THREE ESSAYS ON THE
ECONOMICS OF CLIMATE CHANGE

Faisal Arif

Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
In partial fulfillment of the requirements
For the PhD degree in Economics

Department of Economics
Faculty of Social Sciences
University of Ottawa

© Faisal Arif, Ottawa, Canada, 2012
Thesis Abstracts

Chapter I: Regional burden sharing of GHG mitigation policies – A Canadian perspective. The distribution of the burden of cost arising from the reduction of greenhouse gas (GHG) emissions is a contentious issue in policy discussions; more so among regional jurisdictions in the federalist countries with decentralized authorities over environmental regulations. In this setting, often the policy discussions are focused on the distribution of regional emission reduction targets that, in turn, entails negotiations over the distribution of the scarcity rents and the regional transfers of wealth. The allocation of regional emission entitlements is thus a key factor that could hinder the political feasibility of a national GHG mitigation policy. In this paper, we build a multi-region computable general equilibrium (CGE) model of the Canadian economy to assess the implications of different burden sharing rules governing the national GHG abatement policy with a cap-and-trade system of emission permits. In addition to assessing the impacts of traditional regional emissions allocation rules that involve intra-regional transfers of wealth, we consider a particular emission allocation that avoids such transfers, which may be a more palatable option given the context of likely fierce negotiations over the issue. Our results indicate to differing outcomes depending on the allocation policy in use. The CGE framework is also able to shed light on the transmission mechanisms that drive the results underlying the policy options.

Chapter II: Endogenous technological change and emission allowances. Given the imminent threat of global warming due to GHG emissions, a number of emission mitigation policies have been proposed in the literature. However, they generally suffer from the classical equity-efficiency trade-off. High costs from equity concerns often render environmental policies politically unattractive and thus hard to implement. Recent advancement in the climate policy modeling literature that incorporates endogenous technological change (ETC) into the framework can potentially bring new insights into this debate. Using an inter-temporal, multi-sector CGE approach with ETC incorporated into the framework, this paper builds a model that focuses on the equity-efficiency debate for the policymakers. Canada is chosen as the country of investigation for this purpose. The paper provides a new welfare ranking of four permit allocation policies that address the equity-efficiency trade-off. In a second-best setting
with pre-existing distortions, output-based allocation (OBA) of emission permits is compared to three other policy options: (i) an emissions trading system with grandfathered allocation (GFA), (ii) an auction permit trading system where permit revenue is recycled to lower payroll taxes (RPT), and (iii) a hybrid of OBA and R&D subsidy (O-R&D). We find that adapting OBA, as well as O-R&D, is welfare improving over GFA. The implicit output subsidy, entailed in the OBA policy, mitigates against the rising cost effect in the GFA policy. This is reinforced through added investment incentive in R&D when ETC is incorporated into the framework. With O-R&D, since the R&D subsidy corrects for market imperfections in the knowledge accumulation process, the effect is further bolstered, culminating into mitigation of uneven distributional outcome for energy-intensive industries as a whole. Contrary to previous results, we also find that, in terms of the welfare metric, OBA unequivocally improves the distributional outcome across sectors as compared to the RPT policy. Inclusion of ETC also unequivocally generates a higher welfare ranking for all permit policy schemes.

Chapter III: Emission permit banking and induced technological change. This paper attempts to undertake an exploratory research by integrating two themes in the emission mitigation policy literature, which include: the inter-temporal emission permit banking and borrowing and the role of induced technological change in emission mitigation. Using a simple optimal control approach, we construct a unified framework that evaluates the optimal path of emissions and the optimal trajectory of permit price when both inter-temporal banking and borrowing of permits and the effects of induced technological change (ITC) are present. We find that ITC leads to a declining emission trajectory over time. The effect of ITC on the optimal permit price path, however, is ambiguous and critically depends on the extent of marginal cost saving that emanates from emission-saving technological innovation.
## CONTENTS IN BRIEF

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgement</td>
<td>xi</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 1 Regional burden sharing of GHG mitigation policies – A Canadian perspective</td>
<td>7</td>
</tr>
<tr>
<td>Chapter 2 Endogenous technological change and emissions allowances</td>
<td>72</td>
</tr>
<tr>
<td>Chapter 3 Emission permit banking and induced technological change</td>
<td>148</td>
</tr>
<tr>
<td>References</td>
<td>173</td>
</tr>
</tbody>
</table>
CONTENTS

List of Figures viii

List of Tables ix

Acknowledgement xi

Introduction to the thesis 1

Chapter 1 Regional burden sharing of GHG mitigation policies – A Canadian perspective 7

1.1 Introduction 8
1.2 Review of relevant literature 12
1.3 The model 13
1.3.1 Households 14
1.3.2 Firms 19
1.3.3 Government 22
1.3.4 Trade: Regional and international 23
1.3.5 Equilibrium conditions 28
1.3.5.1 Commodity market equilibrium 28
1.3.5.2 Input market equilibrium 29
1.3.6 Closure rules 29
1.4 Emission permit allocation schemes – description of the simulations 31
1.4.1 Emission based allocation 33
1.4.2 Allocation based on emissions per capita 33
1.4.3 Allocation based on converging carbon intensity 34
1.4.4 Allocation based on efficiency index 35
1.4.5 Allocation based on multi-criteria Index 36
1.4.6 Allocation based on no prior entitlement of emission rights 37
1.5 Description of the baseline data 38
1.6 Results 41
1.6.1 National impacts 41
1.6.2 Regional aggregate impacts 42
## CONTENTS

1.6.3 Regional sectoral impacts 49
1.7 Concluding remarks 56

*Appendices to Chapter Two*

**Appendix A** List of equations 59
**Appendix B** List of variables 65
**Appendix C** List of parameters 69

---

### Chapter 2

**Endogenous technological change and emissions allowances** 72

2.1 Introduction 73
2.2 Review of relevant literature 77
2.3 The model 79
2.3.1 Households: Consumption and labour supply 80
2.3.1.1 Inter-temporal optimization problem 81
2.3.1.2 Intra-temporal optimization problem 85
2.3.2 Firms: Production, investments, and abatement decision with OBA 88
2.3.2.1 Inter-temporal investment optimization problem 88
2.3.2.2 Allocation of capital goods 94
2.3.2.3 Description of the production nests 95
2.3.3 Trade 97
2.3.3.1 Allocation of domestic output 97
2.3.3.2 The optimal combination of composite goods 99
2.3.3.3 Capital market 100
2.3.4 Government 101
2.3.5 Within period equilibrium conditions 102
2.3.5.1 Commodity market equilibrium 102
2.3.5.2 Labour market equilibrium 102
2.3.5.3 Internal-external balance 103
2.3.6 Steady-State Conditions 103
2.3.6.1 Steady-state household budget constraint 103
2.3.6.2 Steady-state investments 103
2.3.6.3 Steady-state shadow value of knowledge and physical capital 104
2.3.6.4 Steady-state condition for internal-external balance 104
2.4 Data, Calibration and Numerical Solution Strategy 105
2.5 Simulation 109
2.5.1 Description of the simulations 109
2.5.2 Results 110
2.5.2.1 Reducing GHG emissions with grandfathered allocation of emission permits (GFA) under a Cap-and-Trade framework 111
2.5.2.1.1 Aggregate impacts 111
2.5.2.1.2 Sectoral impacts 113
2.5.2.2 Reducing GHG emissions with output-based allocation of emission permits (OBA) 120
2.5.2.3 Reducing GHG emissions with auctioning of permits and reduction in payroll taxes (RPT) 123
## CONTENTS

2.5.2.4 Reducing GHG emissions with OBA and R& D subsidy (O-R&D) ........................................ 125  
2.5.3 Sensitivity analysis ........................................................................................................... 126  
2.6 Concluding observations ................................................................................................... 128  

**Appendices to Chapter Three**

Appendix A First order conditions from the representative firm’s optimization problem ........ 131  
Appendix B List of equations .................................................................................................... 133  
Appendix C List of variables .................................................................................................... 140  
Appendix D List of parameters ................................................................................................ 144  

### Chapter 3 Emission permit banking and induced technological change ................................. 148  
3.1 Introduction ....................................................................................................................... 149  
3.2 An overview of the modeling framework .......................................................................... 153  
3.3 Inter-temporal emission allocation with ITC - The social planner’s solution ................. 156  
3.3.1 Economic interpretation of the necessary conditions for optimization of the social planner’s problem ......................................................................................... 158  
3.4 Permit banking with ITC - The decentralized solution ..................................................... 160  
3.4.1 Economic interpretation of the necessary conditions for decentralized optimization .................................................................................................................... 163  
3.5 Implication of ITC on the optimal level of emissions .......................................................... 164  
3.6 Characterizing the optimal time path of output and emissions .......................................... 166  
3.7 Implication of ITC on the optimal path of permit price ..................................................... 169  
3.8 Conclusion .......................................................................................................................... 170  

References ................................................................................................................................ 173  

---
<table>
<thead>
<tr>
<th>Chapter 1</th>
<th>Regional burden sharing of GHG mitigation policies – A Canadian perspective</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Schematic representation of household preference</td>
<td>18</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Schematic representation of firm technology</td>
<td>19</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Schematic representation of imports</td>
<td>25</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Schematic representation of exports</td>
<td>26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 2</th>
<th>Endogenous technological change and emissions allowances</th>
<th>72</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Schematic representation of household preference</td>
<td>86</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Schematic representation of firm technology</td>
<td>89</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

### Chapter 1  Regional burden sharing of GHG mitigation policies – A Canadian perspective  7

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Simulation schemes</td>
<td>31</td>
</tr>
<tr>
<td>Table 2a</td>
<td>Emissions in the BAU (percentage of total emissions in respective categories)</td>
<td>39</td>
</tr>
<tr>
<td>Table 2b</td>
<td>GDP in the BAU (percentage of Canadian GDP)</td>
<td>39</td>
</tr>
<tr>
<td>Table 2c</td>
<td>Values for behavioural parameters</td>
<td>40</td>
</tr>
<tr>
<td>Table 3</td>
<td>Some aggregate variables for Canada (percentage change from benchmark)</td>
<td>41</td>
</tr>
<tr>
<td>Table 4</td>
<td>Percentage change in GDP at market price</td>
<td>42</td>
</tr>
<tr>
<td>Table 5</td>
<td>Percentage change in welfare</td>
<td>43</td>
</tr>
<tr>
<td>Table 6</td>
<td>Permit revenue ($ ml)</td>
<td>44</td>
</tr>
<tr>
<td>Table 7</td>
<td>Permit price ($)</td>
<td>45</td>
</tr>
<tr>
<td>Table 8</td>
<td>Dollar purchase of CO₂ permits ($ ml)</td>
<td>45</td>
</tr>
<tr>
<td>Table 9</td>
<td>Percentage change in share of CO₂ permit purchase</td>
<td>46</td>
</tr>
<tr>
<td>Table 10</td>
<td>Percentage change in total emissions</td>
<td>47</td>
</tr>
<tr>
<td>Table 11</td>
<td>Percentage change in wage</td>
<td>47</td>
</tr>
<tr>
<td>Table 12</td>
<td>Percentage change in labour supply</td>
<td>48</td>
</tr>
<tr>
<td>Table 13a</td>
<td>Total output in Alberta by industry sectors (percentage change)</td>
<td>50</td>
</tr>
<tr>
<td>Table 13b</td>
<td>Total output in Ontario by industry sectors (percentage change)</td>
<td>51</td>
</tr>
<tr>
<td>Table 13c</td>
<td>Total output in Québec by industry sectors (percentage change)</td>
<td>52</td>
</tr>
<tr>
<td>Table 14a</td>
<td>Emissions in Alberta by industry sectors (percentage change)</td>
<td>53</td>
</tr>
<tr>
<td>Table 14b</td>
<td>Emissions in Ontario by industry sectors (percentage change)</td>
<td>53</td>
</tr>
<tr>
<td>Table 14c</td>
<td>Emissions in Québec by industry sectors (percentage change)</td>
<td>54</td>
</tr>
<tr>
<td>Table 15a</td>
<td>Demand for value-added in utilities sector by regions (percentage change)</td>
<td>54</td>
</tr>
<tr>
<td>Table 15b</td>
<td>Demand for value-added in basic chemicals by regions (percentage change)</td>
<td>54</td>
</tr>
<tr>
<td>Table 15c</td>
<td>Demand for value-added in oil &amp; gas by regions (percentage change)</td>
<td>55</td>
</tr>
<tr>
<td>Table 16a</td>
<td>Total household consumption in utilities sectors (percentage change)</td>
<td>55</td>
</tr>
<tr>
<td>Table 16b</td>
<td>Total household consumption in other manufacturing (percentage change)</td>
<td>56</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>Endogenous technological change and output-based emissions allowances</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Table 1a</td>
<td>Some characteristics of the social accounting matrix for Canada in the base period (percent shares)</td>
<td></td>
</tr>
<tr>
<td>Table 1b</td>
<td>Some characteristics of the social accounting matrix for Canada in the base period (percent shares)</td>
<td></td>
</tr>
<tr>
<td>Table 2</td>
<td>Values for behavioural parameters</td>
<td></td>
</tr>
<tr>
<td>Table 3</td>
<td>Sectoral emission shares (%) in the base period</td>
<td></td>
</tr>
<tr>
<td>Table 4</td>
<td>Impacts on selected aggregate variables in the 20th period from various simulations (percentage deviation from BAU unless otherwise mentioned)</td>
<td></td>
</tr>
<tr>
<td>Table 5a</td>
<td>Sectoral impacts on value-added and employment in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 5b</td>
<td>Sectoral impacts on investments in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 6a</td>
<td>Sectoral impacts on supply in various markets in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 6b</td>
<td>Sectoral impacts on total demand, imports, and domestic demand in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 6c</td>
<td>Sectoral impacts on consumption, and consumer prices in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 7</td>
<td>Sectoral impacts on emission in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 8</td>
<td>Impacts on consumer prices of selected fossil energy products in the 20th period from various simulations (percentage deviation from the BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 9</td>
<td>Sectoral GDP share at market price in the 20th period from various simulation</td>
<td></td>
</tr>
<tr>
<td>Table 10</td>
<td>Long-run impacts (30th period) on some aggregate variables from various simulations (percentage deviation from BAU)</td>
<td></td>
</tr>
<tr>
<td>Table 11</td>
<td>Welfare indicators for various sensitivity scenarios (SS)</td>
<td></td>
</tr>
<tr>
<td>Table 12</td>
<td>Welfare improvements (%) in various scenarios relative to the Base Scenario (BS)</td>
<td></td>
</tr>
<tr>
<td>Table 13</td>
<td>Welfare improvements (%) due to R&amp;D Subsidy with various O-R&amp;D scenarios over pure OBA</td>
<td></td>
</tr>
</tbody>
</table>
Acknowledgement

Every thesis purports to intelligently convey an interesting story. However, the stories that bring the thesis into fruition – the stories of invaluable support, compassion and sacrifice – are as fascinating as the thesis itself. While the scarcity of space may not allow me to do justice to reflect upon all of those tales in entirety, I would like to provide my gratefulness to all who have been at the background during the development of this thesis.

First and foremost, I would like to thank the God Almighty, who provided me with the patience and guidance for me to be able to finish writing this thesis.

I sincerely thank Professor Yazid Dissou, my thesis supervisor, not only for unravelling the magnificently addictive beauty of research to me, but also for instilling the belief in me that I can successfully complete my dissertation at a critical time when my internal motivation was at its lowest ebb. I would also like to thank the member of my thesis evaluation committee, Professor Randall Wigle, Professor Marcel Mérette, Professor M. Leslie Shiell and Professor Douglas A. Smith for providing me with excellent comments to make this thesis an improved product. I gratefully acknowledge the partial funding that I have received from the SSHRC research grant on “Canadian Environmental Issues” while writing the thesis.

Thanks to my mother, Farida Easmin Arif, and particularly to my father, Md. Taufiqul Arif, for igniting the desire in me in my childhood to complete the PhD. Simply, I cannot thank you enough for your loving care and for the continued unflagging support in everything that I try to involve myself in. I also thank Tahmina Farah Arif, my twin sister and Md. Fahim Deen Arif, my brother, for their affection and, to my pleasant surprise, for their joyful pride they take-in in recognizing my effort to complete the degree. I thankfully acknowledge the support from my extended family – all my chachas & chachis, phupus & phupas, mamas & mamis and khalas & khalus – who have always been with me in spirit along the way.

I gratefully acknowledge the great deal of support that I received during the invaluable presence of Gulshan Ara Alamgir and Dr. Muhammad Alamgir, my parents-in-law, who helped me go through the critical first workshop stage of this thesis.
My sincere gratitude to the pool of wonderful friends – Dr. Chahreddine Abbes, Meda-Cristina Horacsek, Dr. Teodora Cosac – who inspired me all along the way. I particularly owe to Chahreddine for providing me with constructive criticisms on numerous earlier versions of this work. I also thankfully acknowledge the direct and indirect support I have received from my colleagues – Bob Papanikolaou, Sean Collins, Patricia Brady and Celeste Hicks – that certainly helped me to complete the dissertation.

It should not go without mentioning the kindness and support of close family friends – Marufa and Mazib Rahman, Nahreen Rahman and Farzad Khan, Rebecca Dipa Khan and Abul Salek, Sharmin Mallick, Razib Iqbal as well as Adnan and Dipa Siddiky.

The metamorphosis of a set of ideas into a full-blown thesis only takes place over time. What makes it possible, however, is the continued invaluable support that sustains the process of tiny but everyday progress, eventually culminating into the thesis. I have always held this priceless support from the loved ones with great esteem and believed it to be as important, if not more, as achieving the overall objective of completing the degree. I would like to gratefully acknowledge all the support that I have received from my loving wife, Rizwana Alamgir-Arif, who undoubtedly bore the brunt of the toll during this process. You deserve the gratitude not only because of your phenomenal inspiration that you infused in me during the numerous upheavals of the thesis writing process, but also because you gracefully did so while writing your own doctoral dissertation and perhaps experiencing the ups-and-down of your own.

Most of my accomplishments as a doctoral candidate occurred since the birth of our first child, Ruhaan Aydeen Arif. My two and a half year old’s beautiful smile, gleeful laugh and boisterous energy radiates cheer and energy to our lives. It is this energy that endorses the completion of this thesis.

Last but not the least, I would like to pay homage to the loving memories of my grand-parents – Khaleda Rawshan Ara and Golam Abedin and Jobeda Khatun and Miah Fazlar Rahman – who had the earnest desire for me to achieve this educational feat, long before I could understand the intrinsic value of the achievement! Particularly I would like to mention the contribution of the
greatest two ladies of my life – my paternal grandmother, Khaleda Rawshan Ara, and my maternal grandmother, Jobeda Khatun – who deserve it for providing me with the compassionate care during the first few frail years of my life. Had it not been for them, perhaps I would not be who I am today. I dedicate this thesis to them.
INTRODUCTION
Introduction to the Thesis

The need for necessary action to stabilize atmospheric greenhouse gas (GHG) concentrations to a level that would prevent dangerous anthropogenic interference with the climate system led to the creation of the largest environmental treaty known as the United Nations Framework Convention on Climate Change (UNFCCC) in 1992. Since then, the treaty purported to bring along periodic updates, generally known as the protocols, in order to allow for provisions that can set legally enforceable emission limits for parties to the convention (i.e. the individual nations that ratified the convention). To date, Kyoto Protocol has been the major update to the UNFCCC. Based on a phased approach, Kyoto Protocol set out detailed initial plan on commitments by the parties to reduce GHG emissions relative to a reference level\(^1\). The first phase of the commitment period was set to be 2008-2012, with the room for negotiations for the second period commitment scheduled to commence in 2005 (Grubb and Depledge, 2001).

These negotiations are generally held at what is referred to as the Conference of Parties (COP)\(^2\). The COP provides a podium for the international community to engage into multilateral negotiations. Since 2005, the goal has been to reach a consensual course of actions beyond the first commitment period in order to effectively mitigate the climate change phenomenon. So far, the outcome from these conferences has not been very promising. The hope for greater alignment of climate change mitigation policies among the major world economies, however, remains even though the path to such a promising future seems rocky.

Why is it that reaching a greater consensus is hard to come by? What are the issues that underlie specific policy stance taken by the specific countries? Unfortunately the answers to these questions are far from being simple. For a plethora of reasons, the issues around climate change mitigation policies are complex in nature. These complexities arise from multiple dimensions that not only vary across different regions of the world, but also vary across

\(^1\) The establishment of the national greenhouse gas inventories, the first major task of UNFCCC, led to the creation of the 1990 benchmark level. This benchmark was used in the Kyoto Protocol for the accession of the Annex I countries, where these countries are required to reduce their GHG emissions within the first commitment period 2008-2012 (for further details, see Annex B of the Kyoto Protocol).

\(^2\) At the time of writing this thesis, the last COP (i.e., the 16th COP) was held in Cancun, Mexico during 29 November-10 December, 2010.
different regions or even different sectors within a given country. Consequently, climate change mitigation policies are hard to formalize and difficult to implement.

The purpose of this thesis is to focus on some of these issues, both from theoretical as well as empirical perspectives. Given the enormous breadth of issues that climate change mitigation policies generally encompass, in the present context we narrow down our horizon by focusing on issues relevant to the development of emission mitigation policies of a single country. Consequently, the discussions in the ensuring chapters of this thesis assume a given emissions reduction target for the country of investigation (i.e. the aggregate national emission reduction target is provided exogenously), and evaluate various policy options to attain the targeted national level of emissions for that country. We take up Canada as our country of investigation.

While focusing on the issues specific to the development of country level emissions mitigation policies may seem that the current thesis abstracts away from the international context, in effect it indirectly addresses the international dimensions in two important ways. First, the policy options considered in this thesis are not necessarily relevant to Canada alone. In principle, they can certainly be applied to the context of other countries, particularly the developed nations that have political and macroeconomic features similar to that of Canada’s. Second, and more importantly, two empirical research included in this thesis critically evaluate likely policy options available to Canada for it to credibly comply with its international commitment once such an obligation is identified at the international level (e.g. through a binding UNFCCC protocol at a future date). In the spirit of backward induction analysis, current research therefore sheds light on the formation of Canada’s likely response to the future international commitments for reducing emissions relative to specific benchmarks.

We begin by laying out two empirical investigations in chapter one and two of this thesis. An exploratory theoretical investigation follows in the final chapter.

In chapter one, we begin by investigating the contentious issue of sharing the cost burden that stems from emission mitigation effort. This is linked to the distributional equity issue that typically trades-off against the efficiency aspect of any policy stance. While this
presents one of the major impasses in the international negotiations (as is commonly observed in the COP negotiations), it has the potential to generate similar obstacle in implementing a national emission mitigation target within the context of a federalist country that has decentralized environmental jurisdiction. In a country with such political division of power over environmental regulation, while the federalist government is often in charge of ratifying an international treaty, it is the regional governments which carryout the implementation plans in order to collectively satisfy the federal obligation. This is the case in countries like Canada, U.S. and Belgium (Boucekkine et al, 2008).

