the US case are incompatible or complementary.

While the literature of formal analysis of central bank preferences asymmetry, just briefly reviewed, exposes the need for methodological contributions, it also reveals that the Canadian case has barely been studied to date. Considering these developments, our contribution to this literature is to find new evidence on the revealed preferences of the Canadian monetary policymakers. This evidence is extracted using a framework that allows for testing the relevant asymmetries in the central bank loss function in a given macroeconomic structure. Our framework extend the models of Favero and Rovelli (2003) and Rodriguez (2008) which take into account nonlinearities in the central bank loss function. Such nonlinearities are clearly identified with asymmetries in the policymaker’s loss function. In doing so, we are able to identify and, thus, retrieve the coefficients of the policymaker’s preferences and of the macroeconomic structure. Moreover, to discriminate between recession avoidance preferences and inflation avoidance preferences\(^4\), the framework may also detect asymmetries in interest rate smoothing, which are not taken into account by Cukierman and Muscatelli (2008).

The paper is organized as follows: Section 2 presents the Canadian’s monetary policy framework. Section 3 sets up the model and solves the optimization problem relevant to the central bank and econometric strategy. Section 4 presents econometric specification. Section 5 describes the data and discusses the empirical results. Finally, section 6 offers

\(^4\)see Cukierman and Muscatelli (2008) for more detail.
some concluding remarks.

2. The Canadian Monetary Policy Framework: A Brief History

In response to the persistence of high inflation during the 1970s and the beginning of 1982, the Bank of Canada adopted a narrowly defined monetary aggregate ($M_1$) as its intermediate target variable during the period 1975-1982. When this aggregate became increasingly unreliable and turned out not to have been all that helpful in achieving the desired lessening of inflation pressures, it was eventually dropped as a target in 1982. Because of the long lags and indirect connections between the instrument(s) and the ultimate goal of monetary policy, central banks have found it helpful to make use of intermediate targets or indicators. Over the years, the Bank of Canada moved away from the use of monetary aggregates as intermediate targets because financial market liberalization, deregulation, and innovations had led to changes in the financial structure that weakened the link between monetary aggregates and the ultimate target variable. In addition, the Bank of Canada viewed monetary targeting as impractical because large changes in interest rates were required to effect changes to the monetary aggregates. Subsequently, the Bank embarked on a protracted empirical search for an alternative monetary aggregate target, but no aggregate was found that would be suitable as a formal target. The relationship between monetary aggregates and inflation or economic growth was becoming increasingly unstable, and even subject to reverse causality due to instability in the demand for money, as well as uncertainty about the magnitude of the money multiplier and velocity parameters. Thus, from 1982 to 1991, monetary policy in Canada was carried out with price stability as the longer-term goal and inflation containment as the shorter-term goal, but without intermediate targets or a specified path to the longer-term objective.
Recently, there have been important changes in the way in which the Bank of Canada conducts monetary policy (see Lavoie and Seccareccia, 2006). In particular, the Bank adopted explicit inflation targets and introduced important changes to its operational framework. The Government and the Bank of Canada in February 1991 announced targets for inflation, aiming at reducing CPI inflation to a range of 1 to 3 percent by the end of 1995, and since then the Bank has conducted monetary policy so as to achieve this target. These announcements confirmed price stability as the appropriate long-term objective for monetary policy in Canada and specified a target path to low inflation. Inflation was brought within the target range earlier than expected, and it was announced in December 1993 that the 1 to 3 percent target range would be extended through the end of 1998. The Bank of Canada also has made some important changes to the way in which it implements monetary policy. Since June 1994, the operational objective of monetary policy is to keep the overnight interest rate within a 50-basis point range. This change shifted the focus away from the three-month treasury bill rate toward the overnight interest rate as the Bank’s short-term operational objective. In February 1996, the bank announced that the Bank Rate would be set at the upper limit of the operating band for the overnight rate. Previously, the Bank Rate was set 25 basis points above the tender average for the three-month treasury bill rate. The main objective of these changes was to reduce the degree of uncertainty regarding policy objectives and to increase the transparency with which the Bank of Canada conducts monetary policy (see Tim, 1995). The key near-term aim of the targets was to help firms and individuals see beyond these price shocks and look at the underlying downward trend of inflation at which monetary policy was aiming, and to take this into account in their economic decision-making. The goal of monetary policy is to contribute to solid economic performance and rising living standards for Canadians by keeping inflation low, stable and predictable. So, the Bank of Canada (BoC) uses the
interest rate (the overnight rate) as its monetary policy instrument instead of monetary aggregates.

To gain insight into the brief history of monetary policy carried out by the Bank of Canada, we take a look at interest rates, inflation and the output gap in Canada. The evolution of interest rates and inflation during the period under investigation is shown in Figure 1.1. The inflation rate experienced its highest levels between the 1970s and early 1980s. Over this period, as shown in Figure 1.1, the inflation rate varied between 9% and 12%. Over the same period, the interest rate hit its lowest level. Since adoption of inflation targeting, inflation rates hovered around the 2% target. Inflation rates in Canada did decline to the targets set, and did remain within or below the target ranges. This means that interest rate hikes took place before inflation exceeded the 2% target, while interest rates were cut shortly before inflation had peaked. This pattern is consistent with a forward-looking response of the Bank of Canada to inflation gaps. The output gap in Figure 1.2 is approximated by the deviation of real GDP from its potential value. The output gap was above its trend at the beginning of 1981, with a peak of 2.79%. It fell towards 5.43% below trend in December 1982. From Figure 1.2 it is clear that the interest rate hikes started when the output gap was at elevated levels, while interest rates were cut when the output gap fell. Afterwards, interest rates remained steady as the output gap hovered around zero.
Overall we can conclude that the behaviour of interest rates was consistent with the standard approach to monetary policy.\textsuperscript{5} The Bank of Canada implemented its monetary policy through its control and setting of the short-term interest rate, in this case the Overnight Rate. However, the weight given to the inflation rate target for monetary policy was clearly heavier than that given in other countries that did not adopt inflation targets and guidelines. This institutional change has stimulated Taylor-rule type monetary policy analysis. For example, Rodriguez (2008) analyzed the efficiency of the monetary policy of the Bank of Canada by estimating a Taylor type monetary policy rule following Favero and Rovelli’s (2003) framework. The Bank of Canada has adopted an explicit inflation target since 1991, and thus a study of its asymmetric preferences to inflation and output gap is a natural extension of the previous studies, which assumed quadratic preferences.

\textsuperscript{5}Central banks set interest rates based on inflation considerations, taking into account output gap as well.
3. The Theoretical Framework

The central bank faces a dynamic optimal control problem the solution of which describes its policy actions. These are the optimal response of monetary authorities to the evolution of the economy as captured by the relationships among the state variables. We describe such a dynamics by means of a simple closed economy-two equation framework made up of an aggregate supply and an aggregate demand function, which actually represent the constraints of the policymakers’ optimization problem. We assume that the central bank conducts monetary policy through a targeting rule according to the terminology of Svensson (1999).

3.1 The Central Bank’s model of the economy

The behavior of the economy is characterized by means of a New Keynesian, backward looking\(^6\), sticky prices framework in which inflation and the output gap depend respectively on the expected future values of those variables and in which the policy instrument of the monetary authority is the nominal interest rate. A simple aggregate version of such a framework has recently been summarized compactly by Clarida et al. (1999) and is reproduced in what follows;

Following standard assumptions in the New-Keynesian literature (see among others Gali and Gertler (1999); Gali, Gertler and Lopez-Salido (2005); Moons, C. et al. (2007)), we assume the following specifications for aggregate demand and aggregate supply:

\[
x_{t+1} = \eta_1 x_t - \eta_2 (i_t - E_t \pi_{t+1}) + \epsilon_{t+1}^d
\]  

\(^6\)We note that there is an alternative approach (forward-looking method) but, we use a structural backward-looking model of a closed economy. This backward-looking model is adopted from Rudebusch and Svensson and it is used in several studies, including Rodriguez (2008) and Dennis (2006). Furthermore, we chose backward-looking for reasons of consistency with our methodology. Indeed, to extract the parameters of policy preferences of the central bank we use Favero and Rovelli’s (2003) approach who uses backward-looking framework.
in which $x$ denotes the output gap, $i$ is the short-term nominal interest rate, $\pi$ is inflation rate, and $\varepsilon^d$ is an aggregate demand shock. $E_t$ defines the expectations of inflation taken with respect to the information available at time $t$. All variables are in logarithms and refer to deviations from an initial steady state. Equation (1.1) shows that the output gap depends on the past output gap, the real interest rate and a demand shock. The backward-looking component in the aggregate demand curve can be explained by habit formation in consumption decision.

The specification of aggregate supply is given by:

$$
\pi_{t+1} = \phi x_t + \theta \pi_t + \varepsilon^\pi_{t+1}
$$

(1.2)

where the supply shock $\varepsilon^\pi$ may be interpreted as a shift of the degree of substitutability between inputs in the production of final goods, or an exogenous cost push shocks. Equation (1.2) is a backward-looking NAIRU type Phillips Curve where the change in inflation is a positive function of the lagged output gap and the inflation shock. Such a specification has also been adopted by Ball (1999), Svensson (1997) and Rudesbusch and Svensson (1999). The presence of inflation inertia in the inflation equation implies that disinflations will be costly in terms of output losses, thus there is a short-run trade-off between inflation and output. However, since lagged inflation enters equation (1.2) with a unity coefficient, the model implies a vertical long-run Phillips curve. This process is also consistent with the empirical finding that inflation in the major industrialised countries is so highly persistent that it may indeed contain a unit root as some studies have shown (see e.g. Grier and Perry, 1998). Equation (1.2) posits no role for expected future inflation in the inflation adjustment equation. The parameter $\phi$ is a positive constant which measures the sensitivity of inflation to excess demand.$^7$

$^7$As Clark, Goodhart, and Huang (1999) point out, there are good reasons to believe that $\phi$ is not constant. However, the assumption of linearity in the Phillips curve helps to obtain a closed-form solution.
In empirical applications, more lags of output (in the case of the IS curve) and output and inflation (for the Phillips curve) are often included to improve the empirical fit. Adding these lags will also induce a more persistent and therefore more realistic adjustment to shocks. In empirical studies and monetary policy analysis, sometimes concepts of equilibrium and/or core inflation are added to (1.2), to distinguish short-run fluctuations of inflation from longer term, equilibrium inflation. In our analysis this issue is not dealt with and inflation (as all other variables) is defined in terms of deviations from (possibly non-zero inflation) steady-state (see Vega and Wynne, 2003).

Following the current monetary policy analysis framework, one possible shortcoming of equations (1.1) and (1.2) is their relevance in the context of open economies, where international trade is an important part of the economic activity and therefore, the exchange rate should be considered as a significant argument in policy functions of open economies. However, using a modified version of equations (1.1) and (1.2), Ball (1999) does not find important changes in the interest rate movements for open and closed economies. On the other hand, using a forward-looking perspective, Svensson (2000) finds varied benefits of including the exchange rate in the monetary rule in comparison with the original Taylor rule. In a similar way, Taylor (2001) finds weak evidence for the exchange rate channel. Clarida et al. (1998, 2000) attempt to re-specify Taylor-type rules for small economies using foreign variables. For the cases of Japan and Germany, they use the US interest rate and the exchange rates in the interest rate rule and the results show that the coefficients may be small and significant but in some cases, as for Germany, the inflation coefficient is negative. Taylor (2001) suggests that the inclusion of the exchange rate is not crucial for the monetary policy rule. As Rodriguez (2008), we consider the role of the exchange rate explicitly in the empirical part of the Phillips curve since open economy issues are important for Canada.

for the optimal feedback rule.
3.2 An Asymmetric Specification of the Loss Function

The asymmetric effects of monetary policy have been an important topic for macroeconomic policy research, and they have been studies from both theoretical and empirical perspectives. The analysis here extends the asymmetric effects framework. Asymmetric effects, in the context of monetary policy refer to a situation in which the effects of a given policy are not constant but vary depending on the circumstances. Typically the asymmetries discussed relate to either the phase of the business cycle or to the policy direction.

Moreover, some of the recent literature has considered the possibility that the loss function of monetary policymakers may not be quadratic and consequently the Taylor rules derived from such functions would not necessarily be linear. In fact, despite its analytical convenience, a quadratic loss function that penalizes equally-sized positive and negative output gaps to the same extent does not appear to be realistic. Obviously, policymakers are averse to negative output gaps but it is less clear that they are, given inflation, equally averse to positive output gaps. They may even, for a given inflation rate, be indifferent between different magnitudes of positive output gaps. Cukierman (2000, 2002) shows that in the last case, and in the presence of uncertainty and rational expectations, there is an inflation bias even if the output target of the Central Bank is equal to the potential level.

Following standard assumptions in the empirical literature of monetary policy, the policymaker’s preferences are modeled as an intertemporal loss function in which, at each period, the loss function depends on both inflation and output in relation to their target values, as well as the smoothing interest rate. Future values are discounted at rate $\beta$, and the weights $\lambda$, and $\mu$ are nonnegative. As usual, we assume that monetary policy is conducted by a central bank that chooses the sequence of short-term nominal interest rates in order to minimize the present discounted value of its loss function. Rather than assuming a quadratic form as is usual in the literature (see Svensson, 1997; Favero and Rovelli, 2003
and Rodriguez, 2008), we use a more general specification (nonlinear or asymmetric loss function) of the monetary authorities objectives.

\[
\text{Loss} = E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left[ (\pi_{t+\zeta} - \pi_0)^2 + \lambda x_{t+\zeta}^2 + \mu (i_{t+\zeta} - i_{t+\zeta-1})^2 \right]
\]

(1.3)

A simple way of capturing nonlinearities or asymmetries in policy behaviour is to estimate threshold models whereby the policy reaction function switches into a different regime whenever a certain variable breaches one or more thresholds. Asymmetries or nonlinearities in the policymakers preferences mean that the structural weights depend on the appropriate state of the economy.

\[
\text{Loss} = E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left\{ \left[ (\pi_{t+\zeta} - \pi_0)^2 + \lambda_1 x_{t+\zeta}^2 + \mu_1 (i_{t+\zeta} - i_{t+\zeta-1})^2 \right] I(q_t \leq \gamma) + \left[ (\pi_{t+\zeta} - \pi_0)^2 + \lambda_2 x_{t+\zeta}^2 + \mu_2 (i_{t+\zeta} - i_{t+\zeta-1})^2 \right] I(q_t > \gamma) \right\}
\]

(1.4)

Where \(0 < \beta < 1, \lambda_j, \mu_j \geq 0, j = 1, 2\), and \(E_t\) defines the expectations taken with respect to the information available at time \(t\), \(\pi_0^*\) being the target level of inflation in each regime. With this objective function, the Central Bank is assumed to stabilize annual inflation about \(\pi_j\), while keeping the output gap and making small changes in the nominal interest rate. The policy preference parameters, or weights, \(\lambda_j\) and \(\mu_j\), indicate the relative importance policymakers put on output gap stabilization and on interest rate smoothing relative to inflation stabilization. On the other hand, \(\lambda_j\) and \(\mu_j\) represent the central bank’s aversion to output fluctuations around potential and to interest rate level fluctuations. \(I(.)\) are indicator functions that take a value equal to one if the condition in parentheses is true and zero otherwise. \(q_t\) is the threshold variable and \(\gamma\) the threshold value that can be estimated.

We consider two observed threshold variables \((q_t)\) related to actual economic development: the output gap \((q_t = x_t)\), and the inflation deviation \((q_t = \pi_t)\). We postulate two possible regimes for central bank behaviour depending on whether \(q_t\) is below (first regime) or above
(second regime) the threshold value. Then, if we suppose $\gamma = 0$ and the output gap as the threshold variable, the central bank may modify the magnitude (and the velocity) of its reaction to the output gap and/or inflation deviations during recessions, with respect to what it does in expansions. Alternatively, with the inflation deviation as the threshold variable, the central bank may react differently to output gap and inflation deviations if high-or low-inflation episodes occur. Empirically, we could find $\lambda_1 \neq \lambda_2$, $\mu_1 \neq \mu_2$ and $\pi_1 \neq \pi_2$ in both cases.

In summary, the intertemporal optimization problem is then to minimize (1.4) subject to the restrictions (1.1) and (1.2). The problem is, then,

$$\text{Min } E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left\{ \left[ (\pi_{t+\zeta} - \pi_1)^2 + \lambda_1 x_{t+\zeta} + \mu_1 (i_t + \zeta - i_{t+\zeta-1})^2 \right] I(q_t \leq \gamma) + \left[ (\pi_{t+\zeta} - \pi_2)^2 + \lambda_2 x_{t+\zeta} + \mu_2 (i_t + \zeta - i_{t+\zeta-1})^2 \right] I(q_t > \gamma) \right\}$$  (1.5)

subject to
$$x_{t+1} = \eta x_t - \alpha (i_t - E_t \pi_{t+1}) + \varepsilon_{ad}$$
$$\pi_{t+1} = \phi x_t + \theta \pi_t + \varepsilon^{gs}_t$$

After finding the first-order conditions for optimality and after some manipulations, it is possible to obtain an interest rate rule. The parameters of this monetary rule are convolutions of the coefficients associated with the restrictions under which the loss function has been intertemporally optimized; that is they are convolutions of the parameters associated with the preferences of the central bank ($\lambda_1, \lambda_2, \mu_1, \mu_2, \pi_1, \pi_2$) and the structure of the economy ($\eta, \phi, \theta, \alpha$).

\textsuperscript{8}We limit our analysis to a unique threshold, thus to two regimes, for convenience reason of the presentation.
4. Econometric Specifications

To assess the empirical support of asymmetric framework considered here, we rely upon two alternative econometric strategies which are described in turn.

4.1 An Asymmetric Taylor Rule for Canada

A simple way of capturing nonlinearities or asymmetries in policy behaviour is to estimate threshold regression models\(^9\) whereby the policy rule switches into a different regime whenever a certain variable breaches one or more thresholds. Structure change and threshold effects are two related issues that have motivated considerable empirical and theoretical research in time series econometrics [e.g. Tsay (1989, 1998), Enders and Granger (1998), Hansen (1999, 2000)]. Furthermore, threshold models have some popularity in current applied econometrics practice. The model splits the sample into classes based on the value of an observed variable, which exceeds or not some threshold. The attractiveness of this model stems from the fact that it treats the sample split value (threshold parameter) as unknown.

Since the seminal paper by Taylor (1993), the nominal interest rate set by central banks is often assumed to depend on the output gap and on inflation. However, output gap and inflation, following standard assumptions in the New-Keynesian literature, describe aggregate demand and aggregate supply, which both define the so-called structure of the economy. We then apply the instrumental variable\(^10\) estimation of a threshold model proposed by Caner and Hansen (2004) to avoid the endogeneity problem and to investigate the threshold effect. Following Koustas and Lamarche (2009), we model a possible asymmetric behaviour.

\(^9\)In the empirical literature, there exist two main approaches to estimate nonlinear models: (i)-Markov-Switching Model and (ii)-Threshold Effect Model. The regime switches are driven by an unobserved process whereas, the latter one is flexible and uses different functional forms. Moreover, it allows to have endogenous thresholds and to distinguish between a threshold and a threshold range.

\(^10\)We use the lagged values of the inflation rate and output gap as the instruments in this research.
on the part of the Bank of Canada, where we postulate two possible regimes for central bank behavior depending on whether $q_t$ is below (first regime) or above (second regime) the threshold value. When estimating monetary policy reaction functions, the observed autocorrelation in interest rates has to be accounted for. This is generally done by assuming that the central bank does not adjust the interest rate immediately to its desired level but is concerned about interest rate smoothing. If the central bank adjusts interest rates towards the desired interest rate in a gradual fashion, the dynamics of adjustment of the actual level of the interest rate to the target interest rate is given by: $i_t = (1 - \rho) i^*_t + \rho i_{t-1} + \zeta_t$.

The number of lagged interest rate terms generally is chosen on empirical grounds so that autocorrelation in the residuals is absent. According to this partial adjustment behaviour, the Central Bank within each period adjusts its instrument in order to eliminate only a fraction $(1 - \rho)$ of the gap between its current target level and some linear combination of its past values.\footnote{See details in Taylor and Davradakis 2006.}

We consider two observed threshold variables ($q_t$) related to actual economic development: the lag of the output gap ($q_t = x_{t-j}$), and the lag of the inflation deviation ($q_t = \pi_{t-1}$), where $j$ and $l$ are inflation, and output gap lags respectively. Our threshold regression model takes the form:\footnote{More detailed descriptions of the threshold regressions model of Caner and Hansen (2004) are presented in Appendix A.}

$$i_t = (\alpha_1 \pi_t + \alpha_2 x_t + \theta_1 i_{t-1}) I(q_t \leq \gamma) + (\beta_1 \pi_t + \beta_2 x_t + \theta_2 i_{t-1}) I(q_t > \gamma) + u_t \quad (1.6)$$

### 4.2 An Approach to Detect Central Bank Asymmetric Preferences

The monetary authority minimizes the expected loss from the inflation gap and the output gap facing the economy, which can be characterized by the aggregate demand and supply relations. Therefore, the problem of the central bank is to choose the current interest rate
and the sequence of future interest rates such as to minimize its loss function subject to the behaviour of the economy. Adopting the method of Optimal Control to solve this problem (see Chiang, 1992), we calculate the first-order conditions for the minimization of the loss function, which leads to the following Euler equation:

\[
0 = \left[ E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left[ (\pi_{t+\zeta} - \pi_1^*) \frac{\partial \pi_{t+\zeta}}{\partial t} \right] + \beta^\zeta \lambda_1 \left[ (x_{t+\zeta}) \frac{\partial x_{t+\zeta}}{\partial t} \right] \right] I(q_t \leq \gamma) \quad (1.7)
\]

\[
+ \left[ E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left[ (\pi_{t+\zeta} - \pi_2^*) \frac{\partial \pi_{t+\zeta}}{\partial t} \right] + \beta^\zeta \lambda_2 \left[ (x_{t+\zeta}) \frac{\partial x_{t+\zeta}}{\partial t} \right] \right] I(q_t > \gamma)
\]

Because of the persistence in the structural equations of the economy, the Euler equation has an infinite horizon, and thus cannot be used directly in empirical work. To estimate this equation it is necessary to truncate its lead polynomials at some reasonable temporal horizon. As Favero and Rovelli (2003), we use a 4 quarters lead horizon. Two reasons stand in favour of the lead truncation of the Euler equation. First, as Favero and Rovelli (2001) have argued, a natural cutting point for the future horizon of the Euler equation emerges anyway, even if we consider a theoretical infinite horizon loss function. In fact, the weight attached to expectations of future gaps and inflation decreases as the time-lead increases, meaning that expectations of the state of the economy carry less relevant information for the present conduct of policy as they relate to periods further away in the future. Second, expanding the horizon in the Euler equation would complicate it and bring collinearities to the system, causing great difficulties in making estimations. It is worth noting that our option is consistent with the standard practice in the estimation of forward-looking policy reaction functions. Boivin and Giannoni (2003) truncate the forecast horizon at 1 quarter for output and 2 quarters for inflation, while Muscatelli et al. (2002), and Orphanides (2001 b) truncate the inflation forecast horizon at 4 quarters. Rodriguez (2008) shows that estimated
backward-looking policy reaction functions for the US and Canada strongly indicate that actual policy decisions involve forecast horizons of inflation not beyond 4 quarters ahead.

Once the Euler equation is truncated at 4 quarters ahead, its partial derivatives components can be expressed as functions of the aggregate demand and aggregate supply parameters, thus building into the Euler equation the cross-equation restrictions. This ensures that the loss function is being properly minimized subject to the constraints given by the economy’s structure.

\[ 0 = \begin{cases} 
E_t \sum_{i=0}^{4} \beta^i \left[ (\pi_{t+i,\zeta} - \pi_{t,\zeta}^*) \frac{\partial \pi_{t+i,\zeta}}{\partial x_{t+i,\zeta}} \right] + E_t \sum_{i=0}^{4} \beta^i \lambda_1 \left[ (x_{t+i,\zeta}^*) \frac{\partial x_{t+i,\zeta}}{\partial x_{t+i,\zeta}} \right] & I(q_t \leq \gamma) \tag{1.8} \\
E_t \sum_{i=0}^{4} \beta^i \left[ (s_{t+i,\zeta}) \frac{\partial s_{t+i,\zeta}}{\partial x_{t+i,\zeta}} \right] + \mu_1 (i_t - i_{t-1}) - \mu_1 \beta E_t (i_{t+1} - i_t) & I(q_t > \gamma) 
\end{cases} \]

Expanding the partial derivatives, (1.8) turns into

\[ 0 = \begin{cases} 
\beta E_t (\pi_{t+2} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+2}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial \pi_{t+2}} \right] + \beta^2 E_t (\pi_{t+3} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial \pi_{t+2}} \right] + \beta^3 E_t (\pi_{t+4} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+3}}{\partial x_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \right] + \beta^4 E_t (\pi_{t+5} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+4}}{\partial x_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+4}} \frac{\partial x_{t+2}}{\partial \pi_{t+4}} \frac{\partial x_{t+2}}{\partial \pi_{t+4}} \right] & I(q_t \leq \gamma) \tag{1.9} \\
\beta E_t (\pi_{t+2} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+2}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial \pi_{t+2}} \right] + \beta^2 E_t (\pi_{t+3} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \right] + \beta^3 E_t (\pi_{t+4} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+3}}{\partial x_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+3}} \right] + \beta^4 E_t (\pi_{t+5} - \pi_{t}^*) \left[ \frac{\partial \pi_{t+4}}{\partial x_{t+3}} \frac{\partial x_{t+2}}{\partial \pi_{t+4}} \frac{\partial x_{t+2}}{\partial \pi_{t+4}} \frac{\partial x_{t+2}}{\partial \pi_{t+4}} \right] & I(q_t > \gamma) 
\end{cases} \]

Then, the IS curve equation, Phillips curve equation and Euler equation can be jointly estimated as a system, generating estimates of the structural parameters $c_1$ through $c_9$, as...
well as of the policymakers structural preferences parameters $\lambda_1, \lambda_2, \mu_1, \mu_2, \pi_1^s, \pi_2^s$.\(^{13}\)

\[
\begin{align*}
x_{t+1} &= c_1 + c_2 x_t + c_3 x_{t-1} + c_4 (i_{t-1} - \pi_{t-1}) + c_5 (i_{t-2} - \pi_{t-2}) + \varepsilon_{t+1}^{ad} \\
\pi_{t+1} &= c_6 \pi_t + c_7 \pi_{t-1} + c_8 x_t + c_9 \Delta w_t + \varepsilon_{t+1}^{ax}
\end{align*}
\]

where $\Delta w_t$ is the exchange rate fluctuations. We rearranged Equation (1.9) and substituted derivatives with coefficients from equation (2.10)

\[
0 = \begin{bmatrix}
[\mu_1 (i_t - i_{t-1}) - \mu_1 \beta E_t (i_{t+1} - i_t)] + \beta^3 E_t (\pi_{t+4} - \pi_1^s) [c_8 c_4] \\
+ \beta^4 E_t (\pi_{t+5} - \pi_1^s) [c_4 c_6 c_8 + c_8 (c_5 + c_2 c_4)] \\
+ \lambda_1 \beta^2 E_t (x_{t+3}) [c_4] + \lambda_1 \beta^2 E_t (x_{t+4}) [(c_5 + c_2 c_4)] \\
+ \lambda_1 \beta^4 E_t (x_{t+5}) [c_2 (c_5 + c_2 c_4) + c_3 c_4]
\end{bmatrix} \quad {\text{if}} \ (q_t \leq \gamma) \quad (1.11)
\]

\[
+ \begin{bmatrix}
[\mu_2 (i_t - i_{t-1}) - \mu_2 \beta E_t (i_{t+1} - i_t)] + \beta^3 E_t (\pi_{t+4} - \pi_2^s) [c_8 c_4] \\
+ \beta^4 E_t (\pi_{t+5} - \pi_2^s) [c_4 c_6 c_8 + c_8 (c_5 + c_2 c_4)] \\
+ \lambda_2 \beta^2 E_t (x_{t+3}) [c_4] + \lambda_2 \beta^2 E_t (x_{t+4}) [(c_5 + c_2 c_4)] \\
+ \lambda_2 \beta^4 E_t (x_{t+5}) [c_2 (c_5 + c_2 c_4) + c_3 c_4]
\end{bmatrix} \quad {\text{if}} \ (q_t > \gamma)
\]

Following Favero et Rovelli (2003), the parameters of the structural equations and the loss function are estimated jointly from a system formed by system (1.10) and the Euler equation (1.11). As we want to obtain the preferences implied by the coefficients from the threshold regression model, the dependent variable in the interest rate is the fitted interest rates from the threshold regression model including the lagged interest rates. Furthermore, to cover the different types of asymmetry in the policymaker’s preferences identified in the literature, estimation is carried out sequentially allowing each of the loss function weights $\lambda$
and $\mu$ to vary with the state of the corresponding target variable, and then concludes with a joint test. Statistical inference is based on individual significance tests and Wald tests.

5. Empirical Evidence

5.1 The Data Set

The structural parameters are estimated using quarterly Canadian data on inflation, the output gap, and the nominal interest rate obtained from Statistics Canada and the Bank of Canada. The cover the period 1961:1 to 2008:4, thus they include 48 years which means we have 192 data points. The previous literature employs both monthly and quarterly data frequencies to estimate monetary policy rules. We report results using only quarterly data and show that the main result of the paper is robust to whether one uses monthly or quarterly data estimates. Annual inflation is measured as $100 \times (p_t - p_{t-4})$, where $p_t$ denotes logarithms of the Consumer Price Index (CPI). The nominal interest rate is the annual percentage yield on 3-month Treasury bills.\textsuperscript{14} Several different methods (linear trend, quadratic trend methods, etc.) have been proposed to measure the output gap (see Rodriguez, 2008). Our aim is not to ascertain the way that real output evolves over the long-run. Instead, the goal is to obtain a reasonable measure of the pressure felt by the Bank of Canada to use monetary policy to affect the level of output. Output is measured by the gross domestic product. The natural output level is the Hodrick-Prescott (HP) trend of the current output. The output gap is then computed as the difference between the current output and its HP trend.

The literature on monetary rules has suggested an estimation by subsamples, where the break point is considered exogenous. In a recent paper, Rodríguez (2004) has estimated

\textsuperscript{14}There is a choice that needs to be made between the overnight rate and the 3-month T-bill rate. We chose the 3-month T-bill rate because it was the implicit target of Canadian monetary policy for a longer time period. Furthermore, the overnight rate series was constructed after the fact. In any case the two series move together.
interest rate rules for Canada and the US using endogenous break points selected by the approach suggested by Bai and Perron (1998, 2003). In general, his results show that the selected break dates are consistent with what previous research has used for the US. Since in our paper we have a system of three equations, while the Bai and Perron’s approach is adequate for single equations, the adequacy or possible modification of the approach to the system case is beyond the scope of this empirical paper. Unlike Rodriguez (2008), we decided to use one break date selected for Canada (1991:1). Note that an explicit inflation target has been announced by the Canadian government since 1991:1. The breakdown of the sample into two subperiods is meant to capture potential differences in the reaction function between the first period, in which there was no explicit target, and the second one which was characterized by an explicitly announced inflation target. For the whole sample period, as well as for both subperiods, the implicit inflation target is estimated along with the other parameters.

5.2 Preliminary Analysis

The estimation of the nonlinear reaction function is carried out using a two-step procedures. First, the Taylor empirical rule is estimated by the Caner and Hansen (2004) approach. Then, the estimated threshold values are used in our specification rule estimated by the Generalized Method of Moment (GMM).

Before proceeding with the estimation of the model it is important to consider some issues. First, the summary statistics of the different regimes are given in Table 1.1. Several interesting insights can be drawn from Table 1.1. On the one hand, periods of inflation and output gap deviations above and below target / potential seem to be balanced so we have enough data points for each case in order to get reliable estimates. On the other hand, the descriptive statistics show that the output gap is lower if inflation is above the inflation threshold value and if the output gap is below the output gap threshold value. Inflation
is lower if the output gap is above the output gap threshold value. So accordingly we find the lowest means for inflation where the output gap is above the threshold value and the highest means where both variables are above their threshold values. Furthermore, the lowest means for the output gap are where inflation is above and the output gap is below their threshold values. Therefore, in these situations we consistently find the highest and lowest realizations of the inflation rate and the output gap. However, also the deviations from the thresholds can be considerable. Finally, the interest rate is highest where both variables (inflation and output gap) are above their threshold values. This corresponds to the suggestions of the Taylor rule that the interest rate should be high in periods where inflation and output are above their target. Putting it the other way around, the interest rate is also found to be lowest in cases where both variables are below their threshold values, which is in line with the Taylor framework as well.

Second, it is necessary that the variables included in the estimated model are stationary, a necessary condition for the use of the approach of Caner and Hansen (2004). Unit root and stationarity tests for the variables considered in this study are presented in Table 1.2. We report the results of two different unit root tests (Augmented Dickey-Fuller (ADF) and Elliott-Rothenberg-Stock DF-GLS test statistic) and the results of the KPSS stationarity test to see whether the power is an issue. The power of unit root tests seems not to be an issue. The KPSS test is able to provide evidence of stationarity for all variables. We estimated the ADF, DF-GLS and KPSS tests using only an intercept\textsuperscript{15}. With the ADF and DF-GLS tests, the unit root null can be rejected even at the 10\% level for all variables. With the KPSS test, we cannot reject the null hypothesis of stationarity for all the variables even at 10\% level significance. These findings are consistent with the work of other researchers (see Castro, 2008).

\textsuperscript{15}We note that, with or without trend our test decision not changes.
Table 1.1: Descriptive Statistics.

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>$\pi_t \leq \gamma_\pi$</th>
<th>$\pi_t &gt; \gamma_\pi$</th>
<th>$x_t \leq \gamma_x$</th>
<th>$x_t &gt; \gamma_x$</th>
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</thead>
<tbody>
<tr>
<td>$\pi_{\text{ron}}$</td>
<td>7.046</td>
<td>0.182</td>
<td>1.650</td>
<td>0.998</td>
<td>1.476</td>
</tr>
<tr>
<td>$\sigma_{\text{ron}}$</td>
<td>3.608</td>
<td>0.455</td>
<td>0.637</td>
<td>0.743</td>
<td>0.798</td>
</tr>
<tr>
<td>$\pi_{\text{max}}$</td>
<td>21.017</td>
<td>2.949</td>
<td>2.074</td>
<td>2.949</td>
<td>2.101</td>
</tr>
<tr>
<td>$\pi_{\text{min}}$</td>
<td>0.945</td>
<td>0.692</td>
<td>-1.085</td>
<td>-0.762</td>
<td>-1.085</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>4.155</td>
<td>2.759</td>
<td>8.990</td>
<td>3.778</td>
<td>4.946</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>2.988</td>
<td>1.387</td>
<td>1.725</td>
<td>2.834</td>
<td>3.167</td>
</tr>
<tr>
<td>$\pi_{\text{max}}$</td>
<td>11.952</td>
<td>5.645</td>
<td>11.952</td>
<td>11.489</td>
<td>11.952</td>
</tr>
<tr>
<td>$\pi_{\text{min}}$</td>
<td>-0.0387</td>
<td>-0.039</td>
<td>5.709</td>
<td>0.000</td>
<td>-0.039</td>
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<tr>
<td>$\bar{x}$</td>
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<td>-0.051</td>
<td>-0.701</td>
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</tr>
<tr>
<td>$\sigma_{x}$</td>
<td>1.372</td>
<td>1.220</td>
<td>1.821</td>
<td>1.038</td>
<td>0.612</td>
</tr>
<tr>
<td>$x_{\text{max}}$</td>
<td>2.792</td>
<td>2.608</td>
<td>2.792</td>
<td>0.670</td>
<td>2.792</td>
</tr>
<tr>
<td>$x_{\text{min}}$</td>
<td>-5.429</td>
<td>-3.000</td>
<td>-5.429</td>
<td>-5.429</td>
<td>0.677</td>
</tr>
<tr>
<td>$\text{excha}$</td>
<td>-0.213</td>
<td>0.294</td>
<td>-1.967</td>
<td>-0.721</td>
<td>0.835</td>
</tr>
<tr>
<td>$\sigma_{\text{excha}}$</td>
<td>5.107</td>
<td>5.348</td>
<td>3.707</td>
<td>5.348</td>
<td>4.419</td>
</tr>
<tr>
<td>$\text{excha}_{\text{max}}$</td>
<td>17.888</td>
<td>17.888</td>
<td>5.305</td>
<td>17.887</td>
<td>13.909</td>
</tr>
<tr>
<td>$\text{excha}_{\text{min}}$</td>
<td>-20.744</td>
<td>-20.744</td>
<td>-10.456</td>
<td>-20.744</td>
<td>-7.515</td>
</tr>
<tr>
<td>$N$</td>
<td>192</td>
<td>149</td>
<td>43</td>
<td>130</td>
<td>62</td>
</tr>
</tbody>
</table>

Notest. $\bar{X}$ stands for the mean of the respective variable, $z_{\text{max}}$ and $z_{\text{min}}$ for the maximum and minimum realization, while $\sigma_{z}$ is the standard deviation, $N =$ number of observations; $\gamma_\pi =$5.709 and $\gamma_x =$0.691.

We use the full sample.

Before turning to the estimation of the nonlinear policy rule, there are a number of econometric issues we have to deal with. Our favorite specification has similarities with the Threshold Regression Approach specification in that the endogenous variable is determined over a number of different regimes. In particular, two regimes can be associated with small and large values of the threshold variable relative to the target, and two other regimes can be associated with the movements of the output gap around the threshold value of zero.\footnote{In the empirical literature of the threshold model, there are two families of models: (i) models with exogenous threshold effects; and (ii) models with endogenous threshold effects. Indeed, models with exogenous threshold effects, as their name suggests, are independent of any structure or state of the economy. However, endogenous threshold model takes into account the structure and condition of the economic system as specified in the econometric model.}
Table 1.2: Unit Root and Stationarity tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>ADF</th>
<th>DF-GLS</th>
<th>KPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>-2.618†</td>
<td>-1.665↑↑</td>
<td>0.212*</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>-4.868↑↑↑</td>
<td>-1.688†</td>
<td>0.179*</td>
</tr>
<tr>
<td>Output gap</td>
<td>-4.971↑↑↑</td>
<td>-3.695↑↑↑</td>
<td>0.041*</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>-3.012↑↑</td>
<td>-2.483↑↑</td>
<td>0.189*</td>
</tr>
</tbody>
</table>

1% critical value -3.465 -2.577 0.739
5% critical value -2.877 -1.943 0.463
10% critical value -2.575 -1.616 0.347

ADF= Augmented Dickey-Fuller (1979, 1981) unit root test, Elliott-Rothenberg-Stock (1996) DF-GLS test statistic (DF-GLS is more powerful and more recent); KPSS= Kwiatkowski, Phillips, Schmidt and Shin (1992) stationarity test. The Bandwidth selection procedure is used in the KPSS tests and, in this case, the autocovariances are weighted by the Bartlett Kernel. †††, ††, †, ‡‡‡, ‡‡‡‡, ‡‡‡‡‡, ‡‡‡‡‡‡, ‡‡‡‡‡‡‡, ‡‡‡‡‡‡‡‡, ‡‡‡‡‡‡‡‡‡, ‡‡‡‡‡‡‡‡‡‡, ‡‡‡‡‡‡‡‡‡‡‡ unit root is rejected at a significance level of 1%, 5%, 10% → stationarity. * stationarity is not rejected at a significance level of 10%. Besides, optimal lag length in these tests were selected using Modified Akaike Information Criterion (MAIC) with maximum lag order of 6.

Accordingly, a more vigorous policy response distinguishes the regime operating during high inflation (output contraction) periods from the one in place during low inflation (output expansion) times. Our task consists in estimating the Threshold Regression Approach reaction function in order to evaluate whether the parameters governing the asymmetries in the policy objective are significantly different from zero.

5.3 Empirical Nonlinear Taylor Reaction Functions for Canada

The results of the test of linearity show that the empirical reaction function can be better modelled with a nonlinear model than with a linear model using the same variables. The estimations show an asymmetric behaviour of the Bank of Canada depending on the actual
state of the economy. For each threshold variable considered, $SupW$ tests reject the linear model at least at the 1% significance level (see Table 1.3 and Table 1.4), thereby pointing to asymmetric policy preferences. The estimation of threshold values reveals that the Bank of Canada is more concerned about both inflation deviations and the output gap when the inflation gap is greater than 5.71%. This is true for the two sub-samples. All regressions display the commonly found gradual adjustment of the interest rate to the desired level with a non-adjustment coefficient that varies between 0.79 and 0.98.

Panel A shows the estimates obtained when the inflation deviation is the relevant threshold variable (see Table 1.3). Estimations reveal a higher Bank of Canada’s response to both inflation deviation and output gap when the actual inflation is above the inflation target. Note that when the actual inflation deviation is positive, the interest rate smoothing parameter is lower than one estimated under the first regime (0.85 versus 0.94 in the full sample). This shows that although the long-run reaction to output gap and inflation deviations are similar, the Bank of Canada reacts by quickly adjusting the interest rate when the actual inflation rate evolves above its target, doing so more slowly in the contrary case. This observation is true for the two subperiods. Finally, the estimation of the threshold values reveals that the Bank of Canada is more concerned about both output gap and inflation deviations when inflation is above the threshold value in the three cases.
Table 1.3: Estimation results for the empirical monetary reaction function using Inflation as the threshold variable.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Estimates</td>
<td>5.709</td>
<td>9.040</td>
<td>1.361</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>[1.037 10.078]</td>
<td>[1.824 10.439]</td>
<td>[0.843 2.271]</td>
</tr>
<tr>
<td><strong>Regime 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.105(^c)(0.054)</td>
<td>0.101(^a)(0.031)</td>
<td>0.074(0.101)</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.118(^c)(0.060)</td>
<td>0.172(^a)(0.051)</td>
<td>0.221(0.143)</td>
</tr>
<tr>
<td>Lagged Interest rate</td>
<td>0.939(^a)(0.029)</td>
<td>0.944(^a)(0.023)</td>
<td>0.949(^a)(0.039)</td>
</tr>
<tr>
<td><strong>Regime 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.204(^a)(0.065)</td>
<td>0.231(^b)(0.086)</td>
<td>0.189(^a)(0.063)</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.330(^a)(0.107)</td>
<td>0.407(^b)(0.143)</td>
<td>0.255(^a)(0.059)</td>
</tr>
<tr>
<td>Lagged Interest rate</td>
<td>0.845(^a)(0.067)</td>
<td>0.793(^a)(0.084)</td>
<td>0.837(^a)(0.035)</td>
</tr>
<tr>
<td>(H_0: \alpha = \beta)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SupW (Statistic)</td>
<td>38.240</td>
<td>298.839</td>
<td>44.419</td>
</tr>
<tr>
<td>SupW (P-value)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>No. obs.</td>
<td>146; 42</td>
<td>96; 20</td>
<td>16; 51</td>
</tr>
</tbody>
</table>

\(^{a,b,c}\) denotes significance levels at 1%, 5.0% and 10%, respectively. Standard errors robust to serial correlation (up to 6 lags) in parentheses. Regime 1 occurs when the threshold variable is below the threshold value and Regime 2 occurs when the threshold variable is above this value.

Panel B presents the estimates obtained when the output gap is considered as the threshold variable (see Table 1.4). The estimations reveal that the Bank of Canada responds more aggressively to inflation deviations when the output gap is above the threshold value in the case of the two subperiods but responds more weakly to the output gap in the second subperiod (inflation target period). However, in the full sample case, the Bank of Canada responds more strongly to the output gap when it is above the threshold value, whereas the opposite situation occurs in the case of inflation deviations. This means that the Bank of Canada worries more about the output gap in recession period and cares more about inflation deviations during expansions. When the output gap is considered as the threshold variable, the asymmetric behaviour of the Bank of Canada is also reflected by differences
in the interest rate smoothing parameter.

Table 1.4: Estimation results for the empirical monetary reaction function using the output gap as the threshold variable.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Estimates</td>
<td>0.691</td>
<td>0.073</td>
<td>-0.194</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>[-1.982, 2.024]</td>
<td>[-1.982, 2.160]</td>
<td>[-1.749, 0.343]</td>
</tr>
</tbody>
</table>

**Regime 1**
- Inflation rate: 0.134$^a$(0.037), 0.091$^c$(0.052), 0.215$^b$(0.101)
- Output gap: 0.053(0.073), 0.256(0.120), 0.265(0.185)
- Lagged Interest rate: 0.907$^a$(0.029), 0.978$^a$(0.048), 0.936$^a$(0.051)

**Regime 2**
- Inflation rate: 0.112$^b$(0.054), 0.121$^b$(0.042), 0.223$^c$(0.130)
- Output gap: 0.439$^b$(0.217), 0.492$^a$(0.153), 0.221$^a$(0.069)
- Lagged Interest rate: 0.890$^a$(0.048), 0.889$^a$(0.037), 0.805$^a$(0.069)

$^a,b,c$ denotes significance levels at 1%, 5.0% and 10%, respectively. Standard errors robust to serial correlation (up to 6 lags) in parentheses. Regime 1 occurs when the threshold variable is below the threshold value and Regime 2 occurs when the threshold variable is above this value.
5.4 Policy Asymmetric Preferences of the Corresponding Central Banks

Table 1.5 and Table 1.6 show the estimates of the asymmetric preference parameters\textsuperscript{17}.

Table 1.5 presents the estimates obtained when inflation is considered as the threshold variable. In a full sample, our results reveal that the Bank of Canada’s behaviour changes dramatically. Indeed, $\lambda_1$ and $\mu_1$ are still positive and significant implying that the Bank of Canada’s reaction function is convex in output gap and interest rate smoothing. However, $\lambda_2$ and $\mu_2$ are still negative and significant implying that the Bank of Canada has a concave preferences in output gap and interest rate smoothing. These findings reveal that when inflation is below the threshold value monetary policy was dominated by recession avoidance preferences, i.e., Canadian monetary policy is characterized by a convex rule. Whereas, if inflation is above the threshold, Canadian monetary policy is characterized by a concave rule supporting inflation avoidance preferences (see Cukierman and Muscatelli, 2008).

\textsuperscript{17}All parameter estimates are in Appendix 1.2 and 1.3
Table 1.5: Estimates of the Central Bank’s Preference Parameters: Using Inflation as Threshold Variable.

<table>
<thead>
<tr>
<th>Period</th>
<th>REGIME 1</th>
<th>REGIME 2</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated Coefficients</td>
<td>Estimated Coefficients</td>
<td>Statistics</td>
</tr>
<tr>
<td></td>
<td>( \lambda_1 )</td>
<td>( \mu_1 )</td>
<td>( \pi_1^* )</td>
</tr>
<tr>
<td>1961:1 to 2008:4</td>
<td>0.005 (^a)</td>
<td>0.002 (^a)</td>
<td>2.115 (^a)</td>
</tr>
<tr>
<td>1961:1 to 1990:4</td>
<td>-0.027 (^a)</td>
<td>-0.004 (^a)</td>
<td>3.939 (^a)</td>
</tr>
<tr>
<td>1991:2 to 2008:4</td>
<td>-0.001 (^b)</td>
<td>0.008 (^a)</td>
<td>1.899 (^a)</td>
</tr>
</tbody>
</table>

\(^a,b,c\) denotes significance levels at 1%, 5.0% and 10%, respectively. Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation); J-statistics is Hansen’s test of the model’s overidentifying restrictions, which is distributed as a \( \chi^2 \) variate under the null hypothesis of valid overidentifying restrictions (\( n \) stands for the number of instruments minus the number of freely estimated parameters). Regime 1 is whether the threshold variable is below the threshold value and Regime 2 is where the threshold variable is above this value.

When the total sample period is broken down into two subperiods, the first without an explicit inflation target and the second with it, the type of nonlinearity changes dramatically.

In an intratemporal analysis, in the first period, when inflation is below the threshold value, \( \lambda_1 \) and \( \mu_1 \) are still negative and significant, implying that the reaction function is concave in output gap and interest rate smoothing. This finding is consistent with the existence of a dominant recession avoidance preferences (RAP). However, if inflation is above the threshold value, only \( \mu_2 \) is still positive and significant implying that the reaction function is convex in interest rate smoothing, supporting the existence of dominant inflation avoidance preferences (IAP). Moreover, the estimate of the coefficient \( \pi^* \) reflects clearly that the Canadian monetary authority responds strongly to inflation deviations, i.e., when the actual inflation deviation is positive, the parameter \( \pi^* \) is lower than one estimated under
the first regime (1.77% versus 3.99%). In the second subperiod, a broadly similar picture emerges with respect to $\lambda_1$ which is negative and significant when inflation is below the threshold value and $\lambda_2$ is positive and significant when inflation is above the threshold. The reverse situation is observed in the case of $\mu$.

In an intertemporal analysis, our findings reveal that the central bank behaviour is characterized by a concave function in the output gap supporting the existence of dominant recession avoidance preferences, while its behaviour changes depending on the actual state of the economy. Furthermore, the value of $\pi^*$ in the last subperiod is close to the middle point of the target inflation rate followed by the Bank of Canada. This suggests the fact that monetary policy has been conducted efficiently in the last subperiod (see Rodriguez, 2008).

In short, the Canadian findings suggest that the introduction of inflation targeting in Canada was accompanied by a fundamental change in the objectives of monetary policy and the behaviour of monetary policymakers, not only with respect to the average target, but also in terms of precautions taken to keep inflation in check in the face of uncertainty about the economy. Thus, Canadian monetary authorities react more aggressively to positive than negative inflation deviations. This type of asymmetry is tied to inflation avoidance preferences.

Table 1.6 shows the estimates obtained when the output gap is considered as the threshold variable. As in Table 1.5, similar observations are obtained from these estimates when the full sample period is considered. However, we note some changes in the case of the intratemporal analysis. In fact, in the first subperiod, when the output gap is below the threshold value, $\lambda_1$ and $\mu_1$ are still positive and significant, implying that the reaction function is convex. This finding is consistent with the existence of dominant inflation avoidance preferences. If the output gap is above the threshold value, $\lambda_2$ and $\mu_2$ are still negative
and significant, implying that the reaction function is concave, supporting the existence of dominant recession avoidance preferences. In the second subperiod, similar observations are found from \( \lambda \) estimates.

In an intertemporal analysis, the efficiency of monetary policy is also confirmed in the last subperiod \((\pi_2^* = 2.076)\). In brief, the Bank of Canada reacts more vigorously to negative than to positive output gaps. This type of asymmetry is called recession avoidance preferences.

Table 1.6: Estimates of the Central Bank’s Preference Parameters: Using Output gap as Threshold variable.

<table>
<thead>
<tr>
<th>Period</th>
<th>REGIME 1</th>
<th>REGIME 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated Coefficients</td>
<td>Statistics</td>
</tr>
<tr>
<td></td>
<td>( \lambda_1 )  ( \mu_1 )  ( \pi_1^* )</td>
<td>( \lambda_2 )  ( \mu_2 )  ( \pi_2^* )</td>
</tr>
</tbody>
</table>
| 1961:1 to 2008:4 | 0.112 \(^a\)  0.003 \(^a\)  3.290 \(^a\) | \( J = 21.105 \) | -0.826 \(^a\) -0.004 \(^a\)  3.492 \(^a\) | \( J = 20.674 \)
| 1961:1 to 1990:4 | 0.043 \(^a\)  0.003 \(^a\)  5.146 \(^a\) | \( J = 13.829 \) | -0.733 \(^a\) -0.008 \(^a\)  5.969 \(^a\) | \( J = 14.051 \)
| 1991:2 to 2008:4 | 0.008 \(^a\)  0.001 \(^b\)  2.021 \(^a\) | \( J = 9.437 \) | -0.198 \(^a\)  0.002 \(^a\)  2.076 \(^a\) | \( J = 9.550 \)

\( ^a,b,c \) denotes significance levels at 1%, 5.0% and 10%, respectively. Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation); \( J \)-statistics is Hansen’s test of the model’s overidentifying restrictions, which is distributed as a \( \chi^2_{(n+1)} \) variate under the null hypothesis of valid overidentifying restrictions (\( n \) stands for the number of instruments minus the number of freely estimated parameters). Regime 1 is whether the threshold variable is below the threshold value and Regime 2 is where the threshold variable is above this value.
6. Conclusion

The Bank of Canada has adopted explicit inflation targeting since 1991. Therefore, the change in the operating procedure provides an interesting environment for testing whether the preferences of the Bank of Canada are consistent with the quadratic preference assumption that is perceived to be standard within the monetary policy literature. Using a threshold approach function instead of the standard quadratic function, this paper investigates the asymmetric preferences of the Bank of Canada with regards to the inflation gap and the output gap under the New-Keynesian economic framework. The nonlinear monetary policy reaction function is derived from the optimization behaviour of the central bank.