Negotiations towards an agreement among the regions in this context are generally marked by each region willing to bear the minimum cost while letting the others shoulder the brunt of the national mitigation target. Thus, concerns regarding equity play a major role that often comes at the cost of efficiency in achieving the mitigation target. We focus on this trade-off within a cap-and-trade (CAT) framework of emission permit allocation policy using a multi-region, multi-sector static computable general equilibrium (CGE) model of the Canadian economy. Under the CAT framework the equity issues, generally entailed in any specific permit allocation policy that reflect the burden cost of the policy, take the form of \textit{ex-post} transfers of rents that stem from emission trading among the regions. We present results from investigating five such permit allocation policies and introduce a sixth policy (we call it the No-Prior Entitlement or the NPE policy) that avoids such inter-regional transfer of scarcity rents thus avoiding the politically contentious issue of sharing the “unfair” amount of burden cost. Results derived from this exercise highlights the relative political appeal of the NPE policy vis-à-vis the others policies.

In chapter two of the thesis, we take-up the equity-efficiency debate one more time but within a separate context. We analyze the trade-off with a focus on the Canadian industry sectors, aggregated at the federal level, that compete amongst each other to bear the minimum burden cost of the national mitigation target. Within the context of the CAT framework, for this purpose we have developed a multi-sector dynamic CGE model of Canada. The novelty in the analysis, however, comes from the introduction of endogenous technological change (ETC) into the modeling framework.
Recent advancements in the climate policy modeling literature indicate that environmental and technology policies work best in tandem. When technology enters into the picture, the terms of the trade-off between the marginal cost of emission control and its marginal social benefit is altered (Popp et al., 2009). Typically technology innovations reduce the marginal cost of achieving a given unit of pollution reduction. In most cases, technological change enables a specified level of environmental cleanup to be achieved at lower total cost to society. New developments in this area indicate that ETC has the potential to greatly ameliorate the welfare cost of emission control policy in the long run (Fischer et al. 2003, Popp et al. 2009). Following this intuition, we reevaluate the performance of three existing emission permit allocation policies in the literature and compare it with a fourth one. We call the fourth emission permit allocation policy the output based allocation with R&D subsidy (the O-R&D policy) that not only correct for the production disincentive, but also correct for market imperfection in the knowledge accumulation sector. Our results provide a renewed ranking of the existing policies in terms of the welfare cost. The results also highlight the important role of ETC into the equity-efficiency debate.

In the final chapter of the thesis, we take-up an exploratory research. We reinvestigate an issue that has received an ample attention in the theoretical climate policy modeling literature, which involves evaluating the role of permit banking and borrowing in mitigating emissions across time (Cornshaw and Kruse, 1996 and Leiby and Rubin, 2001). Our analysis adds on to the existing literature by introducing technological change into the picture that has critically relevant, but not yet explored, link to the overall framework.

Inclusion of endogenous technological change into the climate policy models is typically channeled through what is known as the induced technological change (ITC) process. Originally credited to Hicks (1932), the ITC hypothesis highlights the role of price-induced technological change that is directed to promote innovations in order to save the use of relatively expensive input into the production process. Thus, emission control policies that typically act by increasing the cost of polluting inputs ought to interact with the ITC process.

---

3 The disincentive emanates from the higher cost of production due to emission control enforcement.
Under the CAT framework of emission control, on the other hand, the permit banking and borrowing literature indicates that permit price depends on the demand and supply of permits over time. In other words, with banking and borrowing, permit price becomes dependent across time due to the decentralization of the permit supply management process where private firms are allowed to hold permits at their disposal. This has implication for both the ITC process and permit banking and borrowing system.

Intuitively, given the ITC mechanism and the impact that banking and borrowing has on the permit price, it appears that these two issues are inter-linked and need to be studies in a unified framework. We attempt to address this by developing a simple optimal control model. We provide characterization of the optimal path of emissions and the path of permit price under banking and borrowing when ITC is allowed.
CHAPTER ONE
Regional Burden Sharing of GHG Mitigation Policies
- A Canadian perspective

1.1 Introduction:

The threat of global warming has induced the international community to setup a goal towards reducing atmospheric accumulation of greenhouse gases (GHG) caused by anthropogenic reasons. International negotiations such as those held in the United Nations led Conference of the Parties (COP) provide a forum for international community to negotiate a deal aimed at achieving that goal. The hope is that the negotiations will lead the way for countries to arrive at legally binding commitment levels in order to reduce global emissions. What generates heated discussions in these negotiations is the issue of burden sharing that ultimately stems from the issue of distribution of welfare costs among the participating countries. Not surprisingly, the issue of ‘fair’ burden sharing has continued to recur as a key theme in the climate protocol negotiations since the Kyoto round (Ringius et. al, 1998).

The topic remains as contentious, and has the potential to generate as heated a debate, even within the national boundaries; more so, where the specific country under scrutiny has a non-unitary system of government (i.e. a decentralized federalist structure) like Canada, the U.S. and Belgium (Boucekkine et. al, 2008). The scope of this arises since, in some of these countries, while the federal government has the authority to sign international agreements relating to environmental issues, regional jurisdictions have to decide on their implementation plans.

While debates may exist on the specifics of an approach for implementing a plan, a perception that is equally shared by the federal and regional governments is that the reduction of GHG emissions is costly. Implementation of any plan for curbing emissions, thus, requires some form of consensus on burden sharing among the regional governments. In a decentralized system of governance, however, negotiations towards an agreement are generally marked by each region willing to bear the minimum cost while letting the others shoulder the brunt of the national mitigation target (i.e. the free-riding effect).
Moreover, the costs arising from this equity-efficiency trade-off also tend to differ not only over available policy options, but also from one region to another. In other words, heterogeneity in the composition of regional economies plays a critical role in the debates over cost sharing. Thus, for successful adoption of a binding accord and its subsequent implementation, the regional incidences of GHG burden costs must be carefully investigated. Such an investigation not only ought to identify the optimal policy from the set of feasible policy options, but also needs to ground it on equity principles with firm ethical standing\textsuperscript{4}. This is the primary focus of this study. In particular, based on recent equity-efficiency debate in the literature concerning GHG mitigation efforts and its impacts on different industrial sectors, we present an assessment of six policy scenarios, where we explicitly consider the heterogeneity in energy outlook and the associated burden sharing consequences for the regional economies. We use Canada as the country of investigation for this purpose.

In the context of international climate policy negotiations, existing literature offers a wide range of criteria for evaluation of the burden-sharing rules involving efficiency and equity considerations (Torvanger and Ringius, 2002). These rules may generally also be applicable in the regional context. Within the cap-and-trade (CAT) framework, equity in these negotiations typically implies assigning implicit (and often \textit{a priori}) emission entitlements (i.e. assigning emission rights). Except for the unlikely structure of regionally segregated emission permit markets that might exist within the same national boundary\textsuperscript{5}, such entitlement schemes also entail the contentious issue of inter-regional transfer of scarcity rents that occurs through trading of rights in a common (single) national emission permit market. In fact, this issue has long been a feature of the international negotiations on climate change, and was a major factor in the U.S. withdrawal from the Kyoto Protocol (Jacoby and Reiner, 2001 and Sue Wing, 2007).

Given that emission intensity as well as the extent of access to emission-benign technology varies across regions, from a rent-seeking perspective, regional authorities have incentive to lobby for specific permit allocation policies that favour them over the others. Such lobbying efforts will try to keep costs as low as possible or even gain additional rents at the

\textsuperscript{4} For an excellent discussion on equity principles considered in international negotiations, see Rose (1992 and 1998). Barrett (1992) also provides some valuable discussions on the moral and ethical aspects of some widely recognized equity principles.

\textsuperscript{5} We rule out this possibility in this paper.
expense of others. This generally characterizes the existing emission allocation policies (e.g. emission-based allocation of permits or permit allocation based on emissions per capita) that are often highlighted in the literature. The race to rent-seeking, however, not only makes negotiations highly contentious but also largely diminishes political palatability of available emission allocation policy options.

The challenge is then to devise a mechanism that can avoid this contentious issue of inter-regional transfer of emission rents. In addition, the mechanism should be also capable of achieving the national objective while rendering equitable distribution of burden costs across the regional jurisdictions. In this paper, we attempt to devise such a scheme. We call this the no prior entitlement (NPE) scenario in which, as the name suggests, no ex-ante emission entitlements are assigned to the regions. Regions, in turn, receive, in a lump-sum fashion, their share of scarcity rents ex-post based on residual (i.e. equilibrium) levels of regional emissions. We also provide a comparative analysis on the performance of NPE scheme relative to the existing schemes in the literature. In particular, using a static computable general equilibrium model of Canada, we assess welfare costs of five other permit allocation schemes. These are – emission based allocation (EBA or the proportional allocation of rights based on the past levels of emissions); allocation based on: emissions per capita (EPC); converging carbon intensity (CCI); efficiency index (EI); and the multi-criteria index (MI). A detailed discussion on each of these allocation schemes and the underlying equity rationale is provided in section 3.

While existing literature presents several policy instruments that are capable of achieving the targeted reduction in emission levels, market-based instruments (MBI) are usually the most preferred ones. This is because MBI can achieve the specified objective at the least cost (Tietenberg 1985). Furthermore, MBI also possess the dual benefit of being an efficient instrument (since they function via market mechanisms) in addition to being less burdensome due to their ability to incorporate equity considerations. The current policy schemes considered for the proposed scenarios also fall within this category.

---

6 NPE scheme can be thought of as a fictitious auctioning of emission permits where, given demand and supply of permits, the auction determines the equilibrium price of the permit. The federal government then uses the equilibrium price to allocate emission proceeds to individual regions according to their residual level of emissions.

7 Note that each of these schemes renders a specific value judgment on equity and caters to a specific set of considerations.
A salient feature of the MBI is that, depending on the emission rights allocation policy, it may generate scarcity rents for the recipients of emission permits that can be used to mitigate the negative costs of abatement. These revenues come from unabated emissions, i.e., the targeted level of emissions assigned to the country. Thus, in the form of permits, the distribution of the unabated emissions among the regional jurisdictions will have some implications on the regional costs and welfare impacts of GHG mitigation. This is the framework employed in this paper. In particular, within the CAT structure with a single national emission permit market where regions can trade permits freely, we explore policy options/scenarios that consider equity issues in the form of allocation of emission permits to the regions. Emission allowances thus primarily take two forms under these scenarios – entitlement versus no prior entitlement (NPE).

While under the entitlement-based scenarios emission rights are allocated to the regions ex-ante, under the NPE scenario emissions allowances are allocated based on each regions’ residual (i.e. ex-post) level of emissions. For both of these broad allocation schemes, however, a regional government distributes its lump-sum share of scarcity rent ex-post among its economic actors (more specifically, to the representative household of each region).

For the current purpose, we divide Canada into six regions – Alberta, Atlantic Provinces, Ontario, Other Prairie Provinces, Québec, and British Columbia and the Territories. Once regional emissions allowances are identified under the six schemes considered in the paper, regions are free to trade permits in the single national emission permit market. The scarcity rents, thus generated through trading, is then recycled to the representative households using the lump-sum recycling method. Hence, within the model framework, while the allocation principle essentially tackles the equity concerns of the competing regions, permit trading (and the recycling method of scarcity rents) addresses the efficiency aspect of the equation.

The remainder of the paper is organized as follows. In the next section, we provide a review of recent relevant literature. A thumbnail sketch of the model proceeds after that. In particular, this section includes description of various sectors – i.e. households, firms, and the

---

8 The choice of regional segregation, in effect, corresponds to the available set of data.
9 In other words, this ensures efficiency through the functioning of the emission permit market.
10 Each region is assumed to have one representative household.
governments, as well as the equilibrium conditions of the model, and the closure rules. A detailed discussion on the six policy schemes examined in the current paper and their equity implications follows the next. The salient features from the model-runs are presented in the ensuing results section. Finally, the paper wraps-up with some concluding remarks.

1.2 Review of the Relevant Literature:

In recent times, several studies have attempted to probe into the regional incidences of GHG mitigation costs. In the Canadian context, most of these papers, however, focus primarily on various federal proposals (Snoddon and Wigle 2007 and 2008, and MKJA Associates 2009). In the U.S. context, on the other hand, papers focus on both federal and state level policy options (Sue Wing, 2007 and Sue Wing and Kolodziej, 2008). In almost all of these papers, the general emphasis centers around implementing alternative policy packages that aim to ameliorate the uneven burden costs arising from the national carbon emissions control target.

Using a regional static CGE model of the Canadian economy, Snoddon and Wigle (2007) provide estimates of the aggregate and regional costs, measured in terms of the loss in regional GDP, that are associated with different climate change policy packages. They assume that the federal government uses a proportionality rule (i.e. EBA) for allocating Canada’s total international endowment of emission permits to their provincial counterparts, who in turn, also use an EBA scheme to distribute their share of permits. The authors also compare this scheme to Jaccard, Rivers and Horne (2004) plan. The EBA scheme is based on charging all emitters a price equal to the world price of carbon permits. This constitutes the key feature of the model – namely, that all emitters face the same incentive to abate at the margin. While the authors identify that equal incentive serves to achieve cost-effectiveness condition for the prescribed policy packages, they neither consider pre-existing distortions in the economy, nor do they account for uneven burden costs on various industries within the regional economy. We take up these issues in this paper.

Using a static CGE model of U.S., on the other hand, Sue Wing (2007) emphasizes two issues – the effects of declining factor remuneration on welfare, and the welfare impacts of using
different revenue-recycling schemes (i.e. recycling at the federal or state level). Sue Wing shows that diminishing returns to factor arises from the scale and homogeneity of interstate commodity markets. The imperfect factor mobility, captured in the modeling structure, reinforces this process through curtailment of the expansion of import-competing industries in States, which import large quantities of domestically-produced energy. Together, these raise the burden costs and make it uneven for some States at the expense of the others. Sue Wing, however, further demonstrates that the effects of uneven burden costs can be mitigated through appropriate method of revenue recycling when an auction permits scheme is used. He demonstrates that when revenue is recycled at the State level, it generally distributes the burden cost over all the States (especially the large States) via terms-of-trade effects. If, on the other hand, revenue is recycled at the federal level, the burden cost seems to intensify for the carbon based energy-intensive States at the benefit of the large States. Results derived form our model somewhat correspond to this qualitative finding. However, the recycling method and the policy schemes considered in our paper are different from the ones implemented by Sue Wing (2007).

While these papers consider various within-country issues, they ignore one specific issue – namely, the overall impact on welfare and the burden cost when regional governments are accorded no prior emission entitlements versus the case when they are assigned such entitlements ex-ante. We consider this issue in this paper. Using the lump-sum method of revenue recycling to the representative households, we investigate how welfare changes when different Canadian regions use the entitlement approach for allocating emission permits (EBA, EPC, CCI, EI and MI) versus the case when emission permits are allocated based on no prior entitlement (NPE) principle. We use a static CGE model of Canada’s regional economies to address this question in this paper. In-so-doing, we particularly pay heed to the heterogeneous economic structures of the provincial economies and identify its effects on the overall regional welfare.

1.3 The Model:

In this section, we describe the model developed for the analyses presented in this paper. Current model belongs to the family of static CGE models, where Canada is considered a small
open economy vis-à-vis the rest of the world. Within the national boundary, the country is divided into six regions where regional economies are characterized by the free flow of goods and services.

The model presented in this paper incorporates multi-sector, multi-region structure that entails the characteristics of the individual regional economies. Thus, heterogeneity among regional economies is captured into the modeling framework. Nesting of production structure used in the model is somewhat akin to that of Dissou (2006) with modifications being carried out especially in the markets for primary factors of production. Overall, the model generally features a disaggregation of the production and consumption sectors in each region. An interesting feature of the model is that it captures not only the trade flows of each region with the rest of the world but also the flows between and among these regions.

1.3.1 Households

A fixed portion of household’s income is used to pay income taxes to the government, while its savings are a linear function of its disposable income. A representative household in each region chooses its basket of goods by maximizing its utility subject to its budget constraint. Its optimization problem is postulated as follows:

Max \( U_R = \alpha_R \ln C_R + (1 - \alpha_R) \ln (H_R - L_R) \) \hspace{1cm} [1]

Subject to:
\[ w_R L_R = P_{C,R} C_R \quad \text{or} \quad w_R H_R = P_{C,R} C_R + w_R (H_R - L_R) \] \hspace{1cm} [2-1]

where,
\[ Y_R = (1 - \tau_{Y,R}) Y_{H,R} + T_{G,R} - ER_R T_{R}^w \] \hspace{1cm} [2-2]
\[ Y_{H,R} = w_R L_R + Y_{D,R} + \bar{P} R_R \] \hspace{1cm} [2-3]
\[ L_R = \sum_j L_{j,R} \] \hspace{1cm} [2-4]
\[ Y_{D,R} = \beta_{D,R} (1 - \tau_{P,R}) \sum_j R_{K,R} K_{j,R} \]  

and,

\[ S_{H,R} = \beta_{S,R} Y_R \]

Here, \( U_R \) = temporal utility function for region \( R \); \( C_R \) = aggregate consumption in region \( R \); \( H_R \) = total time endowment of a representative household in region \( R \); \( L_R \) = aggregate labour supplied in region \( R \); \( \alpha_R \) = share of expenditure on aggregate consumption in region \( R \); \( w_R \) = wage rate in region \( R \); \( P_{C,R} \) = index price of unit aggregate consumption in region \( R \); \( Y_R \) = aggregate disposable income of a representative household in region \( R \); \( T_{G,R} \) = government transfer to region \( R \) household; \( E_{R} \) = exchange rate (price of foreign currency in terms of domestic currency) in region \( R \); \( T_{G,R}^{W} \) = government transfer to the rest of the world from region \( R \); \( Y_{H,R} \) = aggregate gross income of region \( R \) household; \( \tau_{Y,R} \) = income tax rate of region \( R \) on non-dividend income; \( Y_{D,R} \) = net of tax dividend income of a representative household in region \( R \); \( \bar{P} \) = unit price of emission permit; \( R_{R} \) = total emission rights received by region \( R \); \( L_{j,R} \) = labour supplied to sector \( j \) in region \( R \); \( R_{K,R} \) = gross rate of return from investment in capital in region \( R \); \( K_{j,R} \) = capital stock in sector \( j \) of region \( R \); \( \beta_{D,R} \) = household share of total dividend income in region \( R \); \( \tau_{P,R} \) = tax rate on dividend income in region \( R \); \( S_{H,R} \) = household savings in region \( R \); and \( \beta_{S,R} \) = household marginal propensity to save in region \( R \).

The objective function \( U_R \) in equation [1] is the log-linear version of the Cobb-Douglas (C-D) utility function with aggregate consumption \( (C_R) \) and the amount of leisure enjoyed by the household \( (H_R - L_R) \) as its two arguments. Hence, the household exhibits labour-leisure choice in the model.
Disposable income for consumption \(Y_R\) in equation [2-2] is defined as a function of gross household income \(Y_{H,R}\) and government transfer \(T_{G,R}\) to household net of transfers to the rest of the world \(ER^w_RT_{G,R}\).

Equation [2-3] is the definition of household income \(Y_{H,R}\) that consists of total labour income net of the payroll tax \(w_R L_R\), household dividend income \(Y_{D,R}\) and household income due to recycling of proceeds received from emission permit trading \(PR_R\) derived from the allocation of regional emission rights.

Equation [2-4] signifies that the total regional labour supply is equal to the sum of labour supplied to industry sectors disaggregated into \(j\) industries in that region. Equation [2-5], on the other hand, provides the definition of dividend income \(Y_{D,R}\) for the representative household that is a fixed share of net-of-tax income received from investing in physical capital \(\sum_j R_{K,J,R} K_{J,R}\) across different sectors of the regional economy\(^{11}\).

Finally, equation [3] gives the definition of household savings. This, in turn, implies that the total household expenditure \(EM_R\) and the non-labour income \(Y_{NL,R}\) respectively are given by:

\[
EM_R = (1 - \beta_{S,R})Y_R \tag{3-1}
\]

\[
Y_{NL,R} = EM_R - (1 - \tau_{Y,R})w_R L_R \tag{3-2}
\]

The Lagrangian for the optimization problem can be written as:

\[
\ell_R = \alpha_R \ln C_R + (1 - \alpha_R) \ln (H_R - L_R) + \lambda_R \left[ w_R L_R - P_{C,R} C_R \right]
\]

\(^{11}\) Given the static general equilibrium framework, we model capital market as geographically segmented, where each region receives an exogenously supplied stock of capital. Hence, depending on the demand for capital by the regional industry sectors, rental return from capital varies across regions.
The interior solution yields the aggregate regional consumption and labour supply function that, respectively, take the form:

\[
C_R = \alpha_R \left\{ \frac{(1-\tau_{Y,R})w_R H_R + Y_{NL,R}}{P_{C,R}} \right\}
\]

and, 

\[
L_R = H_R - (1-\alpha_R) \left\{ \frac{(1-\tau_{Y,R})w_R H_R + Y_{NL,R}}{(1-\tau_{Y,R})w_R} \right\}
\]

The aggregate consumption of the representative household, in turn, is allocated over different consumption goods using an expenditure minimization principle. We use the Constant Elasticity of Substitution (CES) aggregator function for the purpose. The corresponding optimization problem, hence, takes the following form:

\[
\text{Min} \left\{ P_{C,R} C_R \right\} = \sum_i P_{C,i,R} C_{i,R}, \text{ where } P_{C,i,R} = \left[ P_{C,i,R} + \bar{P} \xi_{H,i,R} \right] (1+\tau_{C,i,R})
\]

Subject to: 

\[
C_R = \sum_i A_{c,i,R} \left[ \alpha_{c,i,R} C_{i,R} \right] ^{\sigma_{c,i,R} - 1} \sigma_{c,i,R} ^{\sigma_{c,i,R} - 1}, \text{ where } \sum_i \alpha_{c,i,R} = 1
\]

where, 

\( C_{i,R} \) = consumption of good \( i \) in region \( R \); \( P_{C,i,R} \) = gross of emission tax price of consumption good \( i \) in region \( R \); \( P_{C,i,R} \) = net of tax price of consumption good \( i \) in region \( R \); \( \bar{P} \) = unit price of emission permit; \( \xi_{H,i,R} \) = emission factor of good \( i \) used in household consumption in region \( R \); \( \tau_{C,i,R} \) = indirect tax rate on consumption of good \( i \) in region \( R \); \( \sigma_{c,i,R} \) = elasticity of substitution between consumption goods in region \( R \); \( \alpha_{c,i,R} \) = share parameter in the consumption aggregator function in region \( R \); and \( A_{c,i,R} \) = shift parameter in the consumption aggregator function in region \( R \).

The representative household allocates the optimal aggregate consumption \( (C_R) \) across different consumption commodities \( (C_{i,R}) \). Equation [6] indicates that the aggregate consumption expenditure in each period \( (P_{C,R} C_R) \) equals the sum of consumption spending across different commodities \( \sum_i P_{C,i,R} C_{i,R} \) in each region. In equation [7], we specify the
aggregate consumption as an aggregation of different consumption commodities, where the aggregator function takes the CES specification.

![Diagram](image)

**Figure 1: Schematic representation of household preference**

Figure 1 provides a schematic representation of the representative household’s choice structure. As evident in Figure 1, a system of 4 nodes represents various composition of commodity use\(^{12}\). These aggregator functions allow us to mimic the real life substitution behaviour exhibited by the households of the modeled regional economies. Further the nested structure of the model allows for substitution, on the one hand, between energy and non-energy products and, on the other hand, among various energy goods\(^{13}\).

From the optimization behaviour identified in equation [6], we derive the following first order necessary conditions:

\[
A_{c,i,R} \alpha_{c,i,R} C_{c,i,R}^{\sigma_{c,i,R}^{-1}} \left( \sum_i \left[ \alpha_{c,i,R} C_{c,i,R}^{\sigma_{c,i,R}^{-1}} \right]^{\sigma_{c,i,R}^{-1}} \right)^{-1} = \frac{P_{C,i,R}}{\lambda_{C,R}} \tag{O-1}
\]

\[
C_R = \sum_i A_{c,i,R} \left[ \alpha_{c,i,R} C_{c,i,R}^{\sigma_{c,i,R}^{-1}} \right]^{\sigma_{c,i,R}^{-1}} \tag{O-2}
\]

where, \(\lambda_{C,R}\) is the relevant Lagrange multiplier.

---

\(^{12}\) In the model, the economy’s product lines are disaggregated into a total of 26 commodities as depicted in Figure 1.

\(^{13}\) For the elasticity of substitution values, see Table 2c.
Following the concept of Shephard’s Lemma, equations [O-1], [O-2] and [7] are manipulated further to finally get the consumption demand for each commodity \((C_{i,R})\) and the aggregate price level \((P_{C,R})\) as:

\[
C_{i,R} = \frac{\frac{\alpha_{c,i,R} \frac{P_{C,R}}{P_{c,i,R}}}{\sigma_{c,i,R}} \sum_j \sigma_{c,i,j} P_{C,j,R}}{\frac{1}{\alpha_{c,i,R}} \sum_j \left[ \frac{\alpha_{c,i,R} P_{C,j,R}}{\sigma_{c,i,j} \sigma_{c,j,R}} \right]} \tag{8}
\]

\[
P_{C,R} = \frac{1}{\sigma_{c,i,R} \sigma_{c,i,j}} \sum_j \left[ \frac{\alpha_{c,i,R} P_{C,j,R}}{\sigma_{c,i,j} \sigma_{c,j,R}} \right] \tag{9}
\]

### 1.3.2 Firms

In each of the six regions of Canada considered in this study, the production sector is disaggregated into 19 industries producing 26 commodities. In each region, representative firm in each industry has access to a constant-returns-to-scale (CRS) technology. Similar to other general equilibrium models, the production structure is broken down into a sequential decision process that offers some interesting substitution possibilities among inputs.

In the model, production is disaggregated into 8 production nests. While 6 of them are governed by the CES production technology, 1 of each is governed by the C-D and Leontief production technology.
production technique. Figure 2 provides a schematic representation of representative firms’ production nests at various levels.

Foremost is the composite output, which is a CES function of the composite input of capital-labour-energy and the aggregate input of material-mobile factors. Given the assumption of perfectly competitive markets, the zero profit condition would imply that net revenue \((P_{Q,j,R}Q_{j,R})\) equals the sum of input costs for these composite inputs. Therefore, we postulate a cost minimization problem at the top production nest as:

\[
\min P_{Q,j,R}Q_{j,R} = P_{X,j,R}X_{j,R} + P_{M,j,R}M_{j,R}, \quad \text{where } P_{Q,j,R} = P_{Q,j,R} \left(1 - \tau_{Q,j,R}\right)
\]

Subject to,

\[
Q_{j,R} = A_{Q,j,R} \left[ \varphi_{j,R} X_{j,R} J_{j,R} \left[ \sigma_{Q,j,R} - 1 \right] + \left(1 - \varphi_{j,R}\right) M_{j,R} J_{j,R} \left[ \sigma_{Q,j,R} - 1 \right] \right]
\]

where, \(Q_{j,R}\) = quantity of composite output of industry \(j\) in region \(R\); \(P_{Q,j,R}\) = gross of tax price of output \(j\) in region \(R\); \(\tau_{Q,j,R}\) = rate of production tax on output of good \(j\) in region \(R\); \(X_{j,R}\) = quantity of composite capital-labour-energy input of industry \(j\) in region \(R\); \(P_{X,j,R}\) = index price of composite capital-labour-energy input used in industry \(j\) of region \(R\); \(M_{j,R}\) = quantity of aggregate material-mobile factors of industry \(j\) in region \(R\); \(P_{M,j,R}\) = index price of material-mobile factors used in industry \(j\) of region \(R\); \(\sigma_{Q,j,R}\) = elasticity of substitution between composite inputs used in region \(R\); \(\varphi_{j,R}\) = share parameter in the production aggregator function of region \(R\); and \(A_{Q,j,R}\) = shift parameter in the production aggregator function of region \(R\).

The optimality conditions are:

\[
\frac{X_{j,R}}{M_{j,R}} = \left[ \varphi_{j,R} \frac{P_{M,j,R}}{P_{X,j,R}} \right]^{\sigma_{Q,j,R}} \left[ 1 - \varphi_{j,R} \right]^{\sigma_{Q,j,R}}
\]

\[
Q_{j,R} = A_{Q,j,R} \left[ \varphi_{j,R} X_{j,R} J_{j,R} \left[ \sigma_{Q,j,R} - 1 \right] + \left(1 - \varphi_{j,R}\right) M_{j,R} J_{j,R} \left[ \sigma_{Q,j,R} - 1 \right] \right]
\]
Equation [O-3] indicates that the optimal ratio of composite capital-labor-energy input and material-mobile factor is the function of the ratio of their respective prices and the parameters in CES production function.

A further manipulation of equation [O-3], [O-4], [10] and [11] yields equivalent expressions for optimality conditions:

$$P_{Q,j,R} = \frac{1}{A_{Q,j,R}} \left[ (\varphi_{j,R})^{\sigma_{Q,j,R}} \left( P_{X,j,R} \right)^{1-\sigma_{Q,j,R}} + (1-\varphi_{j,R})^{\sigma_{Q,j,R}} \left( P_{M,j,R} \right)^{1-\sigma_{Q,j,R}} \right]^{-\frac{1}{1-\sigma_{Q,j,R}}} \tag{O-5}$$

$$X_{j,R} = \left( A_{Q,j,R} \right)^{\sigma_{Q,j,R}-1} Q_{j,R} \left( \varphi_{j,R} P_{Q,j,R} Q_{j,R} \right)^{\frac{1}{\sigma_{Q,j,R}}} \tag{O-6}$$

$$M_{j,R} = \left( A_{Q,j,R} \right)^{\sigma_{Q,j,R}-1} Q_{j,R} \left( \left( 1-\varphi_{j,R} \right) P_{Q,j,R} \right)^{\frac{1}{\sigma_{Q,j,R}}} \tag{O-7}$$

As depicted in Figure 2, in the remaining 6 production nests, the same cost minimization principle is used to allocate the cost of the composite input bundle into its component input costs. A CES aggregator function is used for almost all the nests except for the composite of value-added–Energy and of the material inputs. Whereas, for the former composite input bundle a C-D aggregator function is used, for the composite of material input bundle a Leontief specification is used.\(^\text{15}\)

\(^{15}\) For CES specification, a further manipulation of equation [O-3] and [O-4] yields the equivalent expressions for quantity and index price:

$$x_{j,R} = A_{X,j,R} x_{j,R} \left[ \alpha_{s,j,R} P_{X,j,R} \right]^{\alpha_{s,j,R}} \tag{O-10}$$

$$P_{X,R} = A_{X,j,R} \sum_j \left[ \alpha_{s,j,R} P_{j,R} \right]^{-\frac{1}{1-\alpha_{s,j,R}}} \tag{O-11}$$

For Leontief specification the equivalent expressions are:

$$x_{j,R} = \alpha_{s,j,R} x_{j,R} ; \text{and} \tag{O-12}$$

$$P_{X,R} = \sum_j \left[ \alpha_{s,j,R} P_{j,R} \right] \tag{O-13}$$

And for C-D specification the equivalent expressions are:

$$x_{j,R} = \alpha_{s,j,R} x_{j,R} P_{X,j,R} ; \text{and} \tag{O-14}$$

$$P_{X,R} = \frac{1}{A_{X,j,R}} \prod_j \left( \frac{P_{j,R}}{\alpha_{s,j,R}} \right)^{\alpha_{s,j,R}} \tag{O-15}$$
More specifically, as shown in Figure 2, the labour input is combined with the aggregate of capital-energy input using a C-D production process to derive the composite of capital-labor-energy input. The aggregate input of capital-energy, in turn, is obtained via a CES aggregation by combining physical capital and the composite input of energy the latter of which is a CES aggregate of electricity and the non-mobile fossil fuels. Refined petroleum products, natural gas, and coal are combined using a CES function to produce the non-mobile fossil fuel energy input bundle. The aggregate intermediate input, on the other hand, is a CES aggregator of material inputs and mobile energy inputs. While a Leontief aggregation is used to produce the former, a CES function of Gasoline and Diesel is used to combine the latter. At all stages, as identified previously, all firms operate in a competitive environment and maximize profits to determine the level of output supply and their respective factor demands.  

1.3.3 Government

The government sector is kept simple in the model. We consider in each region only one consolidated level of government (i.e. the regional and municipal governments). Government expenditure on goods and services \((C_{G,R})\) is held fixed in real terms. Its other outlay consists of transfers to the households \((T_{G,R})\).

\[
Y_{G,R} = \tau_{Y,R}Y_{H,R} + \tau_{D,R}\sum_j R_{K, R}K_{j, R} + \sum_i \tau_{C,i,R}P_{C,i,R}C_{i, R} + \sum_j \tau_{Q,j,R}P_{Q,j,R}Q_{j, R} + \sum_i \tau_{I,i,R}P_{I,i,R}I_{i, R} + \sum_i E_{R,i}R_{IM,j,R}P_{C,i,R}C_{ROW,j,R} \tag{12}
\]

\[
S_{G,R} = Y_{G,R} - \sum_i P_{C,i,R}C_{G,i,R} - T_{G,R} \tag{13}
\]

where, \(\tau_{I,i,R} = \) tax rate on investment of good \(i\) in region \(R\); \(P_{I,j,R} = \) net of tax price of investment of good \(i\) in region \(R\); \(I_{i,R} = \) quantity of investment of good \(i\) in region \(R\);

---

\(^{16}\) The choice of an aggregator function (i.e., C-D, CES or Leontief) reflects varying degrees of substitutability between the input usage and, in part, is motivated by the characteristics of the base period data. While a Leontief aggregator function implies zero substitutability and a C-D aggregator function shows an elasticity of substitution equal to one (i.e., relatively a high degree of substitution), substitutability reflected by a CES aggregator function can vary. In Figure 2, the C-D aggregator function between labour and composite capital-energy input implies a high degree of substitutability. In other words, it is assumed that at that production nest producers have greater flexibility to respond to a change in the wage rate. The Leontief aggregator function for the composite of material inputs, on the other hand, signifies a technological reality where specific combination of material inputs is required for the production of output.
\[ \tau_{IM,j,R} = \text{import tax rate on good } i \text{ in region } R; \quad P^W_{C,j} = \text{net of tariff price of imported good } i \text{ from ROW in region } R; \quad C^IM_{ROW,j,R} = \text{quantity of imported good } i \text{ from ROW in region } R; \text{ and } S_{G,R} = \text{government savings in region } R. \]

Revenues for the government are raised from indirect taxes on regional transactions and from direct taxes on payments to primary factors. In addition, government also raises revenue from indirect taxes on international transactions. Equation [12] identifies the components of tax revenues \( Y_{G,R} \) – direct tax, indirect tax on activities, import tax and value-added tax. Tax and tariff rates are assumed to be exogenous and fixed throughout the model horizon. Equation [13] defines government savings as the sum of tax income minus the sum of government consumption expenditure, and government transfers to households.

1.3.4 Trade: Regional and International

Trade is modeled in multi-stages. Referring to Armington (1969), regional and foreign goods are distinguished by their origins. This specification has the advantage of accommodating the incidence of cross-hauling both regionally and internationally. On the demand side, regional imports from rest of the regions (ROR) are distinguished from imports from the rest of the world (ROW). We consider, imports from ROR as imperfect substitutes.

A three-level nested CES function allows us to distinguish between international and regional imports. This nested structure allows the representative agent’s decision to take place through a multi-step budgeting process. Figure 3 gives a schematic representation of the nesting structure.

At the first level, regional absorption (i.e. the sum of the demands for final and intermediate use of goods and services) is a CES function of regionally produced goods and the aggregate of imports. We use an expenditure minimization principle to determine the optimal level of each of the final demand components, which is postulated as:

\[
\text{Min } P_{C,j,R} C^D_{i,R} = P^D_{C,j,R} C^D_{i,R} + P^{IM}_{C,j,R} C^{IM}_{i,R} \quad [14]
\]
Subject to, $C_{i,R} = A_{m,i,R} \left[ \kappa_{i,R} \left( C_{i,R}^D \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} + \left(1 - \kappa_{i,R} \right) \left( C_{i,R}^{IM} \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} \right]^{\frac{\sigma_{n,i,R}}{\sigma_{m,i,R}}}$ \[15\]

where, $P_{C,i,R} = $ net of sales tax index price of composite of good $i$ in region $R$; $C_{i,R}^D = $ index price of regionally produced quantity of good $i$ in region $R$; $C_{i,R}^{IM} = $ index price of regionally imported quantity of good $i$ in local currency units; $C_{i,R} = $ regionally imported quantity of good $i$ in region $R$; $\sigma_{m,i,R} = $ elasticity of substitution for import of good $i$ in region $R$; $\kappa_{i,R} = $ share parameter in the aggregator function for good $i$ in region $R$; and $A_{m,i,R} = $ shift parameter in the aggregator function of good $i$ in region $R$.

This yields the optimality conditions:

$$\frac{C_{i,R}^D}{C_{i,R}^{IM}} = \left[ \kappa_{i,R} \left( P_{C,i,R}^D \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} + \left(1 - \kappa_{i,R} \right) \left( P_{C,i,R}^{IM} \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} \right]^{\frac{\sigma_{n,i,R}}{\sigma_{m,i,R}}} \quad \text{[O-8]}$$

$$C_{i,R} = A_{m,i,R} \left[ \kappa_{i,R} \left( C_{i,R}^D \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} + \left(1 - \kappa_{i,R} \right) \left( C_{i,R}^{IM} \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} \right]^{\frac{\sigma_{n,i,R}}{\sigma_{m,i,R}}} \quad \text{[O-9]}$$

Equation [O-8] implies that the optimal ratio of imported to locally produced quantities is a function of the ratio of their relative prices.

A further manipulation of equation [O-8], [O-9] and [14] yields equivalent expressions for optimality conditions:

$$P_{C,i,R} = \frac{1}{A_{m,i,R}} \left[ \kappa_{i,R} \left( P_{C,i,R}^D \right)^{1 - \frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} + \left(1 - \kappa_{i,R} \right) \left( P_{C,i,R}^{IM} \right)^{1 - \frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} \right]^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} \quad \text{[O-10]}$$

$$C_{i,R}^D = \left( A_{m,i,R} \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} C_{i,R} \left[ \kappa_{i,R} \frac{P_{C,i,R}^D}{P_{C,i,R}^{IM}} \right]^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} \quad \text{[O-11]}$$

$$C_{i,R}^{IM} = \left( A_{m,i,R} \right)^{\frac{\sigma_{m,i,R}}{\sigma_{n,i,R}}} C_{i,R} \left( 1 - \kappa_{i,R} \right) \frac{P_{C,i,R}^{IM}}{P_{C,i,R}^D} \quad \text{[O-12]}$$
In addition, since import tariff is imposed on the value of import \( P^W_{C,j} \), the import price in local currency unit \( P^{IM}_{C,j,R} \) is defined gross of tax as:

\[
P^{IM}_{C,j,R} = ER_R \left( 1 + \tau_{IM,j,R} \right) P^W_{C,j}
\]  \[16\]

At the second tier, imports are designated as a CES composite of regional imports and imports from ROW. Finally, at the third tier the aggregate of regional imports is modeled as another CES function of imports from all of the regions that make-up the ROR. At both of these stages the same cost minimization principle allows the determination of the optimal level of each component of the final demand.

![Figure 3: Schematic representation of imports](image)
Akin to imports, as shown in Figure 4 above, exports are also differentiated according to their destinations. A revenue maximization rule is used in this context. For each region, we use a three-tier nested constant elasticity of transformation (CET) function to capture the imperfect substitution between different components of the representative firm’s supply of total output. At the initial node, a firm’s total output is a CET composite of regional supply and aggregate exports. A revenue maximization principle dictates the optimization problem that is identified as:

$$\begin{align*}
\text{Max} & \quad \prod_{j, R} \left[ P_{Q, j, R} Q_{j, R}^{D} \prod_{j, R} P_{Q, j, R} Q_{j, R}^{EX} \right] \\
\text{Subject to,} & \quad EX_{j, R} = A_{e, j, R} \left[ \eta_{j, R} \left( Q_{j, R}^{D} \right)^{1+\sigma_{s, j, R}} + \left(1-\eta_{j, R}\right) \left( Q_{j, R}^{EX} \right)^{1+\sigma_{e, j, R}} \right]^{\frac{\sigma_{e, j, R}}{1+\sigma_{e, j, R}}} \\
\end{align*}$$

where, $P_{EX, j, R} = \text{index price of composite of good } j \text{ exported from region } R; \quad P_{Q, j, R} = \text{index price of regionally consumed quantity of good } j \text{ produced and consumed in region } R; \quad P_{Q, j, R}^{EX} = \text{index price of exported quantity of good } j \text{ from region } R \text{ in local currency units}; \quad Q_{j, R}^{D} = \text{quantity of good } j \text{ produced in region } R; \quad Q_{j, R}^{EX} = \text{regionally exported quantity of good } j \text{ in region } R; \quad \sigma_{s, j, R} = \text{elasticity of substitution for export of good } j \text{ from region } R; \quad \eta_{j, R} = \text{share parameter in the aggregator function for good } j \text{ in region } R; \quad \text{and } A_{e, j, R} = \text{shift parameter in the aggregator function of good } j \text{ in region } R.$
This yields the optimality conditions:

\[
\frac{Q_j^D}{Q_j^{EX}} = \left[ \frac{(1-\eta_{j,R}) P_{Q,j,R}}{\frac{P_{Q,j,R}}{E_j R_j Q_j R_j}} \right]^{\sigma_{e,j,R}}
\]

\[
EX_{j,R} = A_{e,j,R} \left[ \eta_{j,R} \left( \frac{Q_j^D}{\sigma_{e,j,R}} \right)^{1+\sigma_{e,j,R}} + (1-\eta_{j,R}) \left( \frac{Q_j^{EX}}{\sigma_{e,j,R}} \right)^{1+\sigma_{e,j,R}} \right]^{\sigma_{e,j,R}}
\]

Equation [O-13] implies that the optimal ratio of exports to domestic sale is a function of the ratio of exports price to regional market price and the parameters in CET function.

A further manipulation of equation [O-13], [O-14] and [17] yields equivalent expressions for optimality conditions:

\[
P_{EX,j,R} = \frac{1}{A_{e,j,R}} \left[ \left( \frac{\eta_{j,R}}{P_{Q,j,R}} \right)^{\sigma_{e,j,R}} \left( \frac{P_{Q,j,R}}{Q_j^{EX}} \right)^{1+\sigma_{e,j,R}} \right]^{\frac{1}{1+\sigma_{e,j,R}}}
\]

\[
Q_j^D = \left( A_{e,j,R} \right)^{-\sigma_{e,j,R}} \frac{EX_{j,R}}{\eta_{j,R} P_{EX,j,R}} \left( \frac{P_{Q,j,R}}{E_j R_j Q_j R_j} \right)^{\sigma_{e,j,R}}
\]

\[
Q_j^{EX} = \left( A_{e,j,R} \right)^{-\sigma_{e,j,R}} \frac{EX_{j,R}}{\left( 1-\eta_{j,R} \right) P_{EX,j,R}} \left( \frac{P_{Q,j,R}}{E_j R_j Q_j R_j} \right)^{\sigma_{e,j,R}}
\]

More specifically, using the dual approach, equation [O-15] – [O-17] provide the index price of the composite of good \( j \) along with the quantities of the good that are exported and supplied domestically.

In addition, since all domestic regional firms are price-takers, export price of each commodity in local currency unit \( P_{ROW,j,R}^{EX} \) is the product of exchange rate \( ER_j \) and its world market price in foreign currency unit \( P_{Q,j}^{w} \):

\[
P_{ROW,j,R}^{EX} = ER_j P_{Q,j}^{w}
\]
Again, a CET function is used at the second tier to allocate total exports between ROR and ROW. Finally, a third CET aggregator function allows firms to re-allocate regional exports among the remaining regions.

While a small country assumption is used to characterize Canada’s relationship with the ROW signifying that the world price of imports and exports are held fixed, the prices of the bilaterally traded goods among the regions are fully endogenous. These prices are determined by the market clearing conditions for inter-regional trade between any two regions. Finally, given the national structure, the model does not require current account balance to hold for each region individually; rather, it is ensures that the sum of the current accounts of all the regions be balanced for Canada as a whole with the ROW.

1.3.5 Equilibrium Conditions

The general equilibrium of this model is represented by a static allocation of goods and factors supported by a vector of prices such that the following conditions are met:

i) equilibrium in regional goods markets for all regions; and

ii) equilibrium in regional factor markets (i.e. labour and capital) for all regions.

1.3.5.1 Commodity Market Equilibrium

In each region, aggregate demand for each commodity is composed of consumption demand \((C_{i,R})\), investment demands \(\left(\sum_i I_{i,R}\right)\), government consumption demand \(\left(\sum_i C_{G,i,R}\right)\), and intermediate inputs demand \(\left(\sum_i V_{i,j,R}\right)\). Commodity market equilibrium condition in equation [20] requires that the aggregate demand for each commodity be equal to its composite supply \((Q_{j,R})\).

\[
Q_{j,R} = C_{i,R} + \sum_i I_{i,R} + \sum_i C_{G,i,R} + \sum_i V_{i,j,R} \quad \text{[20]}
\]

\[
C_{i,R}^D = Q_{j,R}^D \quad \text{[21]}
\]
Equation [21] also requires that, in equilibrium, demand for regionally produced commodities \((C^D_{i,R})\) be equal to the quantity of regional output supplied to the local market \((Q^D_{j,R})\).

### 1.3.5.2 Input Market Equilibrium

Equation [23] and [24] provide the equilibrium condition for labour and capital markets. The equilibrium requires that the sum of labour demand across production sectors \((L^D_R = \sum_j L_{j,R})\) in each region be equal to the labour supply \((L_R)\).

\[
L^D_R = L_R \quad \quad \quad \quad [23]
\]

\[
K^D_R = \bar{K}_R \quad \quad \quad \quad [24]
\]

In the capital market, on the other hand, the equilibrium dictates that the sum of capital demand across production sectors \((K^D_R = \sum_j K_{j,R})\) in each region be equal to the exogenously supplied capital stock \((\bar{K}_R)\).

### 1.3.6 Closure Rules

The models numéraire is the nominal exchange rate with the ROW. We consider three closure rules in the model. The first rule applies to the regional government accounts. We keep government savings in each region constant to its benchmark value. Government transfer to households is the equilibrating variable that adjusts to respect the constraint on government savings.

The second closure rule pertains to the equilibrium condition between expenditures on investment goods and savings, which is achieved at the national level, thanks to the regional nature of the model\(^{17}\).