We estimate the reaction function and the asymmetric preference parameters on the inflation gap and the output gap for the sample period 1961:1 to 2008:4. The empirical results show that the asymmetry parameter of the inflation gap is statistically significant. Thus, our result indicates that the Bank of Canada has placed more weight on positive deviations of inflation from the target than on negative deviations. In others words, our results reveal that in the presence of uncertainty about future shocks Canadian policymakers react more vigorously to positive than to negative inflation gaps. This type of asymmetry is called inflation avoidance preferences. Furthermore, the empirical evidence suggests, without any doubt, the fact that the monetary policy has been successful since the Canadian authorities have adopted inflation targeting. We find strong statistical support for this decline and the result is consistent with previous findings by Favero and Rovelli (2003) and Rodriguez (2008). In addition, we interpret the parameters associated with the preferences of the central bank, something that Rodriguez (2008) did not achieve. The main contribution of this paper is to successfully analyze the presence of nonlinearities and asymmetries in the Canadian reaction function.
A natural extension of this paper in future research would consist in estimating the policy parameters directly, using Caner and Hansen’s (2004) threshold approach. Even though this is technically challenging, it would enable us to estimate the system over a longer sample period and to investigate the implications of those switches for the macro dynamics. Other future avenues for research include the possibility to use both variables (inflation and the output gap) as determinants of threshold variables. Finally, another direction for future research will be to introduce stock market prices in the central bank preferences. Indeed, in times of financial turmoil, monetary policy responses are of special interest for the academic community. This was especially the case after the Asian crisis of 1997/98, when the Bank of Japan (BoJ) cut interest rates down to zero. In the US, first the bursting dot-com bubble in 2000 and now the subprime market crisis justified sharp interest rate cuts to stabilize markets, which have brought world interest rates close to zero. In this regard, an increasing number of authors argue that in the past interest rates were not increased symmetrically when markets stabilized and were booming. Accordingly, interest rates remained too low for too long. This provided low-cost liquidity to flourishing markets (Belke et al. 2008, Taylor 2008, Schnabl and Hoffmann 2008, Weber 2008). Therefore, the aim of the next paper is to test for asymmetric monetary policy with respect to asset markets.

The outline of this appendix follows closely Caner and Hansen (2004).

A.1. Model

In order to avoid the endogeneity problem, an instrumental variable estimation threshold model introduced by Caner and Hansen (2004) is used.

The observed sample is \( \{y_i, z_i, x_i\}_{i=1}^n \), where \( y_i \) is real valued, \( z_i \) is a \( m \)-vector, and \( x_i \) is a \( k \)-vector, with \( k \geq m \). The threshold variable, \( q_i = q(x_i) \), is an element or a function of the vector, and must have a continuous distribution. The data are either a random sample or a weakly dependent time series, so that unit roots and stochastic trends are excluded.

The structural equation of interest is

\[
\begin{align*}
y_i &= \beta_1' z_i + \epsilon_i & q_i \leq \gamma \\
y_i &= \beta_2' z_i + \epsilon_i & q_i > \gamma
\end{align*}
\] (A.1)

which may also be written in the form

\[
y_i = \alpha' z_i I(q_i \leq \gamma) + \beta' z_i I(q_i > \gamma) + \rho w_i + \epsilon_i
\] (A.2)

where, for each \( i \), \( y_i \) is the dependent variable, \( z_i \) is an \( m \)-vector of explanatory variables, \( w_i \) is explanatory variable that do not depends on the threshold value, \( q_i \) is the threshold variable assumed to be strictly exogenous, \( \gamma \) is the threshold parameter, \( \alpha \) and \( \beta \) are \( m \)-vectors of slope parameters that may differ depending on the value of \( q_i \), and \( \epsilon_i \) is a random disturbance term. The vector of explanatory variables is partitioned into a \( m_1 \) dimensional subset, \( z_{1i} \), of exogenous variables uncorrelated with \( \epsilon_i \), and a \( m_2 \) dimensional subset of endogenous variables, \( z_{2i} \), correlated with \( \epsilon_i \). \( I(q_i \leq \gamma) \) is an indicator variable that is one
if $q_t \leq \gamma$ and zero otherwise. In addition, to structure equation (A.2) the model requires a suitable set of $k \geq m$ instrumental variables, $x_i$, that includes $z_{1i}$.

A.2. Estimation of parameters

Following Caner and Hansen (2004), we estimate the parameter sequentially.

First, we use OLS to estimate the reduced form parameter $\pi$,

$$z_i = \pi' x_i + u_i$$ (A.3)

and obtained the predicted value $\hat{z}_i = \hat{\pi}' x_i$.

Second, we turn to estimation of the threshold $\gamma$. For each $\gamma$, let $Y$, $Z_1$ and $Z_2$ denote the matrices of stacked vectors $y_i$, $z_1^I (q_i \leq \gamma)$ and $z_2^I (q_i > \gamma)$ respectively. By regressing $Y$ on $Z_1$ and $Z_2$, we can obtain the LS sum of squared residuals $S_n (\gamma)$. The 2SLS estimator for $\gamma$ is the minimizer of the sum of squared residuals,

$$\hat{\gamma} = \arg \min_{\gamma \in \mathbb{R}} S_n (\gamma)$$ (A.4)

Third, we estimate the slope parameters $\alpha$ and $\beta$ by 2SLS or GMM on the split sample implied by $\hat{\gamma}$. Let $\hat{X}_1, \hat{X}_2, \hat{Z}_1$ and $\hat{Z}_2$ denote the matrices of stacked vectors $x_1^I (q_i \leq \gamma)$, $x_1^I (q_i > \gamma)$, $z_1^I (q_i \leq \gamma)$ and $z_1^I (q_i > \gamma)$ respectively. The 2SLS estimators for $\alpha$ and $\beta$ are:

$$\tilde{\alpha} = \left[ \hat{Z}_1 \hat{X}_1 \left( \hat{X}_1 \hat{X}_1 \right)^{-1} \right]^{-1} \left[ \hat{Z}_1 \hat{X}_1 \left( \hat{X}_1 \hat{X}_1 \right)^{-1} \hat{X}_1 Y \right]$$

$$\tilde{\beta} = \left[ \hat{Z}_2 \hat{X}_2 \left( \hat{X}_2 \hat{X}_2 \right)^{-1} \right]^{-1} \left[ \hat{Z}_2 \hat{X}_2 \left( \hat{X}_2 \hat{X}_2 \right)^{-1} \hat{X}_2 Y \right]$$

with the residual from of this regression being $\tilde{e}_i = y_i - z_i^\dagger \alpha I (q_i \leq \hat{\gamma}) - z_i^\dagger \beta I (q_i > \hat{\gamma})$. We use these residuals to construct the weight matrices,

$$\tilde{\Omega}_1 = \sum_{i=1}^{n} x_i x_i^\dagger \tilde{e}_i^2 I (q_i \leq \hat{\gamma}) \quad \text{and} \quad \tilde{\Omega}_2 = \sum_{i=1}^{n} x_i x_i^\dagger \tilde{e}_i^2 I (q_i > \hat{\gamma})$$
Then the GMM estimators for $\alpha$ and $\beta$ are

$$\tilde{\alpha} = \left( \tilde{Z}_1 \tilde{X}_1 \tilde{\Omega}_1^{-1} \tilde{X}_1 \tilde{Z}_1 \right)^{-1} \left( \tilde{Z}_1 \tilde{X}_1 \tilde{\Omega}_1^{-1} \tilde{X}_1 Y \right) \quad (A.5)$$

$$\tilde{\beta} = \left( \tilde{Z}_2 \tilde{X}_2 \tilde{\Omega}_2^{-1} \tilde{X}_2 \tilde{Z}_2 \right)^{-1} \left( \tilde{Z}_2 \tilde{X}_2 \tilde{\Omega}_2^{-1} \tilde{X}_2 Y \right)$$

and the corresponding (estimated) variance-covariance matrices for the GMM estimators are

$$\tilde{V}_1 = \left( \tilde{Z}_1 \tilde{X}_1 \tilde{\Omega}_1^{-1} \tilde{X}_1 \tilde{Z}_1 \right)^{-1} \quad \text{and} \quad \tilde{V}_2 = \left( \tilde{Z}_2 \tilde{X}_2 \tilde{\Omega}_2^{-1} \tilde{X}_2 \tilde{Z}_2 \right)^{-1} \quad (A.6)$$

### A.3. Testing for the threshold effect

It is important to test model linearity ($H_0 : \alpha = \beta$ versus $H_1 : \alpha \neq \beta$). To do this, Hansen and Caner (2004) propose using the $SupW$ statistic. Since under the null hypothesis the threshold value is unknown, this statistic has a non conventional distribution that can be replicated by bootstrap. The $SupW$ statistic is obtained by finding the threshold value ($\gamma$) that maximizes the Wald statistic for $H_0$:

$$W_n (\gamma) = \left[ \tilde{\alpha} (\gamma) - \tilde{\beta} (\gamma) \right] \left[ \tilde{V}_1 (\gamma) - \tilde{V}_2 (\gamma) \right]^{-1} \left[ \tilde{\alpha} (\gamma) - \tilde{\beta} (\gamma) \right]$$

(1.12)

where $\tilde{V}_1 (\gamma)$ and $\tilde{V}_2 (\gamma)$ are the variance-covariance matrices of $\tilde{\alpha} (\gamma)$ and $\tilde{\beta} (\gamma)$ respectively. The $SupW$ statistic is contrasted with the resulting ones from artificial data ($SupW^*$) to obtain the p-value of the test:

$$\frac{1}{B} I_{pv} (SupW^* > SupW) \quad (1.13)$$

where $B$ is the number of bootstrap replications and $I_{pv}$ is an indicator function that takes a value of one if the condition prevails and zero otherwise. The asymptotic distribution of this test statistic is nonstandard due to the presence of nuisance parameter $\gamma$ (not identified under the null hypothesis). Thus, we follow the suggestion of Hansen (1996) and

18 where $\alpha = [\alpha_1, \alpha_2, \theta_1]$, $\beta = [\beta_1, \beta_2, \theta_2]$. The null hypothesis corresponds to the joint test of equality of parameters between both regimes.
Caner and Hansen (2004) to obtain the p-value by simulation (bootstrapping). First, collect the estimated residual under the unrestricted model for each $\gamma$, that is, $\hat{e}_i(\gamma)$, and define $y_i^* = \hat{e}_i(\gamma) \eta_i$, where $\eta_i$ is a random draw from i.i.d. $N(0, 1)$. Second, using this pseudo-dependent variable in place of $y_i$ to repeat the calculation above, the resulting $SupW^*$ statistic has the same asymptotic distribution as $SupW$. Thus, by repeated simulation draws, the asymptotic p-value of the test statistic $SupW$ can be calculated with arbitrary accuracy.

The results of this test show that, the $SupW$ tests reject the linear model at the 1% significance level (see Table 1.3 and Table 1.4), thereby pointing to asymmetric policy preferences.
Appendix B.1: Descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>Interest rate</th>
<th>Inflation rate</th>
<th>Output gap</th>
<th>Exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.0457</td>
<td>4.1552</td>
<td>0.0000</td>
<td>-0.2128</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>3.6075</td>
<td>2.9879</td>
<td>1.3724</td>
<td>5.1067</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.9450</td>
<td>-0.0387</td>
<td>-5.4295</td>
<td>-20.7438</td>
</tr>
<tr>
<td>Maximum</td>
<td>21.0167</td>
<td>11.9524</td>
<td>2.7919</td>
<td>17.8876</td>
</tr>
<tr>
<td>Jacques Bera</td>
<td>36.4621</td>
<td>32.1990</td>
<td>15.6266</td>
<td>28.6247</td>
</tr>
<tr>
<td>Prob.</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0004</td>
<td>0.0000</td>
</tr>
<tr>
<td>Obs.</td>
<td>192</td>
<td>192</td>
<td>192</td>
<td>192</td>
</tr>
</tbody>
</table>

Appendix B.2: Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>Interest rate</th>
<th>Inflation rate</th>
<th>Output gap</th>
<th>Exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrest rate</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation rate</td>
<td>-0.9088</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output gap</td>
<td>-0.2687</td>
<td>0.0907</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Exchange rate</td>
<td>0.3241</td>
<td>-0.0955</td>
<td>0.1387</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Appendix B.3: Estimates of the preferences of monetary policy using inflation as threshold variable.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regime 1</td>
<td>Regime 2</td>
<td>Regime 1</td>
</tr>
<tr>
<td>v1</td>
<td>0.005(0.016)</td>
<td>0.002(0.012)</td>
<td>0.001(0.020)</td>
</tr>
<tr>
<td>v2</td>
<td>1.085a(0.022)</td>
<td>1.067a(0.020)</td>
<td>0.997a(0.026)</td>
</tr>
<tr>
<td>v3</td>
<td>-0.275a(0.023)</td>
<td>-0.277a(0.017)</td>
<td>-0.230a(0.022)</td>
</tr>
<tr>
<td>v4</td>
<td>0.185a(0.015)</td>
<td>-0.030a(0.002)</td>
<td>0.004a(0.001)</td>
</tr>
<tr>
<td>v5</td>
<td>-0.094a(0.016)</td>
<td>0.046a(0.003)</td>
<td>-0.018a(0.002)</td>
</tr>
<tr>
<td>v6</td>
<td>0.888a(0.009)</td>
<td>1.081a(0.009)</td>
<td>0.585a(0.012)</td>
</tr>
<tr>
<td>v7</td>
<td>0.058a(0.008)</td>
<td>-0.11a(0.009)</td>
<td>0.102a(0.008)</td>
</tr>
<tr>
<td>v8</td>
<td>0.016a(0.002)</td>
<td>0.175a(0.009)</td>
<td>0.251a(0.020)</td>
</tr>
<tr>
<td>v9</td>
<td>0.007a(0.002)</td>
<td>-0.013a(0.002)</td>
<td>-0.009b(0.004)</td>
</tr>
<tr>
<td>β</td>
<td>0.975e</td>
<td>0.975e</td>
<td>0.975e</td>
</tr>
<tr>
<td>π*</td>
<td>2.115a(0.075)</td>
<td>2.120a(0.163)</td>
<td>3.990a(0.062)</td>
</tr>
<tr>
<td>λ</td>
<td>0.005a(0.002)</td>
<td>-0.575a(0.065)</td>
<td>-0.027a(0.007)</td>
</tr>
<tr>
<td>μ</td>
<td>0.002a(0.000)</td>
<td>-0.006b(0.003)</td>
<td>-0.004a(0.000)</td>
</tr>
<tr>
<td>σ(εd)</td>
<td>0.720</td>
<td>0.721</td>
<td>0.852</td>
</tr>
<tr>
<td>σ(εh)</td>
<td>0.952</td>
<td>0.937</td>
<td>1.632</td>
</tr>
<tr>
<td>σ(εm)</td>
<td>0.005</td>
<td>0.033</td>
<td>0.005</td>
</tr>
</tbody>
</table>

*a,b,c* denotes significance levels at 1%, 5.0% and 10%, respectively. *e* indicates that the coefficient has been imposed in the estimation. Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation); J-statistics is Hansen’s test of the model’s overidentifying restrictions, which is distributed as a $\chi^2_{(n+1)}$ variate under the null hypothesis of valid overidentifying restrictions ($n$ stands for the number of instruments minus the number of freely estimated parameters). Regime 1 is whether the threshold variable is below the threshold value and Regime 2 is where the threshold variable is above this value.
Appendix B.4: Estimates of the preferences of monetary policy using output gap as threshold variable.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regime1</td>
<td>Regime2</td>
<td>Regime1</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-0.074$^a$ (0.012)</td>
<td>0.147$^a$ (0.009)</td>
<td>-0.000 (0.019)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.818$^a$ (0.026)</td>
<td>0.936$^a$ (0.019)</td>
<td>0.816$^a$ (0.020)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>-0.094$^a$ (0.027)</td>
<td>-0.242$^a$ (0.020)</td>
<td>-0.099$^a$ (0.019)</td>
</tr>
<tr>
<td>$c_4$</td>
<td>-0.006$^a$ (0.002)</td>
<td>-0.014$^a$ (0.002)</td>
<td>-0.011$^a$ (0.001)</td>
</tr>
<tr>
<td>$c_5$</td>
<td>0.030$^a$ (0.005)</td>
<td>0.014$^a$ (0.002)</td>
<td>0.043$^a$ (0.005)</td>
</tr>
<tr>
<td>$ c_6 $</td>
<td>1.151$^a$ (0.021)</td>
<td>1.193$^a$ (0.020)</td>
<td>1.188$^a$ (0.030)</td>
</tr>
<tr>
<td>$c_7$</td>
<td>0.145$^a$ (0.022)</td>
<td>-0.216$^a$ (0.020)</td>
<td>-0.173$^a$ (0.032)</td>
</tr>
<tr>
<td>$c_8$</td>
<td>0.192$^a$ (0.014)</td>
<td>0.146$^a$ (0.013)</td>
<td>0.208$^a$ (0.016)</td>
</tr>
<tr>
<td>$c_9$</td>
<td>0.014$^a$ (0.003)</td>
<td>0.008$^a$ (0.003)</td>
<td>0.024$^a$ (0.004)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.975$^e$</td>
<td>0.975$^e$</td>
<td>0.975$^e$</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>3.290$^a$ (0.146)</td>
<td>3.492$^a$ (0.129)</td>
<td>5.146$^a$ (0.118)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.112$^a$ (0.024)</td>
<td>-0.826$^a$ (0.138)</td>
<td>0.043$^a$ (0.007)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.003$^a$ (0.001)</td>
<td>-0.004$^a$ (0.001)</td>
<td>0.003$^a$ (0.001)</td>
</tr>
<tr>
<td>$\sigma (\varepsilon^d)$</td>
<td>0.598</td>
<td>0.496</td>
<td>0.602</td>
</tr>
<tr>
<td>$\sigma (\varepsilon^s)$</td>
<td>0.650</td>
<td>0.661</td>
<td>0.599</td>
</tr>
<tr>
<td>$\sigma (\varepsilon^m)$</td>
<td>0.007</td>
<td>0.013</td>
<td>0.006</td>
</tr>
</tbody>
</table>

$^a,^b,^c$ denotes significance levels at 1%, 5.0% and 10%, respectively. $^e$ indicates that the coefficient has been imposed in the estimation. Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation); J-statistics is Hansen’s test of the model’s overidentifying restrictions, which is distributed as a $\chi^2_{(n+1)}$ variate under the null hypothesis of valid overidentifying restrictions ($n$ stands for the number of instruments minus the number of freely estimated parameters). Regime 1 is whether the threshold variable is below the threshold value and Regime 2 is where the threshold variable is above this value.
Essay 2

Efficiency of Monetary Policy and Asymmetric Preferences of Central Bank with Respect to Asset Markets
1. Introduction

The study of Central Bank behaviour has attracted considerable interest in recent years. Attention has focused on two rather different issues. One has to do with whether Taylor’s rule (Taylor, 1993) adequately describes Central bank behaviour. However, empirical evidence suggests that actual interest rate policy appears more cautious than might be expected based on Taylor rule specifications (e.g., see Clarida et al. 1999; Rudebusch and Svensson, 1999). The other issue is whether it is appropriate to have monetary policy responses to asset price movements. Should the Central Bank care about financial instability associated with large asset price fluctuations? This question of whether central banks should or can target asset prices in their conduct of monetary policy has regained the interest of policymakers and academics after the Japanese asset price bubble in the late 1980s, the new technology stock market boom in the late 1990s, and the recent financial and economic crisis (2007). In fact, the recent financial crisis has shown that the economic consequences of financial instability can be devastating. The pre-crisis consensus that asset prices should only affect monetary policy decisions insofar as they affect inflation or the output gap, has come under discussion.

Recent analyses of central bank behaviour begin with a policy objective function and construct policy rules by optimizing the objective function subject to a system of constraints. In fact, central banks set interest rates based on inflation considerations, taking into account growth developments as well. This standard approach to monetary policy implies that stock prices only enter the deliberations of central banks insofar as asset prices affect inflation or GDP. An alternative policy approach is that the central bank actively tries to stabilize asset prices around fundamental values or attempts to pick certain asset price bubbles. To contribute to this discussion, we ask whether the basic Taylor rule could instead be augmented with an alternative variable that collects and synthesises the infor-
mation from the asset and financial markets, i.e. whether central banks are targeting the relevant economic information contained in a group of financial variables and not simply targeting each financial variable.

Two reasons might lead central banks to respond to asset price movements. First, monetary policy and its conduct require credible inflation forecasts. It is worth nothing that, some financial variables (stock prices, real exchange rates, composite indices of asset prices, etc...) may constitute a repelling force behind the conduct of monetary policy. It is well known that asset prices adjust more responsively to current and future economic conditions. Their integration into the reaction of a monetary policy rule would expand the information that could lead to better states of nature in the future. The informational scope of asset pricing contributes moreover to the resurgence of the arguments developed four decades ago by Alchian and Klein (1973), for whom the asset purchase constitutes a deferral of consumption over time. The preservation of purchasing power could justify that asset pricing should be considered in the measure of inflation. On the other hand, it is not clear that volatility in financial markets has increased; financial instability which in turn has worsened with the rise of institutional management. As the exposure of households, businesses and banks to capital markets has increased, frequent and persistent disconnections of asset prices from their respective core values help make economies more vulnerable to financial cycles. For example, the experience of Japan and the Scandinavian countries showed the impact of stock market bubbles and explosions of real estate financing systems and their consequences on the macroeconomic balance. Therefore, there is no need to mention the systemic risk associated with brick changes in financial flows and the consequences of volatile asset prices and their correlation with the real cycle. The strong linkages between the financial and real spheres are a sufficient condition to consider what attitude the monetary authorities should observe vis-à-vis the financial movements. In this vein, a number of studies have contributed
in recent years to determine whether or not central banks should respond to movements in asset pricing. Specifically, following the lead of the vast literature on monetary policy rules, this work is to determine if it is optimal for central banks to give to asset prices the same weight as that given to inflation and the output gap.

The role of asset prices is an important issue considered in some studies. However, no consensus has been reached about whether the central bank should or not target this kind of variables. Cecchetti et al. (2000), Borio and Lowe (2002), Goodhart and Hofmann (2002), Chadha et al. (2004) and Rotondi and Vaciago (2005) consider important that central banks target asset prices and provide strong support and evidence in that direction. On the contrary, Bernanke and Gertler (1999, 2001) and Bullard and Schaling (2002) do not agree with an ex-ante control over asset prices. They consider that, once the predictive content of asset prices for inflation has been accounted for, monetary authorities should not respond to movements in asset prices. Instead, central banks should act only if it is expected that they affect inflation forecast or after the burst of a financial bubble in order to avoid damages to the real economy. Moreover, Drifill et al. (2006) analyse the interactions between monetary policy and the futures market in the context of a linear reaction function. They find evidence supporting the inclusion of futures prices in the central bank’s reaction function as a proxy for financial stability. The issue of financial stability is also investigated by Montagnoli and Napolitano (2005). They build and use a financial conditional indicator that includes the exchange rate, share prices and housing prices in the estimation of a Taylor rule for some central banks. Their results show that this indicator can be helpful in modelling the conduct of monetary policy.

Considering these developments, our first contribution is simply to estimate a Taylor rule for Canada, where the information from some financial variables is accounted for to shed some more light on its importance. Despite, the good macroeconomic record of the
past decade, there has been a growing concern among academics and policymakers that the achievement of price stability may be associated with an increased risk of financial instability. To get a deeper understanding of central bank’s behaviour, this paper investigates whether that reaction function can be augmented with a financial conditions index containing information from several asset prices. At the same time, it also asks whether central banks are indeed following a linear reaction function or a nonlinear reaction function. In this paper we address the question of how central bankers ought to respond to asset price volatility, in the context of an overall strategy for monetary policy.

In most of the studies that were carried out, the Taylor rule is considered a simple linear interest rate rule that represents an optimal policy rule that is derived under two assumptions namely, a linear Phillips curve and quadratic loss function for the preferences of the central bank. Further, in imposing time-invariant reaction functions, the parameters of the Phillips curve and the loss function are assumed to be constant over time. However, in reality, this may not be the case and the central bank can have asymmetric preferences and, therefore, follow a nonlinear Taylor rule. If the central bank is indeed assigning different weights to negative and positive inflation and output gaps in its loss function, then a nonlinear Taylor rule seems to be more adequate to explain the behaviour of monetary policy (see, e.g. Bordo and Jeanne, 2002 and Chadha et al., 2004).

From a theoretical point of view, non-linearity in the policy rule can be motivated in three different ways. First, the underlying aggregate supply schedule might be nonlinear, leading to a nonlinear adjustment of the policy rate [see Nobay and Peel (2000), Ruge-Murcia (2004) and Dolado et al. (2005)]. Second, the policymaker might face uncertainty [see Meyer et al. (2001) and Swanson (2006)] so that nonlinearities might stem from uncertainty about the natural rate of unemployment, formalized by a non-Gaussian prior distribution and a non-linear updating rule. Third, the preferences of the policymaker might
not be quadratic in output and inflation. Bec et al. (2002), Kim et al. (2004), Martin and Milas (2004), Petersen (2007) and Surico (2007), among others, find empirical support for the presence of nonlinearity in central bank interest rate reaction functions. The resulting non-linear interest rate rule performs well in the pre-Volcker period but shows less signs of asymmetry in the post-Volcker era. As a result of the signal extraction problem, the central bank is more careful about adjusting interest rates in response to small output gaps than in a standard Taylor rule but more aggressive when they reach a certain threshold (see Ruge-Murcia (2003), Nobay and Peel (2003), and Cukierman and Muscatelli (2008) for the implications of non-standard, i.e., asymmetric, preferences in monetary policy models). Furthermore, indicator and monitoring models that use market-determined asset prices are being developed for policy advice. To date, linear indicator models using asset prices as leading indicators of output and inflation tend to be unstable and are therefore of limited use for policy analysis. Non-linear models using simple measures of asset-price misalignments, however, are more promising, but they are underdeveloped.