\(^{17}\) In contrast, in a multi-country model, equilibrium between savings and investment expenditures will be achieved in each country.
\[ P_{i,R} I_{i,R} = \beta_{i,R} (S_F + S_D) \]  
\[ S_D = \sum_R (S_R + S_{G,R} + S_{Q,R}) \]  
\[ S_{Q,R} = \beta_{Q,R} (1 - \tau_{Q,R}) \sum_j R_{K,j,R} K_{j,R} \]  
\[ S_F = \sum_R E R_{G,R} S_{F,R} \]

where,

\[ P_{i,R} = P_{i,R} (1 + \tau_{i,R}) \]

and where,

\[ P_{i,R} = \text{gross of tax unit price of investment of good } i \text{ in region } R; \]
\[ I_{i,R} = \text{investment demand of good } i \text{ in region } R; \]
\[ \beta_{i,R} = \text{region } R \text{’s share of national savings; } S_F = \text{national foreign savings; } \]
\[ S_D = \text{national domestic savings; } S_{Q,R} = \text{corporate savings in region } R; \]
\[ \beta_{Q,R} = \text{marginal propensity of corporate savings in region } R; \]
\[ T_{G,R}^w = \text{government transfer to rest of the world by region } R. \]

The model is savings-driven in the sense that investment expenditures in each region are endogenous and are determined by the amount of total available national savings. As evident in equation [25], total regional expenditure on quantity demanded of investment is equal to its share of total national savings that is given by the sum of foreign and domestic savings across all regions. Domestic savings, in turn, is equal to the sum of the savings of households, firms, and the governments over all regions (equation [26]). Regional corporate savings is defined in equation [27] that equals a fixed portion of net of tax regional dividend income. Total Canadian foreign savings, on the other hand, is the sum of the foreign savings of all regions (equation [29]), which in turn is the sum of total value of net export and regional government’s transfer to the ROW (equation [28]). Finally, it is worth noting that, in the context of current general equilibrium model, the sum of intra-regional savings (regional trade balances) is expectedly zero.
Hence, no attempt was made to ensure balance of payments for trades occurring among the regions.

The final closure rule of the model deals with Canada’s external account. We assume that Canada’s total current account is equal to its foreign savings, which we maintain as fixed. Note that this does not preclude external account of each region to vary but only requires the aggregate to be constant. This equilibrium condition is achieved by an adjustment of the real exchange rate, which is defined by the relative prices of traded and non-traded goods.

1.4 Emissions Permit Allocation Schemes – Description of the Simulations:

Policy experiments within two broad scenarios – *entitlement* versus *no prior entitlement* (NPE) to emission rights – are considered, which constitute the focus of the paper. Table 1 provides a snapshot of the counterfactual simulations performed in this paper. The impacts of the NPE principle in allocating emission permits are compared with those of five other entitlement based allocation system, namely – (i) emission based allocation (EBA); allocation based on: (ii) emissions per capita (EPC); (iii) converging carbon intensity (CCI); (iv) efficiency index (EI); and (v) multi-criteria index (MI). Once an allocation scheme is chosen, based on which the regions trade amongst themselves, the resulting scarcity rents generated through emissions trading are then distributed by the regional governments within their own jurisdiction in a lump-sum fashion.

<table>
<thead>
<tr>
<th>Table 1: Simulation schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenarios</td>
</tr>
<tr>
<td>Allocation schemes</td>
</tr>
<tr>
<td>(Once entitlements and regional emission permit needs are known, regions trade residual permits among themselves in the national emission permit market).</td>
</tr>
</tbody>
</table>

We, hence, present a total of six allocation schemes each representing a specific policy option for allocating Canada’s carbon emission rights. The equilibrium condition on emissions, as identified in equation [30] below, confirms that Canada’s total level of carbon emissions,
which is the sum of regional emissions over all regions \( E = \sum_R E_R \), does not exceed its national target \( \bar{E} \).

\[
E = \bar{E}
\]

where \( E_R = E_{H,R} + \sum_j E_{Q,j,R} \) \[30\]

and, \( R_R = \Phi_{*,R} \bar{E} \) \[31\]

Here, \( E_{H,R} \) = household emissions in region \( R \); \( E_{Q,j,R} \) = industrial (combustible) emissions (i.e. emissions related to the use of inputs) produced from good \( j \) in region \( R \); \( R_R \) = total emission rights received by region \( R \); and \( \Phi_{*,R} \) = exogenous parameter that reflects the allocation principle used for initial distribution of emission permits to a region.

Since regional level of emissions are endogenously linked to the level of economic activity in each region, endogenously determined scale-back factor is used to ensure that all region collectively do not exceed exogenously allocated level of emission rights that correspond to the national emissions target\(^{18}\).

Note that while five of these schemes are based on a prior distribution of permits each reflecting a specific equity criterion, one scheme – NPE – does not entail any prior distribution of emission permits. Notwithstanding the differences, NPE reflects its own equity principle that is akin to the outcome-based allocation of emission permits. This is since, under this scheme, emission rights are distributed among the regions based on their residual (or equilibrium), as opposed to anticipated, level of emissions. The model is, therefore, designed to answer a specific, yet crucial, question that has not been explored in the literature: whether prior allocation of emission permits serves the equity consideration relatively better than a scheme where no such prior allocation is allowed? In order to contextualize this question within the policy schemes studied in this paper, we now provide some discussions on the simulation scenarios performed in this paper.

\(^{18}\) See the list of equations provided in the Appendix A for further details.
1.4.1 Emissions Based Allocation

Allocation of emission rights based on proportional level of historic emissions is the most commonplace principle cited in the literature. In essence, this relates to the sovereignty principle, which implies that each individual, or an economic entity, is guaranteed some rights and resources. The principle of sovereignty is commonly observed in international environmental treaty-making and institution-building, and is adopted in the United Nations Framework Convention on Climate Change (UNFCCC)\(^\text{19}\).

\[
\Phi_{EBA,R} = \frac{E_{R,Base}}{\sum_{R} E_{R,Base}} \tag{[33]}
\]

here, \(E_{R,Base}\) = regional emissions at the base period

One way to interpret the principle is to reduce GHG emissions in proportion to existing levels of emissions (Ringius et al., 1998). The EBA policy in the context of the current paper reflects regional entitlement of emission permits in proportion to each region's historic level of emissions, where aggregate (i.e. national) reduction target of GHG emissions is distributed following the proportionality rule (equation [33]). In terms of the equity and fairness, this implies that all regions are essentially left at their status-quo state.

1.4.2 Allocation Based on Emissions Per Capita

The EPC criterion is another frequently cited allocation principle in the literature. This rule reflects the egalitarian approach – the principle of equal rights for all persons – and is mostly concerned with the concept of equality. In the context of climate change, the principle implies that every individual has the same right to use the atmosphere and is allowed the same right to emit GHG irrespective of their geographic location.

\[
\Phi_{Pop,R} = \frac{Pop_{R,Base}}{\sum_{R} Pop_{R,Base}} \tag{[34]}
\]

\(^{19}\) According to one of the UNFCCC’s preambles: “States have, in accordance with the Charter of the United Nations and the principles of international law, the sovereign right to exploit their own resources pursuant to their own environmental and developmental policies, and the responsibility to ensure that activities within their jurisdiction or control do not cause damage to the environment of other States or of areas beyond the limits of national jurisdiction.”
here, $Pop_{R, Base} = \text{regional population at the base period}$

The concept essentially entails the underlying notion that emission permits belong to the individuals, not to the governments, where each individual is entitled to the same amount of permits. In terms of operationalization (equation [34]), this means that regional emissions targets are distributed in proportion to population, where the aggregate reduction target of GHG emissions is respected as the binding constraint.

1.4.3 Allocation Based on Converging Carbon Intensity

While neither EBA nor EPC reflects a region’s carbon consumption behaviour, the CCI, proposed by Rowlands (1997), presents an allocation scheme that recognizes this phenomenon. The underlying principle for this allocation scheme is related to the concept of horizontal equity, which calls for all persons in the same group to be treated equally (resembling, to some degree, the principle of sovereignty). The principle of horizontal equity implies equal treatment being accorded to all members that belong to a group. This principle is motivated by the requirement to equalize the burden of abatement costs across all regions.

The allocation scheme corresponding to the CCI is operationalized based on converging carbon intensity of GDP (i.e. carbon emissions per unit of regional GDP), which is used as an indicator of efficiency (i.e. the lower the carbon intensity of an economy, the higher its efficiency). The underlying idea essentially implies that more stringent emission reductions should be prescribed for economies where actual efficiency is low. Emission entitlements under this scheme, therefore, mandate high initial reductions for economies with the high carbon emissions levels. Emission reductions are mandated until the region’s carbon intensity of GDP equals that of the region with the next lowest score. This is reflected in equation [35] - [37] below.

\[
R_R = \nu \Phi_{CCI} \text{ } GDP_R \\
\sum R_R = \bar{E}
\]
$$\Phi_{CCI} = \frac{E_{R,Base}}{GDP_{R,Base}} = \min \left\{ \frac{E_{R,Base}}{GDP_{R,Base}} \right\} \text{ for } R = 1, 2, 3,... \quad [37]$$

here, $\upsilon$ = endogenously determined scale-back factor that ensures the attainment of the national emissions mitigation target; $E_{R,Base}$ = regional level of emissions at the base period; $E_{R,Base}^*$ = lowest regional level of emissions at the base period; $GDP_{R,Base}$ = regional GDP at the base period; and $\Phi_{CCI}$ = lowest regional carbon intensity of GDP at the base period.

Whereas this allocation policy recognizes efficiency aspect in regional level of carbon emissions, it does so relative to the national rather than the regional backdrop. In a decentralized federalist economic union like Canada, this remains at odds since regional economies enjoy rather a larger degree of autonomy in the union. In addition, while this allocation policy reflects principles of horizontal equity, it does not provide sufficient incentives for already efficient regions (i.e. regions with low regional carbon intensity) for achieving the efficiency in the past. An equitable distribution of emission rights would require recognition of past efficiency in terms of special allowances be allotted to such regions, which can also be expected to have lower marginal abatement costs.

1.4.4 Allocation Based on Efficiency Index

Proposed by Gupta and Bhandari (1999), allocation scheme based on the EI approach addresses the issue identified above. Their approach, in a way, is a hybrid of the EBA (that includes proportional percentage reduction of emissions) and the CCI approach (which incorporates the efficiency cost criterion). Under this policy scheme, regions that have achieved high efficiency (reflected in terms of the low regional carbon intensity of GDP relative to the national carbon intensity of GDP) in the past are accorded more emission permits. Therefore, the following adjustment is suggested:

$$R_R = \upsilon \times E_{R,Base} \left(1 - \text{GAP}^* \Phi_{EI,R} \right) \quad [38]$$

$$\sum_R R_R = E \quad \text{[same as 36]}$$

$$\Phi_{EI,R} = \frac{E_{R,Base}}{\sum_R E_{R,Base} / \sum_R GDP_{R,Base}} \quad [39]$$
where $\Phi_{EI,R}$ represents the regional efficiency index that is applied to the percentage reduction in emissions rather than the level of emissions.

The efficiency index is thus defined as the carbon intensity of a region, divided by the carbon intensity of the country. Unlike CCI approach, the EI approach hence measures regional carbon intensity (i.e. relative to its own economy) vis-à-vis the national carbon intensity. It, therefore, addresses both horizontal and vertical equity.

Although EI based allocation performs better in addressing equity concerns, it still suffers from one specific criticism – this criterion entails rather a narrow economic base in that it only considers regional carbon intensity and ignores any other criteria that capture the characteristics of region-specific economic composition. This criticism is more relevant in the context of Canadian economic union since different regions in the union seem to entail different economic structures (i.e. manifested through its industrial composition) that differs significantly from one another.

1.4.5 Allocation Based on Multi-Criteria Index

The above criticism is addressed in the permit allocation scheme that is based on the MI approach. Proposed by Ringius et al. (1998), the approach combines a set of six indicators (population, CO$_2$ emissions, CO$_2$ emissions per GDP, CO$_2$ emission per capita, regional GDP, and regional GDP per capita) to calculate emission reduction target for each region.

As indicated in equation [41] below, the index defines emission reduction for each region as a percentage of emissions in the reference year. The formula includes the following indicators: CO$_2$ emission per capita ($A$), GDP ($B$), emissions per unit of GDP ($C$), and GDP per capita ($D$):

\[
R_R = \nu \frac{E_{R,Base}}{\sum R_R \Phi_{MI,R}} \left(1 - \text{GAP} \Phi_{MI,R}\right)
\]

\[
\sum R_R = \bar{E}
\] [same as 36]
These are understood as indicators for emission entitlement, size of a region, regional energy efficiency, and a region’s ability to pay, respectively. The resulting $\Phi_{MI,R}$ is the index for region $R$ and $\omega_i$ to $\omega_d$ are the weights that add up to 1\(^2\). The parameter $\upsilon$ is a scaling factor, determined endogenously such that the emission reductions by all regions become equal to the national target in the aggregate.

The MI approach addresses several equity principles at the same time by combining a subset of four indicators in the formula. CO\(_2\) emissions per capita can be considered to represent the egalitarian equity; GDP per capita addresses the vertical equity; emissions per GDP reflects the horizontal equity; and the GDP represents the size of economy while the overall level of CO\(_2\) emissions in a base year reflect the proportional equality. The MI approach, thus, characterizes a high degree of flexibility and is able to incorporate characteristics of the regional economic composition.

1.4.6 Allocation Based on No Prior Entitlement of Emission Rights

Depending on their implications on a region’s own economy, while all five entitlement-based schemes identified above address various equity concerns with some degree of satisfaction, they all involve inter-regional transfer of scarcity rents generated through trading of emission rights in the national emission permit market. Thus, they have the potential to engender fierce political debates depending on the outcome of any specific permit allocation policy.

In light of the importance of political appeal for likely implementation of any allocation policy vis-à-vis the contentious issue of inter-regional transfer of scarcity rents, we propose the sixth allocation scheme – NPE – and evaluate its performance relative to the others.

\(^{20}\) No objective numerical values are proposed for the weight of each indicator, therefore leaving the scope of possible negotiations among the regions. For the current purpose, we chose $\omega_i = 0.25$ for $i = A, B, C, D$. 

$\Phi_{MI,R} = \left[ \omega_A \left( \frac{A_R}{\sum R A_R} \right) + \omega_B \left( \frac{B_R}{\sum R B_R} \right) + \omega_C \left( \frac{C_R}{\sum R C_R} \right) + \omega_D \left( \frac{D_R}{\sum R D_R} \right) \right]$ [41]
\[ R_R = E_{R,\text{Residual}} \] 

As evident in equation [42], unlike the other five schemes, this scheme does not involve prior allocation of emission rights and hence avoids any transfer of emission rents across regions. Under this policy scheme, emission rights are distributed based on residual level of emissions \( (E_{R,\text{Residual}}) \). The scheme, hence, reflects outcome-based allocation principle and entails welfare implications that yield results that are different than that of the ones from other five entitlement-based allocation schemes.

In all the six simulations, however, we consider a cap-and-trade emissions trading framework where permits are traded in a single domestic market across the regions\(^{21}\). For an illustrative purpose, we consider a 25% reduction of total emissions in Canada in comparison to the benchmark\(^{22}\). For a good understanding of the differences among the simulations, the ensuing discussions on results are focused on the basic mechanisms at play for the NPE simulation, and on how results change for the other five policy simulations relative to the base period.

1.5 Description of the Baseline Data:

The model is calibrated to a customized dataset that is sourced from Statistics Canada for the year 2005. The business-as-usual (BAU) dataset entails detailed disaggregation both at the regional and sectoral levels such that the confidentiality provisions are respected. Emissions data by fuel type are built using information supplied by Statistics Canada and conform to the emissions forecasts contained in the Analysis and Modeling Branch report (1999).

---

\(^{21}\) It is important to understand that, under NPE, no actual trading of emission rights occurs. The structure of the model, however, endogenously determines the equilibrium price of emission rights that is identical to the price that would have resulted had the permits been physically traded in the national emission trading market. To ensure the successful attainment of reduced national target of emissions level, we use an endogenously determined scale-back factor that guarantees a fixed aggregate supply of emission permits. Coupled with the aggregate regional demands for emission permits, the capped aggregate supply, thus, determines the equilibrium price of emission rights.

\(^{22}\) 2005 is the benchmark year.
Using these data we calculate emission factors for different fossil fuels and emission intensities by region and by industry. Values used for various behavioural parameters, on the other hand, are borrowed from previous studies on Canada, such as Ab Iorwerth et al. (2000) and Wigle (2001) and are not very different from the values used in many other general equilibrium models of Canada or the United States. The calibration of the model involves using the SAM, exogenous parameters, first-order conditions, and steady state conditions to recover other parameters in the behavioural functions and the values of the non-observed variables to reproduce the reference situation. To this end, we use the calibration procedures frequently employed in static and dynamic general equilibrium models as explained in Dissou (2002), Keuschnigg and Kohler (1994), and Mansur and Whalley (1984).
Table 2c: Values for behavioural parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution elasticity</td>
<td></td>
</tr>
<tr>
<td>between value added-energy and intermediate inputs</td>
<td>0.2-0.7</td>
</tr>
<tr>
<td>between labour &amp; capital-energy</td>
<td>1.0</td>
</tr>
<tr>
<td>between capital &amp; energy</td>
<td>0.2-0.6</td>
</tr>
<tr>
<td>between electricity &amp; fossil energy</td>
<td>0.3-0.6</td>
</tr>
<tr>
<td>among stationary fossil fuels</td>
<td>0.5-0.9</td>
</tr>
<tr>
<td>between other intermediate inputs &amp; mobile fossil fuels</td>
<td>0.3-0.6</td>
</tr>
<tr>
<td>among mobile fossil fuels</td>
<td>0.5-0.9</td>
</tr>
<tr>
<td>between imports and domestic goods</td>
<td>0.9-4.0</td>
</tr>
<tr>
<td>between exports and domestic goods</td>
<td>2.0-4.0</td>
</tr>
<tr>
<td>among same industry products*</td>
<td>2.0-4.0</td>
</tr>
</tbody>
</table>

Sources: Various studies
* For multi-products industries

Table 2a and 2b above provide a general snapshot of the share of emissions and GDP levels of six Canadian regions in 2005, while Table 2c provides the values used for the elasticity of substitution parameters. Data indicate that while Alberta’s share of national CO₂ emissions equals that of Ontario’s (both roughly at 30%), Ontario (at 38.4%) contributes more than twice as much to national GDP as Alberta (at 17%). This implies that Alberta has an emission intensity level that is more than twice the emission intensity level of Ontario. The BAU data also indicate that Québec is the third most polluting region of Canada, although it contributes less than half of the pollution level as compared to Alberta or Ontario (at 12%). Québec also contributes roughly 20% to Canada’s national GDP, which signifies an emission intensity level that is smaller than that of Ontario’s. The BAU data thus reveal contrasting positions of these regions in terms of the level of emissions vis-à-vis their level of economic activity.

The data reveal that the political response to any uniform permit allocation policy that can achieve the national emission mitigation target would vary from one region to another. This fact serves as the primary motivation for this paper. As indicated above, in this paper we perform an independent evaluation of six such allocation policies that corresponds to different equity criteria. We now turn to the next section that provides the results of this evaluation.
1.6 Results:

Unless otherwise mentioned, all the results presented in this paper are expressed as a percentage deviation from the reference situation (i.e. the BAU scenario). The results from different simulation are contained in Tables 3-16. The permit price affects the prices of polluting goods and thus all relative prices in the economy. It has both direct and indirect effects, characterized by the changes in regional production costs, composition of aggregate demand, and household welfare.

1.6.1 National Impacts

Table 3 presents some aggregate results for Canada. We observe that all indicators register a percentage decline under all policies except for household disposable income and aggregate consumption. Given the revenue recycling (i.e. lump-sum\(^{23}\)) system used by each regional government to redistribute its share of emission rents to households, it is not surprising to see that household disposable income, and consequently the real aggregate consumption, rise. Disposable income increases due to the emission rent transfer that the households receive because of the lump-sum recycling method.

<table>
<thead>
<tr>
<th>Table 3: Some aggregate variables for Canada (percentage change from benchmark)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
</tr>
<tr>
<td>GDP at market price</td>
</tr>
<tr>
<td>Disposable income</td>
</tr>
<tr>
<td>Household real consumption</td>
</tr>
<tr>
<td>Exports</td>
</tr>
<tr>
<td>Imports</td>
</tr>
<tr>
<td>Investment demand</td>
</tr>
<tr>
<td>Industrial CO(_2) emissions</td>
</tr>
<tr>
<td>Household CO(_2) emissions</td>
</tr>
<tr>
<td>Total CO(_2) emissions</td>
</tr>
</tbody>
</table>

Source: Simulation results

For the remainder of indicators – GDP at market price, exports, imports, investment demands and CO\(_2\) emissions by household and industry sources – at the aggregate level, the

\(^{23}\)Lump-sum revenue recycling amounts to transferring the value of permits to the households.
differences from one policy scenario to another is somewhat minor. All emission permit
distribution policies typically reduce industrial emissions by a large amount (around 28%).
Households bear fewer burdens in the overall effort to reduce emissions, thanks primarily to the
lump-sum revenue recycling policy. Larger industrial brunt of emissions reduction seems to
translate into lower demand for investments (around 4%). National emissions reduction also
takes toll on exports and imports (down by 2%) in addition to registering a reduction of 0.6% in
the overall GDP of the economy.

These aggregate results, which summarize the impact of the policy change on Canadian
economy as a whole, however do not provide information on the variety of regional
adjustments. The regional impacts that shed light on the equity aspects are discussed below.
We first highlight some salient aggregate results at the regional level followed by results that
elucidate adjustments in some of the industry sectors of interest in individual regions. In-so-
doing, we offer intuitive explanations and highlight the main transmission mechanisms at work.

1.6.2 Regional Aggregate Impacts

Table 4 provides results regarding the impact of various policy schemes on regional GDP. Out
of the six Canadian regions, results indicate that the economy of the Atlantic region (i.e.
Newfoundland and Labrador, Prince Edward Island, Nova Scotia, and New Brunswick) gener-ally contracts the most followed by that of Alberta's under almost all policy schemes. Québec, on the other hand, experiences the least contraction closely followed by Ontario and British Columbia and the Territories (BCT).

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-0.5</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>EBA</td>
<td>-1.2</td>
<td>-1.1</td>
<td>-0.4</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>EPC</td>
<td>-0.9</td>
<td>-1.1</td>
<td>-0.5</td>
<td>-0.7</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>CCI</td>
<td>-0.9</td>
<td>-1.0</td>
<td>-0.5</td>
<td>-0.7</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>EI</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-0.5</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>MI</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-0.5</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Source: Simulation results
When comparing NPE policy with rest of the entitlement-based policy schemes, all regions tend to suffer almost the same amount of GDP losses under the former relative to the latter schemes. Québec, however, seems to do slightly better under NPE relative to the EPC and the CCI schemes (-0.4% as oppose to -0.5%) while Alberta experiences the opposite (-1.1% versus -0.9%).

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>1.7</td>
<td>0.2</td>
<td>0.3</td>
<td>1.5</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>EBA</td>
<td>2.8</td>
<td>0.8</td>
<td>0.1</td>
<td>1.5</td>
<td>-0.1</td>
<td>-0.03</td>
</tr>
<tr>
<td>EPC</td>
<td>-2.5</td>
<td>0.0</td>
<td>0.7</td>
<td>0.2</td>
<td>1.4</td>
<td>0.9</td>
</tr>
<tr>
<td>CCI</td>
<td>-1.2</td>
<td>-0.6</td>
<td>0.8</td>
<td>-0.1</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>EI</td>
<td>0.7</td>
<td>0.4</td>
<td>0.5</td>
<td>1.1</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>MI</td>
<td>1.1</td>
<td>1.0</td>
<td>0.3</td>
<td>1.6</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Source: Simulation results

Interestingly the story reverses when we consider percentage change in welfare. While, due to the lump-sum revenue recycling method, all regions generally experience improvements in (household) welfare, Alberta does so relatively more under the NPE scheme compared to the remaining regions. Québec, on the other hand, gains the least (0.1%) followed by the Atlantic provinces and the BCT (0.2%). This is linked to both the policy schemes (which render varying amounts of emission rent transfers to each region) and the recycling mechanism (i.e. lump-sum) that typically benefit Alberta and Other Prairie Provinces (OPP comprises of Manitoba and Saskatchewan) relative to rest of the Canadian regions.