In view of these arguments, the motivation of this study is twofold: First of all, instead of simply relying on a linear model, à la Surico (2007b), where the asymmetries are accounted for by using products and cross products of inflation and output gap or by a separate analysis for inflation above or below the target, this paper estimates a nonlinear model for monetary policy where the presence of asymmetries is taken into account directly in the structure of the model. Our study adopts a different perspective. Instead of focusing on nonlinearities only from periods of recession and expansion, we allow for monetary policy changes between periods of high and low inflation. Note that inflation is actually the most important variable targeted by the central banks analyzed in this study, especially for Canada whose main objective is precisely to promote price stability. Furthermore, contrary to Bec et al. (2000), we allow for the possibility of interest rate smoothing. Besides analysing
monetary policy asymmetries, this procedure will also permit an answer to the question of whether a central bank follows a point target or a target range for inflation. Second, this paper also extends the nonlinear specification of the Taylor rule with the stock market price index. This is to check whether, after controlling for nonlinearities, the Bank of Canada is still (or not) reacting to the information contained in this index. In fact, several studies have documented the role of asset prices as forecasters of inflation and output and their relevance as instrumental variables in Taylor rules (Siklos & Bohl, 2009). However, most studies about the Bank of Canada’s monetary policy neglect the direct influence of stock prices on interest rates, while this has been an important research topic with respect to other central banks, such as the Fed (e.g. Chadha, Sarno, & Valente, 2004; Fuhrer & Tootell, 2008). Our paper aims to fill this gap and improve knowledge about the ambiguous role of stock prices in influencing monetary policy decisions.

This paper proceeds as follows. Section 2 presents asset prices perspective in Canada. Section 3 extends the models. First, the specification of the threshold regression model and the estimation procedure are presented. It presents the methodology used to estimate the nonlinear reaction function. It assumes that if the central bank is assigning different weights to negative and positive inflation and output gaps in its loss function, then a nonlinear Taylor rule seems to be more adequate to explain the behaviour of the monetary policy. Second, we try to isolate the central bank’s preference parameters from the estimated reaction function. In this spirit, we consider an Euler equation specification, in line with the influential approach by Favero and Rovelli (2003), to capture the performance of a policy rule describing the evolution of a continuously adjusted short-term interest rate. Rudebusch and Svensson (1999, 2002) describe the economy by an aggregate demand and a Phillips curve or aggregate supply and a central bank’s loss function. Under the assumption that the economic structure is constant over time but that the parameters of the loss function
may shift, this model is used to derive the weights the central bank assigns to the different arguments. Section 4 provides the results of the estimations. Section 5 offers the main findings of this paper and concludes.

2. Monetary Policy and Asset Prices: A Canadian Example

The main reason to include asset price fluctuations in monetary policy rules is that an asset price bubble is socially unwanted, due to its disruptive effects on consumption and investment and in the soundness of the financial system. Policy analysts at the Bank of Canada introduce movements in asset prices into their analysis in many ways. According to Selody and Wilkins (2004) there are three ways: “First, fundamental asset-price values are implicit in the calculations that determine the value of wealth in the main structural model used for policy advice and through their direct effect on the CPI. Second, indicator and monitoring models that use market determined asset prices are being developed for policy advice. Third, descriptions and analysis of the evolution of market determined asset prices are included in the regular briefings to policymakers that precede policy decisions”. These authors find that the Canadian case monetary policy should therefore aim for temporary deviations from its target only under rare and extreme circumstances. Moreover, housing-price bubbles should be a greater concern for Canadian monetary policy than equity-price bubbles, since rising housing prices are more likely to reflect excessively easy domestic credit conditions than are equity prices, which are largely determined in global markets. Choudhry (1996) investigated the relationship between stock prices and the long-run money demand function in Canada and US during 1955-1989. He finds that stock prices play a significant role in the determination of stationary long-run demand functions in both countries. Recently Daisy et al. (2010) examined whether the sensitivity of stock returns to unexpected changes in monetary policy vary across different economies. They empirically
investigate the relationship between monetary policy shocks and stock prices using VAR models for the Canadian and US economies from the period 1988. They find that, in Canada, the immediate response of stock prices to a domestic contractionary monetary policy shock is small and the dynamic response is brief, whereas in the U.S., the immediate response of stock prices to a similar shock is relatively large and the dynamic response is relatively prolonged. These differences refer probably to the differences in financial market openness between the two countries. Although the Bank of Canada admits that asset prices are considered in its policy deliberations because of their effects on inflation or output gap, the Bank of Canada denies trying to stabilize asset prices around fundamental values. However we have seen booms as well as busts in the stock market. Are we to believe that the Bank of Canada did not react to these stock market fluctuations, apart from their consequences on the economy? To be clear, we agree that monetary policy is not by itself a sufficient tool to contain the potentially damaging effects of booms and busts in asset prices. We examine this issue by using Favero and Rovelli’s (2003) framework that is augmented by a stock market variable. We use an econometric method that allows us to distinguish the direct effect of asset prices on Bank of Canada policy rates from indirect effects via inflation or GDP.

To gain insight into the history of monetary policy carried out by the Bank of Canada, we take a look at asset prices (dsmpi), interest rates (ron) in Canada since 1961. The evolution of stock prices and interest rates during the period covered our study is shown in Figure 2.1. We show the stock prices level in deviation from its average over the sample period, to get an idea of the size of peaks and troughs. The stock market peaked in 1983 with a 2.9 point deviations from the sample average and reached its trough in the end of 2007 with a -1.8 points deviation from average. Thus a boom as well as a bust in stock prices occurred. Volatility in stock prices was considerable during the period of this research. The
question is how did the Bank of Canada respond to these stock price fluctuations? Figure 2.1 also shows the interest rates in the Bank of Canada that are effectively targeted by Bank of Canada. The interest rate moves closely around the overnight rate, which is the policy rate of the Bank of Canada. It is apparent that the central bank raised interest rates at the beginning of 1981, while interest rates decreased from the end of 2004 until 2008. This implies that monetary policy was tightened during the stock market boom while it was eased afterwards. This behaviour of the Bank of Canada is consistent with the alternative policy approach that includes stabilizing stock prices.

![Graph showing asset prices and interest rates](image)

Figure 2.1: Asset prices and interest rates

### 3. Theoretical Framework

We use a structural backward-looking model of a closed economy that allows for the effect of asset prices on aggregate demand. The model augments the standard Ball (1999) and Svensson (1997) specification by taking into account asset prices. Aggregate supply is the result of firms that set the prices for their products so as to maximize profits in a monopolistic competition setting.
3.1 Structure of the Economy

Svensson (1999) shows that the traditional Taylor rule is an optimal reaction function for a central bank which targets inflation in a simple backward-looking two equations model of the economy, with the coefficients in the Taylor rule being a convolution of policymaker’s preferences and the parameters in the aggregate demand and aggregate supply functions. One key finding of the Svensson model is that policymakers react to the output gap even if they are strict inflation targeters. In this study, we try to extend that model by including two equations for the determination of the economic structure and asset prices, showing that policymakers react also to asset prices fluctuations. Therefore, we use an extension framework developed by Ball (1997) and Svensson (1997, 1999), applied recently by Kontonikas and Montagnoli (2005) and Nisticò (2006). It will be a very simple model consisting of three optimal behavioural equations:

\[ \pi_{t+1} = \phi x_t + \theta \pi_t + \varepsilon_{t+1} \]  
\[ x_{t+1} = \eta_1 x_t - \eta_2 (i_t - E_t \pi_{t+1}) + \eta_3 s_{t+1} + \mu_{t+1} \]  
\[ s_{t+1} = \gamma_1 \varepsilon_{t+1} + \gamma_2 s_t - \gamma_3 (i_t - E_t \pi_{t+1}) + \vartheta_{t+1} \]

where \( x \) denotes the output gap, \( i \) is the monetary policy instrument (the short-term nominal interest rate), \( \pi \) the inflation rate, \( s \) the stock market price index (asset prices), measured as the deviations from their trend. This last variable appears here to incorporate the asset price effects on aggregate demand. Finally, \( \mu_{t+1} \) is an aggregate demand shock, \( \varepsilon_{t+1} \) is the supply shock, which may be interpreted as a shift of the degree of substitutability between inputs in the production of final goods, or an exogenous cost push shocks.
and $\theta_{t+1}$ represents exogenous random shocks to asset prices. It can be interpreted as a shock to the equilibrium real stock price value. All variables are in logarithms and refer to deviations from an initial steady state. The structural parameters can be interpreted as partial elasticities. Equation (2.1) is an inflation dynamics equation that links inflation positively to the output gap and the inflation shock. The larger is $\theta$, the stronger is the adjustment of prices to deviations of output from full potential. The backward-looking term $(\pi_t)$ reflects the existence of firms that employ a rule of thumb approach to set their prices. This process is also consistent with the empirical finding that inflation in the major industrialised countries is so highly persistent that it may indeed contain a unit root as some studies have shown (see e.g., Grier and Perry, 1998). The parameter $\phi$ is a positive constant which measures the sensitivity of inflation to excess demand.\(^1\) Equation (2.2) is consistent with the specification employed by Walsh (1998), Ball (1999), and Svensson (1997) with one important difference: the aggregate demand equation allows for a transmission lag of monetary policy; it relates the output gap inversely to the real interest rate and positively to asset prices. The parameters $\eta$ are assumed to be positive, and $0 \leq \eta_1 \leq 1$. The rationale for including a lag of the output gap is to account for habit persistence in consumption wealth effects and investment balance sheet effects. For example, a persistent decrease in the level of stock prices increases the perceived level of households’ financial distress causing a reduction in consumption spending. The balance sheet channel implies a positive relationship between the firms’ ability to borrow and their net worth, which in turn depends on asset valuations. There is a vast amount of empirical evidence indicating that asset price movements are strongly correlated with aggregate demand in most major economies.\(^2\) In

\(^1\)As Clark, Goodhart, and Huang (1999) point out, there are good reasons to believe that $\alpha$ is not constant. However, the assumption of linearity in the Phillips curve helps to obtain a closed-form solution for the optimal feedback rule.

\(^2\)See among others, Kontonikas and Montagnoli (2005) for relevant empirical evidence concerning the UK economy, and Goodhart and Hofmann (2000) for international evidence. A recent study by the IMF (2003) points out that equity price reductions are associated with heavy GDP losses.
our model, the central bank takes into account the effect of wealth on aggregate demand, that is, it is fully aware of the effect of \( s_t \) on \( x_t \) and its magnitude. Furthermore, parameter \( \eta_3 \) in the aggregate demand equation is of crucial interest since it indicates the magnitude of the effects of asset price movements on output. If there are no wealth effects/balance sheet effects then \( \eta_3 = 0 \) and Eq. (2) resembles a traditional dynamic IS curve. Equation (2.3) has its roots in the standard dividend model of asset pricing. Asset prices deviations are a function of the dividends of the next period (assumed to depend on productivity shocks) and the real interest rate. We also add a backward-looking term in the equation. As we see, the real interest rate affects output with a one-period lag, and hence inflation with a two period lags. On the other hand, asset price increases widen the output gap through wealth effects on consumption and raise inflationary threats, affecting inflation with a one-period lag. Asset prices don’t directly affect the future path of inflation. Nevertheless, they are predictors of future inflation. Inflation expectations in year \( t \) are, by equation (2.1)\(^3\):

\[
\pi_{t+1} = \phi x_t + \pi_t
\]

Using (2.4) and (2.2), we obtain the following reduced form aggregate demand equation:

\[
x_{t+1} = \sigma x_t - \eta_2 (i_t - \pi_t) + \eta_3 s_{t+1} + \mu_{t+1}
\]

where \( \sigma = \eta_1 + \phi \)

\[
s_{t+1} = \gamma_2 s_t - \gamma_3 (i_t - \pi_t) + \delta x_t + \xi_{t+1}
\]

where \( \tau = \phi \gamma_3 \) and \( \xi_{t+1} = \gamma_1 \varsigma_{t+1} + \theta_{t+1} \)

In short, the structure of economy is presented as follow:

\(^3\)We consider that the coefficient on \( \pi_t \) is equal to one, which signifies that last period’s inflation is very important for the formation of current inflation. According to Peersman and Smets (1998), this coefficient is equal to 0.92 for five European countries and Rudebusch and Svensson (1999), as ourselves in the estimates of the last section, impose the restriction that the sum of the lagged coefficients of inflation equals one.
Finally, asset prices deviations depend on contemporaneous shocks to inflation, the output gap and the real interest rate, which is more appealing since asset market participants presumably look at all available and relevant information when determining the appropriate price of the assets.

In empirical applications, more lags of output and asset prices (in the case of the IS curve) and output and inflation (for the Phillips curve) are often included to improve the empirical fit. Adding these lags will also induce a more persistent and therefore more realistic adjustment to shocks. In empirical studies and monetary policy analysis concepts of equilibrium and/or core inflation are sometimes added to (2.1), to distinguish short-run fluctuations of inflation from longer term, equilibrium inflation. In our analysis this issue is not dealt with and inflation (as all other variables) is defined in terms of deviations from (possibly non-zero inflation) steady-state (see Vega and Wynne, 2003).

Following the current monetary policy analysis framework, one possible shortcoming of equations (2.1) and (2.2) is their relevance in the context of open economies, where international trade is an important part of the economic activity and therefore, the exchange rate should be considered as a significant argument in policy functions of open economies. However, using modified versions of equations (2.1) and (2.2), Ball (1999) does not find important changes in the interest rate movements for open and closed economies. On the other hand, using a forward-looking perspective, Svensson (2000) finds varied benefits of including the exchange rates in the monetary rule in comparison with the original Taylor rule. In a similar way, Taylor (2001) finds weak evidence for the exchange rate channel. Clarida et al. (1998, 2000) attempt to re-specify Taylor-type rules for small economies using foreign variables. For the cases of Japan and Germany, they use the US interest rate
and the exchange rates in the interest rate rule and the results show that the coefficients may be small and significant but in some cases, as for Germany, the inflation coefficient is negative. Taylor (2001) suggests that the inclusion of the exchange rate is not crucial for the monetary policy rule. As Rodriguez (2008), we consider the role of the exchange rate explicitly in the empirical part of the Phillips curve.

3.2 The Specification of Asymmetric Objective Functions

Moreover, some of the recent literature has considered the possibility that the loss function of monetary policymakers may not be quadratic and consequently the Taylor rules derived from such functions would not necessarily be linear. In fact, despite its analytical convenience, a quadratic loss function that penalizes equally positive and negative output gaps does not appear to be realistic. Obviously, policymakers are averse to negative output gaps but it is less clear that they are, given inflation, equally averse to positive output gaps. They may even, given inflation, be indifferent between different magnitudes of positive output gaps. Cukierman (2000, 2002) shows that in the last case, and in the presence of uncertainty and rational expectations, there is an inflation bias even if the output target of the Central Bank is equal to the potential level.

Following standard assumptions in the empirical literature of monetary policy, the policymaker’s preferences are modeled as an intertemporal loss function in which, at each period, the loss function depends on both inflation and output in relation to their target values, as well as the smoothing interest rate and other potential variables (e.g., asset prices). Future values are discounted at rate $\beta$, and the weights $\lambda$, $\mu$, and $\delta$ are nonnegative. As usual, we assume that monetary policy is conducted by a central bank that chooses the sequence of short-term nominal interest rates in order to minimize the present discounted value of its loss function. Rather than assuming a quadratic form as is usual in the literature (see Svensson, 1997; Favero and Rovelli, 2003 and Rodriguez, 2008), we use a more
general specification (nonlinear or asymmetric loss function) of the monetary authorities objectives.

\[
\text{Loss} = E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left[ (\pi_{t+\zeta} - \pi^*)^2 + \lambda x_{t+\zeta}^2 + \mu (i_{t+\zeta} - i_{t+\zeta-1})^2 + \delta s_{t+\zeta}^2 \right] \tag{2.8}
\]

A simple way of capturing nonlinearities or asymmetries in policy behaviour is to estimate threshold models whereby the policy reaction function switches into a different regime whenever a certain variable breaches one or more thresholds. Asymmetries or nonlinearities in the policymakers' preferences mean that the structural weights depend on the appropriate state of the economy. Indeed, the types of nonlinearities or asymmetric preferences suggested in the literature may be written as follows, reflecting Cukierman's (2000) precautionary demand for expansions (the threshold variable is the output gap), or Goodhart's (1999) precautionary demand for price stability (the threshold variable is the inflation rate) and asset price asymmetry (the threshold variable is asset prices).

\[
\text{Loss} = E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left\{ \begin{array}{c}
(\pi_{t+\zeta} - \pi^*_j)^2 + \lambda_1 x_{t+\zeta}^2 + \mu_1 (i_{t+\zeta} - i_{t+\zeta-1})^2 + \delta_1 s_{t+\zeta}^2 \cdot I(q_t \leq \gamma) \\
+ (\pi_{t+\zeta} - \pi^*_2)^2 + \lambda_2 x_{t+\zeta}^2 + \mu_2 (i_{t+\zeta} - i_{t+\zeta-1})^2 + \delta_2 s_{t+\zeta}^2 \cdot I(q_t > \gamma)
\end{array} \right\} \tag{2.9}
\]

Where \(0 < \beta < 1, \lambda_j, \mu_j, \delta_j \geq 0, j = 1, 2,\) and \(E_t\) defines the expectations taken with respect to the information available at time \(t\), \(\pi^*_j\) being the target level of inflation in each regime. With this objective function, the Central Bank is assumed to stabilize annual inflation about \(\pi^*_j\), while keeping the output gap and the fluctuations in asset prices close to zero and making small changes in the nominal interest rate. The policy preference parameters, or weights, \(\lambda_j, \mu_j\) and \(\delta_j\), indicate the relative importance policymakers put on output gap stabilization, on asset price and on interest rate smoothing relative to inflation stabilization. On the other hand, \(\lambda_j, \mu_j\) and \(\delta_j\) represent the central bank’s aversion to output fluctuations around potential and to interest rate and asset price level fluctuations.
$I(.)$ are indicator functions that take a value equal to one if the condition in parentheses is true and zero otherwise. $q_t$ is the threshold variable and $\gamma$ the threshold value that can be estimated. We consider two observed threshold variables ($q_t$) related to actual economic development: the output gap ($q_t = x_t$), and the inflation deviation ($q_t = \pi_t$). We postulate two possible regimes for central bank behaviour depending on whether $q_t$ is below (first regime) or above (second regime) the threshold value. Then, if we suppose $\gamma = 0$ and the output gap is the threshold variable, the central bank may modify the magnitude (and the velocity) of its reaction to the output gap and/or inflation deviations during recessions, with respect to what it does in expansions. Alternatively, with the inflation deviation as the threshold variable, the central bank may react differently to output gap and inflation deviations if high-or low-inflation episodes occur. Empirically, we could find $\lambda_1 \neq \lambda_2$, $\mu_1 \neq \mu_2$, $\delta_1 \neq \delta_2$ and $\pi_1^* \neq \pi_2^*$ in both cases.

### 3.3 The Policy Process and the Policy Rule

An important aspect of monetary policymaking is that the interest rate has to be chosen before the realization of economic shocks is known with certainty by monetary authorities. This fact is captured here by assuming that the realization of the shocks are unknown at the time policymakers pick the nominal interest rates. The policy rule can be found by minimizing the expected value of the loss function, equation (2.9), subject to the behaviour of the economy as given by equations (2.7). The problem of the Central bank is to choose the current interest rate and the sequence of the future interest rates. In summary, the intertemporal optimization problem is then:

$$\min_{i_t} \mathbb{E}_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left\{ \begin{array}{c} (\pi_{t+\zeta} - \pi_1^*)^2 + \lambda_1 x_{t+\zeta}^2 + \mu_1 (i_{t+\zeta} - i_{t+\zeta-1})^2 + \delta_1 s_{t+\zeta}^2 \right\} I(q_t \leq \gamma) \\
+ \left\{ (\pi_{t+\zeta} - \pi_2^*)^2 + \lambda_2 x_{t+\zeta}^2 + \mu_2 (i_{t+\zeta} - i_{t+\zeta-1})^2 + \delta_2 s_{t+\zeta}^2 \right\} I(q_t > \gamma) \end{array} \right\}$$

(2.10)
Subject to \[
\begin{align*}
x_{t+1} &= \sigma x_t - \eta_2 (i_t - \pi_t) + \eta_3 s_{t+1} + \mu_{t+1} \\
\pi_{t+1} &= \phi x_t + \theta \pi_t + \varepsilon_{t+1} \\
s_{t+1} &= \gamma_2 s_t - \gamma_3 (i_t - \pi_t) + \tau x_t + \xi_{t+1}
\end{align*}
\]

After finding the first-order conditions for optimality and after some manipulations, it is possible to obtain an interest rate rule. The parameters of this monetary rule are convolutions of the coefficients associated with the restrictions under which the loss function has been intertemporally optimized; that is they are convolutions of the parameters associated with the preferences of the central bank \((\lambda_1, \lambda_2, \mu_1, \mu_2, \delta_1, \delta_2, \pi_1^*, \pi_2^*)\) and the structure of the economy \((\eta_2, \eta_3, \phi, \theta, \gamma_2, \gamma_3, \tau)\).

Adopting the method of Optimal Control to solve this problem (see Chiang, 1992), we calculate the first-order conditions for the minimization of the loss function, which leads to the following Euler equation:

\[
0 = \left[ E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \left( (\pi_t + \zeta - \pi_1^*) \frac{\partial \pi_{t+\zeta}}{\partial i_t} \right) + E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \lambda_1 \left( (x_t + \zeta) \frac{\partial x_{t+\zeta}}{\partial i_t} \right) \right] I(q_t \leq \gamma) (2.11)
\]

\[
+ \left[ E_t \sum_{\zeta=0}^{\infty} \beta^\zeta \delta_1 \left( (s_t + \zeta) \frac{\partial s_{t+\zeta}}{\partial i_t} \right) + [\mu_1 (i_t - i_{t-1}) - \mu_1 \beta E_t (i_{t+1} - i_t) \right] I(q_t > \gamma)
\]

Because of the persistence in the structural equations of the economy, the Euler equation has an infinite horizon, and thus cannot be used directly in empirical work. To estimate this equation it is necessary to truncate its lead polynomials at some reasonable temporal horizon. As Favero and Rovelli (2003), we use a 4 quarters lead horizon. Two reasons stand in favour of the lead truncation of the Euler equation: First, as Favero and Rovelli (2001) have argued, a natural cutting point for the future horizon of the Euler equation emerges anyway, even if we consider a theoretical infinite horizon loss function. In fact, the weight
attached to expectations of future gaps and inflation decreases as the time-lead increases, meaning that expectations of the state of the economy carry less relevant information for the present conduct of policy as they relate to periods further away in the future. Second, expanding the horizon in the Euler equation would complicate it and bring collinearities to the system, causing great difficulties in making estimations. It is worth noting that our option is consistent with the standard practice in the estimation of forward-looking policy reaction functions. Boivin and Giannoni (2003) truncate the forecast horizon at 1 quarter for output and 2 quarters for inflation, while Muscatelli et al. (2002), and Orphanides (2001 b) truncate the inflation forecast horizon at 4 quarters. Rodriguez (2008) shows that estimated backward-looking policy reaction functions for US and Canada, strongly indicate that actual policy decisions involve forecast horizons of inflation not beyond 4 quarters ahead.