In terms of the entitlement-based schemes, with EPC and CCI, while Alberta heavily suffers from welfare losses (-2.5% and -1.2% respectively), Québec gains the most under those policies (1.4% and 1.0%). This is not surprising since with relatively large population base, Québec receives larger share of emission rights under the EPC scheme. With the CCI policy, on the other hand, while other regions receive less number of rights due to their high emission intensity relative to their respective regional GDP level, because of Québec’s improved

---

Note that the index of welfare change does not take into account the environmental improvement due to the reduction in GHG emissions. The measure is an indication of the percentage change in the household consumption in the BAU situation that would yield the same utility level as the one with the policy change.
performance in historical level of emissions, it again receives more permits and consequently scores high in the welfare ranking.

This somewhat consolidates with EI and MI policies that not only look into the efficiency aspect, but also take into account the economic burden (under the EI) and the regional characteristics (under the MI). With both of these policies, welfare performance of the oil-rich Prairie Provinces improves while Ontario’s welfare levels deteriorate only marginally relative to the EPC and CCI policies. Québec, on the other hand, suffers from substantial welfare losses under EI and MI schemes compared to the EPC and CCI policies. However, this does not imply that absolute welfare metrics fall in these regions. In fact, under MI policy, our model calculations reveal that even for Québec the absolute welfare level increases by 0.2%, while it increases by 0.3% and 1.1% for Ontario and Alberta respectively. To put these measures into the context, calculation shows that the corresponding increase in weighted welfare level in Canada is 0.5%.

With the EBA policy that distributes emission rights based on past levels of emissions, however, Prairie Provinces (i.e. OPP plus Alberta) benefit the most at the expense of Québec and Ontario since they receive a high share of permits relative to the others, which they are able to trade as a net seller in the national emission trading market. This brings in net transfer of scarcity rents into Alberta and OPP from rest of the regions.

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>5,934</td>
<td>1,670</td>
<td>7,155</td>
<td>2,100</td>
<td>2,922</td>
<td>2,197</td>
</tr>
<tr>
<td>EBA</td>
<td>6,914</td>
<td>1,969</td>
<td>6,446</td>
<td>2,112</td>
<td>2,595</td>
<td>1,981</td>
</tr>
<tr>
<td>EPC</td>
<td>2,185</td>
<td>1,606</td>
<td>8,480</td>
<td>1,482</td>
<td>5,154</td>
<td>2,930</td>
</tr>
<tr>
<td>CCI</td>
<td>3,341</td>
<td>1,283</td>
<td>8,697</td>
<td>1,354</td>
<td>4,407</td>
<td>2,785</td>
</tr>
<tr>
<td>EI</td>
<td>4,989</td>
<td>1,785</td>
<td>7,664</td>
<td>1,899</td>
<td>3,228</td>
<td>2,379</td>
</tr>
<tr>
<td>MI</td>
<td>5,329</td>
<td>2,062</td>
<td>7,122</td>
<td>2,151</td>
<td>3,037</td>
<td>2,265</td>
</tr>
</tbody>
</table>

Source: Simulation results

---

25 Regional share of aggregate consumption is used as the weight for calculating change in the overall Canadian welfare level.
Table 6 provides a snapshot of the value of permit revenues received by various Canadian regions. Table 7, on the other hand, provides the unit price of emission permits under various policy schemes. Evidently, Alberta and Ontario receive the largest share of permit revenues under different allocation schemes while Atlantic and Other Prairie Provinces receive the least. Two factors influence the size of a region’s share of emission revenues – allocation scheme in use (i.e. entitlement-based versus no entitlement-based schemes) and the scale of the economy. While Alberta’s share is due largely to the allocation scheme itself, Ontario’s share is influence by both of these factors. Price of emission permits, on the other hand, seems not to vary considerably from one policy to another. It hovers around approx. $49 per ton of CO₂.

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Price in the national market</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>49.47</td>
</tr>
<tr>
<td>EBA</td>
<td>49.57</td>
</tr>
<tr>
<td>EPC</td>
<td>49.15</td>
</tr>
<tr>
<td>CCI</td>
<td>49.22</td>
</tr>
<tr>
<td>EI</td>
<td>49.40</td>
</tr>
<tr>
<td>MI</td>
<td>49.45</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 8: Dollar purchase of CO₂ permits ($ ml)

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBA</td>
<td>-963</td>
<td>-295</td>
<td>718</td>
<td>-9</td>
<td>331</td>
<td>219</td>
</tr>
<tr>
<td>EPC</td>
<td>3689</td>
<td>56</td>
<td>-1361</td>
<td>605</td>
<td>-2246</td>
<td>-743</td>
</tr>
<tr>
<td>CCI</td>
<td>2549</td>
<td>379</td>
<td>-1570</td>
<td>735</td>
<td>-1496</td>
<td>-596</td>
</tr>
<tr>
<td>EI</td>
<td>930</td>
<td>-116</td>
<td>-518</td>
<td>198</td>
<td>-310</td>
<td>-184</td>
</tr>
<tr>
<td>MI</td>
<td>598</td>
<td>-390</td>
<td>29</td>
<td>-52</td>
<td>-116</td>
<td>-69</td>
</tr>
</tbody>
</table>

Note: A negative number implies the receipt of permit revenues
Source: Simulation results

It is, however, also evident that under the entitlement-based schemes the dollar amount of emission permit purchase varies considerably among the regions depending on the chosen policy option. While attempting to identify the relative position of each region as a net buyer or seller of emission permits when an entitlement based policy is used, as indicated in Table 8, we observe that all Prairie and Atlantic Provinces emerge as the net seller where the remaining
regions turn out to be the net buyer of emission permits in the national market under the EBA policy. The scenario drastically reverses under the policy schemes EPC, CCI and EI where Québec and Ontario emerge as the net seller of permits to the remaining regions. Nonetheless, when the MI policy, which takes into account of the regional characteristics, is implemented the intensity of the situation withers away. Interestingly, with the MI policy while Québec continues to be a net seller, Ontario becomes a net buyer of permits like Alberta.

These results are inspired by the emission rent transfer mechanisms that is entailed in each policy option. EBA, being based on the historical emissions level, naturally provides more permits to Prairie and Atlantic Provinces thus making these regions a net seller of permits. EPC scheme, on the other hand, provides more permits to Québec and Ontario as it is based on per-capita emissions level. CCI and EI also render similar outcomes since these policy schemes reward small emitters over the large emitter like Alberta. MI scheme reduces the impact of EI policy since it also takes into account the regional economic structure and the emissions history in addition to rewarding the efficient emitters.

Table 9: Percentage change in share of CO₂ permit purchase

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBA</td>
<td>-13.9</td>
<td>-15.0</td>
<td>11.1</td>
<td>-0.4</td>
<td>12.7</td>
<td>11.0</td>
</tr>
<tr>
<td>EPC</td>
<td>168.9</td>
<td>3.5</td>
<td>-16.1</td>
<td>40.8</td>
<td>-43.6</td>
<td>-25.4</td>
</tr>
<tr>
<td>CCI</td>
<td>76.3</td>
<td>29.6</td>
<td>-18.0</td>
<td>54.3</td>
<td>-34.0</td>
<td>-21.4</td>
</tr>
<tr>
<td>EI</td>
<td>18.7</td>
<td>-6.5</td>
<td>-6.8</td>
<td>10.4</td>
<td>-9.6</td>
<td>-7.7</td>
</tr>
<tr>
<td>MI</td>
<td>11.2</td>
<td>-18.9</td>
<td>0.4</td>
<td>-2.4</td>
<td>-3.8</td>
<td>-3.0</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 9 provides a snapshot of the compliance burden experienced by the regions that emanates from the rent transfer aspect. In particular, it provides the percentage change in the share of CO₂ permit purchase relative to a region’s own permit endowment under each policy scheme. Evidently, Alberta and Atlantic Provinces suffer under the EBA policy (14% and 15% respectively) whereas BCT, Ontario and Québec gain (11%, 11% and 13% respectively). Alberta, however, suffers the most from EPC policy (169%) while Québec is the largest beneficiary under the scheme (-44%). Not surprisingly, with EPC policy Alberta and OPP along with the Atlantic Provinces, benefit the least when Québec and Ontario benefit the most. With
CCI, MI and EI policies Alberta’s and Atlantic Provinces’ burden gradually reduces while that of Québec’s and Ontario’s increases with mixed results being observed for the OPP.

Results of the policy impacts in terms of percentage change in total emissions are presented in Table 10. For Prairie and Atlantic Provinces being historically the large emitting regions, it is not surprising to observer that, with the implementation of (any) policies, reduction in their emissions levels drift around 26-36% relative to the pre-implementation stage. Emission reduction in Québec, Ontario and BCT, on the other hand, floats around 16-17%.

Table 10: Percentage change in total emissions

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>-35.5</td>
<td>-36.3</td>
<td>-16.6</td>
<td>-25.3</td>
<td>-15.4</td>
<td>-16.7</td>
</tr>
<tr>
<td>EBA</td>
<td>-35.4</td>
<td>-36.2</td>
<td>-16.6</td>
<td>-25.3</td>
<td>-15.4</td>
<td>-16.7</td>
</tr>
<tr>
<td>EPC</td>
<td>-35.8</td>
<td>-36.2</td>
<td>-16.5</td>
<td>-25.3</td>
<td>-15.3</td>
<td>-16.5</td>
</tr>
<tr>
<td>CCI</td>
<td>-35.7</td>
<td>-36.3</td>
<td>-16.5</td>
<td>-25.3</td>
<td>-15.3</td>
<td>-16.6</td>
</tr>
<tr>
<td>EI</td>
<td>-35.6</td>
<td>-36.2</td>
<td>-16.6</td>
<td>-25.3</td>
<td>-15.4</td>
<td>-16.6</td>
</tr>
<tr>
<td>MI</td>
<td>-35.6</td>
<td>-36.2</td>
<td>-16.6</td>
<td>-25.3</td>
<td>-15.4</td>
<td>-16.7</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 11: Percentage change in wage

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>-1.5</td>
<td>-2.1</td>
<td>-1.0</td>
<td>-0.8</td>
<td>-1.0</td>
<td>-1.2</td>
</tr>
<tr>
<td>EBA</td>
<td>-1.1</td>
<td>-1.8</td>
<td>-1.0</td>
<td>-0.8</td>
<td>-1.1</td>
<td>-1.3</td>
</tr>
<tr>
<td>EPC</td>
<td>-3.3</td>
<td>-2.2</td>
<td>-0.9</td>
<td>-1.4</td>
<td>-0.7</td>
<td>-1.0</td>
</tr>
<tr>
<td>CCI</td>
<td>-2.8</td>
<td>-2.4</td>
<td>-0.9</td>
<td>-1.5</td>
<td>-0.8</td>
<td>-1.0</td>
</tr>
<tr>
<td>EI</td>
<td>-2.0</td>
<td>-2.0</td>
<td>-0.9</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.2</td>
</tr>
<tr>
<td>MI</td>
<td>-1.8</td>
<td>-1.8</td>
<td>-1.0</td>
<td>-0.8</td>
<td>-1.0</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 12: Percentage change in labour supply

<table>
<thead>
<tr>
<th>Allocation rule</th>
<th>Alberta</th>
<th>Atlantic Provinces</th>
<th>Ontario</th>
<th>Other Prairie Provinces</th>
<th>Québec</th>
<th>British Columbia and the Territories</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPE</td>
<td>-1.1</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.9</td>
<td>-0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>EBA</td>
<td>-1.3</td>
<td>-0.9</td>
<td>-0.4</td>
<td>-0.9</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>EPC</td>
<td>-0.2</td>
<td>-0.8</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.6</td>
<td>-0.5</td>
</tr>
<tr>
<td>CCI</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>EI</td>
<td>-0.9</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.8</td>
<td>-0.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>MI</td>
<td>-0.9</td>
<td>-1.0</td>
<td>-0.4</td>
<td>-0.9</td>
<td>-0.3</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Source: Simulation results

As explained in the model description section, we treat labour market in the model as geographically segmented. An upshot of this is that the equilibrium wage varies across the regions even though it is identical across the industry sectors of a given region. Table 11 and 12 provide results regarding the impacts on regional wage rates and labour supply, respectively. Overall the results indicate a mirror reflection of the qualitative results subsumed in policy impacts on regional GDP. More than anything, perhaps the information in these tables underscores the importance of incorporating regional economic characteristics into the policy stance.

On the superficial level, we observe that with all policy schemes, regional wage rate and consequently the labour supply fall. When observed more closely, however, results show that the impact of policy implementation is more subdued for large regional economies like Ontario and Québec relative to the smaller ones. This occurs since with the contraction of regional economy stemming from the implementation of (any) CO₂ emission reduction policy, demand for labour falls. Hence the fall in the wage rate. However, that makes labour relatively cheaper. Through feedback loops then the productive sectors of the economy substitutes labour for more expensive inputs into the production process, giving a boost in the wage rate. Consequently, the fall in the wage rate moderates a little. The regional economic characteristics come into play when we, however, concentrate on the relative intensity of these impacts. Irrespective of the chosen policy, regions which have more CO₂ intensive productive sectors (such as those of the Prairie and Atlantic Provinces) suffer from larger fall in wage rate and labour supply since they are unable to make greater substitution in the production process due to the nature of their industry composition relative to the other regions.
1.6.3 Regional Sectoral Impacts

The multi-region, multi-sector structure of our model yielded large quantity of data for the six policy simulations performed in the paper. In light of the basic focus of this paper and for brevity’s sake, however, in this section we present a subset of the results that bring forth the salient features in terms of the impacts of each policy scheme on some regional sectors of interests.

In particular, we concentrate on three big regions – Alberta, Ontario, and Québec – as these regions relatively get more affected by the choice of any particular policy scheme. Our choice of regions is further motivated by regional governments’ disparate views on how to mitigate climate change issues in these jurisdictions. We also look into the performance of some of the industry sectors, in particular those of – oil and gas, utilities, basic chemicals, and other manufacturing sector – within these regions in order to bring out further insights at the industry level. Again the choice of these industry sectors is inspired by the high share of emissions of these sectors in the base period. Arguably, implementation of policy schemes studied in this paper can relatively put more burdens on these sectors, thus, creating an uneven distribution of mitigation costs across the industry sectors. Given the focus of the paper, these sectors, therefore, become natural candidates of interest and hence assist in highlighting the impacts of different policy schemes vis-à-vis the brunt of the climate change mitigation costs.
Table 13a: Total output in Alberta by industry sectors (percentage change)

<table>
<thead>
<tr>
<th>Industry</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-3.02</td>
<td>-3.65</td>
<td>-0.44</td>
<td>-1.25</td>
<td>-2.39</td>
<td>-2.62</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>0.76</td>
<td>0.39</td>
<td>2.16</td>
<td>1.74</td>
<td>1.12</td>
<td>0.99</td>
</tr>
<tr>
<td>Mining</td>
<td>-5.00</td>
<td>-5.35</td>
<td>-3.66</td>
<td>-4.07</td>
<td>-4.66</td>
<td>-4.79</td>
</tr>
<tr>
<td>Construction</td>
<td>-4.34</td>
<td>-4.54</td>
<td>-3.54</td>
<td>-3.79</td>
<td>-4.14</td>
<td>-4.22</td>
</tr>
<tr>
<td>Food</td>
<td>-1.12</td>
<td>-1.64</td>
<td>1.04</td>
<td>0.39</td>
<td>-0.58</td>
<td>-0.80</td>
</tr>
<tr>
<td>Wood</td>
<td>-6.29</td>
<td>-7.93</td>
<td>0.69</td>
<td>-1.45</td>
<td>-4.60</td>
<td>-5.26</td>
</tr>
<tr>
<td>Paper</td>
<td>-35.32</td>
<td>-36.50</td>
<td>-30.31</td>
<td>-31.87</td>
<td>-34.11</td>
<td>-34.58</td>
</tr>
<tr>
<td>Printing</td>
<td>-2.15</td>
<td>-2.31</td>
<td>-1.59</td>
<td>-1.77</td>
<td>-2.01</td>
<td>-2.06</td>
</tr>
<tr>
<td>Basic chemicals</td>
<td>-45.24</td>
<td>-45.59</td>
<td>-43.91</td>
<td>-44.31</td>
<td>-44.91</td>
<td>-45.04</td>
</tr>
<tr>
<td>Plastic &amp; Rubber</td>
<td>-8.82</td>
<td>-9.52</td>
<td>-5.98</td>
<td>-6.86</td>
<td>-8.11</td>
<td>-8.37</td>
</tr>
<tr>
<td>Metal &amp; Fabrics</td>
<td>-9.33</td>
<td>-10.06</td>
<td>-6.41</td>
<td>-7.31</td>
<td>-8.60</td>
<td>-8.87</td>
</tr>
<tr>
<td>Machineries</td>
<td>-14.00</td>
<td>-15.15</td>
<td>-9.33</td>
<td>-10.80</td>
<td>-12.87</td>
<td>-13.29</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>-3.87</td>
<td>-4.82</td>
<td>-0.02</td>
<td>-1.26</td>
<td>-2.92</td>
<td>-3.25</td>
</tr>
<tr>
<td>Furniture and related manufact. products</td>
<td>-1.13</td>
<td>-1.91</td>
<td>1.99</td>
<td>0.99</td>
<td>-0.36</td>
<td>-0.63</td>
</tr>
<tr>
<td>Trade</td>
<td>-0.42</td>
<td>-0.21</td>
<td>-1.27</td>
<td>-1.02</td>
<td>-0.64</td>
<td>-0.56</td>
</tr>
<tr>
<td>Transport</td>
<td>-1.33</td>
<td>-1.45</td>
<td>-0.90</td>
<td>-1.04</td>
<td>-1.23</td>
<td>-1.26</td>
</tr>
<tr>
<td>Services</td>
<td>-0.15</td>
<td>0.04</td>
<td>-0.90</td>
<td>-0.67</td>
<td>-0.34</td>
<td>-0.27</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 13a to 13c provide sectoral impacts of change in the total output in the regions of Alberta, Ontario, and Québec. In general, the largest fall in output is experienced in basic chemicals under all policy schemes across all there regions. While the paper industry turns out to be the sector with second largest fall in Alberta, for Ontario and Québec it appears to be the oil and gas and mining industries, respectively. Whereas in the utility producing sector Québec seems to contract the least followed by Ontario and Alberta, in the other manufacturing sector Alberta experiences the largest decline in output followed by Québec and Ontario. Interestingly results indicate that Alberta’s output in oil and gas sector, in fact, slightly increases under all policy schemes while for Ontario it decreases substantially. This seemingly surprising outcome signifies, relative to the other regions, Alberta’s ability to make greater adjustments (e.g. through value-added substitutions) in this sector. Overall these differences in the impacts underscore varying composition of regional economies and validate the necessity of incorporating regional characteristics in designing the policy schemes.

26 A significant part of emissions in Oil and Gas sector tend to be process emissions, which are not easily abated. The results obtained for Alberta, therefore, can also be due to the fact that the current model only captures combustible emissions.
<table>
<thead>
<tr>
<th>Industry</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-0.61</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.63</td>
<td>-0.62</td>
<td>-0.61</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>-11.56</td>
<td>-11.46</td>
<td>-11.78</td>
<td>-11.78</td>
<td>-11.64</td>
<td>-11.58</td>
</tr>
<tr>
<td>Utilities</td>
<td>-6.11</td>
<td>-6.19</td>
<td>-5.95</td>
<td>-5.95</td>
<td>-6.06</td>
<td>-6.11</td>
</tr>
<tr>
<td>Construction</td>
<td>-3.02</td>
<td>-2.99</td>
<td>-3.11</td>
<td>-3.10</td>
<td>-3.05</td>
<td>-3.03</td>
</tr>
<tr>
<td>Food</td>
<td>0.96</td>
<td>0.95</td>
<td>0.98</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Wood</td>
<td>-1.95</td>
<td>-1.78</td>
<td>-2.16</td>
<td>-2.24</td>
<td>-2.06</td>
<td>-1.94</td>
</tr>
<tr>
<td>Paper</td>
<td>-5.41</td>
<td>-5.27</td>
<td>-5.58</td>
<td>-5.67</td>
<td>-5.50</td>
<td>-5.39</td>
</tr>
<tr>
<td>Printing</td>
<td>0.34</td>
<td>0.35</td>
<td>0.32</td>
<td>0.31</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>Plastic &amp; Rubber</td>
<td>-1.89</td>
<td>-1.54</td>
<td>-2.51</td>
<td>-2.63</td>
<td>-2.14</td>
<td>-1.87</td>
</tr>
<tr>
<td>Metal &amp; Fabrics</td>
<td>-3.53</td>
<td>-3.21</td>
<td>-4.17</td>
<td>-4.24</td>
<td>-3.76</td>
<td>-3.52</td>
</tr>
<tr>
<td>Machineries</td>
<td>-6.81</td>
<td>-6.49</td>
<td>-7.30</td>
<td>-7.47</td>
<td>-7.04</td>
<td>-6.78</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>3.79</td>
<td>4.48</td>
<td>2.32</td>
<td>2.23</td>
<td>3.29</td>
<td>3.79</td>
</tr>
<tr>
<td>Furniture and related</td>
<td>1.14</td>
<td>1.26</td>
<td>0.98</td>
<td>0.91</td>
<td>1.06</td>
<td>1.15</td>
</tr>
<tr>
<td>manufact. products</td>
<td>Trade</td>
<td>0.05</td>
<td>0.01</td>
<td>0.12</td>
<td>0.13</td>
<td>0.07</td>
</tr>
<tr>
<td>Transport</td>
<td>-0.35</td>
<td>-0.35</td>
<td>-0.34</td>
<td>-0.34</td>
<td>-0.35</td>
<td>-0.35</td>
</tr>
<tr>
<td>Services</td>
<td>0.27</td>
<td>0.24</td>
<td>0.34</td>
<td>0.35</td>
<td>0.30</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Source: Simulation results

When we compare these industries over all policy schemes, results indicate that NPE policy performs better relative to the entitlement-based policies across most of the industry sectors. In particular, while the MI policy, (relative to other entitlement-based policies) mostly aligned with the regional economic structure, performs quite the same as the NPE policy, the latter has the added benefit of being able to avoid politically contentious issue of inter-regional transfer of scarcity rents. This makes the NPE policy politically more appealing and, hence, a better candidate among all the policy options.
### Table 13c: Total output in Québec by industry sectors (percentage change)