Once the Euler equation is truncated at 4 quarters ahead, its partial derivatives components can be expressed as functions of the aggregate demand and aggregate supply parameters, thus building into the Euler equation the cross-equation restrictions. This ensures that the loss function is being properly minimized subject to the constraints given by the economy’s structure.

\[
0 = \left[ \sum_{\zeta=0}^{\infty} E_t \sum_{\zeta=0}^{4} \beta^\zeta \left[ (\pi_{t+\zeta} - \pi_1^*) \frac{\partial \pi_{t+\zeta}}{\partial \pi_1^*} \right] + E_t \sum_{\zeta=0}^{4} \beta^\zeta \lambda_1 \left[ (x_{t+\zeta}) \frac{\partial x_{t+\zeta}}{\partial x_t} \right] \right] I(q_t \leq \gamma_{2.12}) \\
+ \sum_{\zeta=0}^{\infty} E_t \sum_{\zeta=0}^{4} \beta^\zeta \left[ (s_{t+\zeta}) \frac{\partial s_{t+\zeta}}{\partial s_t} \right] + \left[ \mu_1 (i_t - i_{t-1}) - \mu_1 \beta E_t (i_{t+1} - i_t) \right] \\
+ \left[ \sum_{\zeta=0}^{\infty} E_t \sum_{\zeta=0}^{4} \beta^\zeta \delta_1 \left[ (\pi_{t+\zeta} - \pi_2^*) \frac{\partial \pi_{t+\zeta}}{\partial \pi_2^*} \right] + E_t \sum_{\zeta=0}^{4} \beta^\zeta \lambda_2 \left[ (x_{t+\zeta}) \frac{\partial x_{t+\zeta}}{\partial x_t} \right] \right] I(q_t > \gamma)
\]
Expanding the partial derivatives, (2.11) turns into

\[
0 = \begin{bmatrix}
\beta E_t \left( \pi_{t+2} - \pi_t^* \right) \frac{\partial \pi_{t+2}}{\partial t} + \beta^2 E_t \left( \pi_{t+3} - \pi_t^* \right) \frac{\partial \pi_{t+3}}{\partial t} + \beta^3 E_t \left( \pi_{t+4} - \pi_t^* \right) \frac{\partial \pi_{t+4}}{\partial t}
+ \lambda_1 \beta E_t \left( x_{t+2} \right) \frac{\partial x_{t+2}}{\partial t} + \lambda_2 \beta^2 E_t \left( x_{t+3} \right) \frac{\partial x_{t+3}}{\partial t} + \lambda_3 \beta^3 E_t \left( x_{t+4} \right) \frac{\partial x_{t+4}}{\partial t}
+ \lambda_1 \beta E_t \left( x_{t+2} \right) \frac{\partial x_{t+2}}{\partial t} + \lambda_2 \beta^2 E_t \left( x_{t+3} \right) \frac{\partial x_{t+3}}{\partial t} + \lambda_3 \beta^3 E_t \left( x_{t+4} \right) \frac{\partial x_{t+4}}{\partial t}
+ \lambda_1 \beta E_t \left( x_{t+2} \right) \frac{\partial x_{t+2}}{\partial t} + \lambda_2 \beta^2 E_t \left( x_{t+3} \right) \frac{\partial x_{t+3}}{\partial t} + \lambda_3 \beta^3 E_t \left( x_{t+4} \right) \frac{\partial x_{t+4}}{\partial t}
\end{bmatrix}
\]

Then, the IS curve equation, Phillips curve equation and Euler equation can be jointly estimated as a system, generating estimates of the structural parameters \(c_1\) through \(c_{15}\), as well as of the policymakers structural preferences parameters \(\lambda_1, \lambda_2, \mu_1, \mu_2, \delta_1, \delta_2, \pi_1^*, \pi_2^*\):

\[
x_{t+1} = c_1 + c_2 x_t + c_3 x_{t-1} + c_4 \left( i_{t-1} - \pi_{t-1} \right) + c_5 \left( i_{t-2} - \pi_{t-2} \right) + c_6 s_t + \mu_{t+1}
\]

\[
\pi_{t+1} = c_7 \pi_t + c_8 \pi_{t-1} + c_9 x_t + c_{10} \Delta w_t + \varepsilon_{t+1}
\]

\[
s_{t+1} = c_{11} s_t + c_{12} s_{t-1} + c_{13} \left( i_{t-1} - \pi_{t-1} \right) + c_{14} \left( i_{t-2} - \pi_{t-2} \right) + c_{15} x_t + \xi_{t+1}
\]

We rearranged Equation (2.12) and substituted derivatives with coefficients from equa-
tion (2.13) to obtain

\[
0 = \begin{bmatrix}
[\mu_1 (i_t - i_{t-1}) - \mu_1 \beta E_t (i_{t+1} - i_t)] + \beta^2 E_t (\pi_{t+3} - \pi^*_t) [c_9c_4] \\
+ \beta^3 E_t (\pi_{t+4} - \pi^*_t) [c_9 (c_5 + c_2c_4) + c_7c_9c_4] \\
+ \lambda_1 \beta E_t (x_{t+2}) [c_4] + \lambda_1 \beta^2 E_t (x_{t+3}) [c_5 + c_2c_4 + c_12c_6] \\
+ \lambda_1 \beta^3 E_t (x_{t+4}) [c_2 (c_5 + c_2c_4) + c_6 (c_12 + c_12c_4)] \\
+ \delta_1 \beta E_t (s_{t+2}) [c_{12} + c_4c_11] + \delta_1 \beta^2 E_t (s_{t+3}) [c_{13} + c_2c_14 + c_{11}c_2c_4] \\
+ \delta_1 \beta^3 E_t (s_{t+4}) [c_{14} (c_{13} + c_12c_{14}) + c_5c_9c_{11}]
\end{bmatrix}
\]

\[I(q_t \leq \gamma)
\]

\[
0 + \begin{bmatrix}
[\mu_2 (i_t - i_{t-1}) - \mu_2 \beta E_t (i_{t+1} - i_t)] + \beta^2 E_t (\pi_{t+3} - \pi^*_t) [c_9c_4] \\
+ \beta^3 E_t (\pi_{t+4} - \pi^*_t) [c_9 (c_5 + c_2c_4) + c_7c_9c_4] \\
+ \lambda_2 \beta E_t (x_{t+2}) [c_4] + \lambda_2 \beta^2 E_t (x_{t+3}) [c_5 + c_2c_4 + c_12c_6] \\
+ \lambda_2 \beta^3 E_t (x_{t+4}) [c_2 (c_5 + c_2c_4) + c_6 (c_12 + c_12c_4)] \\
+ \delta_2 \beta E_t (s_{t+2}) [c_{12} + c_4c_11] + \delta_2 \beta^2 E_t (s_{t+3}) [c_{13} + c_2c_14 + c_{11}c_2c_4] \\
+ \delta_2 \beta^3 E_t (s_{t+4}) [c_{14} (c_{13} + c_12c_{14}) + c_5c_9c_{11}]
\end{bmatrix}
\]

\[I(q_t > \gamma)
\]

Following Favero et Rovelli (2003), the parameters of the structural equations and the loss function are estimated jointly from a system formed by system (2.14) and the Euler equation (2.15). As we want to obtain the preferences implied by the coefficients from the threshold regression model, the dependent variable in the interest rate is the fitted interest rates from the threshold regression model including the lagged interest rates. Furthermore, to cover the different types of asymmetry in the policymaker’s preferences identified in the literature, estimation is carried out sequentially allowing each of the loss function weights \( \lambda, \mu \) and \( \delta \) to vary with the state of the corresponding target variable, and then concludes with a joint test. Statistical inference is based on individual significance tests and Wald tests.

4. Estimation Strategy and Data Set

4.1 Estimation strategy

To assess the empirical support of the departure from the linear-quadratic framework considered here, we rely upon two alternative econometric strategies which are described in turn.
First of all, to determine the endogenous threshold values, we consider the Taylor empirical rule and use Caner and Hansen’s (2004) approach. Indeed, Caner and Hansen (2004) estimate a model by developing a two-stage least squares estimator of the threshold parameter and a generalized method of moments estimator of the slope parameters. Specifically, the estimator is based on estimation of a reduced form regression for the endogenous variables as a function of exogenous instruments. This requires the development of a model of the conditional mean of the endogenous variables as a function of the exogenous variables. Based on the reduced form, predicted values for the endogenous variables are formed and substituted into the structural equation of interest. Least-squares (LS) minimization yields the estimate of the threshold. Estimation of the slope parameters of this equation occurs in the third step, where the sample is split based on the estimated threshold, and then conventional two-stage least squares (2SLS) or generalized method of moments (GMM) estimation is performed on the subsamples. Moreover, they propose a supremum Wald statistic to test for the existence of a threshold effect and suggest using bootstrap approaches to obtain the correct (asymptotic) p-value.\(^4\)

This is an important feature of the model and it limits potential applications. In some circumstance, specifically in empirical monetary policy rules, the key variables for economic structure are likely to be endogenous. Endogeneity can arise as a result of measurement error, autoregression with auto correlated errors, simultaneity, omitted variables and sample selection error. The problem of endogeneity occurs when one or more regressors are correlated with the error term in a regression model, which implies that the regression coefficients in an OLS regression are biased. In order to avoid the endogeneity problem, an instrumental variable estimation threshold model introduced by Caner and Hansen (2004) is used.

\(^4\)More detailed descriptions of the threshold regressions model with instrumental variables are presented in Appendix A, chapter one.
It is widely perceived that, since the seminal paper by Taylor (1993), the nominal interest rate set by central banks is often assumed to depend on the output gap and on inflation. However, output gap and inflation, following standard assumptions in the New-Keynesian literature, describe aggregate demand and aggregate supply, which both define the so-called structure of economy. We then apply the instrumental variable estimation of a threshold model proposed by Caner and Hansen (2004) to avoid the endogeneity problem and to investigate the threshold effect. Following Koustas and Lamarche (2009), and extended to others variables (asset prices), we model a possible asymmetric behaviour on the part of the Bank of Canada, where we postulate two possible regimes for central bank behavior depending on whether $q_t$ is below (first regime) or above (second regime) the threshold value. We assume that the Central Bank adjusts interest rates in a cautious way through smoothing in the form of a partial adjustment as follows: 

$$i_t = (1 - \rho) i_t^* + \rho i_{t-1} + \varsigma_t$$

. According to this partial adjustment behaviour, the Central Bank within each period adjusts its instrument in order to eliminate only a fraction $(1 - \rho)$ of the gap between its current target level and some linear combination of its past values.5

We consider two observed threshold variables ($q_t$) related to actual economic development: the lag of the output gap ($q_t = x_{t-j}$), and the lag of the inflation deviation ($q_t = \pi_{t-l}$), where $j$, and $l$ are inflation, output gap lags respectively. Our threshold regression model takes the form:

$$i_t = (\alpha_1 \pi_t + \alpha_2 x_t + \theta_1 i_{t-1}) I (q_t \leq \gamma) + (\beta_1 \pi_t + \beta_2 x_t + \theta_2 i_{t-1}) I (q_t > \gamma) + u_t$$

(2.16)

To estimate our specification model for Canada, we use the GMM procedure and the result of the empirically estimated Taylor rule. The latter appears highly adequate for our purposes because at the time of its interest rate setting decision, the central banks cannot observe the ex-post realized right-hand side variables. That is why central banks have to

\[\text{5see details in Taylor and Davradakis 2006.}\]
base their decisions on lagged values only. We decided to use the first six lags of inflation and the output gap and - whenever it is added to the regression equation - the first six lags of the “additional” variable as instruments. Moreover, we perform a J-test to test for the validity of over-identifying restrictions to check for the appropriateness of our selected set of instruments. As the relevant weighting matrix we choose, as usual, the heteroskedasticity and autocorrelation consistent HAC matrix by Newey and West (1987).

4.2 The Data Set

The estimation is conducted on quarterly data for the Canadian economy, obtained from Statistics Canada and the Bank of Canada, that spans the period from 1961:Q1 to 2008:Q4. Several different methods have been proposed to measure the output gap (see Rodriguez, 2008). Our aim is not to ascertain the way that real output evolves over the long-run. Instead, the goal is to obtain a reasonable measure of the pressure felt by the Bank of Canada to use monetary policy to affect the level of output. Potential output is obtained from the Hodrich-Prescott (HP) trend of the Canadian real GDP. The output gap is then constructed as the percentage difference between the logarithm of real GDP and its HP trend. Annual inflation is measured as $100 \times (p_t - p_{t-4})$, where $p_t$ denotes logarithms of the Consumer Price Index (CPI). The nominal interest rate is the annual percentage yield on 3-month Treasury bills. Financial variables represent another group of variables that have been recently considered in the specification of the Taylor rule for the analysis of the behaviour of the Central Bank. In this paper, we consider the effects of S&P/TSX. In fact, the S&P/TSX composite index is an index of the stock prices of the largest companies on the Toronto Stock Exchange as measured by market capitalization. We choose this index because the TSX listed companies in this index comprise about 70% of the market capitalization for all Canadian based companies listed on the TSX, thus it is the best financial index which contains the information that can help the Bank of Canada when
making policy decisions.

Figure 2.2 shows the evolution of the main variables (the short-term interest rate, the annual inflation rate, the output gap, and the stock market index price) considered in this study. Inflation and interest rates decline markedly during the 1990s. These series indicate some volatility and this volatility is generally higher starting in the 1970s.

5. Empirical Evidence and implications

5.1 The statistical validity of the model

The descriptive statistics are presented in table (2.1) of the appendix. In short, data vary enough so that one can apprehend relevant correlations between the dependent variable and explanatory variables. Moreover the matrix of correlations between explanatory variables (Table (2.2) of the appendix) suggests that the inclusion of all these variables in the same model poses no problem of multicollinearity. Indeed, coefficients of correlation appear quite low on the whole.

Knowledge of the integrational properties of the variables is important for the specifi-
cations of the econometric model. Given the implications for econometric modelling, we formally test for unit roots, a necessary conditions for the use of the approach of Caner and Hansen (2004). In order to investigate the stationarity of each time series that we are considering in this study, many tests exist. Apart from the conventional augmented Dickey and Fuller (1979, 1981) there is the nonparametric test proposed by Phillips and Perron (PP) test (1988), the ADF statistic based on the Generalized Least Squares detrending procedure proposed by Elliott, Rothenberg and Stock (DF-GLS) (1996), and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test (1992). In contrast with previous studies, we decide to run ADF, DF-GLS and KPSS tests because they are more powerful. The results of the tests for the variables, reported in Table 2.1 provide evidence against the unit root hypothesis. For all the variables, we estimated the ADF, DF-GLS and KPSS tests using only an intercept. With the ADF and DF-GLS tests, the unit root null can be rejected even at the 10% level for all variables. With the KPSS test, we cannot reject the null hypothesis of stationarity for all of the variables even at 10% level significance. These findings are consistent with the work of other researchers, and constitute a benchmark consistent with a unit root in the variables (see Rodriguez, 2004).
Table 2.1: Unit Root and Stationarity tests.

<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
<th>DF-GLS</th>
<th>KPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Test</td>
<td>Test</td>
</tr>
<tr>
<td>Interest rate</td>
<td>-2.618†</td>
<td>-1.665††</td>
<td>0.212*</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>-4.868†††</td>
<td>-1.688†</td>
<td>0.179*</td>
</tr>
<tr>
<td>Output gap</td>
<td>-4.971†††</td>
<td>-3.695†††</td>
<td>0.041*</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>-3.012††</td>
<td>-2.483††</td>
<td>0.189*</td>
</tr>
<tr>
<td>Asset price index</td>
<td>-4.927††</td>
<td>-3.610†††</td>
<td>0.045*</td>
</tr>
<tr>
<td>1% critical value</td>
<td>-3.465</td>
<td>-2.577</td>
<td>0.739</td>
</tr>
<tr>
<td>5% critical value</td>
<td>-2.877</td>
<td>-1.943</td>
<td>0.463</td>
</tr>
<tr>
<td>10% critical value</td>
<td>-2.575</td>
<td>-1.616</td>
<td>0.347</td>
</tr>
</tbody>
</table>

ADF= Augmented Dickey-Fuller (1979, 1981) unit root test, Elliott-Rothenberg-Stock (1996) DF-GLS test statistic (DF-GLS is more powerful and more recent); KPSS= Kwiatkowski, Phillips, Schmidt and Shin (1992) stationarity test. The Bandwidth selection procedure is used in the KPSS tests and, in this case, the autocovariances are weighted by the Bartlett Kernel. †††, ††, †, unit root is rejected at a significance level of 1%, 5%, 10% → stationarity. * stationarity is not rejected at a significance level of 10%. Besides, optimal lag length in these tests were selected using Modified Akaike Information Criterion (MAIC) with maximum lag order of 6.

5.2 Empirical Nonlinear Extended Taylor Reaction Functions for Canada

The results of this test show that, excluding the pre-inflation targeting period (1961:1 to 1990:4) when the output gap is considered as a threshold variable (where we find weak evidence to reject the linear model), the SupW tests reject the linear model at the 1% significance level (see Table 2.2), thereby pointing to asymmetric policy preferences.

In general, the results of the estimation are very interesting. First, the results show that the reaction function can be better modelled with a nonlinear model than with a linear model using the same variables. However, we find weak evidence to reject the linear model in the pre-inflation targeting period when the output gap is used as a threshold variable. Second, estimation results from this augmented Taylor rule give new relevant insights into
the influence of stock market index prices on monetary policy in Canada. These findings about the role of stock market index prices for the Bank of Canada provide insights regarding the opportunities and limitations of incorporating financial indicators in monetary policy decision making. They also give financial market participants, such as analysts, bankers and traders, a better understanding of the impact of stock market index prices on the Bank of Canada policy. Third, we find that over time, the Bank of Canada has assigned changing weights to inflation, the output gap and the stock market index price.

Table 2.2 (Panel A) and Table 2.3 (Panel B) present the results using inflation and the output gap as threshold variables respectively. For comparison, in each panel, we include the estimation using the total sample. For each panel, statistical significance is denoted by superscripts $a, b$ and $c$ meaning 1, 5, and 10% levels of significance, respectively. Panel A presents the estimates obtained when the inflation rate is considered as a threshold variable. The estimates of threshold values reflect clearly the evolution of this parameter through the different samples. The threshold value in the inflation targeting period (1991:2 - 2008:4) is close to the middle point of the target inflation band set by the Bank of Canada. The important point to note is that the coefficients on inflation, output gap and stock market index price are much greater in the high inflation regime than in the low inflation regime over the whole period as well as during the two subperiods. Moreover, the nominal interest rate smoothing coefficient is greater when inflation is low than when it is high. These results imply that the Bank of Canada is far more responsive to contemporaneous movements in inflation, output gap and stock market index price whenever inflation exceeds the threshold value. When inflation crosses the threshold into the high inflation regime in each subperiod, there is a switch in the behaviour of the Bank of Canada. Following Cukierman and Muscatelli (2007), these results indicate that in the first subperiod and in both regimes the coefficient of inflation is still positive and significant, implying that the
reaction function is convex in inflation. However, it is negative and not significant during
the first regime of the inflation targeting period but in the second regime it is still positive
and significant. A picture emerges with respect to the coefficient of the output gap which is
positive and significant in both subperiods and positive during the inflation targeting period.
These imply that the reaction function of the Bank of Canada is characterized by a convex
rule supporting inflation avoidance preferences on the part of the monetary policymakers. In
particular, observing the threshold value, it seems that the first subperiod is characterized
by a poor conduct of monetary policy. During periods of inflation stabilization in which
monetary policymakers are trying to build credibility up, they may be more averse to
positive than to negative inflation gaps of equal size. In the presence of uncertainty about
future shocks, this leads policymakers to react more vigorously to positive than to negative
inflation gaps.
Table 2.2: Estimation results for the empirical monetary reaction function using Inflation as the threshold variable.

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Inflation as the Threshold variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Estimates</td>
<td>7.480</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>[1.037 9.965]</td>
</tr>
</tbody>
</table>

**Regime 1**
- Inflation rate: 0.088\(^b\) (0.031) 0.075\(^b\) (0.035) -0.154 (0.167)
- Output gap: 0.169\(^b\) (0.063) 0.320\(^a\) (0.083) 0.112 (0.170)
- Stock Market Index: 0.014\(^a\) (0.004) 0.031\(^a\) (0.007) 0.010 (0.010)
- Lagged Interest rate: 0.926\(^a\) (0.021) 0.925\(^a\) (0.024) 1.041\(^a\) (0.073)

**Regime 2**
- Inflation rate: 0.174\(^b\) (0.071) 0.220\(^b\) (0.088) 0.284\(^a\) (0.042)
- Output gap: 0.357\(^a\) (0.084) 0.328\(^b\) (0.126) 0.151\(^a\) (0.043)
- Stock Market Index: 0.031\(^a\) (0.010) 0.033\(^a\) (0.014) 0.016\(^a\) (0.003)
- Lagged Interest rate: 0.835\(^a\) (0.076) 0.791\(^a\) (0.088) 0.761\(^a\) (0.023)

\(H_0 : \alpha = \beta\)
- \(\text{SupW (Statistic)}\): 129.608 165.552 516.034
- \(\text{SupW (P-value)}\): 0.000 0.000 0.000
- No. obs.: 154; 34 91; 25 44; 23

\(^a,b,c\) denotes significance levels at 1%, 5.0% and 10%, respectively. Standard errors robust to serial correlation (up to 6 lags) in parentheses.

Table 2.3 shows the results obtained when the output gap has been used as threshold. Estimations show that the Bank of Canada responds more strongly to inflation deviations when the output gap is above the threshold value but responds more weakly to stock market index prices in both subperiods. The results indicate that the \(\text{SupW} \) test cannot reject the linear model for the first subperiod, when there was no explicit target. Furthermore, the results show that in the first subperiod the coefficient of inflation and output gap are still positive and significant (except the coefficient of inflation in the first regime). This implies that the reaction function is convex in both inflation and the output gap. In the second subperiod, the coefficient of inflation is still negative in both regimes but significant only in
the first regime, implying that the reaction function is concave in inflation. The behaviour of the Bank of Canada depends on the threshold value. In fact, in the second subperiod, when the output gap is above the threshold value, the reaction function is characterized by a concave rule and it is followed by a convex rule when the output gap is below the threshold value. This means that the Bank of Canada worries more about the output gap in a recession period and cares more about inflation deviations during expansions.

Table 2.3: Estimation results for the empirical monetary reaction function using the Output gap as the threshold variable.

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Output gap as the Threshold variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Estimates</td>
<td>1.940</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>[-1.850 1.991]</td>
</tr>
<tr>
<td><strong>Regime 1</strong></td>
<td></td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.137 (0.032)</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.225 (0.064)</td>
</tr>
<tr>
<td>Stock Market Index</td>
<td>0.016 (0.005)</td>
</tr>
<tr>
<td>Lagged Interest rate</td>
<td>0.895 (0.027)</td>
</tr>
<tr>
<td><strong>Regime 2</strong></td>
<td></td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.226 (0.084)</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.227 (0.281)</td>
</tr>
<tr>
<td>Stock Market Index</td>
<td>0.037 (0.012)</td>
</tr>
<tr>
<td>Lagged Interest rate</td>
<td>0.879 (0.064)</td>
</tr>
</tbody>
</table>

H0 : \( \alpha = \beta \)

| SupW (Statistic) | 236.272 | 0.031 | 651.881 |
| SupW (P-value) | 0.000 | 0.999 | 0.000 |
| No. obs. | 173; 15 | 48; 68 | 7; 60 |

\( a,b,c \) denotes significance levels at 1%, 5.0% and 10%, respectively. Standard errors robust to serial correlation (up to 6 lags) in parentheses.
5.3 Derivation of the preference parameters

Equation (2.15) is jointly estimated with the system (2.14), generating estimates of the coefficients describing the monetary policy regime $\mu, \lambda, \delta$ and $\pi^*$ as well as of the aggregate demand, the aggregate supply and the dynamic evolution of the stock market prices coefficients. Because expectations are replaced by actual observations, estimation uses GMM, as it seems reasonable to assume that policymakers use efficiently the information available when forming expectations. As we want to obtain the preferences implied by the coefficients from the threshold regression model, the variables used are the fitted variables from the threshold regression model, including their lagged values. For comparison, in each estimation table, we include the estimation using the total sample. The discount factor $\beta$ is set to 0.975 for quarterly data, as is common in the literature (see Dennis, 2001; Favero and Rovelli, 2003; Rodriguez, 2008). Notice that the sample size constraints the number of instruments used in these cases and the estimates obtained are the best considering these restrictions.

Table 2.3 summarizes the results of estimation when the inflation rate is considered as the threshold variable. Firstly, the change observed in the value of $\pi^*$ among regimes reflects a successful monetary policy. On the other hand, the value of the coefficient $\mu$ indicates a reduction in the smoothing of interest rates between both regimes for the pre-inflation targeting period (1961:1-1990:4), while it indicates an increase for the targeting inflation period. The value of $\lambda$ implies a significant increase between between regimes of both subperiods but it shows a significant decrease when the inflation rate is above its threshold value in both subperiods. The value of $\delta$ indicates a significant reduction of the weight assigned to the financial market in the conduct of monetary policy for the second subperiod.

---

\footnote{Estimating $\beta$ together with the model parameters leads to a slightly lower value between 0.94 and 0.96 without changing the results. This accords with Favero and Rovelli (2003) who also found that qualitative results are not sensitive to variations in the discount factor.}
This means that the central bank uses an indirect mechanism, that of aggregate demand, to take into account the stock market in its decision making. Moreover, the standard deviations of aggregate demand and supply suggest that the economic conditions related to aggregate supply have been favourable in comparison with those related to aggregate demand. The standard deviations of the monetary rules indicate that monetary policy has been more successful in the inflation targeting period. In particular, observing this parameter, it seems that the pre-inflation targeting period has been characterized by a bad conduct of monetary policy.

Table 2.4 shows the results when the output gap is considered as the threshold variable. Some results are similar to those found in the case where inflation is considered as the threshold variable. For example, the value of $\pi^*$ indicates that the implicit target has been reduced significantly in the second subperiod. The value of $\lambda$ indicates a significant increase of the weight assigned to the output gap in the conduct of monetary policy between both subperiods. What is more interesting is that the standard deviations of the monetary rule is close to zero in the last subperiod, indicating that monetary policy has been successful in this subperiod. In contrast to the result in Table 2.3, the economic conditions related to aggregate demand have been favourable in comparison with those related to aggregate supply. Furthermore, the value of $\delta$ goes from zero to a significant positive one indicating that, in the pre-inflation targeting period, the monetary authority has given an important weight to the stock market index price in the conduct of monetary policy, while, in the inflation targeting period, the evidence suggests that the monetary authority does not give directly any weight to the stock market index price. The main difference in preferences for both subperiods across regimes relates to a higher preference for interest rate smoothing when the output gap is above its threshold value.

This analysis demonstrates the extreme sensitivity of the estimates to the different
variables used as threshold variable. It is particularly the cases for the parameters $\mu$, $\lambda$ and $\delta$. Another point is the fact that preferences of the monetary authorities have changed drastically in the inflation targeting period. It is clearly reflected in the estimates of $\pi^*$. We have found empirical support to an asymmetric behaviour of the Bank of Canada depending on actual economic conditions like the output gap level and the inflation deviation from its target level. Better macroeconomic conditions are also observed from the side of aggregate demand in comparison with those from aggregate supply only in the case when the output gap is considered as the threshold variable. The reverse situation is observed when the inflation rate plays the role of the threshold variable. The empirical evidence suggests, without any doubt, that monetary policy has been conducted efficiently in the last subperiod.