<table>
<thead>
<tr>
<th>Industry sectors</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>0.09</td>
<td>0.10</td>
<td>-0.21</td>
<td>-0.08</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Mining</td>
<td>-7.95</td>
<td>-7.81</td>
<td>-8.92</td>
<td>-8.60</td>
<td>-8.06</td>
<td>-7.96</td>
</tr>
<tr>
<td>Utilities</td>
<td>-0.13</td>
<td>-0.20</td>
<td>0.31</td>
<td>0.17</td>
<td>-0.06</td>
<td>-0.11</td>
</tr>
<tr>
<td>Construction</td>
<td>-3.08</td>
<td>-3.05</td>
<td>-3.27</td>
<td>-3.20</td>
<td>-3.11</td>
<td>-3.10</td>
</tr>
<tr>
<td>Food</td>
<td>0.33</td>
<td>0.32</td>
<td>0.30</td>
<td>0.32</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>-7.92</td>
<td>-7.85</td>
<td>-8.53</td>
<td>-8.31</td>
<td>-7.99</td>
<td>-7.95</td>
</tr>
<tr>
<td>Wood</td>
<td>-0.43</td>
<td>-0.27</td>
<td>-1.86</td>
<td>-1.31</td>
<td>-0.61</td>
<td>-0.53</td>
</tr>
<tr>
<td>Paper</td>
<td>-2.39</td>
<td>-2.21</td>
<td>-4.01</td>
<td>-3.40</td>
<td>-2.58</td>
<td>-2.48</td>
</tr>
<tr>
<td>Printing</td>
<td>0.08</td>
<td>0.10</td>
<td>-0.14</td>
<td>-0.05</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Basic chemicals</td>
<td>-33.24</td>
<td>-33.21</td>
<td>-33.69</td>
<td>-33.51</td>
<td>-33.29</td>
<td>-33.27</td>
</tr>
<tr>
<td>Plastic &amp; Rubber</td>
<td>-1.85</td>
<td>-1.61</td>
<td>-3.32</td>
<td>-2.85</td>
<td>-2.06</td>
<td>-1.92</td>
</tr>
<tr>
<td>Machineries</td>
<td>-7.99</td>
<td>-7.49</td>
<td>-11.59</td>
<td>-10.36</td>
<td>-8.46</td>
<td>-8.18</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>22.60</td>
<td>23.96</td>
<td>14.80</td>
<td>17.16</td>
<td>21.39</td>
<td>22.20</td>
</tr>
<tr>
<td>Furniture and related manufact. products</td>
<td>0.36</td>
<td>0.41</td>
<td>-0.15</td>
<td>0.05</td>
<td>0.30</td>
<td>0.33</td>
</tr>
<tr>
<td>Trade</td>
<td>-0.11</td>
<td>-0.17</td>
<td>0.28</td>
<td>0.15</td>
<td>-0.05</td>
<td>-0.09</td>
</tr>
<tr>
<td>Transport</td>
<td>-0.39</td>
<td>-0.39</td>
<td>-0.40</td>
<td>-0.40</td>
<td>-0.38</td>
<td>-0.38</td>
</tr>
<tr>
<td>Services</td>
<td>-0.04</td>
<td>-0.08</td>
<td>0.24</td>
<td>0.15</td>
<td>0.00</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 14a to 14c present results in emissions reduction across the industry sectors. For Alberta, complying with the national emissions reduction target translates into the largest reduction in emissions in the utility producing sector followed by the basic chemicals and the paper industries. This, however, differs for other two regions. In Ontario while utilities and basic chemicals industries shoulder most of the emissions reduction efforts followed by the oil and gas sector, in Québec basic chemicals contribute almost twice as much in emissions reduction efforts as in the utilities and other manufacturing industries. When we compare the results over the policy schemes, by and larger, similar qualitative inferences emerge as it does for the output indicator. We find that the MI policy scheme generates impacts that are similar to that of the NPE scheme. However, the latter scheme precludes inter-regional transfer of scarcity rents and, hence, appears to be a superior policy candidate.
Table 14a: Emissions in Alberta by industry sectors (percentage change)

<table>
<thead>
<tr>
<th>Industry</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil &amp; Gas</td>
<td>-11.75</td>
<td>-12.08</td>
<td>-10.50</td>
<td>-10.87</td>
<td>-11.43</td>
<td>-11.55</td>
</tr>
<tr>
<td>Mining</td>
<td>-18.27</td>
<td>-18.40</td>
<td>-17.81</td>
<td>-17.94</td>
<td>-18.16</td>
<td>-18.21</td>
</tr>
<tr>
<td>Utilities</td>
<td>-57.50</td>
<td>-57.39</td>
<td>-57.93</td>
<td>-57.79</td>
<td>-57.61</td>
<td>-57.58</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>-35.46</td>
<td>-35.63</td>
<td>-34.81</td>
<td>-35.00</td>
<td>-35.30</td>
<td>-35.36</td>
</tr>
<tr>
<td>Paper</td>
<td>-47.87</td>
<td>-48.75</td>
<td>-44.12</td>
<td>-45.29</td>
<td>-46.97</td>
<td>-47.32</td>
</tr>
<tr>
<td>Printing</td>
<td>-17.95</td>
<td>-17.88</td>
<td>-18.29</td>
<td>-18.19</td>
<td>-18.04</td>
<td>-18.01</td>
</tr>
<tr>
<td>Basic chemicals</td>
<td>-49.45</td>
<td>-49.77</td>
<td>-48.26</td>
<td>-48.62</td>
<td>-49.16</td>
<td>-49.28</td>
</tr>
<tr>
<td>Trade</td>
<td>-17.64</td>
<td>-17.30</td>
<td>-18.97</td>
<td>-18.56</td>
<td>-17.98</td>
<td>-17.86</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 14b: Emissions in Ontario by industry sectors (percentage change)

<table>
<thead>
<tr>
<th>Industry</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utilities</td>
<td>-34.01</td>
<td>-34.10</td>
<td>-33.78</td>
<td>-33.80</td>
<td>-33.94</td>
<td>-34.00</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>-17.38</td>
<td>-17.34</td>
<td>-17.37</td>
<td>-17.44</td>
<td>-17.40</td>
<td>-17.36</td>
</tr>
<tr>
<td>Wood</td>
<td>-11.66</td>
<td>-11.55</td>
<td>-11.75</td>
<td>-11.84</td>
<td>-11.73</td>
<td>-11.64</td>
</tr>
<tr>
<td>Paper</td>
<td>-17.60</td>
<td>-17.52</td>
<td>-17.61</td>
<td>-17.71</td>
<td>-17.64</td>
<td>-17.56</td>
</tr>
<tr>
<td>Printing</td>
<td>-6.52</td>
<td>-6.55</td>
<td>-6.44</td>
<td>-6.45</td>
<td>-6.50</td>
<td>-6.52</td>
</tr>
<tr>
<td>Basic chemicals</td>
<td>-33.78</td>
<td>-33.75</td>
<td>-33.73</td>
<td>-33.81</td>
<td>-33.80</td>
<td>-33.76</td>
</tr>
<tr>
<td>Metal &amp; Fabrics</td>
<td>-10.14</td>
<td>-9.87</td>
<td>-10.66</td>
<td>-10.73</td>
<td>-10.33</td>
<td>-10.13</td>
</tr>
<tr>
<td>Machineries</td>
<td>-12.30</td>
<td>-12.03</td>
<td>-12.70</td>
<td>-12.85</td>
<td>-12.49</td>
<td>-12.27</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>-5.68</td>
<td>-5.10</td>
<td>-6.92</td>
<td>-7.01</td>
<td>-6.11</td>
<td>-5.68</td>
</tr>
<tr>
<td>Trade</td>
<td>-10.27</td>
<td>-10.34</td>
<td>-10.09</td>
<td>-10.10</td>
<td>-10.21</td>
<td>-10.26</td>
</tr>
<tr>
<td>Transport</td>
<td>-5.89</td>
<td>-5.92</td>
<td>-5.82</td>
<td>-5.83</td>
<td>-5.87</td>
<td>-5.88</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 14c: Emissions in Québec by industry sectors (percentage change)

<table>
<thead>
<tr>
<th>Industry</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-10.25</td>
<td>-10.26</td>
<td>-10.37</td>
<td>-10.30</td>
<td>-10.26</td>
<td>-10.27</td>
</tr>
<tr>
<td>Mining</td>
<td>-17.73</td>
<td>-17.63</td>
<td>-18.45</td>
<td>-18.22</td>
<td>-17.80</td>
<td>-17.73</td>
</tr>
<tr>
<td>Construction</td>
<td>-8.01</td>
<td>-8.01</td>
<td>-8.01</td>
<td>-8.00</td>
<td>-8.01</td>
<td>-8.02</td>
</tr>
<tr>
<td>Food</td>
<td>-20.46</td>
<td>-20.51</td>
<td>-20.22</td>
<td>-20.28</td>
<td>-20.41</td>
<td>-20.44</td>
</tr>
<tr>
<td>Printing</td>
<td>-10.06</td>
<td>-10.09</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-10.04</td>
<td>-10.05</td>
</tr>
<tr>
<td>Basic chemicals</td>
<td>-43.22</td>
<td>-43.22</td>
<td>-43.51</td>
<td>-43.38</td>
<td>-43.24</td>
<td>-43.24</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>7.78</td>
<td>8.93</td>
<td>1.13</td>
<td>3.15</td>
<td>6.75</td>
<td>7.44</td>
</tr>
<tr>
<td>Furniture and related</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>manufact. products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transport</td>
<td>-6.93</td>
<td>-6.95</td>
<td>-6.81</td>
<td>-6.85</td>
<td>-6.91</td>
<td>-6.92</td>
</tr>
<tr>
<td>Services</td>
<td>-11.73</td>
<td>-11.80</td>
<td>-11.29</td>
<td>-11.43</td>
<td>-11.66</td>
<td>-11.70</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 15a: Demand for value-added in utilities sector by regions (percentage change)

<table>
<thead>
<tr>
<th>Region</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alberta</td>
<td>-16.97</td>
<td>-16.75</td>
<td>-17.79</td>
<td>-17.53</td>
<td>-17.18</td>
<td>-17.11</td>
</tr>
<tr>
<td>Ontario</td>
<td>-7.02</td>
<td>-7.10</td>
<td>-6.85</td>
<td>-6.85</td>
<td>-6.97</td>
<td>-7.02</td>
</tr>
<tr>
<td>Québec</td>
<td>-2.81</td>
<td>-2.77</td>
<td>-3.03</td>
<td>-2.95</td>
<td>-2.84</td>
<td>-2.83</td>
</tr>
<tr>
<td>British Columbia and the Territories</td>
<td>-1.00</td>
<td>-1.03</td>
<td>-0.90</td>
<td>-0.93</td>
<td>-0.98</td>
<td>-0.99</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 15b: Demand for value-added in basic chemicals by regions (percentage change)

<table>
<thead>
<tr>
<th>Region</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alberta</td>
<td>-46.47</td>
<td>-46.80</td>
<td>-45.18</td>
<td>-45.57</td>
<td>-46.15</td>
<td>-46.27</td>
</tr>
<tr>
<td>Atlantic Provinces</td>
<td>-40.68</td>
<td>-41.03</td>
<td>-40.51</td>
<td>-40.19</td>
<td>-40.78</td>
<td>-41.09</td>
</tr>
<tr>
<td>Ontario</td>
<td>-29.15</td>
<td>-29.10</td>
<td>-29.14</td>
<td>-29.22</td>
<td>-29.18</td>
<td>-29.13</td>
</tr>
<tr>
<td>Québec</td>
<td>-1.47</td>
<td>-1.23</td>
<td>-2.95</td>
<td>-2.48</td>
<td>-1.68</td>
<td>-1.53</td>
</tr>
<tr>
<td>British Columbia and the Territories</td>
<td>-40.38</td>
<td>-40.26</td>
<td>-40.78</td>
<td>-40.72</td>
<td>-40.48</td>
<td>-40.42</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 15c: Demand for value-added in oil & gas by regions (percentage change)

<table>
<thead>
<tr>
<th>Region</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alberta</td>
<td>1.13</td>
<td>0.79</td>
<td>2.42</td>
<td>2.03</td>
<td>1.46</td>
<td>1.34</td>
</tr>
<tr>
<td>Atlantic Provinces</td>
<td>11.65</td>
<td>11.43</td>
<td>11.63</td>
<td>11.88</td>
<td>11.54</td>
<td>11.33</td>
</tr>
<tr>
<td>Other Prairie Provinces</td>
<td>0.93</td>
<td>0.92</td>
<td>1.22</td>
<td>1.27</td>
<td>1.03</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 15a to 15c provide results for changes in regional demand for value-added for three specific industries – utilities, basic chemicals, and oil and gas. As the results indicate, Alberta experiences the largest fall in value-added demand in basic chemicals and the second largest fall in utilities among all the regions. This explains the corresponding large fall in outputs of these sectors in Alberta. It is interesting to observe that while Ontario also experiences similar decline (though of lesser magnitude), the corresponding reduction in value-added demand in Québec for these industry sectors are very small. This is due largely to the structure of industry composition in Québec. It also signifies Québec’s lesser ability to substitute for value-added inputs relative to other regions in order to comply with the national emissions reduction target. The picture reverses for the oil and gas sector, where Alberta seems to demand more of value-added inputs relative to a decline in the same in Ontario. Contrary to Québec’s situation, this indicates Alberta’s ability to substitute for value-added inputs in oil and gas sector that can help the region to attain its emission reduction goal. When we consider all policy schemes, again the results indicate relatively better performance of the NPE scheme as compared to the other entitlement-based policies.

Table 16a: Total household consumption in utilities sectors (percentage change)

<table>
<thead>
<tr>
<th>Region</th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ontario</td>
<td>-4.93</td>
<td>-5.16</td>
<td>-4.48</td>
<td>-4.42</td>
<td>-4.76</td>
<td>-4.94</td>
</tr>
<tr>
<td>Québec</td>
<td>-0.56</td>
<td>-0.77</td>
<td>0.91</td>
<td>0.42</td>
<td>-0.35</td>
<td>-0.48</td>
</tr>
<tr>
<td>British Columbia and the Territories</td>
<td>-2.52</td>
<td>-2.76</td>
<td>-1.73</td>
<td>-1.89</td>
<td>-2.33</td>
<td>-2.45</td>
</tr>
<tr>
<td>Canada</td>
<td>-5.02</td>
<td>-5.05</td>
<td>-4.81</td>
<td>-4.90</td>
<td>-4.97</td>
<td>-4.97</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 16b: Total household consumption in other manufacturing (percentage change)

<table>
<thead>
<tr>
<th></th>
<th>NPE</th>
<th>EBA</th>
<th>EPC</th>
<th>CCI</th>
<th>EI</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alberta</td>
<td>1.80</td>
<td>2.98</td>
<td>-2.73</td>
<td>-1.32</td>
<td>0.66</td>
<td>1.06</td>
</tr>
<tr>
<td>Atlantic Province</td>
<td>0.22</td>
<td>0.87</td>
<td>0.09</td>
<td>-0.61</td>
<td>0.48</td>
<td>1.08</td>
</tr>
<tr>
<td>Ontario</td>
<td>0.38</td>
<td>0.15</td>
<td>0.81</td>
<td>0.88</td>
<td>0.55</td>
<td>0.37</td>
</tr>
<tr>
<td>Other Prairie Provinces</td>
<td>1.76</td>
<td>1.77</td>
<td>0.40</td>
<td>0.10</td>
<td>1.31</td>
<td>1.88</td>
</tr>
<tr>
<td>Québec</td>
<td>0.03</td>
<td>-0.17</td>
<td>1.42</td>
<td>0.96</td>
<td>0.22</td>
<td>0.10</td>
</tr>
<tr>
<td>British Columbia and the Territories</td>
<td>0.15</td>
<td>-0.08</td>
<td>0.94</td>
<td>0.78</td>
<td>0.35</td>
<td>0.22</td>
</tr>
<tr>
<td>Canada</td>
<td>0.51</td>
<td>0.52</td>
<td>0.49</td>
<td>0.48</td>
<td>0.50</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Source: Simulation results

Finally, Table 16a and 16b presents household consumption of output produced by two industry sectors – utilities and other manufacturing. As revealed in the benchmark data, since the utility producing sector emits large quantities of effluents, implementing any emission reduction policy scheme essentially translates into raising the cost of production for this sector, which in turn causes price of utilities to sore, ultimately leading to a decline in the output demand. The results confirm this and indicate that Albertans shrink their demand the most while residents of BCT show least contraction of utility demands. In terms of consumption of other manufacturing products, results reveal a general rise in the demand across all regions under all policy schemes. This is largely due to the lump-sum method of recycling emission rents as it essentially enhances purchasing power of the households, who are the owner of productive factors in the economy.

Overall a comparison of all six policy schemes generally demonstrates the relative appeal of the NPE policy over the others. Particularly, even though the former scheme performs similar to the MI scheme, it precludes the politically contentious issue of the inter-regional transfer of emission rents.

1.7 Concluding Remarks:

In this paper we examine two broad categories of emission permit allocation policies, namely the entitlement versus no prior entitlement (NPE) based policies. Most commonly cited policies in
the literature (e.g. emission based allocation of permits or permit allocation based on emission per capita) generally fall within the former category. An upshot of the entitlement based policies is that they entail the contentious issue of transfer of scarcity rents among economic units that occurs through the permit trading system. Consequently, even though these policies may possess excellent ethical properties, they may not have sufficient political appeal for them to be put to implementation. Non-entitlement based policies, on the other, may not suffer from this lack of political palatability as they preclude such transfers of scarcity rents. The issue is then to devise such a policy and examine its properties vis-à-vis the entitlement based policies. The paper does so by providing detailed comparison of five entitlement based policies with a newly devised non-entitlement based policy that we call the NPE policy.

More specifically, using a static computable general equilibrium model of six Canadian regions (i.e. Alberta, Atlantic Provinces, Ontario, Other Prairie Provinces, Québec and British Columbia and the Territories), in this paper we assess welfare cost of six allocation schemes. While five of these policy schemes (i.e. emission based allocation of permits, EBA; allocation based: on emission per capita, EPC; converging carbon intensity, CCI; efficiency index, EI and multi-criteria index, MI) are based on prior entitlement of emission rights, the NPE scheme does not entail any prior allocation of emission permits.

These results, derived from the six simulations performed in this paper, reflect both equity considerations as well as the extent of emission rent transfers that are entailed in each policy option. The EBA policy, being based on the historical emission levels, provides more permits to the Prairie and Atlantic Provinces, thus, making these regions a net seller of permits. EPC scheme, on the other hand, provides more permits to Québec and Ontario as it is based on per-capita emission level. CCI and EI also render similar outcomes since these policy schemes reward small emitters over the large emitter like Alberta. MI scheme reduces the impact of EI policy since it also takes into account the heterogeneity in the composition of regional economies along with the emissions history and the reward for the efficient emitters. Entitlement-based allocation policies, thus, inevitably benefit some regions at the expense of others.
In contrast, under the NPE policy since emission rights are distributed based on the residual (i.e. equilibrium) level of emissions, no inter-regional transfer of scarcity rents occurs. This significantly raises the political appeal of this policy.

From the welfare perspective, model results further indicate that under all six policy schemes, Alberta benefits the most while Québec bears the brunt of the emissions control target. Nonetheless, when reduction in regional GDP is taken into account, Alberta’s economy experiences the most contraction relative to the other regional economies. The magnitude of the overall burden, however, varies from one scheme to another.

When considering different criteria of evaluation – such as welfare, household consumption, GDP, dollar purchase of emission permits and others – results corresponding to the set of policy scenarios offer one strikingly consistent yet interesting revelation: allocation scheme based on no prior entitlement of emission right (i.e. the NPE policy) tends to dominate the entitlement-based schemes. In other words, regional economies perform better under the NPE policy, which also has the greatest political appeal. This result is the centerpiece of the qualitative findings derived in this paper, which brings in new insights into the equity-efficiency debate in the exiting literature.
Appendices to Chapter One