The intuition and policy implications become clearer if aggregate demand is affected by the evolution of asset prices; then the monetary authorities should include asset price fluctuations in their optimal feedback rule and there should be a change in the distribution of the relevant interest rate weights. This allows for asset prices to be considered as an element of the authorities’ reaction function without necessarily implying tighter policy since the response to inflation and output will be less aggressive. In other words, our results imply that first, asset prices should have an independent role instead of being considered as instruments to help forecast output and inflation; and second, there should be a shift in the magnitude of reaction, away from the traditional variables (inflation, output gap) and towards a direct response to financial instability.
Table 2.4: Estimates of the preferences of monetary policy. Estimates using Inflation as the Threshold variable

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>Regime 2</td>
<td>Regime 1</td>
<td>Regime 2</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.975</td>
<td>0.975</td>
<td>0.975</td>
</tr>
<tr>
<td>( \pi^* )</td>
<td>4.829</td>
<td>4.130</td>
<td>6.129</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-0.003</td>
<td>-0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.016</td>
<td>-0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.000</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>( \sigma(\underline{u}^d) )</td>
<td>0.676</td>
<td>0.694</td>
<td>0.803</td>
</tr>
<tr>
<td>( \sigma(\underline{u}^o) )</td>
<td>1.141</td>
<td>1.188</td>
<td>1.213</td>
</tr>
<tr>
<td>( \sigma(\underline{u}^a) )</td>
<td>15.799</td>
<td>14.999</td>
<td>15.514</td>
</tr>
<tr>
<td>( \sigma(\underline{w}^a) )</td>
<td>0.817</td>
<td>0.025</td>
<td>0.006</td>
</tr>
</tbody>
</table>

\( a,b,c \) denotes significance levels at 1%, 5.0% and 10%, respectively. \( e \) indicates that the coefficient has been imposed in the estimation. J -statistic reports Hansen’s test for over-identifying restrictions. Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation)
Table 2.5: Estimates of the preferences of monetary policy. Estimates using Output gap as the Threshold variable

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No Breaks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.975^e</td>
<td>0.975^e</td>
<td>0.975^e</td>
<td>0.975^e</td>
</tr>
<tr>
<td>(\pi^*)</td>
<td>2.410^e (0.073)</td>
<td>1.853^e (0.075)</td>
<td>3.783^e (0.051)</td>
<td>4.231^e (0.056)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-0.018^a (0.001)</td>
<td>0.029^a (0.001)</td>
<td>-0.003^a (0.000)</td>
<td>0.002^a (0.000)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0.004^a (0.001)</td>
<td>0.016^a (0.001)</td>
<td>0.001^a (0.000)</td>
<td>0.005^a (0.001)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.000^b (0.000)</td>
<td>0.001^b (0.008)</td>
<td>0.000 (0.001)</td>
<td>0.002^a (0.000)</td>
</tr>
<tr>
<td>(\sigma(u_d))</td>
<td>1.933</td>
<td>2.964</td>
<td>2.426</td>
<td>2.472</td>
</tr>
<tr>
<td>(\sigma(u_o))</td>
<td>0.670</td>
<td>0.647</td>
<td>0.634</td>
<td>0.626</td>
</tr>
<tr>
<td>(\sigma(u_s))</td>
<td>15.293</td>
<td>15.021</td>
<td>15.129</td>
<td>14.641</td>
</tr>
<tr>
<td>(\sigma(u_m))</td>
<td>0.651</td>
<td>0.123</td>
<td>0.017</td>
<td>0.051</td>
</tr>
</tbody>
</table>

The superscript a, b, c denotes significance levels at 1%, 5.0% and 10%, respectively. The superscript e indicates that the coefficient has been imposed in the estimation. J-statistic reports Hansen’s test for over-identifying restrictions. Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation).
6. Conclusion

This paper discusses two important issues. First, should the central bank care about the financial instability associated with large asset price fluctuations when setting interest rate? In other words, it asks whether the central bank should take into account asset price fluctuations when setting interest rates. Second, it investigates the view that, due to asymmetric central bank preferences, Taylor rules are often nonlinear and that the nature of those asymmetries changes over different policy regimes.

Using a threshold regression model, estimation results from this augmented Taylor rule give new relevant insights into the influence of stock market prices on monetary policy in Canada. These findings about the role of stock market prices on Canadian monetary policy provide relevant insights regarding the opportunities and limitations of incorporating financial indicators in monetary policy decision making. They also give financial market participants, such as analysts, bankers and traders, a better understanding of the impact of stock market prices on Bank of Canada policy. This is not the case in the U.S. (see Castro, 2008). Indeed, it would seem that the Fed leaves those markets free from any direct control. This difference in the behaviour of the two central banks might be one of the causes for the credit crunch that arose recently in the US housing market and that affected the real economy, with important repercussions in the world economy, but to which Canada remained less exposed. Thus, the first main contribution of this paper is that targeting financial conditions might be a solution to avoid the financial and asset market instabilities and, consequently, to help to avoid sharp economic slowdowns. However, we acknowledge that it may be difficult to interpret asset price movements and distinguish between fundamental and non-fundamental components, but the same type of uncertainty exists when policy makers are faced with stochastic trend output. Hence, there is scope for the monetary authorities to take into account asset price fluctuations in the conduct of monetary policy.
despite the measurement errors that they might face. Excluding the pre-inflation targeting period in the case when the output gap is considered as the threshold variable, the evidence supports the existence of nonlinearities in interest rate reaction functions under all subperiods. Hence, the reaction function can be better modelled with a nonlinear model than with a linear model using the same variables. We conclude that the augmented nonlinear Taylor rule is the policy rule that best represents the Bank of Canada’s behaviour.

To derive the central bank preferences, we use a loss function that includes an additional stock market index price target. The preferences of monetary policy are estimated jointly with the structural parameters in the model. There is strong evidence of change in the central bank’s target rate of inflation towards the end of the 1970s. In the pre-inflation targeting period, we estimate the target rate of inflation to be between 5.37 and 6.13 percent per annum when inflation is considered as the threshold variable; and to be between 3.78 and 4.25 percent per annum when the output gap is considered as the threshold variable. Since the inflation targeting period, the target rate of inflation seems to be between 1.78 and 1.99 percent per annum when inflation is considered as the threshold variable; and to be around 1.39 and 1.95 percent per annum when the output gap is considered as the threshold variable. This implies, without any doubt, that monetary policy has been conducted efficiently in the last subperiod. We find strong statistical support for this decline and the result is consistent with previous findings by Favero and Rovelli (2003) and Rodriguez (2008). Whether the relative weight that the central bank gives to output stabilization has increased (fallen) since the inflation targeting period becomes certain when inflation (the output gap) is considered as the threshold variable. Our estimates for \( \lambda \) and \( \delta \) lead us to conclude that the central bank does care about output and stock market price stabilization (in addition to inflation stabilization). Better macroeconomic conditions are also observed from the side of aggregate demand in comparison with those from aggregate supply only in the case when the output
gap is considered as the threshold variable, while, the reverse situation is observed when
the inflation rate plays the role of the threshold variable. An natural extension of this
research would be to make out of sample forecasts of interest rates, during the 2009-2011
crisis period, on the basis of our asymmetric augmented reaction function.
Appendix A: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Interest rate</th>
<th>Inflation rate</th>
<th>Stock Market Index price</th>
<th>Output gap</th>
<th>Exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.0457</td>
<td>4.1552</td>
<td>6.5498</td>
<td>0.0000</td>
<td>-0.2128</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>3.6075</td>
<td>2.9879</td>
<td>17.1211</td>
<td>1.3724</td>
<td>5.1067</td>
</tr>
<tr>
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Appendix B: Correlation Matrix

<table>
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<th>Interest rate</th>
<th>Inflation rate</th>
<th>Stock Market Index price</th>
<th>Output gap</th>
<th>Exchange rate</th>
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<td>Interest rate</td>
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<td></td>
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<tr>
<td>Inflation rate</td>
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<td>1.0000</td>
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<td>Stock Market Index Price</td>
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<td>Exchange rate</td>
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<td>-0.0955</td>
<td>0.1888</td>
<td>0.1387</td>
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</table>
Essay 3

Inflation and Economic Growth with Cross-Section Dependency: An Empirical Analysis in Terms of Relative Price Variability
1. Introduction and Motivation

One of the objectives of macroeconomic policies is to achieve high and sustainable economic growth rates along with low, stable and predictable inflation rates. For that reason, the relationship between economic growth and inflation is fundamental for policymakers. If these two variables are interrelated, then policymakers would like to control them depending on the structure of such relationship in order to reach policy targets. Considering its importance, the inflation-growth relationship has attracted much attention from economists, both in academia and in the world of central banking.

Several theoretical and empirical studies explored this issue and reached several conclusions. Indeed, if one agrees that the origins of inflation are triple (money, cost and demand), the channels through which it affects economic growth are subject to controversy. There are three possible results regarding the impact of inflation on economic activity: i) none; ii) positive; and iii) negative. Sidrauski (1967) established the first result, showing that money is neutral and superneutral\(^1\) in an optimal control framework, claiming that an increase in the inflation rate does not affect economic growth. Tobin (1965), who assumed money is as substitute to capital, established the positive impact of inflation on growth; his result is being known as the Tobin effect. The negative impact of inflation on growth is associated mainly with cash-in-advance models (e.g., Stockman, 1981) which consider money as complementary to capital. Clearly, inflation could have different impacts on economic growth. Indeed, inflation imposes negative externalities on the economy when it interferes with an economy’s efficiency. Inflation can lead to uncertainty about the future profitability of investment projects. This leads to more conservative investment strategies than would

\(^1\)Money is neutral when an increase in money supply leads to an equal increase in all prices and no real variables are affected. Money is superneutral when changes in the growth rate of money supply have no effect on the real variables of the economy (Romer, 1996). This role of money is very close to the one described by the quantity theory of money in the long run (see Lucas, 1996).
otherwise be the case. Higher inflation may also reduce a country’s international competitiveness, by making its exports relatively more expensive, thus impacting on the balance of payments. Moreover, inflation can interact with the tax system to distort borrowing and lending decisions.

As important as the possible relation between inflation and growth may be, Ragan’s (2000) review of the research shows the difficulty in finding clear and convincing evidence that a linkage exists. Following Blanchard et al. (2010), the effects of inflation on growth are difficult to discern, so long as inflation remains in the single digits. As a consequence, they suggest that an inflation target of 4 percent might be more appropriate because it leaves more room for expansionary monetary policy in case of adverse shocks. What is clear is that, in the countries that have experienced very high inflation, there is a significant and negative effect on economic growth.² What is much less clear from the empirical research is whether there exists any relationship between economic growth and inflation in countries that already have low, or even moderate, inflation. According to Ragan (2005), in a country with an inflation rate of, say, 5 percent per year, it is not clear from the data that a policy decision to reduce inflation would have any positive impact on long-run economic growth. We assume that this result depends on particular assumptions about the shape of the estimated model (linear or nonlinear model) or of the policy variable considered in the model (inflation target or relative price variability) or the nature of the data used. Therefore, following the earlier literature and recent studies at the Bank of Canada which suggest that price level targeting is a welfare-improving policy relative to inflation targeting, besides looking at the relation between inflation and economic growth, we investigate the relationship between relative price variability and economic growth. We also assume a nonlinear model specification.

²When inflation becomes very high, money is losing its value so quickly that it soon ceases to be useful as either a medium of exchange or as a store of value.
This has given a renewed interest in the debate on relative price variability. Variability in relative prices is known to be a major channel through which inflation can induce welfare costs by impeding an efficient allocation of resources in the economy. As Friedman (1977) made clear in his Nobel lecture, relative price variability is a direct means by which inflation can induce welfare-diminishing resource misallocation. Since the influential paper by Parks (1978), several studies, using Parks’s (1978) framework, have provided evidence in favour of a significant impact of inflation on relative price variability. Consequently, substantial effort has been devoted to the examination of the link between relative price variability (RPV) and aggregate inflation. Although much of the existing theoretical and empirical literature points to a positive monotonic relationship, newer contributions suggest that the relationship between inflation and RPV is more complicated, particularly in terms of its sensitivity to the inflation regime.

In Canada, the dual problem of the relationship between inflation and growth and between inflation and relative price variability has given rise to few empirical investigations. Monetary authorities have an ongoing interest in the relationship between inflation and economic growth through relative price variability in Canada. Binette and Martel (2005) investigate empirically the relationship between different aspects of inflation and relative price dispersion in Canada using a Markov regime-switching Phillips curve. They observe a strong asymmetry regarding the impact of positive and negative unexpected inflation on relative price dispersion using total inflation, but this asymmetry is not observed for core inflation. This suggests that the strong asymmetry arises mainly from the presence of components typically associated with supply shocks, and not from the presence of downward nominal rigidities. Omay and Kan (2010) analyze the empirical relationship between inflation and output growth using a panel smooth transition regression model for six in-

dustrialized countries including Canada. Their model takes into account the control of cross-section dependency and unobserved heterogeneity at both country and time levels. They find a stronger negative relationship between inflation and growth for inflation rates above a critical threshold level (2.5%).

Our paper aims to improve our understanding of this relationship. Our work differs from the recent literature in the following way: Rather than asking how well inflation or relative price variability affect economic growth for various countries, as has been done in previous empirical research on this topic, we ask how this relationship holds across Canadian provinces. On the one hand, it is not expected that the analysis would hold at the provincial level since provinces respond to the same monetary policy and have the same central bank. It is possible that different provinces are hit by different shocks and therefore relative prices have to adjust. Another possibility is that provinces react differently to common shocks. The Bank of Canada is committed to aggregate inflation and has one instrument for this purpose. Even if there are deviations from the target in some provinces, the Bank cannot react to all of them, due to the limitations of the one instrument-one target link, as long as overall inflation is not affected. By analyzing provincial data, we take a look at possible asymmetries in the economic structure of provinces. We are not the first to analyze provincial disparities in Canada (Chaban and Voss, 2010). What is novel in our approach is to use the dynamics panel data with cross-section dependence to improve the design of monetary policy in Canada.

If the inflation-targeting framework in Canada is believed to increase the credibility of monetary policy, to enhance the anchoring of expectations of future inflation, and to reduce uncertainty in decision making by economic agents, the analysis of the effect of inflation on economic growth in Canada raises two recurring questions. On the one hand, what is important, the threshold of inflation, or relative price variability? On the other hand, as
the side effects of inflation on output may be different depending on the province due to the heterogeneity of the transmission mechanism of shocks (Fernandez Valdovinos and Gerling, 2011), to what extent do the Canadian monetary authorities worry about the contagion effects due to inflation in one of the provinces?

Most of previous empirical studies focus on using aggregate level data without cross-section dependence to investigate the relationship between inflation, relative price variability and economic growth. The study of the effects of inflation and relative-price variability on the real side of the economy is a clear prerequisite for determining the welfare and efficiency implications of the inflationary process. Moreover, some of these studies make use of time series data which is supposed or known to yield unreliable and inconsistent results due to the low power of the unit root test. The use of panel data may increase the sample size allowing for more accurate and reliable statistical tests. There are very few empirical works on this topic that have made an attempt to gain statistical power through the pooling of information across units. The few that have, have neglected to account for the presence of cross-section dependencies of the data. As has been shown in the literature, failure to adequately account for the presence of cross-section dependence in panel data study could lead to a serious bias problem.\textsuperscript{4}

This paper contributes to the literature by providing new evidence on the relation between inflation, relative price variability and economic growth to a panel of Canadian provinces over the period 1981-2008. The major purpose of this paper is to econometrically assess the effects of provincial inflation and relative-price variability on economic growth by taking into consideration the assumption of cross-section dependence. To the best of our knowledge, there has never been an attempt to investigate the relationship between these important macroeconomic variables by employing disaggregated level panel data and con-

\textsuperscript{4}Andrews (2005), Pesaran (2006) and Bai and Ng (2010).
sidering the presence of cross-section dependence. This paper tries to fill this gap. In this spirit, we argue that the investigation of the relationship between inflation and economic growth carried out in this paper may be of great importance for monetary authorities to better understand whether it is possible to improve the design of monetary policy in Canada given the presence of heterogeneity across the provinces and thereby contribute to achieve its primary main goal (solid economic performance and rising living standards for Canadians by keeping inflation low, stable, and predictable).

The remainder of the paper proceeds as follows: Section 2 presents a brief literature review and section 3 describes the data used in the current study. Section 4 outlines the econometric methodology. Section 5 analyzes results of the relationship between the inflation rate and economic growth while section 6 examines the results of the impact of inflation variation on economic growth. Section 7 provides a new technique which eliminates cross-section dependency from the non-linear panel estimation. Section 8 concludes.

2. Related Literature

The issue of the nature of the relationship between the levels of inflation and output growth has been one of the most researched topics in macroeconomics both on the theoretical and empirical front. Economists and policymakers need a clear understanding of the major channels through which inflation may affect the real economy, if they aim to minimize the adverse economic consequences and welfare costs of increases in the price level.

2.1 The link between inflation and relative price variability

The relationship between inflation rates and relative price variability varies across different classes of models. Firstly, menu-cost models (Rotemberg, 1983) predict inflation increases relative price variability, distorts the information content of prices, and, thereby, impedes the efficient allocation of resources. Secondly models which incorporate the signal extraction
mechanism of Lucas (1973, 1994) predict a non-negative relationship between RPV and the absolute value of unanticipated inflation. This group includes Barro (1976), Hercowitz (1981) and Cukierman (1983). On the theoretical side, several different types of model can be used to interpret a correlation between RPV and aggregate inflation. Thirdly, the search models state that consumers accumulate information only on a subset of existing prices, but because of the deterioration in consumers’ price information during inflationary periods, the stock of information a person holds declines and consequently the dispersion of prices widens (Caglayan and Filiztekin, 2003).

On the empirical side, Vining and Elwertowski (1976) have shown that the variance of the inflation rate and relative price variability are positively related over time for postwar U.S. data. Parks (1978), Fischer (1981), and others have regressed relative price variability on linear and either quadratic or absolute value terms involving the inflation rate, unanticipated inflation, or the change in the inflation rate. Following menu cost models, most empirical work on the relationship between inflation and relative price variability typically focuses on linear regressions of relative price variability on inflation. Recent researchers, namely Parsley (1996), Debelle and Lamont (1997), Aarstol (1999) and others find a significant positive impact of inflation. However, Lastrapes (2006) find that the relationship between US inflation and relative price variability breaks down in the mid-1980s, whereas Reinsdorf (1994) proves that this relationship is negative during the desinflationary period of the early 1980s. A first attempt to analyze the Canadian inflation relative price variability nexus is provided by Amano et al. (1997), who examine the empirical support for the predictions of the menu-cost model using Canadian data. They find that the Canadian data, both in the context of partial correlations and standard price Phillips curve equations, are highly supportive of the predictions that arise from the menu-cost models. Lending support to search models, for example, more empirical studies suggest that the link between inflation
and relative price variability is nonlinear. Several studies find that the impact of inflation on relative price variability changes between high and low inflation periods and between countries with different inflationary situations (Caglayan and Filiztekin, 2003; Becker and Nautz, 2009 and Choi, 2010). Recently, others studies apply panel threshold models and find evidence of threshold effects in the U.S. (Bick and Nautz, 2008) and in the European area (Becker and Nautz, 2010).

Staff at the Bank have an ongoing interest in the relationship between inflation and relative price variability in Canada. Since the early 1990s, Canada has had lower trend inflation and lower inflation uncertainty. Research on the effects of these changes on relative price variability can provide evidence of the welfare cost of inflation (as described by Friedman 1977).

2.2 Inflation and economic growth

The relationship between inflation and economic growth is perhaps one of the most investigated yet controversial issues in macroeconomics on both theoretical and empirical grounds. Several theoretical studies argued that depending on its level, inflation can either promote or harm economic growth. For instance, Lucas (1973) explained that inflation allows to overcome the rigidity of nominal prices and wages. In addition, inflation can realign relative prices in response to structural changes in production during fast modernization periods. In this case inflation is quite important for economic growth. Another strand of the literature argued that inflation is detrimental for output growth in the long run. Inflation might affect output growth negatively through different channels. Friedman (1977) argues that higher inflation rates may cause a reallocation of scarce resources to unproductive activities and thus reduce output growth. Furthermore, faster inflation increases inflation uncertainty and distorts economic efficiency and thus reduces employment.

Existing empirical studies reflect different views on the relationship between inflation and
economic growth. The findings differ depending on data periods and countries, suggesting that the linkage between inflation and growth is not stable. On the one hand, using a panel for 100 countries over the period 1960-1990, Barro (1995) found clear evidence that a negative relationship exists only when high inflation data is included in the sample. But there is not enough information to argue that the same conclusion holds for lower inflation rates. The linearity of the relation between inflation and growth has been questioned by the empirical growth literature. Still, economists now widely accept the existence of a nonlinear and concave relationship between these two variables. They show that the link between inflation and growth is significant only for certain levels of inflation. Based on cross-country growth regressions, the relevance of inflation thresholds in the inflation-growth nexus has already been suggested by Fisher (1993) and Bruno and Easterly (1998). Indeed, Fisher (1993) was the first to identify a non-linear relationship where low inflation rates have a positive impact on growth, an impact that turns negative as inflation increases. Bruno and Easterly (1998) confirm the finding of a negative effect for high inflation rates but doubt the growth-enhancing effect of low inflation. They argued that in this case inflation and growth are influenced jointly by different demand and supply shocks and thus no stable pattern exists. In these papers, the inflation threshold is not estimated but imposed exogenously. More recent contributions adopt panel econometric techniques where the number and location of thresholds are not imposed but estimated from the data. For instance, Ghosh and Philips (1998) found that although inflation and growth are positively correlated at very low inflation rates (about 2 to 3 percent a year), the relationship is reversed at higher rates. Furthermore, the relationship is convex, so that the decline in growth associated with an increase in inflation from 10 to 20 percent is much larger than that associated with an increase from 40 to 50 percent. Following Blanchard, et al. (2010), the effects of inflation on growth are difficult to discern, so long as inflation remains in
the single digits. As a consequence, they suggest that an inflation target of 4% might be more appropriate because it leaves more room for expansionary monetary policy in case of adverse shocks.

In Canada, Talan and Osberg (1998) and Vitek (2002) are among the first to address the relationship between inflation and economic growth. Talan and Osberg (1998) examine the relationship between the level of inflation and sectorial output growth variability. They main finding is that during this period there was, overall, no significant relationship between output growth variability and inflation in Canada. Vitek (2002) conducts an empirical investigation of dynamic interrelationships among inflation, inflation uncertainty, relative price dispersion, and output growth within a trivariate GARCH-M model. One limitation of these studies is that they have used the sectoral output growth that are not necessarily representative of the overall economy. From a social welfare perspective, the important issue is whether inflation affects output variability. Hence, in this paper we examine the link between economic growth and inflation using data of the Canadian economy.

2.3 Relative price variability and economic growth

Variability in relative prices is known to be a major channel through which inflation can induce welfare costs by impeding an efficient allocation of resources in the economy. Consequently, substantial effort has been devoted in the literature to examine the link between relative price variability (RPV) and aggregate inflation. Higher inflation uncertainty induces higher relative price variability, which, in turn, generates more variability in investment and output growth. Friedman (1977) argues that the higher the variability of inflation, the harder it becomes to extract information about relative prices from absolute prices. In his Nobel Lecture, Friedman (1977) argued that inflation may have a negative effect on output growth by increasing inflation uncertainty. Therefore, by reducing economic efficiency, greater relative price variability reduces economic growth. Blejer and Leiderman (1980)
evaluate the effect of relative price variability on economic activity in the US economy and find that increased relative price variability is correlated with a decrease in the level of output and employment. Fisher (1993) argued that inflation hampers the efficient allocation of resources due to harmful changes of relative prices. Relative prices appear to be one of the most important channels in the process of efficient decision-making.

The link between RPV and economic growth is explained by the theory of rational expectations and incomplete information models. According to these two theories, a large total output and employment volatility implies greater price variability. Indeed, using an econometric model consistent with sticky prices and rational expectations, Taylor (1979) found a second-order Phillips curve tradeoff between fluctuations in output and fluctuations in inflation rates. He argued that this tradeoff is downward sloping, and concluded that business cycle fluctuations could be reduced only by increasing inflation variability. Cecchetti and Ehrmann (1999) argued that aggregate shocks create a tradeoff between output and inflation variability. In addition, a large variability in the price level is likely to generate monetary policies the consequences of which would be to increase uncertainty over inflation, thus affecting economic growth.

3. Data and Preliminary Analysis

3.1 The Data Set

3.1.1 Data construction and classification

In this paper, we consider annual data from ten Canadian provinces.\(^5\) All data are (if necessary) seasonally adjusted and directly collected from Cansim (Statistics Canada) and the Bank of Canada. They cover the period 1981 to 2008; thus they include 28 years which means we have 280 points. We begin modelling the relative price variability-inflation

\(^5\)Newfoundland and Labrador, Nova Scotia, New Brunswick, Prince Edward Island, Quebec, Ontario, Manitoba, Saskatchewan, Alberta and British Columbia.
relationship by estimating a balanced panel data model for relative price variability ($r_{vit}$) using inflation ($\pi_{it}$). The relative price variability ($r_{vit}$) is then constructed by the weighted average of subaggregate inflation for double-digit consumer price sectors using the standard deviation. The inflation ($\pi_{it}$) is measured as a percentage change of consumer price index. The other explanatory variables include openness and investment. Openness ($opens_{it}$), a measure of international trade, is believed to affect growth through several channels, such as access to technology from abroad, greater access to a variety of inputs for production and access to broader markets that raise the efficiency of domestic production through increased specialization. In this paper, we define openness as the ratio of exports of goods and services to GDP. We measure the level of investment in an economy as gross fixed capital formation as a share of GDP. The investment ratio ($igdp_{it}$) is generally considered to be the best variable measure of the level of investment in cross country studies since this ratio accounts for country size. Economic growth ($dgdp_{it}$) is measured as an annual percentage growth rate of GDP.

Having constructed the data we can now separate them into the different states by simply introducing the threshold effects. For the threshold value, we assume two alternatives: a known (exogenous) value and an unknown (endogenous estimate). The summary statistics of these different states together with those for each threshold and the linear specification are given in Table 3.1.

The descriptive statistics show that economic growth is lower if inflation is above target and it becomes higher if inflation is below this target. As far as the relative price variability is concerned, it is higher if inflation is higher. So accordingly we find that lower inflation means high economic growth and lower relative price variability which could mean low inflation leads to an improvement of economic growth.
Table 3.1: Descriptive Statistics

<table>
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<th>Variables</th>
<th>Linear</th>
<th>Endogenous threshold $\gamma_\pi \in [1.61% ; 3.32%]$</th>
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<td></td>
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<td>$d_1$</td>
</tr>
<tr>
<td>$dgd\rho$</td>
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<td>0.088</td>
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<td>$dgd\rho_{min}$</td>
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<td>-0.009</td>
</tr>
<tr>
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<tr>
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<tr>
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<td>0.061</td>
</tr>
<tr>
<td>$\tau p\nu_{min}$</td>
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<td>0.005</td>
</tr>
<tr>
<td>$\overline{\pi}$</td>
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<td>0.011</td>
</tr>
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<tr>
<td>$\pi_{min}$</td>
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<td>-0.005</td>
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<td>$op\epsilon$</td>
<td>0.515</td>
<td>0.541</td>
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<td>$\sigma_{op\epsilon}$</td>
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<td>0.038</td>
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<tr>
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<td>0.314</td>
</tr>
<tr>
<td>$lgd\rho_{min}$</td>
<td>0.13</td>
<td>0.143</td>
</tr>
<tr>
<td>$N$</td>
<td>280</td>
<td>45</td>
</tr>
</tbody>
</table>

Note. $\overline{\pi}$ stands for the mean of the respective variable, $x_{max}$ and $x_{min}$ for the maximum and minimum realization, while $\sigma_x$ is the standard deviation, $N = \text{number of observations}$. $d_1$ represents $\pi_{it} \leq 1.61$ percent, $d_2$ is when $1.61 < \pi_{it} \leq 3.32$ and $d_3$ corresponds $\pi_{it} > 3.32$. 
Canadian inflation has been low and stable over the last years. Since 1991 the average inflation rate across Canadian provinces has fluctuated around 2 percent. Figure 3.1 further displays the minimum and the maximum of province-specific inflation rates, indicating that inflation in Canadian provinces exceeded 6 percent and went below zero, at least for some provinces in some periods. This illustrates that inflation differentials between Canadian provinces have been modest but far from negligible. Typically, inflation rates varied in a range between 1 percent to 3.5 percent. The persistence of inflation differentials between Canadian provinces has been relatively low. Given the observed inflation differentials across provinces, it is an important feature of the panel threshold model that at each point of time different provinces are allowed to be in different inflation regimes. Figure 3.2 reveals more information about the distribution of the provincial inflation rates from a timeless perspective. In particular, 25 percent of the observed inflation rates were below 1.5 percent or above 3.5 percent.