Appendix A: List of equations

Household:
\[ Y_R = (1 - \tau_{Y,R}) Y_{H,R} + T_{G,R} - E R \tau^{w}_{G,R} \]
\[ Y_{H,R} = w_R L_R + Y_{D,R} + P R_R \]
\[ Y_{D,R} = \beta_{D,R} (1 - \tau_{P,R}) \sum_j R_{K,R} K_{j,R} \]
\[ S_{H,R} = \beta_{S,R} Y_R \]
\[ EM_R = (1 - \beta_{S,R}) Y_R \]
\[ Y_{NL,R} = EM_R - (1 - \tau_{Y,R}) w_R L_R \]
\[ C_R = \alpha_R \left( \frac{1}{P_{C,R}} \right) \]
\[ L_R = H_R - (1 - \alpha_R) \frac{\left(1 - \tau_{Y,R}\right) w_R H_R + Y_{NL,R}}{(1 - \tau_{Y,R}) w_R} \]
\[ P_{C,R} = \frac{1}{A_{C,R}} \left[ \left( \theta_{C,R} \right)^{\sigma_{i,R}} \left( \frac{P_{P,C,R}}{P_{HET,R}} \right)^{\frac{1}{1 - \sigma_{j,R}}} + \left(1 - \theta_{C,R} \right)^{\sigma_{i,R}} \left( \frac{P_{P,HOG,R}}{P_{HOG,R}} \right)^{1 - \sigma_{j,R}} \right]^{\frac{1}{1 - \sigma_{j,R}}} \]
\[ HET_R = \left( \frac{A_{C,R}}{P_{HET,R}} \right)^{\sigma_{i,R}} C_R \left( \frac{\theta_{C,R} P_{C,R}}{P_{HET,R}} \right)^{\sigma_{j,R}} \]
\[ HOG_R = \left( \frac{A_{C,R}}{P_{HOG,R}} \right)^{\sigma_{i,R}} C_R \left( \frac{\left(1 - \theta_{C,R}\right) P_{P,C,R}}{P_{HOG,R}} \right)^{\sigma_{j,R}} \]
\[ P_{HET,R} = \frac{1}{A_{HET,R}} \left[ \sum_i (\theta_{HET,i,R})^{\sigma_{HET,i,R}} \left( \frac{P_{P,HET,i,R}}{P_{HET,i,R}} \right)^{1 - \sigma_{HET,i,R}} \right]^{\frac{1}{1 - \sigma_{HET,i,R}}} \]
\[ HET_{I,R} = \left( \frac{A_{HET,R}}{P_{HET,i,R}} \right)^{\sigma_{HET,I,R}} HET_R \left( \frac{\theta_{HET,i,R} P_{P,HET,R}}{P_{HET,i,R}} \right)^{\sigma_{HET,R}} \]
\[ P_{HOG,R} = \frac{1}{A_{HOG,R}} \left[ \sum_i (\theta_{HOG,i,R})^{\sigma_{HOG,i,R}} \left( \frac{P_{P,HOG,i,R}}{P_{HOG,i,R}} \right)^{1 - \sigma_{HOG,i,R}} \right]^{\frac{1}{1 - \sigma_{HOG,i,R}}} \]
\[ HOG_{I,R} = \left( \frac{A_{HOG,R}}{P_{HOG,i,R}} \right)^{\sigma_{HOG,I,R}} HOG_R \left( \frac{\theta_{HOG,i,R} P_{P,HOG,R}}{P_{HOG,i,R}} \right)^{\sigma_{HOG,R}} \]
\[
\begin{align*}
P_{Q,j,R} &= \frac{1}{A_{Q,j,R}} \left[ (\varphi_{j,R})^{\sigma_{Q,j,R}} \left( P_{X,j,R} \right)^{1-\sigma_{Q,j,R}} + (1 - \varphi_{j,R})^{\sigma_{Q,j,R}} \left( P_{M,j,R} \right)^{1-\sigma_{Q,j,R}} \right]^{-1} \\
X_{j,R} &= (A_{Q,j,R})^{\sigma_{Q,j,R}^{-1} - 1} Q_{j,R} \left[ \frac{\varphi_{j,R} P_{Q,j,R}}{P_{X,j,R}} \right]^{\sigma_{Q,j,R}} \\
M_{j,R} &= (A_{Q,j,R})^{\sigma_{Q,j,R}^{-1} - 1} Q_{j,R} \left[ (1 - \varphi_{j,R}) \frac{P_{Q,j,R}}{P_{M,j,R}} \right]^{\sigma_{Q,j,R}} \\
P_{X,j,R} &= \frac{1}{A_{X,j,R}} \left[ (\varphi_{X,j,R})^{\sigma_{X,j,R}} \left( P_{KE,j,R} \right)^{1-\sigma_{X,j,R}} + (1 - \varphi_{X,j,R})^{\sigma_{X,j,R}} \left( P_{KE,j,R} \right)^{1-\sigma_{X,j,R}} \right]^{-1} \\
KE_{j,R} &= \varphi_{X,j,R} \frac{P_{X,j,R} X_{j,R}}{P_{KE,j,R}} \\
L_{j,R}^D &= (1 - \varphi_{X,j,R}) \frac{P_{X,j,R} X_{j,R}}{W_R} \\
P_{KE,j,R} &= \frac{1}{A_{KE,j,R}} \left[ (\varphi_{KE,j,R})^{\sigma_{KE,j,R}} \left( R_{K,R} \right)^{1-\sigma_{KE,j,R}} + (1 - \varphi_{KE,j,R})^{\sigma_{KE,j,R}} \left( P_{EST,j,R} \right)^{1-\sigma_{KE,j,R}} \right]^{-1} \\
K_{j,R}^D &= (A_{KE,j,R})^{\sigma_{KE,j,R}^{-1} - 1} KE_{j,R} \left[ \frac{\varphi_{KE,j,R} P_{KE,j,R}}{R_{K,R}} \right]^{\sigma_{KE,j,R}} \\
EST_{j,R} &= (A_{KE,j,R})^{\sigma_{KE,j,R}^{-1} - 1} KE_{j,R} \left[ (1 - \varphi_{KE,j,R}) \frac{P_{KE,j,R}}{P_{KE,j,R}} \right]^{\sigma_{KE,j,R}} \\
P_{EST,j,R} &= \frac{1}{A_{EST,j,R}} \left[ (\varphi_{EST,j,R})^{\sigma_{EST,j,R}} \left( P_{EST,j,R} \right)^{1-\sigma_{EST,j,R}} + (1 - \varphi_{EST,j,R})^{\sigma_{EST,j,R}} \left( P_{EST,j,R} \right)^{1-\sigma_{EST,j,R}} \right]^{-1} \\
ESF_{j,R} &= (A_{EST,j,R})^{\sigma_{EST,j,R}^{-1} - 1} EST_{j,R} \left[ \frac{\varphi_{EST,j,R} P_{EST,j,R}}{P_{EST,j,R}} \right]^{\sigma_{EST,j,R}} \\
ELEC_{j,R} &= (A_{EST,j,R})^{\sigma_{EST,j,R}^{-1} - 1} EST_{j,R} \left[ (1 - \varphi_{EST,j,R}) \frac{P_{EST,j,R}}{P_{ELEC,j,R}} \right]^{\sigma_{EST,j,R}} \\
P_{ESF,j,R} &= \frac{1}{A_{ESF,j,R}} \left[ \sum_{i \in V_{S_{j}}} (\varphi_{ESF,j,i,R})^{\sigma_{ESF,j,R}} \left( P_{ESF,j,i,R} \right)^{1-\sigma_{ESF,j,R}} \right]^{-1} \\
VS_{j,j,R} &= (A_{ESF,j,R})^{\sigma_{ESF,j,R}^{-1} - 1} ESF_{j,R} \left[ \frac{\varphi_{ESF,j,i,R} P_{ESF,j,i,R}}{P_{VS,j,i,R}} \right]^{\sigma_{ESF,j,R}} \\
P_{M,j,R} &= \frac{1}{A_{M,j,R}} \left[ (\varphi_{M,j,R})^{\sigma_{M,j,R}} \left( P_{M,j,R} \right)^{1-\sigma_{M,j,R}} + (1 - \varphi_{M,j,R})^{\sigma_{M,j,R}} \left( P_{MATT,j,R} \right)^{1-\sigma_{M,j,R}} \right]^{-1}
\end{align*}
\]
\[
EMF_{j,R} = \left( A_{M,j,R} \right)^{\sigma_{m,j,R}^{-1}} M_{j,R} \left[ \frac{\varphi_{M,j,R} P_{M,j,R}}{P_{EMF,j,R}} \right]^{\sigma_{m,j,R}} \\
MATT_{j,R} = \left( A_{M,j,R} \right)^{\sigma_{m,j,R}^{-1}} M_{j,R} \left[ \left( 1 - \varphi_{M,j,R} \right) P_{M,j,R} \right]^{\sigma_{m,j,R}} \\
P_{EMF,j,R} = \frac{1}{A_{EMF,j,R}} \left[ \sum_{i \in VM_{i,j}} \left( \varphi_{EMF,j,i,R} \right)^{\sigma_{EMF,j,R}} \left( P_{VM,j,i,R} \right)^{1-\sigma_{EMF,j,R}} \right]^{-1} \\
VM_{i,j,R} = \left( A_{EMF,j,R} \right)^{\sigma_{EMF,j,R}^{-1}} EMF_{j,R} \left[ \frac{\varphi_{EMF,j,R} P_{EMF,j,R}}{P_{VM,i,j,R}} \right]^{\sigma_{EMF,j,R}} \\
P_{MATT,j,R} = \sum_{i \in VMAT}_{i,j,R} P_{VMAT,j,i,R} \\
VMATT_{i,j,R} = \gamma_{i,j,R} MATT_{i,j,R} \\
S_{Q,R} = \beta_{Q,R} \left( 1 - \tau_{P,R} \right) \sum_{j} R_{K,j,R} K_{j,R} \\

Government:
\]
\[
Y_{G,R} = \tau_{Y,R} Y_{H,R} + \tau_{D,R} \sum_{i} R_{K,i,R} K_{j,R} + \sum_{i} \tau_{C,i,R} P_{C,i,R} C_{i,R} + \sum_{j} \tau_{Q,j,R} P_{Q,j,R} Q_{j,R} \\
+ \sum_{i} \tau_{I,i,R} P_{I,i,R} I_{i,R} + \sum_{i} \tau_{ER,i,R} \sum_{j} \tau_{IM,j,R} P_{IM,j,R} C_{j,R}^{IM} \\
S_{G,R} = Y_{G,R} - \sum_{i} P_{C,i,R} C_{G,i,R} - T_{G,R} \\

Trade – International and Regional:
\]
\[
P_{C,i,R} = \frac{1}{A_{m,i,R}} \left[ \left( \kappa_{i,R} \right)^{\sigma_{m,i,R}} \left( P_{C,i,R}^{D} \right)^{1-\sigma_{m,i,R}} + \left( 1 - \kappa_{i,R} \right)^{\sigma_{m,i,R}} \left( P_{C,i,R}^{IM} \right)^{1-\sigma_{m,i,R}} \right]^{-1} \\
C_{i,R}^{D} = \left( A_{m,i,R} \right)^{\sigma_{m,i,R}^{-1}} C_{i,R} \left( \kappa_{i,R} P_{C,i,R}^{D} \right)^{\sigma_{m,i,R}} \\
C_{i,R}^{IM} = \left( A_{m,i,R} \right)^{\sigma_{m,i,R}^{-1}} C_{i,R} \left( \left( 1 - \kappa_{i,R} \right) P_{C,i,R}^{IM} \right)^{\sigma_{m,i,R}} \\
P_{IM,j,R} = \frac{1}{A_{m,j,R}} \left[ \left( \kappa_{IM,j,R} \right)^{\sigma_{IM,j,R}} \left( P_{IM,ROR,j,R}^{IM} \right)^{1-\sigma_{IM,j,R}} + \left( 1 - \kappa_{IM,j,R} \right)^{\sigma_{IM,j,R}} \left( P_{IM,ROR,j,R}^{IM} \right)^{1-\sigma_{IM,j,R}} \right]^{-1} \\
C_{ROR,j,R}^{IM} = \left( A_{m,j,R} \right)^{\sigma_{IM,j,R}^{-1}} C_{i,R}^{IM} \left( \kappa_{IM,j,R} P_{C,i,R}^{IM} \right)^{\sigma_{IM,j,R}} \\
\]
\[ C_{\text{IM}, j,R}^{\text{ROW}, j,R} = (A_{{\text{IM}, j,R}}) r^{-1} C_{i,R} \left( \left( 1 - \kappa_{\text{IM}, j,R} \right) \frac{P_{\text{IM}, j,R}}{P_{\text{ROW}, j,R}} \right) \]

\[ P_{\text{ROR}, i,R} = \frac{1}{A_{\text{IMR}, i,R}} \left[ \sum_{j} \left( \kappa_{\text{IMR}, j,R} \right) \left( \frac{P_{\text{IMR}, j,R}^{\text{ROW}, j,R}}{P_{\text{ROR}, j,R}^{\text{ROW}, j,R}} \right)^{1 - \sigma_{\text{IMR}, j,R}} \right] \]

\[ C_{\text{IMR}, j,R}^{\text{IMR}, j,R} = \left( \frac{\kappa_{\text{IMR}, j,R} P_{\text{IMR}, j,R}}{P_{\text{ROR}, j,R}^{\text{ROW}, j,R}} \right) \sigma_{\text{IMR}, j,R} \]

\[ P_{\text{EX}, j,R} = \frac{1}{A_{\text{EX}, j,R}} \left[ \left( \frac{\eta_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{1 + \sigma_{\text{EX}, j,R}} + \left( 1 - \eta_{\text{EX}, j,R} \right)^{1 - \sigma_{\text{EX}, j,R}} \right] \left( \frac{P_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{1 + \sigma_{\text{EX}, j,R}} \]

\[ Q_{\text{EX}, j,R} = \left( \frac{\eta_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{1 - \sigma_{\text{EX}, j,R}} \left( \frac{P_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{\sigma_{\text{EX}, j,R}} \]

\[ P_{\text{ROR}, j,R} = \frac{1}{A_{\text{EX}, j,R}} \left[ \sum_{j} \left( \eta_{\text{EX}, j,R} \right)^{1 - \sigma_{\text{EX}, j,R}} \left( \frac{P_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{1 + \sigma_{\text{EX}, j,R}} \right] \]

\[ Q_{\text{ROW}, j,R} = \left( \frac{\eta_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{1 - \sigma_{\text{EX}, j,R}} \left( \frac{P_{\text{EX}, j,R}}{P_{\text{Q}, j,R}} \right)^{\sigma_{\text{EX}, j,R}} \]

Capital account:

\[ S_{F,R} = \sum_{i} P_{\text{C}, j,R}^{\text{IM}, j,R} - \sum_{j} P_{\text{Q}, j,R}^{\text{EX}, j,R} - T_{G,R}^{\text{W}} \]

\[ S_{F} = \sum_{R} E_{R} S_{F,R} \]

Demand for investment commodities and total demand:

\[ P_{j,R} = \beta_{j,R} \left( S_{F} + S_{D} \right) \]
\[ S_D = \sum_R \left( S_{H,R} + S_{G,R} + S_{Q,R} \right) \]
\[ C_{i,R}^D = C_{i,R} + C_{G,i,R} + I_{i,R} + \sum_j V_{i,j,R} \]

**Prices:**
\[ P_{C,i,j,R} = P_{C,i,R} + \bar{P} \xi_{H,i,j,R} \left( 1 + \tau_{C,i,j,R} \right) \]
\[ P_{Q,i,j,R} = P_{Q,i,R} \left( 1 - \tau_{Q,i,j,R} \right) \]
\[ P_{I,i,j,R} = P_{I,i,R} \left( 1 + \tau_{I,i,j,R} \right) \]
\[ P_{V,i,j,R} = \left[ P_{C,i,R} + \bar{P} \xi_{Q,i,j,R} \right] \]
\[ P_{ROR,i,j,P,R} = P_{ROR,i,P,R} \]
\[ Q_{ROR,i,j,P,R} = \alpha_{ROR,i,P,R} \]
\[ P_{ROW,i,j,R} = ER_R \left( 1 + \tau_{IM,i,j,R} \right) P_{C,i}^W \]
\[ P_{ROW,i,j,R} = ER_R P_{Q,i,j}^W \]
\[ \overline{P}_R = \bar{P} \]

**Equilibrium conditions:**
\[ C_{i,R}^D = Q_{j,R}^D \]
\[ \sum_j I_{j,R}^D = L_R \]
\[ \sum_j K_{j,R}^D = \overline{K}_R \]
\[ \sum_R E_R = \overline{E} \]

**Emissions:**
\[ E_{H,R} = \sum_i \left( \xi_{H,i,R} C_{i,R} \right) \]
\[ E_{Q,R} = \sum_i \left( \xi_{Q,i,R} V_{i,j,R} \right) \]
\[ E_R = E_{H,R} + \sum_{j} E_{Q,j,R} \]
\[ \overline{E} = \sum_R \overline{E}_R \]
\[ PR_R = \overline{P}_R R_R \]

**Emission based allocation of permits:**
\[ R_R = \Phi_{EBA,R} \overline{E} \]

Allocation based on emissions per capita:
\( R_R = \Phi_{Pop,R} \bar{E} \)

Allocation based on converging carbon intensity:
\[
R_R = \nu \frac{\Phi_{CCI} \text{GDP}_R}{\sum_R R_R} = \frac{E_{R,\text{Base}}}{\text{GDP}_{R,\text{Base}}} = \min \left\{ \frac{E_{R,\text{Base}}}{\text{GDP}_{R,\text{Base}}} \right\} \quad \text{for } R = 1, 2, 3, \ldots
\]

\( \text{GDP}_R = \sum_j T_{j,R}^D + \sum_{k,R} K_{j,R}^D \)

Allocation based on efficiency index:
\[
R_R = \nu E_{R,\text{Base}} \left( 1 - \text{GAP}^* \Phi_{EI,R} \right)
\]

where \( \text{GAP} = \frac{\left( \sum_R E_R - \bar{E} \right) \Phi_{EI,R}}{\sum_R E_R} \)

\[
\sum_R R_R = \bar{E}
\]

\[
\Phi_{EI,R} = \frac{E_{R,\text{Base}}}{\sum_R E_{R,\text{Base}}} \left( \sum_R \text{GDP}_{R,\text{Base}} \right)
\]

Allocation based on multi-criteria index:
\[
R_R = \nu E_{R,\text{Base}} \left( 1 - \text{GAP}^* \Phi_{MI,R} \right)
\]

where \( \text{GAP} = \frac{\left( \sum_R E_R - \bar{E} \right) \Phi_{MI,R}}{\sum_R E_R} \)

\[
\sum_R R_R = \bar{E}
\]

\[
\Phi_{MI,R} = \left[ \omega_A \left( \sum_R A_R \right) + \omega_B \left( \sum_R B_R \right) + \omega_C \left( \sum_R C_R \right) + \omega_D \left( \sum_R D_R \right) \right]
\]

Allocation based on no prior entitlement:
\( R_R = E_{R,\text{Residual}} \)
Appendix B: List of variables (subscript \( R \) in any variable denotes a region)

\( Y_R \): aggregate disposable income of the representative household in region \( R \)
\( Y_{H,R} \): gross aggregate income of household
\( T_{G,R} \): government transfer to region \( R \) household
\( T_{G,W} \): government transfer to rest of the world by region \( R \)
\( w_R \): net rate of return to labour (i.e. wage rate net of (payroll) tax)
\( L_R \): aggregate supply of labour
\( Y_{D,R} \): disposable (i.e. net of tax) dividend income
\( \bar{PR}_R \): permit revenue received by the household
\( R_{K,R} \): gross rate of return to capital (i.e. gross rate of return from investment in capital)
\( S_{H,R} \): household savings
\( EM_R \): total household expenditure
\( Y_{NL,R} \): non-labour income
\( C_R \): composite consumption
\( P_{C,R} \): index price of aggregate consumption bundle
\( HET_R \): composite of (non-mobile) energy goods
\( P_{HET,R} \): index price of composite (non-mobile) energy goods
\( HET_{i,R} \): quantity of (non-mobile) energy good \( i \)
\( P_{HET,i,R} \): price of (non-mobile) energy good \( i \)
\( HOG_R \): composite of materials-mobile energy commodities (i.e. other household goods)
\( P_{HOG,R} \): index price of composite materials-mobile energy commodities
\( HOG_{i,R} \): quantity of materials-mobile energy good \( i \)
\( P_{HOG,i,R} \): price of materials-mobile energy good \( i \)
\( Q_{j,R} \): quantity of composite output produced by firm \( j \)
\( P_{Q,j,R} \): net of tax price of output of firm \( j \)
\( P_{Q,j,R} \): gross of tax price of output \( j \)
\( X_{j,R} \): quantity of composite capital-labour-energy input of industry \( j \)
\( P_{X,j,R} \): index price of composite capital-labour-energy input used in industry \( j \)
\( M_{j,R} \): quantity of aggregate material-mobile factor used in firm \( j \)
\( P_{M,j,R} \): index price of material-mobile factor used in firm \( j \)
\( KE_{j,R} \): quantity of composite capital-energy factor used in industry \( j \)
\( P_{KE,j,R} \): index price of capital-energy factor used in industry \( j \)

\( L^D_{j,R} \): quantity of labour demanded in firm \( j \)

\( EST_{j,R} \): quantity of composite of energy factors used in industry \( j \)

\( P_{EST,j,R} \): index price of composite of energy factors used in industry \( j \)

\( K^D_{j,R} \): quantity of capital demanded in firm \( j \)

\( ESF_{j,R} \): quantity of composite of non-mobile energy factors used in industry \( j \)

\( P_{ESF,j,R} \): index price of composite of non-mobile energy factors used in industry \( j \)

\( ELEC_{j,R} \): amount of electricity input used in firm \( j \)

\( P_{ELEC,j,R} \): price of electricity input used in firm \( j \)

\( VS_{i,j,R} \): quantity of non-mobile factor \( i \) used as intermediate input in industry \( j \)

\( P_{VS,i,j,R} \): price of non-mobile factor \( i \) used as intermediate input in industry \( j \)

\( EMF_{j,R} \): quantity of composite of mobile energy factors used in firm \( j \)

\( P_{EMF,j,R} \): index price of composite of mobile energy factors used in firm \( j \)

\( MATT_{j,R} \): quantity of composite of material factors used in industry \( j \)

\( P_{MATT,j,R} \): index price of composite of material factors used in industry \( j \)

\( VM_{i,j,R} \): quantity of mobile factor \( i \) used as intermediate input in firm \( j \)

\( P_{VM,i,j,R} \): price of mobile factor \( i \) used as intermediate input in firm \( j \)

\( VMATT_{i,j,R} \): quantity of material factor \( i \) used as intermediate input in industry \( j \)

\( P_{VMATT,i,j,R} \): price of material factor \( i \) used as intermediate input in industry \( j \)

\( S_{Q,R} \): corporate savings by firm \( j \)

\( Y_{G,R} \): government income

\( S_{G,R} \): government savings

\( I_{i,R} \): quantity of investment of good \( i \)

\( P_{I_{i,R}} \): net of tax price of investment of good \( i \)

\( C_{G,i,R} \): government consumption of good \( i \)

\( P_{C_{i,R}} \): gross of tax price of good \( i \)

\( C_{i,R} \): quantity of composite of good \( i \)

\( P_{C_{i,R}} \): net of tax price of good \( i \)
$C_{i,R}^D$: quantity consumed of good $i$ sourced from domestic market

$P_{C_i,R}^D$: index price of good $i$ sourced from domestic market

$C_{i,R}^{IM}$: quantity of composite of total imports of good $i$

$P_{C_i,R}^{IM}$: index price of composite of total imports of good $i$

$C_{i,R}^{IM_ROR}$: quantity of composite imports of good $i$ sourced from rest of the region (ROR)

$P_{C_i,R}^{IM_ROR}$: index price of composite imports of good $i$ sourced from ROR

$C_{i,R}^{IM ROW}$: quantity of composite imports of good $i$ sourced from rest of the world (ROW)

$P_{C_i,R}^{IM ROW}$: index price of composite imports of good $i$ sourced from ROW

$C_{i,R}^{IM ROR}$: quantity imported of good $i$ from ROR

$P_{C_i,R}^{IM ROR}$: price of imported good $i$ from ROR

$EX_{j,R}$: quantity of composite of total exports from industry $j$

$P_{EX_{j,R}}^D$: index price of composite of total exports from industry $j$

$Q_{j,R}^D$: quantity supplied to domestic market from firm $j$

$Q_{j,R}^{EX}$: quantity supplied to export markets from industry $j$

$P_{Q_{j,R}^{EX}}$: index price of export composite from industry $j$

$Q_{j,R}^{EX ROR}$: quantity supplied to composite of rest of the region (ROR) export markets from firm $j$

$P_{Q_{j,R}^{EX ROR}}$: index price of export composite for ROR markets from firm $j$

$Q_{j,R}^{EX ROW}$: quantity supplied to composite of rest of the world (ROW) export markets from industry $j$

$P_{Q_{j,R}^{EX ROW}}$: index price of export composite for ROW from industry $j$

$Q_{j,R}^{EX ROR}$: quantity supplied to ROR export market from firm $j$

$P_{Q_{j,R}^{EX ROR}}$: price of ROR export from firm $j$

$S_{F,R}$: foreign savings in region $R$

$S_F$: national foreign savings

$P_{i,j,R}$: gross of tax price of investment of good $i$

$S_D$: total domestic savings

$V_{i,j,R}$: quantity of factor $i$ used as intermediate input in firm $j$

$P_{V_{i,j,R}}$: price of good $i$ used as intermediate good in industry $j$

$\overline{P}$: endogenously determined (at the national emission permit market) price of emission permit
\( P_R \): price of permit in region \( R \)

\( E_{H,R} \): household emissions in region \( R \)

\( E_{Q,j,R} \): industrial (process) emissions produced from good \( j \) in region \( R \)

\( E_R \): total emissions in region \( R \)

\( E \): national emissions

\( R_R \): total emission rights received by region \( R \)
Appendix C: List of parameters (subscript $R$ in any variable denotes a region)

$ER_R$: exchange rate (price of foreign currency in terms of domestic currency); model numéraire

$\beta_{D,R}$: household share of total dividend income

$\tau_{Y,R}$: non-dividend income tax rate in region $R$

$\tau_{P,R}$: tax rate on dividend income

$\beta_{S,R}$: household marginal propensity to save

$\alpha_R$: share of expenditure on aggregate consumption

$H_R$: total time endowment

$\xi_{H,i,R}$: emission factor of good $i$ used in household consumption in region $R$

$\xi_{Q,i,j,R}$: emission factor of good $i$ used as intermediate (in industry $j$) input in production in region $R$