Figure 3.1: Inflation Rates Across Canadian Provinces
Notes: Minimum, mean and maximum of yearly CPI inflation rates of 10 Canadian Provinces from 1991 to 2008. Source: Statistics Canada

Figure 3.2: Distribution of Provincial Inflation Rates in Canada
Note: Yearly CPI inflation rates of 10 Canadian Provinces from 1981 to 2008. Source: Statistics Canada
Finally, Figure 3.3 displays the relationship between inflation and economic growth across Canadian provinces. It is clear that the relation between the two variables is not linear and potentially depends on the economic states.

Figure 3.3: Relationship between Inflation and Economic growth across Canadian provinces

### 3.1.2 Correlation Analysis

Suspecting strong collinearity between some regressors, Table 3.2 reports the pairwise correlation coefficients between all the candidate variables of the models. As can be seen, economic growth is strongly negatively correlated with the inflation rate and relative price variability; but it is positively correlated with trade openness. And the correlation between relative price variability and growth is smaller than that between inflation and growth,
suggesting that economic growth is best explained by inflation. Table 3.2 also shows that inflation and relative price variability are strongly positively correlated at the provincial level, suggesting that aggregate shocks (such as monetary factors) are not entirely responsible for the correlation.

Table 3.2: Correlation Matrix across Canadian Provinces, 1981-2008

<table>
<thead>
<tr>
<th>Variables</th>
<th>DGDP</th>
<th>IGDP</th>
<th>INF</th>
<th>RPV</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGDP</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IGDP</td>
<td>-0.011(0.86)</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INF</td>
<td>-0.354a(0.00)</td>
<td>-0.007(0.90)</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPV</td>
<td>0.197a(0.00)</td>
<td>0.117b(0.05)</td>
<td>0.380a(0.00)</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>OPEN</td>
<td>0.102c(0.08)</td>
<td>0.192a(0.00)</td>
<td>-0.357a(0.00)</td>
<td>-0.033(0.59)</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: The value in the parentheses is p-value. a, b and c a significance level of 1%, 5%, and 10%.

3.1.3 Relative Price Variability

To construct our measure of Relative Price Variability (RPV), we follow the recent literature and use the measure proposed by Parks (1978). Our measure of RPV for province $i$ ($i = 1, 2, \ldots, 10$) in period $t$ from 1981 to 2008 ($t = 1, 2, \ldots, 28$) is calculated as the square root of the weighted sum of square deviations of subcategory inflation around the average inflation for province $i$:

$$RPV_{it} = \sqrt{\sum_{h=1}^{10} \omega_{iht} (\pi_{iht} - \pi_{it})^2}$$

(3.1)

where $\pi_{iht} = \ln(P_{iht}) - \ln(P_{iht-12})$ is the yearly inflation rate for subcategory $h = 1, 2, \ldots, 10$ and $P_{iht}$ is the level of price index of the $h^{th}$ subcategory in province $i$ in period $t$. $\omega_{iht}$ denotes the province-specific weight of the $h^{th}$ subcategory in the aggregate index so that
\[ P_{it} = \sum_{h=1}^{10} \omega_{ih} P_{ih} \] gives the aggregate price level in province \( i \) and in period \( t \). Hence, the inflation rates are defined as the annualized quarterly change in the relevant seasonally adjusted consumption deflator from 1981 to 2008: \[ \pi_{it} = \ln(P_{it}) - \ln(P_{it-12}). \]

### 3.2 Preliminary Data Analysis

#### 3.2.1 Panel unit root tests

All the asymptotic theories for the threshold Regression Models are for stationary regressors (see Hansen, 1996, 1999, 2000). Therefore, our specification procedures rely on the assumption that output growth, relative price variability, inflation, investment, and openness to trade are \( I(0) \) processes. In order to analyze the stationarity proprieties of the data, prior to the estimation of the linear model, we first investigate whether or not the variables appear to contain panel unit roots. Non-stationary panels have become extremely popular and have attracted much attention in both theoretical and empirical research over the last decade. A number of panel unit root tests have been proposed in the literature which include Levin et al. (2002), Breitung (2000), Im et al. (2003), Maddala and Wu (1999) and Choi (2001).

The Breitung (2000) and Levin et al. (2002) panel unit root tests assume a homogeneous autoregressive unit root under the alternative hypothesis whereas Im et al. (2003) allows for a heterogeneous autoregressive unit root under the alternative hypothesis. Fundamentally, the Im et al. (2003) test averages the individual augmented Dickey-Fuller (ADF) test statistics. Both the Levin et al. (2002) and Im et al. (2003) tests suffer from a dramatic loss of power when individual specific trends are included, which is due to the bias correction. However, the Breitung (2000) panel unit root test does not rely on bias correction factors. Monte Carlo experiments showed that the Breitung (2000) test yields substantially higher power and smallest size distortions compared to Levin et al. (2002) and Im et al. (2003). Maddala and Wu (1999) and Choi (2001) suggest comparable unit root tests to be performed
using the non-parametric Fisher statistic. The Fisher type test neither requires a balanced panel nor identical lag lengths in the individual regressions. The downside of the Fisher test is that the probability values need to be drawn from Monte Carlo simulations. Maddala and Wu (1999) argued that a Fisher type test with bootstrapped probability values is also an excellent choice for testing cointegration in addition to non-stationarity tests in panels. Table 3.3 displays the results of panel unit root tests in levels for all the variables. All tests reject the null hypothesis of a unit root in the examined series. As regards to openness to trade and investment, the tests failed to reject the null hypothesis of unit root. According to Omay and Kan (2010), this result may be due to the fact that the tests have a low power against nonlinear stationary process. From the nonlinear unit root test, we can conclude that all the variables in the paper are $I(0)$.

Table 3.3: Panel Unit Root Test Results

<table>
<thead>
<tr>
<th></th>
<th>INFL</th>
<th>RPV</th>
<th>OPEN</th>
<th>IGD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intercept</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levin, Lin and Chu</td>
<td>-10.42$^a$(0.00)</td>
<td>-8.41$^a$(0.00)</td>
<td>-1.72$^b$(0.04)</td>
<td>2.20(0.98)</td>
</tr>
<tr>
<td>Breitung</td>
<td>0.99(0.84)</td>
<td>-3.06$^a$(0.00)</td>
<td>2.90(0.99)</td>
<td>-0.46(0.32)</td>
</tr>
<tr>
<td>Im, Pesaran and Shin</td>
<td>-11.02$^a$(0.00)</td>
<td>-8.43$^a$(0.00)</td>
<td>0.52(0.69)</td>
<td>0.50(0.69)</td>
</tr>
<tr>
<td>Fisher-ADF</td>
<td>141.60$^a$(0.00)</td>
<td>115.41$^a$(0.00)</td>
<td>12.85(0.88)</td>
<td>25.67(0.18)</td>
</tr>
<tr>
<td>Fisher-PP</td>
<td>196.30$^a$(0.00)</td>
<td>154.48$^a$(0.00)</td>
<td>12.63(0.89)</td>
<td>14.73(0.79)</td>
</tr>
<tr>
<td><strong>Intercept + trend</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levin, Lin and Chu</td>
<td>-4.81$^a$(0.00)</td>
<td>-10.96$^a$(0.00)</td>
<td>1.22(0.89)</td>
<td>-0.34(0.39)</td>
</tr>
<tr>
<td>Breitung</td>
<td>1.39(0.92)</td>
<td>-1.32$^c$(0.09)</td>
<td>-5.16$^a$(0.00)</td>
<td>1.69(0.95)</td>
</tr>
<tr>
<td>Im, Pesaran and Shin</td>
<td>-5.88$^a$(0.00)</td>
<td>-9.28$^a$(0.00)</td>
<td>-1.41$^c$(0.08)</td>
<td>-0.62(0.27)</td>
</tr>
<tr>
<td>Fisher-ADF</td>
<td>77.90$^a$(0.00)</td>
<td>107.51$^a$(0.00)</td>
<td>25.77(0.17)</td>
<td>24.59(0.22)</td>
</tr>
<tr>
<td>Fisher-PP</td>
<td>115.93$^a$(0.00)</td>
<td>125.32$^a$(0.00)</td>
<td>15.44(0.75)</td>
<td>29.08$^c$(0.69)</td>
</tr>
</tbody>
</table>

Note: Figures in square brackets are probability values. $a$, $b$, and $c$ represent significance at 1%, 5%, and 10% respectively. The maximum number of lags is set to be three. AIC is used to select the lag length. The bandwidth is selected using the Newey-West method. Barlett is used as the spectral estimation method.
3.2.2 Panel Causality Tests

As our variables are stationary (inflation, relative price variability and economic growth), to determine the direction of causality between them, we use simple Granger causality test.\textsuperscript{6} Table 3.4 displays the results of Granger-causality tests. It is interesting to note that when the causal relation between inflation and growth or inflation and relative price variability are investigated, the existence and nature of the relation changes depending on the inflation regime. With the full sample (no regime), results reveal that there is a double directional Granger causality involving both inflation and economic growth on the one hand and inflation and relative price variability on the other hand. However, in the low inflation regime, there is a unidirectional Granger causality running from both growth to inflation and relative price variability to economic growth, while there is double directional Granger causality running from both inflation to relative price variability. In the moderate inflation regime, the direction of causality in the case of inflation and growth changes. There is a unidirectional Granger causality running from economic growth to inflation but growth and to relative price variability remaining the same. The most surprising finding is that, in the high inflation regime there is a double directional Granger causality running from inflation to economic growth and from relative price variability to economic growth respectively.

\textsuperscript{6}According to Granger’s definition of causality, a stationary time series $Y_t$ is said to ‘cause’ another stationary time series $X_t$ if – under the assumption that all other information is irrelevant – the inclusion of past values of $Y_t$ significantly reduces the predictive error variance of $X_t$. In econometric practice, Granger-causality tests are carried out by regressing $X_t$ on its own lags and on lags of $Y_t$. If the lags of $Y_t$ are found to be jointly statistically significant, then the null hypothesis that $Y_t$ does not Granger-cause $X_t$ can be rejected.
### Table 3.4: Panel causality tests results.

<table>
<thead>
<tr>
<th>Inflation regime</th>
<th>Source of causation</th>
<th>Growth</th>
<th>Inflation</th>
<th>RPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth</td>
<td></td>
<td>2.659b</td>
<td>0.516</td>
<td>1.331</td>
</tr>
<tr>
<td>growth</td>
<td></td>
<td>(0.034)</td>
<td>(0.724)</td>
<td>(0.259)</td>
</tr>
<tr>
<td>no regime</td>
<td>inflation</td>
<td>4.337a</td>
<td>5.045a</td>
<td>2.877</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.000)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>d1</td>
<td>growth</td>
<td>2.043c</td>
<td>0.534</td>
<td>3.431</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
<td>(0.711)</td>
<td>(0.009)</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>1.086</td>
<td>3.826</td>
<td>3.401</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.364)</td>
<td>(0.005)</td>
<td>(0.010)</td>
</tr>
<tr>
<td></td>
<td>rpv</td>
<td>3.431</td>
<td>3.401</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.010)</td>
<td></td>
</tr>
<tr>
<td>d2</td>
<td>growth</td>
<td>0.644</td>
<td>0.697</td>
<td>2.640</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.632)</td>
<td>(0.595)</td>
<td>(0.035)</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>2.915</td>
<td>1.617</td>
<td>2.915</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.022)</td>
<td>(0.171)</td>
<td>(0.022)</td>
</tr>
<tr>
<td></td>
<td>rpv</td>
<td>2.640</td>
<td>0.665</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.035)</td>
<td>(0.617)</td>
<td></td>
</tr>
<tr>
<td>d3</td>
<td>growth</td>
<td>4.161</td>
<td>3.208</td>
<td>3.324</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.003)</td>
<td>(0.014)</td>
<td>(0.011)</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>3.662</td>
<td>11.496</td>
<td>11.347</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>rpv</td>
<td>3.324</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Partial F-statistics reported with respect changes in the independent variables. Figures in square brackets are probability values. a, b, and c represent significance at 1%, 5%, and 10% respectively. The maximum number of lags is set to be four. AIC is used to select the lag length. $d_1$ represents $\pi_{it} \leq 1.61\%$, $d_2$ is when $1.61 < \pi_{it} \leq 3.32$ and $d_3$ corresponds $\pi_{it} > 3.32$.

### 4. Econometric Methodology

The main purpose of this paper is to use a threshold variable to investigate whether the relationship between inflation and economic growth through the relative price variability is different in each sample grouped on the basis of certain thresholds. The endogenous determination of threshold effects between variables is different from the traditional approach in which the threshold level is determined exogenously. If the threshold level is chosen arbitrarily, or is not determined within an empirical model, it is not possible to derive confidence intervals for the chosen threshold. The robustness of the results from the conventional approach is likely to be sensitive to the level of the threshold. The economet-
ric estimator generated on the basis of exogenous sample splitting may also pose serious inferential problems (for further details, see Hansen (1999, 2000)).

4.1 Econometric Framework: Panel Threshold Models

Hansen (1999) developed the econometric techniques appropriate for threshold regression with panel data. Allowing for fixed individual effects, the panel threshold model divides the observations into two or more regimes, depending on whether each observation is above or below the threshold level. The general specification threshold model takes the following form:

$$y_{it} = \mu_i + \sum_{k=0}^{K-1} \beta_{k+1} x_{it} I(\gamma_k < q_{it} \leq \gamma_{k+1}) + \beta_{K+1} x_{it} I(\gamma_K < q_{it} \leq \gamma_{K+1}) + \varepsilon_{it}$$  \hspace{1cm} (3.3)

where subscripts $i$ stands for the cross-sections with $(1 \leq i \leq N)$ and $t$ indexes time $(1 \leq t \leq T)$. $\mu_i$ is the province-specific fixed effect and the error term $\varepsilon_{it}$ is independent and identically distributed (iid) with mean zero and finite variance $\sigma^2$. $I(\cdot)$ is the indicator function indicating the regime defined by the threshold variable $q_{it}$, the threshold parameter $\gamma$. $y_{it}$ is dependent variable and $x_{it}$ the vector of explanatory variables. $\gamma_0 = -\infty$, $\gamma_{K+1} = +\infty$. Equation (3.2) allows for $K$ threshold values and, thus, $(K + 1)$ regimes. In each regime, the marginal effect of $x_{it}$ ($\beta_k$) on $y_{it}$ may differ.

Following the modified version of Hansen’s (1999) panel threshold model proposed by Bick and Nautz (2008), we consider a discriminator constant which is not individual specific but captures a common effect for all cross-sections. According to these authors, ignoring regime dependent intercepts ($\delta_k$) can lead to biased estimates of both the thresholds and the corresponding marginal impacts.
\[\begin{align*}
y_{it} &= \mu_i + \sum_{k=0}^{K-1} (\delta_{k+1} + \beta_{k+1}) x_{it} I(\gamma_k < q_{it} \leq \gamma_{k+1}) + \\
&\quad \beta_{K+1} x_{it} I(\gamma_K < q_{it} \leq \gamma_{K+1}) + \epsilon_{it} \tag{3.4}
\end{align*}\]

This formulation assumes that the difference in the regime intercepts, represented by \((\delta_k)\), is not individual specific but the same for all cross-sections. According to Bick and Nautz (2008), omission of any variable correlated with at least one regressor and the dependent variable causes biased estimates, but regime intercepts are a particularly interesting case. First, the bias can be clearly interpreted. Second, availability of regime intercepts as regressors is not an issue since they are as easily constructed as the regime-dependent exogenous regressors for a given threshold.

### 4.2 Estimation and Tests Strategy

Estimation of the panel threshold model involves several stages. First, estimation of the parameters model requires eliminating the individual effects \(\mu_i\) by removing individual-specific means and then applying the least squares sequential procedure (see Hansen (1999) for more details). Indeed, the individual specific effects are eliminated using the standard fixed-effects transformation implying for the identification of \(\beta_k\) and \(\beta_{k+1}\) that the elements of \(x_{it}\) are neither time-invariant nor adding up to a vector of ones. This case applies to regime intercepts which are usually included in each regime in threshold models in pure cross-sectional or time-series contexts. For example, in the case of two regimes, even in the presence of fixed effects it is possible to control for differences in the regime intercepts by including them in all but one regime as in the extension of the following equation:\(^7\)

\(^7\)There is no reason to limit our analysis to just two regimes. Hence, the estimation approach proposed by Hansen (1999) and extended by Bick and Nautz (2008) allows a more general specification with K thresholds (i.e. K + 1 regimes)
\[ y_{it} = \mu_i + (\delta_1 + \beta_1) x_{it} I(q_{it} \leq \gamma) + \beta_2 x_{it} I(q_{it} > \gamma) + \varepsilon_{it} \] (3.5)

The seminal contribution of Hansen (2000) allows us to estimate and make valid statistical inferences on the threshold. There are three statistical issues that need to be addressed in a threshold model: (1) how to jointly estimate the threshold value \( \gamma \) and the slope parameters; (2) how to test the hypothesis that a threshold exists and; (3) how to construct confidence intervals for \( \gamma \) and \( \beta \). We briefly discuss each in turn. Hansen (2000) recommends obtaining the least squares estimate \( \hat{\gamma} \) as the value that minimises the concentrated sum of squared errors, \( S_1(\gamma) \). The sum of the squared error function depends on \( \gamma \) only through the indicator function. Hence, the minimisation problem is a step procedure where each step occurs at distinct values of the observed threshold variable \( (\pi_{it}) \). After the threshold value \( \gamma \) is estimated, it is important to determine whether the threshold effect is statistically significant. In order to test the statistical significance of a threshold effect typically we would want to test the null hypothesis of no threshold effect, \( H_0 : \beta_1 = \beta_2 \). However, since \( \gamma \) is only identified under the alternative \( (H_1 : \beta_1 \neq \beta_2) \), the distribution of classical test statistics, such as the Wald and Likelihood ratio tests, are not asymptotically Chi-squared. In essence this is because the likelihood surface is flat with respect to \( \gamma \), consequently the information matrix becomes singular and standard asymptotic arguments no longer apply. There are methods for handling hypothesis testing within these contexts. In some instances, we are able to bound the asymptotic distribution of likelihood ratio statistics (Davis, 1977 and 1987); alternatively their asymptotic distribution must be derived by bootstrap methods (see Hansen, 2000). The appropriate test statistic is \( F_1 = \frac{S_0 - S_1(\hat{\gamma})}{\sigma^2} \) where \( S_0 \) and \( S_1 \) are, respectively, the residual sum of squares under the null hypothesis \( H_0 \) and the alternative \( H_1 \) with \( \sigma^2 \) the residual variance under the alternative hypothesis. Once the threshold effect exists, the next question is whether or not the threshold value
can be known. The null hypothesis of the threshold value is \( H_0 : \gamma = \gamma_0 \), and the likelihood ratio statistics is \( LR_1 (\gamma) = \frac{S_1(\gamma) - S_1(\hat{\gamma})}{\sigma^2} \) where \( S_1(\gamma) \) and \( S_1(\hat{\gamma}) \) are the residual sum of squares from equation (3.3) given the true and estimated value, respectively. The null hypothesis is rejected for large value of \( LR_1 \). The asymptotic distribution of \( LR_1 (\gamma_0) \) can be used to form valid asymptotic confidence interval about the estimated threshold values. The statistics of \( LR_1 (\gamma_0) \) are not normally distributed and Hansen (2000) computed their no-rejection region, \( c(\alpha) \), with \( \alpha \) the given asymptotic level. He proves that the distribution function has the inverse \( c(\alpha) = -2 \ln (1 - \sqrt{1 - \alpha}) \) from which it is easy to compute the critical values. The test rejects the null hypothesis at the asymptotic level \( \alpha \) if \( LR_1 (\gamma_0) \) exceeds \( c(\alpha) \). The asymptotic \((1 - \alpha)\) confidence interval for \( \gamma \) is set of values of \( \gamma \) such that \( LR_1 (\gamma_0) \leq c(\alpha) \).

5. **Empirical Analysis**

This paper examines the relationship between inflation, relative price variability and economic growth in Canadian provinces. The sample is derived from a disaggregated price data from 1981 to 2008 including ten commodities. Since Friedman (1977) made clear in his Nobel lecture that relative price variability is a direct means by which inflation can induce welfare-diminishing resource misallocation, the first part of this empirical analysis attempts to find if relative price is a channel through which inflation affects economic growth in a nonlinear fashion. Indeed, we investigate the relationship between relative price variability and inflation by using a model with threshold effects. Once this relationship is established, the second part tries to examine the relationship between inflation and economic growth. Then we carry out some investigations on the relationship between relative price variability and economic growth. In the last part, in order to improve our results, we release the

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\(^8\)For a detailed review of the general estimation and inference strategy and the treatment of multiple thresholds the reader is referred to Hansen (1999).
hypothesis of cross-sectional independence.

5.1 Inflation and Threshold Effects on Relative Price Variability

The application of this model enables us to test for the number of inflation regimes and to estimate both the threshold levels as well as the marginal impact of inflation on RPV in the various regimes. Specifically, we consider the following threshold model for the inflation-relative price variability relationship:

\[
rpv_{it} = \mu_i + \sum_{k=0}^{K-1} (\delta_{k+1} + \beta_{k+1}) \pi_{it} I\left(\gamma_k < \pi_{it} \leq \gamma_{k+1}\right) + \beta_{K+1} \pi_{it} I\left(\gamma_K < \pi_{it} \leq \gamma_{K+1}\right) + \varepsilon_{it}
\]

5.1.1 The Number of Inflation Thresholds

In a first step, we applied Hansen’s (1999) sequential testing procedure for determining the number of inflation thresholds. To ensure that the threshold estimation strategy includes sufficient observations in any one of the regimes, we restrict the minimization problem to values of \(\gamma\) such that at least 5 percent of the observations lie in both regimes.

\(LR_1, LR_2\) and \(LR_3\) give the observed value of the likelihood ratios for testing the hypothesis of no threshold, at most one threshold, at most two thresholds. The significance levels followed by \(LR_1, LR_2\) and \(LR_3\) have been computed by using the bootstrap distributions of \(LR_1, LR_2\) and \(LR_3\). The results indicate a clear rejection of a no-threshold effect between relative price variability and inflation in favour of a double threshold model for both without (no regime intercepts and regime intercepts) and with cross-section dependence. Specifically, the null hypothesis of a single inflation threshold in the inflation-relative price variability equation can be rejected at the 1 percent significance level but at the 5 percent significance level for no regime intercepts. Thus, the data least strongly support the existence of threshold effects. According to the p-value associated to \(LR_2\), the null hypothesis
### Table 3.5: Test Procedure Establishing the Number of Thresholds

<table>
<thead>
<tr>
<th></th>
<th>Without CSD*</th>
<th>With CSD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No regime intercepts</td>
<td>Regime intercepts</td>
</tr>
<tr>
<td>$H_0$: no threshold</td>
<td>14.247</td>
<td>31.605</td>
</tr>
<tr>
<td>LR$_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.019</td>
<td>0.000</td>
</tr>
<tr>
<td>(10%, 5%, 1% critical values)</td>
<td>(9.017, 11.645, 15.583)</td>
<td>(10.573, 11.929, 16.252)</td>
</tr>
<tr>
<td>$H_0$: at most one threshold</td>
<td>13.569</td>
<td>24.065</td>
</tr>
<tr>
<td>LR$_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.012</td>
<td>0.000</td>
</tr>
<tr>
<td>(10%, 5%, 1% critical values)</td>
<td>(9.314, 10.449, 13.742)</td>
<td>(11.103, 12.734, 17.027)</td>
</tr>
<tr>
<td>$H_0$: at most two thresholds</td>
<td>17.785</td>
<td>6.607</td>
</tr>
<tr>
<td>LR$_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.015</td>
<td>0.573</td>
</tr>
</tbody>
</table>

Notes. The threshold variable $\pi_{it}$ is annually inflation. The sequential test procedure indicates that the number of the threshold is one. 1000 bootstrap replications were used to obtain the p-values. Following Hansen (1999), each regime is required to contain at least 5% of all observations. (*) CSD= Cross-section dependence.

...of a double thresholds can be rejected at the 1 percent significance level. However, the test statistic of the null hypothesis of a three thresholds cannot be rejected at 10 percent significance level for the regime intercepts, while it can be rejected at the 5 percent significance level in the no regime intercepts. This result is consistent when taking into account the cross-section dependency for double thresholds at the 5 percent significance level. Our results show that threshold effects of inflation can be confirmed for the nonlinear linkage between inflation and relative price variability in the Canadian provinces. Therefore, the sequential test procedure implies two thresholds and, thus, three inflation regimes in the inflation and relative price variability relationship for the Canadian provinces. Hence when the threshold variable takes on values less than the estimated threshold values, we call this regime a low inflation regime and when the threshold variable exceeds specified threshold...
values, we call this regime a high inflation regime. When the inflation rates are between the two threshold values, we call this regime the moderate inflation regime.

5.1.2 Estimating the Inflation Threshold and the Slope Coefficients

The data suggest the presence of two thresholds in the function relating relative price variability and inflation. We thus estimated the following double threshold model:

\[
r_{p,v_{it}} = \mu_i + (\beta_1 + \beta_1 \pi_{it}) I(\pi_{it} < \gamma_1) + (\beta_2 + \beta_2 \pi_{it}) I(\gamma_1 \leq \pi_{it} \leq \gamma_2) + \beta_3 I(\pi_{it} > \gamma_2) + \varepsilon_{it}
\] (3.7)

Table 3.6 presents the results for both specifications: without (column 2) and with (column 3) regime intercepts. Our results suggest that the two thresholds are estimated at \((\gamma_1 = 2.27\% \text{ and } \gamma_2 = 5.64\%)\) for the no-regime intercepts and \((\gamma_1 = 1.61\% \text{ and } \gamma_2 = 3.32\%)\) for regime intercepts. These findings show that inclusion of regime intercepts decreases the threshold estimate at \(\gamma_1\) from 2.27 percent to 1.61 percent and for \(\gamma_2\) from 5.64 percent to 3.32 percent. In addition, the upper bound of the 95% confidence interval decreases also from 6.57 percent to 3.42 percent. The most remarkable observation is that, in a presence of regime intercepts, inflation rates inside the moderate inflation regime have a positive impact (0.689), and a statistically significant one, while in the no-regime intercepts this impact is negative and not statistically significant. In the low inflation regime, the marginal impact of inflation on relative price variability, without and with regime intercepts, is significantly negative \((\beta_1 = -0.441)\) and \((\beta_1 = -1.219)\) respectively. Thus, a further decline of inflation would increase relative price variability significantly. According to Jaramillo (1999), this impact of inflation rates close to zero may point to the presence of nominal downward wage and price rigidities. In the middle-inflation regime, which contains the observations for the regime of inflation rates between 2.27\% and 5.64\% for the no regime intercepts and 1.61\% and 3.32\% for regime intercepts, the impact of inflation on relative price variability
is negative and insignificant for no regime intercepts while in the regime intercepts, the inflation’s effect is positive and significantly weaker in absolute terms. In the high inflation regime, which has the observations with inflation rates exceeding 5.64 percent (no regime intercepts) and 3.32 percent (regime intercepts), the effect is still significantly positive at the 1 percent level. Our results from the specification with a regime intercepts are in line with those by Bick and Nautz (2008) who affirm: “When inflation exceeds an upper threshold, it seems that relative price variability-increasing aspects of inflation (including e.g. menu costs and imperfect information about the price level) become eventually dominant while relative price variability-decreasing aspects of inflation have faded out”.

These findings suggest that for no regime intercepts, relative price variability is the channel through which inflation can affect economic performance when inflation is below 2.27 percent and above 5.64 percent, and otherwise, it is not a good channel. However, for the regime intercepts, the evidence strongly supports the view that relative price variability is, for all three inflation regimes, an important channel through which inflation adversely affects economic performance, hence affects our standards living. From these findings, controlling for differences in the regime intercepts, has important implications. For example, the point estimate and upper bound of confidence interval are both substantially changed. For these reasons, we subsequently concentrate on the possibility of using the regime intercepts specification in the remainder of the paper.
Table 3.6: Threshold Effects of Inflation on the Relative Price Variability

<table>
<thead>
<tr>
<th>Threshold estimates and confidence intervals</th>
<th>No regime intercepts</th>
<th>Regime intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>2.27%</td>
<td>1.61%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[1.60% 2.42%]</td>
<td>[1.36% 1.87%]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>5.64%</td>
<td>3.32%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[1.61% 6.57%]</td>
<td>[3.20% 3.42%]</td>
</tr>
</tbody>
</table>

| Regime dependent inflation coefficients     |                       |                   |
| $\beta_1$                                  | -0.411$^a$ (0.106)    | -1.219$^a$ (0.238) |
| $\beta_2$                                  | -0.064 (0.048)        | 0.689$^a$ (0.136)  |
| $\beta_3$                                  | 0.066$^a$ (0.022)     | 0.168$^a$ (0.022)  |

| Regime dependent intercepts                 |                       |                   |
| $\delta_1$                                 | 0.017$^a$ (0.003)     |                   |
| $\delta_2$                                 | -0.009$^b$ (0.004)    |                   |

Notes. Standard errors are given in parentheses. a, b and c indicate significance at the 1%, 5% and 10% level. Intuitively, when Central Bank sets an inflation target, it implies that beyond this target economic agents can not set their expectations. According to the theory beyond a threshold, the variability relative price increases slowly that below the threshold. Therefore, the results of $\beta_2$ may suggest the existence of a higher threshold.

5.2 Analysis of the Economic Growth-Inflation Relationship

As discussed in section two, some theoretical models and empirical data predict that threshold effects are associated with a rate of inflation exceeding some critical value or below some critical value. We now allow discrete slopes to differentiate high, middle and low rates of inflation. We then estimate the equation:

$$dgdp_{it} = \mu_i + (\delta_1 + \beta_1 \pi_{it}) I(\pi_{it} < \gamma_1) + (\delta_2 + \beta_2 \pi_{it}) I(\gamma_1 \leq \pi_{it} \leq \gamma_2) + \beta_3 I(\pi_{it} > \gamma_2) + \theta Z_{it} + \varepsilon_{it},$$

(3.8)

where $Z_{it}$ the vector of control variable (see Data set in above). $I(\pi_{it} < \gamma_1)$, $I(\gamma_1 \leq \pi_{it} \leq \gamma_2)$ and $I(\pi_{it} > \gamma_2)$ are indicator functions which take the value of one if the term between parentheses is true, and are zero otherwise. This model specifies the effects of inflation with three coefficients: $\beta_1$, $\beta_2$ and $\beta_3$. $\beta_1$ denotes the effect of inflation below the first threshold.
level $\gamma_1$, $\beta_2$ denotes the effect of inflation between $\gamma_1$ and $\gamma_2$, and $\beta_3$ denotes the effect of inflation exceeding the second threshold level $\gamma_2$.

The estimation results are presented in Table 3.7. Our investigation shows that, low to moderate inflation regime has a strongly positive effect on the economic growth rate, however, this positive relationship is not statistically significant. This result is consistent with Ragan’s (2005) analysis which states that in countries that have experienced low and stable inflation, there is no significant impact on long run economic growth. Therefore, it is clear that, in a low to moderate inflation regime an increase in inflation rates has no adverse long-run impact on Canadian living standards. At high rates of inflation, the marginal impact of additional inflation on the economic growth diminishes rapidly but is still significantly negative. In particular, our empirical results suggest that inflation distorts economic growth provided it exceeds a certain critical value. Barro (1995) estimated that a shift in monetary policy that raises the long-term average inflation by 10 percentage points per year lowers the level of the real GDP after 30 years by 4 to 7 percent. Therefore, high inflation in the long run is very harmful to Canadian provincial economies. Intuitively, our finding implies that any policy of targeting inflation rates that exceed the second threshold value will be detrimental to the economic performance of Canada. Furthermore, investment is not significant, hence there is no apparent relationship between these regressors. Openness of economy is strongly significant and positive, which corroborates received theory. This finding is confirmed by Omay and Kan (2010) as well as Drukker et al. (2005) who find a similar result in the case of industrialized countries.
Table 3.7: Threshold Effects of Inflation on Economic Growth

<table>
<thead>
<tr>
<th>Regime intercepts</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold estimates and confidence intervals</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>1.61%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[1.36% 1.87%]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>3.32%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[3.20% 3.42%]</td>
</tr>
</tbody>
</table>

Coefficient estimates from Equation 3.8

<table>
<thead>
<tr>
<th>Regime-dependent regressors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.135 (0.786)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.038 (0.394)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.569a (0.133)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.028b (0.011)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.024b (0.011)</td>
</tr>
</tbody>
</table>

Regime independent regressors

| openness                                      | 0.175a (0.064) |
| igdp                                          | 0.075 (0.085) |

| R$^2$                                         | 0.168       |

Observations in regime 1                     | 45          |
Observations in regime 2                     | 135         |
Observations in regime 3                     | 100         |

Notes. Standard errors are given in parentheses. a, b and c indicate significance at the 1%, 5% and 10% level.

5.3 Analysis of the Growth-Relative Price Variability Relationship

As we mentioned in Section 2 of the literature review, the theoretical and empirical literature has suggested that the level of relative price variability might be the channel that explains the link from inflation to real economic growth. In addition, the preliminary data of Table 3.1 show that an increase in the inflation average corresponds to a reduction in economic growth and an increase of relative price variability. This shows a similar pattern to that of the growth rates as inflation rises across quartiles. Furthermore, the preliminary analysis of the inflation-relative price variability relationship suggests that for Canadian provinces, inflation probably affects economic growth through the level of relative price variability.
Hence, we discuss the impact of price variability on economic growth. To do this, we substitute the variable inflation by the square root of the relative price variability associated with each inflation regime.\footnote{Taking into account the square root (the standard deviation) is preferred to the variance for good highlight possible thresholds. See Bick and Nautz (2008).}

\[
dgdp_{it} = \mu_i + (\delta_1 + \beta_1 \text{rpv}_{it}) I(\pi_{it} < \gamma_1) + (\delta_2 + \beta_2 \text{rpv}_{it}) I(\gamma_1 \leq \pi_{it} \leq \gamma_2) + \beta_3 I(\pi_{it} > \gamma_2) + \theta' Z_{it} + \varepsilon_{it},
\]

(3.9)

Table 3.8 presents the estimation results of the inflation-investment relationship with threshold effects. The most striking difference between this result and the previous one is that the impact of relative price variability in the low inflation regime becomes negative, while the coefficient of relative price variability is positive for the moderate inflation regime and negative for the high inflation regime. Indeed, this finding suggests that except the first threshold, under which relative price variability has either no significant impact or even a negative effect on growth, the relative price variability is the important channel through which inflation adversely affects economic performance in high inflation regime. Moreover, the consideration of relative price variability as the main control variable makes our results more consistent with respect to other control variables (regime independent regressors) in terms of magnitude (from 0.175 to 0.203 for openness and 0.075 to 0.111 for investment) and significance level.

6. Cross-section Dependency

Recently, the importance of taking into account cross-section correlation for testing the unit roots hypothesis has been emphasized. Pesaran’s (2007) simulations show that the tests assuming cross-section independence tend to over-reject the null if cross-section correlation is
Table 3.8: Threshold Effects of Relative Price Variability on Economic Growth

<table>
<thead>
<tr>
<th>Regime intercepts</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold estimates and confidence intervals</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>1.61%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[1.36% 1.87%]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>3.32%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[3.20% 3.42%]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficient estimates from Equation 3.9</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime-dependent regressors</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.125 (0.329)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.010 (0.254)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-1.252$^a$ (0.322)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.022$^b$ (0.009)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.020$^b$ (0.008)</td>
</tr>
<tr>
<td>Regime independent regressors</td>
<td></td>
</tr>
<tr>
<td>openness</td>
<td>0.203$^a$ (0.061)</td>
</tr>
<tr>
<td>igdp</td>
<td>0.111 (0.085)</td>
</tr>
</tbody>
</table>

| $R^2$ | 0.150 |
| Observations in regime 1 | 45 |
| Observations in regime 2 | 135 |
| Observations in regime 3 | 100 |

Notes. Standard errors are given in parentheses. $a$, $b$ and $c$ indicate significance at the 1%, 5% and 10% level.

present and Baltagi et al. (2007) find that when spatial auto regression is present, while first generation tests become oversized, the tests explicitly allowing for cross sectional dependence yield a lower frequency of type I error. Indeed, the panel unit root tests employed so far have been constructed under the assumption of cross-sectional independence. However, it has been shown in the literature that failure to account for cross-sectional dependence may cause severe size distortions and thereby invalidate estimation and inference. To overcome the bias in the panel unit root investigation, we first need to test for the presence of cross-section dependence. If cross-section dependence is confirmed in the data, stationarity should be checked by so-called second generation panel unit root tests. Several tests for cross section
dependence have been proposed in the econometric literature.

To diagnose the presence of cross-section dependence, we used the Lagrange Multiplier (LM) test, test suggested by Breusch and Pagan (1980), which is based on the squared pairwise Pearson’s correlation coefficient of the residuals. It is well known that when $T > N$ (as is the case in this paper), these tests enjoy highly desirable statistical properties (relative to other tests) and can be used with balanced and unbalanced panel. The Breusch and Pagan (1980) test statistic is 292.225, revealing that the null hypothesis of cross-section independence is rejected at the 1 percent significance level. Thus, the test rejects the null hypothesis of cross-sectional independence.

Having established that the series are cross-sectionally correlated, the next step is to implement a panel unit root test that accounts for the presence of cross-section dependence. One such test is the cross-sectionally augmented version of the Im et al. (2003) test proposed by Pesaran (2007). Pesaran’s test is favoured over all others for its simplicity and clarity. Table 3.9 displays the Pesaran (2007) statistics with an optimal number of lags (4). It is clear-cut that after accounting for cross-section dependence, the hypothesis that the series contain a unit root is confirmed at the 1 percent significance level.

In the presence of cross-section dependence, traditional OLS-based estimations are biased and not valid. To eliminate the cross-section dependency problem, we use a method proposed by Pesaran (2006) and Omay and Kan (2010) to correct for cross-section dependency in nonlinear panel threshold regression models. This method consists in treating the equations from the different cross-section units as a system of Seemingly Unrelated Regression Equations (SURE) and then estimate the system by a Generalized Least Squares

\[ y_{it} = \alpha_i + \beta_i y_{it-1} + \delta_i y_{it-j} + \epsilon_{ij}, \]

where $\bar{y}_{it-1} = \frac{1}{N} \sum_{i=1}^{N} y_{it-1}$ and $\Delta \bar{y}_t = \frac{1}{T} \sum_{t=1}^{T} y_{it}$. The test is obtained as $CIPS = \frac{1}{N} \sum_{i=1}^{N} t_i (N, T)$ refers to the t-ratio of the OLS estimate of $b_i$.\textsuperscript{10}
Table 3.9: Pesaran (2007) test statistics

<table>
<thead>
<tr>
<th>Series in Level</th>
<th>Series in first differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
</tr>
<tr>
<td>INF</td>
<td>-4.281&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>OPENS</td>
<td>-1.405</td>
</tr>
<tr>
<td>IGDP</td>
<td>-1.308</td>
</tr>
<tr>
<td>RPV</td>
<td>-4.722&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Note: Critical values for the Pesaran (2007) test are -2.56(1%), -2.33(5%) and -2.21(10%) for the case of intercept only and -3.11(1%), -2.86(5%) and -2.73(10%) for the intercept and a linear trend. See Table II(b,c) in Pesaran (2007). Besides, optimal lag length in these tests were selected using AIC with maximum lag order of 4. a, b, c denote significance at 1%, 5% and 10% levels.

(GLS) technique. Two thresholds are detected and proved to be significant. These thresholds are estimated at 1.82 percent and 4.16 percent, which are robust to model specification and estimation approaches. At inflation rates below this threshold (1.82%), inflation has a significantly positive effect on relative price variability, while the magnitude of this positive impact diminishes as inflation exceeds 4.16%. Taking into account the cross-section dependence improves the model’s explanatory power (see the coefficient of correlation \( R^2 \)). Moreover, the results present in Table 3.10 reveal that the impact of inflation on RPV is hump-shaped. Inflation increases relative price variability if inflation is either very low (< 1.82%) or very high (> 4.16%). Between these two thresholds inflation has no real effects on the economy via its impact on relative price variability. Indeed, for example, when inflation is moving in this interval, the Canadian monetary authorities do not consider it necessary to intervene. Therefore, threshold effects of inflation provide a further rationale for the announcement of critical levels of inflation and inflation target zones. In all these variants of the threshold model, the general conclusion remains: there is only a significant impact of inflation on relative price variability if inflation is either very low or very high,
supporting price level stability as an outcome of optimal monetary policy. These findings suggest that during a period of moderate inflation, relative price variability is not an important channel through which inflation affects economic growth in Canadian provinces.

Table 3.10: Threshold Effects of Inflation on Relative Price Variability taking account cross-section dependence

<table>
<thead>
<tr>
<th>Regime intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Threshold estimates and confidence intervals</strong></td>
</tr>
</tbody>
</table>
| $\gamma_1$ | 1.82% 
| 95% confidence interval | [1.72% 1.83%] 
| $\gamma_2$ | 4.16% 
| 95% confidence interval | [3.38% 4.97%] 
| **Regime dependent inflation coefficients** |  
| $\beta_1$ | 0.345 (0.042) 
| $\beta_2$ | 0.069 (0.176) 
| $\beta_3$ | 0.042 (0.012) 
| **Regime dependent intercepts** |  
| $\delta_1$ | 0.001 (0.001) 
| $\delta_2$ | -0.009 (0.002) 

Notes. Standard errors are given in parentheses. a, b and c indicate significance at the 1%, 5% and 10% level.
Table 3.11 indicates the results obtained with respect to the inflation and economic growth relationship when we consider cross-section dependence. An interesting finding is that for the low-inflation regime (in which the inflation rate is below 1.82 percent per year) and the middle-inflation regime (which contains the observations for the regime with inflation rates between 1.82 percent and 4.16 percent), the coefficient of inflation ($\beta_1, \beta_2$) are strongly positive. This result shows that a 1-percentage-point increase in inflation will cause a 0.057 to 0.098-percentage-point increase in economic growth. However, this positive relationship is only significant when inflation rates are between 1.82 percent and 4.16 percent. In the high inflation regime, which has the observations with inflation rates exceeding 4.16 percent, the coefficient of inflation is still significantly negative at the 1 percent level. The major improvement in our estimation result with a cross-section dependence correction is that the regime independent regressors not only become statistically significant with the expected sign, but also they are more consistent in terms of magnitude (from 0.175 to 0.189 for openness and 0.075 to 0.090 for investment).
Table 3.11: Threshold Effects of Inflation on Economic Growth taking account of cross-section dependence

<table>
<thead>
<tr>
<th>Regime intercepts</th>
<th>Threshold estimates and confidence intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_1 )</td>
<td>1.82%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[1.72%  1.83%]</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>4.16%</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[3.38%  4.97%]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficient estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime-dependent regressors</td>
</tr>
<tr>
<td>( \beta_1 )</td>
</tr>
<tr>
<td>( \beta_2 )</td>
</tr>
<tr>
<td>( \beta_3 )</td>
</tr>
<tr>
<td>( \delta_1 )</td>
</tr>
<tr>
<td>( \delta_2 )</td>
</tr>
<tr>
<td>Regime independent regressors</td>
</tr>
<tr>
<td>openness</td>
</tr>
<tr>
<td>igdp</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
<tr>
<td>Observations in regime 1</td>
</tr>
<tr>
<td>Observations in regime 2</td>
</tr>
<tr>
<td>Observations in regime 3</td>
</tr>
</tbody>
</table>

Notes. Standard errors are given in parentheses. \(^a\), \(^b\) and \(^c\) indicate significance at the 1%, 5% and 10% level.

Table 3.12 shows the results obtained with respect to the relative price variability and economic growth relationship, when we consider cross-section dependence. The existence of two threshold levels implies that inflation can be divided into three parts. As inflation rises from 0 to 1.82 percentage point, the effect on economic growth is negligible or even negative. As inflation crosses the low threshold level, it has no significant and positive impact on economic growth, up to a certain level. When inflation crosses the second threshold level (4.16 percent), the marginal impact is negative and significant. Unlike in the case of inflation-growth, the major difference is the regime independent regressors have not only achieved a significance level and respect the expected sign but are consistent in terms of
Table 3.12: Threshold Effects of Relative Price Variability on Economic Growth taking account of cross-section dependence

| Regime intercepts |  
|-------------------|---|
| Threshold estimates and confidence intervals |  
| $\gamma_1$ | 1.82% |
| 95% confidence interval | [1.72% 1.83%] |
| $\gamma_2$ | 4.16% |
| 95% confidence interval | [3.38% 4.97%] |

Coefficient estimates

| Regime-dependent regressors |  
|-----------------------------|---|
| $\beta_1$ | -0.036 (0.126) |
| $\beta_2$ | 0.071 (0.089) |
| $\beta_3$ | -0.824a (0.194) |
| $\delta_1$ | 0.001 (0.002) |
| $\delta_2$ | 0.003b (0.001) |

Regime independent regressors

| openness | 0.182a (0.029) |
| igdp | 0.090b (0.035) |

| R^2 | 0.295 |

Observations in regime 1 99
Observations in regime 2 135
Observations in regime 3 46

Notes. Standard errors are given in parentheses. a, b and c indicate significance at the 1%, 5% and 10% level.
7. Conclusion

This paper provides new evidence on the nonlinear relationship between inflation and economic growth through relative price variability. Recent empirical analysis suggests a negative nonlinear inflation-growth relationship. Many papers addressing this issue explain this nonlinearity with threshold effects. That is, below a specific threshold value, inflation is found to have a statistically non significant small negative or positive effect on economic growth, whereas above it, the effect becomes negative and statistically significant. To confirm or not the general consensus among economists that inflation produces welfare costs and price stability should be the prior goal of monetary policy, we follow Hansen (1999) and Omay and Kan (2010) and developed a panel threshold model with cross-section dependence. We first investigate the relationship between inflation and relative price variability. This allows us to verify if relative price variability should be an important channel through which inflation adversely affects Canadian provinces economic performance. The empirical results show the existence of a double threshold (1.61 percent and 3.32 percent), that divides the inflation range into three categories, i.e., low inflation, moderate inflation and high inflation. These threshold effects of inflation can be confirmed for the inflation-relative price variability nexus in the Canadian provinces. Examining the relationship between inflation and economic performance, we find that for low and moderate inflation regimes the marginal effect of inflation is strongly positive; however this positive relationship is not significant. In the extremely high inflation regime the marginal effect is significantly negative at the 1 percent level. Using relative price variability as a channel through which inflation affects economic growth, we find the same result except that for the low inflation regime the marginal effect is negative. In addition, our results are more consistent with respect to other control variables (openness and investment) in terms of magnitude and significance level.
All these findings are based on the assumption of independence over the cross-section units. However, we see from our analysis that this assumption is violated. Therefore, we do a Breusch and Pagan (1980) test to diagnose for the presence of cross-section dependence. This test reveals the presence of cross-section dependence, thus the traditional estimations are not efficient and not valid. To overcome this cross-section dependence problem, we apply the SURE-GLS approach. The results of our estimation indicate the existence of a double threshold (1.82 percent and 4.16 percent). As Freidman (1977), our findings suggest that the relative price variability is an important channel through which inflation adversely affects economic performance in Canadian provinces. Furthermore, taking into account cross-section dependence improves the model’s explanatory power (see the coefficient of correlation $R^2$) and all the control variables are more robust in terms of significance level. Inflation below the first threshold (1.82 percent) effects economic growth insignificantly and positively; at moderate rates of inflation, between the two threshold levels (1.82 percent and 4.16 percent), the effect of inflation is significant and strongly positive and at high rates of inflation, above the second threshold (above 4.16 percent), the marginal impact of additional inflation on economic growth is significantly negative.

These findings provide some policy implications. On the basis of this study, it is desirable to keep inflation in the moderate inflation regime and therefore the Bank of Canada should concentrate on those policies which keep the inflation rate between 1.82 percent and 4.16 percent because it may be helpful for the achievement of sustainable economic growth and to improve the living standards of Canadian provinces. This information gives a very important signal for Canadian policymakers to impose new policies to provide economic stabilization to the Canadian provinces. Our results confirm the claims of Blanchard et al. (2010, page 11) who argue that: "Should policymakers therefore aim for a higher target inflation rate in normal times, in order to increase the room for monetary policy to react
to such shocks. To be concrete, are the net costs of inflation much higher at, say, 4 percent than at 2 percent, the current target range? Is it more difficult to anchor expectations at 4 percent than at 2 percent?" In other words, the effects of inflation on growth are difficult to discern, so long as inflation remains in the single digit range. As a consequence, this would suggest that an inflation target of 4 percent might be more appropriate because it leaves more room for expansionary monetary policy in the case of adverse shocks.
Appendix A: The Lagrange Multiple Test

Breusch and Pagan (1980) proposed a Lagrange Multiplier (LM) statistic for testing the null of zero cross equation error correlations, which is defined as

$$LM = T \sum_{i}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij}^2$$

where $\hat{\rho}_{ij}$ is the sample estimate of the pair-wise correlation of the residuals $\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} e_{it}e_{jt}}{\left(\sum_{t=1}^{T} e_{it}^2\right)^{1/2}\left(\sum_{t=1}^{T} e_{jt}^2\right)^{1/2}}$, where $e_{it}$ is the OLS estimate of terms of error. LM is asymptotically distributed as Chi-squared with $\frac{N(N-1)}{2}$ degrees of freedom under the null hypothesis as $T > N$.

Appendix B: Distribution of observations of both sides of the thresholds.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Endog. thres.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1.61%  3.32%]</td>
</tr>
<tr>
<td></td>
<td>d₁  d₂  d₃</td>
</tr>
<tr>
<td>AB</td>
<td>4  13  11</td>
</tr>
<tr>
<td>BC</td>
<td>5  14  9</td>
</tr>
<tr>
<td>MA</td>
<td>3  14  11</td>
</tr>
<tr>
<td>NB</td>
<td>7  12  9</td>
</tr>
<tr>
<td>NFL</td>
<td>6  14  8</td>
</tr>
<tr>
<td>NS</td>
<td>3  15  10</td>
</tr>
<tr>
<td>PEI</td>
<td>5  13  10</td>
</tr>
<tr>
<td>ON</td>
<td>3  14  11</td>
</tr>
<tr>
<td>QC</td>
<td>6  11  11</td>
</tr>
<tr>
<td>SAS</td>
<td>3  15  10</td>
</tr>
<tr>
<td>Total (Panel)</td>
<td>45 135 100</td>
</tr>
</tbody>
</table>

Notes. Alberta=AB; British Columbia=BC; Manitoba=MA; New Brunswick=NB; Newfoundland and Labrador=NFL; Nova Scotia=NS; Prince Edward Island=PEI; Ontario=ON; Quebec=QC and Saskatchewan=SAS. Source: Author’s calculation. $d₁$ represents $\pi_{it} \leq 1.61$ percent, $d₂$ is when $1.61 < \pi_{it} \leq 3.32$ and $d₃$ corresponds $\pi_{it} > 3.32$. 
References