$A_{C,R}$: shift parameter in the consumption aggregator function

$\theta_{C,R}$: share parameter in the consumption aggregator function

$\sigma_{C,R}$: elasticity of substitution between consumption goods

$A_{HET,R}$: shift parameter in the energy composite good

$\theta_{HET,i,R}$: share parameter for energy good $i$

$\sigma_{HET,R}$: elasticity of substitution among energy goods

$A_{HOG,R}$: shift parameter in the non-energy composite commodity

$\theta_{HOG,i,R}$: share parameter for non-energy good $i$

$\sigma_{HOG,R}$: elasticity of substitution among non-energy commodities

$\beta_{Q,R}$: corporate marginal propensity to save

$\tau_{Q,j,R}$: rate of production tax on output of good $j$

$A_{Q,j,R}$: shift parameter in the production aggregator function of firm $j$

$\phi_{j,R}$: share parameter in the production aggregator function of firm $j$

$\sigma_{Q,j,R}$: elasticity of substitution between inputs used to produce composite output of firm $j$

$A_{X,j,R}$: shift parameter in capital-energy composite function of industry $j$
\( \varphi_{X,j,R} \): share parameter in capital-energy composite function of industry \( j \)

\( A_{KE,j,R} \): shift parameter in energy composite function of firm \( j \)

\( \varphi_{KE,j,R} \): share parameter in energy composite function of firm \( j \)

\( \sigma_{KE,j,R} \): elasticity of substitution between capital and composite of energy inputs of firm \( j \)

\( A_{EST,j,R} \): shift parameter in aggregator function of non-mobile energy and electricity inputs of industry \( j \)

\( \varphi_{EST,j,R} \): share parameter in aggregator function of non-mobile energy and electricity inputs of industry \( j \)

\( \sigma_{EST,j,R} \): elasticity of substitution between electricity and composite of non-mobile energy inputs used in industry \( j \)

\( A_{ESF,j,R} \): shift parameter in aggregator function of non-mobile energy inputs of firm \( j \)

\( \varphi_{ESF,j,R} \): share parameter in aggregator function of non-mobile energy inputs of firm \( j \)

\( \sigma_{ESF,j,R} \): elasticity of substitution among non-mobile energy inputs used in firm \( j \)

\( A_{M,j,R} \): shift parameter in aggregator function of material inputs and mobile energy factors used in industry \( j \)

\( \varphi_{M,j,R} \): share parameter in aggregator function of material inputs and mobile energy factors used in industry \( j \)

\( \sigma_{M,j,R} \): elasticity of substitution between composite of mobile energy factors and material inputs used in industry \( j \)

\( A_{EMF,j,R} \): shift parameter in aggregator function of mobile energy inputs of firm \( j \)

\( \varphi_{EMF,j,R} \): share parameter in aggregator function of mobile energy inputs of firm \( j \)

\( \sigma_{EMF,j,R} \): elasticity of substitution among mobile energy inputs used in firm \( j \)

\( \Upsilon_{i,j,R} \): Leontief co-efficient for composite of material inputs used in industry \( j \)

\( P_{C,i}^W \): net of tariff price of imported good \( i \) in foreign currency units

\( P_{Q,j}^W \): price of exported good \( j \) in world market in foreign currency units

\( \tau_{C,i,R} \): tax rate on consumption good \( i \)

\( \tau_{I,i,R} \): tax rate on investment of good \( i \)

\( \tau_{IM,i,R} \): import tariff rate on good \( i \)
$A_{m,i,R}$: shift parameter in total import aggregator function for good $i$

$\kappa_{i,R}$: share parameter in total import aggregator function for good $i$

$\sigma_{m,i,R}$: elasticity of substitution between domestic and import demands for good $i$

$A_{IM,i,R}$: shift parameter in aggregator function for import composite for good $i$

$\kappa_{IM,i,R}$: share parameter in aggregator function for import composite for good $i$

$\sigma_{IM,i,R}$: elasticity of substitution between rest of the region (ROR) and rest of the world (ROW) import demands for good $i$

$A_{IMR,i,R}$: shift parameter in aggregator function for regional import composite for good $i$

$\kappa_{IMR,i,R}$: share parameter in aggregator function for regional import composite for good $i$

$\sigma_{IMR,i,R}$: elasticity of substitution for imports sourced from ROR markets for good $i$

$A_{e,j,R}$: shift parameter in total export aggregator function from firm $j$

$\eta_{j,R}$: share parameter in total export aggregator function from firm $j$

$\sigma_{e,j,R}$: elasticity of substitution between domestic and export supplies from firm $j$

$A_{EX,j,R}$: shift parameter in aggregator function for export composite from industry $j$

$\eta_{EX,j,R}$: share parameter in aggregator function for export composite from industry $j$

$\sigma_{EX,j,R}$: elasticity of substitution between rest of the region (ROR) and rest of the world (ROW) export supplies from industry $j$

$A_{EXR,j,R}$: shift parameter in aggregator function for regional export composite from firm $j$

$\eta_{EXR,j,R}$: share parameter in aggregator function for regional export composite from firm $j$

$\sigma_{EXR,j,R}$: elasticity of substitution for exports among ROR from firm $j$

$\beta_{i,R}$: region $R$’s share of national savings

$\kappa$: exogenous capital stock

$E$: exogenous national emissions target

$\Phi_{EBA,R}$: parameter reflecting emission based allocation

$\Phi_{Pop,R}$: parameter reflecting allocation based on per capita emissions

$\Phi_{CCI}$: parameter reflecting allocation based on converging carbon intensity

$R_{Base}$: level of regional emissions the base period

$\Phi_{EI}$: parameter reflecting allocation based on efficiency index

$\Phi_{MI}$: parameter reflecting allocation based on multi-criteria index
CHAPTER TWO
Endogenous Technological Change and Emissions Allowances

2.1 Introduction:

With increasing amount of scientific evidence, climate change is now recognized as an imminent threat to environmental stability. Emission mitigation policies aimed at curbing escalating greenhouse gas emissions, however, greatly suffer from the classical trade-off between equity and efficiency. From a policy implementation perspective, this trade-off often determines the direction and the extent of public policy intervention. Whereas efficiency criteria relate to achieving environmental goals in the most cost-effective manner, equity considerations pertain to achieving prescribed objectives while reducing the uneven burden cost among various competing sectors of the economy. High costs from equity concerns often render environmental policies politically unattractive and thus hard to implement.

Since environmental measures, such as permit allocation or emission tax, not only minimize social cost of emission (i.e. the efficiency aspect) but also redistribute income among various sectors (i.e. the equity aspect), losing sectors oppose an efficient scheme and possibly prevent its realization, provided their political influence is sufficiently strong (Felder and Schleiniger, 2002). From the perspective of economic efficiency, this appears to be disappointing. From a political standpoint, however, this is not surprising as far as the political appeal of any specific policy option is concerned.

In the context of domestic economy, for an environmental policy to be politically attractive and be feasible for legislative adoption and subsequent implementation, this trade-off must be addressed. It is, therefore, of interest to conduct a welfare analysis of available emission mitigation policies in light of the debate concerning the equity-efficiency trade-off among the various competing sectors of the economy.

A few recent studies attempt to do this. Under the cap-and-trade (CAT) framework for allocation of emission permits, most often the policies that undergo such a scrutiny are –
grandfathered allocation (GFA), output-based allocation (OBA), and some combination of both with revenue recycling options\textsuperscript{27}. However, these studies generally yield poor welfare ranking of the available policy options in that the constraint – stemming from the equity consideration – results in greater loss of economic efficiency. Therefore, the question is whether it is possible to improve the existing welfare ranking of available policy instruments by bringing in new insights into the analysis? This paper aims to address this.

In doing so, we draw upon recent advancements in climate policy modeling. In particular, we use a general equilibrium model to present new welfare metric of four specific policy scenarios for permit allocation – GFA, OBA, auction permit policy with revenue recycling option that addresses pre-existing distortions in the economy (RPT), and the policy option that corrects for market failures in an important relevant market, namely the market for R&D, while retaining the OBA policy (O-R&D).

In a separate, albeit relevant blooming stream of climate policy modeling literature, researchers are working to build models that are increasingly incorporating endogenous technological change (ETC) into the modeling framework. In this context, studies evaluating the effectiveness of various policy options find that environmental and technology policies work best in tandem. This is because when technology enters the scenario, the terms of the trade-off between the marginal cost of pollution control and its marginal social benefit is altered (Popp et al., 2009). Typically technology innovations reduce the marginal cost of achieving a given unit of pollution reduction. In most cases, technological change enables a specified level of environmental cleanup to be achieved at lower total cost to society. New developments in this area indicate that ETC has the potential to greatly ameliorate the welfare cost of emission control policy in the long run (Fischer et al. 2003, Popp et al. 2009). Intuitively then, if incorporated into the efficiency-equity modeling framework, ETC can provide additional insights leading to improvements in the welfare ranking of available policy options.

Two existing papers – Goulder and Schneider (G&S, 1999), and Sue Wing (2003) – provide such frameworks with ETC embedded into the modeling structure. Both of these papers essentially build on the approach that parallels the endogenous-growth literature by

\textsuperscript{27} See Fischer and Fox (2004), Dissou and Robichaud (2005), and Dissou (2006).
including a stock of “knowledge capital” that is generated through investments in R&D activities. In these models, knowledge capital serves as an input into the ETC process\textsuperscript{28}. Indeed, the knowledge stock has been introduced into climate policy models in a variety of ways. While the theoretical basis for change in relative prices in inducing technological change in a particular direction is quite well-developed, exactly how the knowledge stock accumulates and influences production possibilities is not completely settled in the literature.

In this paper, we attempt to introduce a modeling structure regarding the knowledge accumulation process and bring new insights into the literature. The novelty in our approach lies in providing richer and more realistic assumptions regarding the knowledge (capital) accumulation process that parallels the accumulation process in physical capital.

While our model follows the same tradition of modeling ETC as in G&S and Sue Wing, we carry out significant modifications to improve upon the existing frameworks. In particular, unlike the recursive dynamic model as in Sue Wing (2003), ours is a perfect-foresight, rational expectation model that allows for labour-leisure dynamics on the household side, and investment dynamics on the production front. We also bring methodological improvements in the modeling framework in two ways as compared to G&S (1999). We introduce convex adjustment cost into the knowledge accumulation process where, following Buonanno et al. (2003), the stock is held to depreciate at a constant rate. Second, unlike G&S (1999) where investment in R&D is held to be 20\% of the investment in physical capital somewhat in an ad-hoc manner, we use a consistent framework to capitalize R&D investment using the approach developed by Statistics Canada\textsuperscript{29} \textsuperscript{30}. Thus our model provides a consistent structure for data management process that supplies baseline information into the model.

With these modifications, using a multi-sector dynamic general equilibrium set-up we derive interesting results. On one hand, the results conform to the received wisdom that policies, such as R&D subsidy, that correct for market imperfections in the knowledge

\textsuperscript{28}This way of modeling endogenous economic growth has been pioneered by Lucas (1988), and further developed by Romer (1990), Grossman and Helpman (1994), Aghion and Howitt (1998), Acemoglu (1998), and Eaton and Kortum (2001).

\textsuperscript{29}See Salem and Siddiqui (2006).

\textsuperscript{30}In recent times, statistical agencies across the globe are putting considerable efforts to capitalize R&D investment into the system of national accounts (SNA). For an overview of these efforts please see Zuzana Krášiková (2011).
accumulation sector\textsuperscript{31} help to improve the welfare metric of specific environmental policy tools, such as the OBA. On the other hand, they also stand somewhat in contrast to the concept of double dividend hypothesis\textsuperscript{32} that predicts improved welfare ranking for revenue-neutral environmental policies. More specifically, contrary to the double dividend hypothesis, we find that auction permits, where revenue is recycled to reduce payroll tax, may lead to a lower welfare ranking.

While at the onset this result seems startling, on a deeper reflection it turns out to be not surprising. There are two explanations for this. First, this happen since under the auction permit policy with payroll subsidy while the production sectors benefit from higher labour supply, when labour is a poor substitute for other inputs, welfare improvements may not be realized. This occurs since with poor labour substitutability payroll subsidy may not lead to higher production levels, thus precluding output prices from falling and consequently preventing welfare improvements. Under such a scenario, households do not benefit from increased real income (provided through the payroll subsidy) leading to the ramifications of low welfare performance. Secondly, and more plausibly, this result is derived due our modeling framework that allows for ETC. While payroll subsidy provides incentives primarily to households and secondarily to all production sectors, unlike OBA, it does not provide targeted impetus to sectors that particularly suffer from uneven burden costs. The impact of the targeted subsidy provided through the OBA and O-R&D policy further reinforces when technology is endogenized into the framework that helps emission intensive sectors to ameliorate the rising cost of production from emission control. The upshot is greater amount of outputs being produced by these sectors that lower the prices of these outputs, and consequently increases the welfare.

With ETC embedded into the modeling framework, a stand-alone payroll subsidy policy also suffers from the lack of incentive for undertaking R&D investments due to the

\textsuperscript{31} In this context, market imperfections arise from the fact that under a decentralized solution private firms do not internalize the positive externalities from knowledge spillover effects that emanate from R&D activities and hence invest too little in R&D.

\textsuperscript{32} The double dividend hypothesis states that the introduction of environmental or green taxes yields two benefits – first, it compels firms to internalize negative environment externalities thus yields the obvious dividend from improved environmental quality; second, the revenue collected from taxes can be used to cut other pre-existing taxes in the economy, leading to a second dividend from reducing the distortion due to those other taxes. The weak form of this hypothesis states that tax revenues collected from a revenue-neutral green tax reform can be used to cut distorting taxes thus lowering the welfare cost of the green tax reform (Schöb, 2003 and Bovenberg, 1999).
imperfections in the knowledge sector. With a pure OBA policy the former is rectified through implicit production subsidy, yielding an overall improved welfare metric for OBA. When a combination of OBA and R&D subsidy policy is used (i.e. the O-R&D policy), both the disincentive on production and the imperfection in knowledge accumulation are addressed leading to further improvements in the welfare performance.

Although climate policy models are increasingly getting sophisticated in terms of the modeling structure that can facilitate ETC for evaluating benefit-cost of specific emission control policies, current stock of policy evaluation tools tend to emphasize more on efficiency issues compared to the equity considerations. From implementation perspective, the latter criterion is important and often plays a pivotal role. Our paper is specifically motivated by this. In particular, we attempt to provide welfare assessment of four emission permit allocation policies where attempts are made to mitigate costs arising from the equity-efficiency trade-off by bringing in insights from the growing ETC literature into the current modeling framework.

2.2 Review of the Relevant Literature:

Existing literature on equity-efficiency trade-off offers competing views. Goulder (2001) argues that when less than one-tenth of emission permits are distributed using grandfathered allocation to the affected industries, it could yield desirable equity outcome with relatively less cost arising from the efficiency losses. Smith et al. (2002), however, contests this view citing real-world regularities. With real world carbon trading schemes taken into consideration, they find that high cost of policy could obliterate any opportunity to recycle revenue while also compensating affected industries. This view is also shared by Dissou (2005) while evaluating an alternative non-market based regulatory policy instrument, namely the performance standard system of reducing GHG emission. Dissou finds that the cost of designing an optimal performance standard system could be prohibitive and as such may be rendered infeasible to implement.

In a separate vein, Dissou and Robichaud (2003) numerically evaluate impacts of using OBA in a tradable permit system along with other non-market instruments. They, however, do not address the trade-off between efficiency and uneven sectoral distributional effects. Fischer and Fox (2004), on the other hand, address the trade-off using a computable general equilibrium
model. While their model provides a multi-country analysis, they address the trade-off using a static framework. They find that OBA could address the uneven distributional impact of an emission trading system with less efficiency compared to a system where permit proceeds are used to mitigate pre-existing distortionary taxes. In a separate paper Dissou (2006) analyzes the effects from heterogeneity among carbon-intensive industries (energy-intensive and/or fossil-energy-producing industries). He shows that auction permits, where revenue is recycled to address payroll tax distortions, yields 70% lower welfare cost in terms of efficiency-equity trade-off compared to an OBA scheme where firm heterogeneity is explicitly accounted for.

In the other fast growing stream of climate policy literature, the concept of ETC has proved to elicit some interesting aspects that may usher in new insights into the equity-efficiency debate. With ETC incorporated into the modeling framework Sue Wing (2003) has shown that the welfare cost of emission mitigation policy can be significantly reduced. Using U.S. data, Sue Wing finds that relative price effects of an emission tax induce significant intra-sectoral substitution and inter-sectoral reallocation of knowledge inputs that leads to an increase in gross input substitutability in the supply side of the economy. When this (positive) “substitution effect” is taken into consideration along with the (empirically argued negative) knowledge “accumulation effect” to generate efficiency enhancing induced technological innovations of fossil-based fuel use, ETC can more than compensate associated welfare cost of the policy and yield a net positive return. While Sue Wing provides the welfare impact of ETC with a cap-and-trade (CAT) emission mitigation policy, he does not consider equity-efficiency trade-off among the competing sectors of the economy in his framework. We combine these two separate, albeit closely relevant, streams of literature in this current paper thereby bringing in interesting dynamics into the framework. In our model, we use the insight from ETC’s impact on emission policies to further evaluate the efficiency-equity debate for four policy options.

More specifically, we analyse how well OBA could address the trade-off for carbon abatement policies when ETC, triggered by the change in relative price of carbon-rich inputs, is incorporated into the framework. Intuitively, the inclusion of ETC should enhance substitution of carbon-rich input for less carbon-intensive inputs due to innovations in carbon-saving technology that, consequently yield a better welfare metric for the available policy options.
The development and the modification of the model presented in this paper build upon recent contributions to the literature on general equilibrium modeling and on environmental policies, such as Bovenberg and Goulder (2000), Fischer (2001), Fischer and Fox (2004) and Goulder et al. (1999). The remainder of the paper is organized as follows. Next section presents a description of the model, while the fourth section discusses data and model calibration process. In the fifth section, simulation results are presented, and conclusions are drawn in the final section.

2.3 The Model:

In this section, we describe the model developed for the analyses presented in this paper. The current model belongs to the family of dynamic CGE models with prefect foresight of a small-open economy with an extension to incorporate the adjustment costs of investment. Unlike the traditional models with investment dynamics (where usually physical capital is the only accumulable factor), the novelty of the current paper lies in introducing investments in two accumulable factors – physical and knowledge capital. While a few other models (e.g. G&S (1999), and Sue Wing (2003)) attempt to introduce two accumulable factors of production, however. Their treatment for accumulation of knowledge capital is somewhat *ad-hoc*. We overcome this problem by introducing a convex adjustment cost to both accumulable factors of production.

Following Buonanno, Carraro and Galeotti (2003) and unlike G&S (1999), we further introduce an exogenous rate of depreciation to knowledge capital. Following the convention, in the model physical capital stock depreciates at a deterministic rate. Depreciation of knowledge capital is based on the premise that knowledge becomes obsolete overtime. For example, a production technology becomes obsolete with the innovation of a new technology that replaces the old one.

The current model draws upon the Ramsey theoretical model, as described in Blanchard and Fischer (1989). Its core dynamics is taken from Devarajan and Go (1998) with modifications to take into account multi-investment sectors and multi-consumption goods. Its dynamic module is characterized by inter-temporal decisions and forward-looking behaviour of domestic agents. Investments and consumption are the result of separate but simultaneous
optimal behaviour. At any point in time, investments are increasing functions of the ratio of the present value of profit stream to the replacement costs of capital (Tobin’s Q) and are subject to convex adjustment costs of capital as in Hayashi (1982). Consumption is an increasing function of wealth. Overall, its dynamic module features the neoclassical Ramsey-Cass-Koopmans growth theory with the inclusion of investment Q theory.

The model’s core static framework is similar to that of Dissou (2006). We assume a single representative consumer and close the saving-investment gap by foreign borrowing, which is supplied at a given world interest rate. In addition, the model solves simultaneously for intra- and inter-temporal equilibria in all markets. This implies that the number of unknown variables increases with the size of the model horizon. Without the loss of generality we use the current model to simulate 150 years from the base period of 2004.

The production sector in the model has 11 production nests. Overall, the model identifies 15 production and 14 investment sectors with a single representative household. The structure thus allows for the relevant degree of details for production and investment diversity as well as represents the heterogeneity in energy consumption within the production sectors. Overall the model can be divided into six blocks – consumption, investment and production, foreign sector, government, equilibrium conditions, and terminal conditions.

2.3.1 Households: Consumption and Labour Supply

As in the standard economic theory, households are the owners of factors of production and maximize their utility by choosing the optimal level of consumption and leisure (thus, determining the optimal supply of labour). Households, in the model, are faced with two levels of optimal decision: inter-temporal decision to locate the time path of optimal aggregate consumption and saving; and intra-temporal decision to choose optimal amount of leisure and to allocate aggregate consumption in each period across different consumption goods. In addition, for the purpose of model implementation, we define households’ budget constraint at the national scope such that it is consistent with the national income identity.
2.3.1.1 Inter-temporal Optimization Problem

All households in the economy are represented by a single infinitely-lived, forward-looking representative agent. The representative household is endowed with one (normalized) unit of labour, and is the owner of the stock of physical and knowledge capital. The representative household chooses its inter-temporal consumption by maximizing its lifetime utility subject to its lifetime budget constraint. Its inter-temporal optimization problem is postulated as follows:

Max \( U_t = \sum_{s=t}^{\infty} (1 + \rho)^{t-s} \left[ \alpha \ln C_s + (1 - \alpha) \ln (1 - L_s) \right] \) \[1\]

Subject to:

\( (1 + n) A_{s+1} = (1 + r) A_s + Y_s - P_{c,s} C_s \) \[2-1\]
\( Y_s = (1 - \tau_y) Y_{H,s} + (1 - \tau_D) Y_{D,s} + S_{G,s} \) \[2-2\]
\( Y_{H,s} = \sum_i w_i L_{i,s} + T_{G,s} + ER_i S_{E,s} + \beta_{H,s} \bar{P}_s EM_s \) \[2-3\]

The transversality condition: \( \lim_{s \to \infty} \left( \frac{1}{1 + r} \right)^s A_{s+1} = 0 \) \[2-4\]

where, \( A_0 \) given \[2-5\]
\( L_{S,s} = \sum_j L_{j,s} \) \[2-6\]
\( N_{s+1} = (1 + n) N_s \) \[2-7\]

\[3^4\]For the representative household, the dividend income \( Y_{D,s} \) comprises of the sum of (gross) returns received from investing in physical and knowledge capital across different sectors of the economy. The full expression for \( Y_{D,s} \), which is further described in the representative firm’s optimization problem, is given by:

\( Y_{D,s} = \sum_j p_{KET,j} RR_{K,j} K_{j,s} - \sum_j p_{k,j} J_{K,j} - \sum_j \left( p_{G,j} - \bar{P}_s e_{j,s} + \alpha \beta_{H,j} \bar{P}_s \right) RR_{H,j} H_{j,s} - \sum_j (1 - \nu_{j,s}) \bar{P}_s \) \[2-8\]

where, \( J_{K,j,t} \) = gross investment in physical capital in sector \( j \) at time \( t \); \( J_{K,j,t} \) = investment in knowledge capital in sector \( j \) at time \( t \); \( p_{k,j} \) = unit cost of physical capital investment at time \( t \); \( p_{k,j} \) = unit cost of knowledge capital investment at time \( t \); \( \bar{P}_s \) = price of emission permit at time \( t \); \( p_{KET,j} \) = index price of capital-energy factor used in industry \( j \) at time \( t \); \( p_{G,j} \) = gross (of production tax) price of industry \( j \)’s output at time \( t \); \( RR_{K,j} \) = marginal product of physical capital in sector \( j \) at time \( t \); \( RR_{H,j} \) = marginal product of knowledge capital in sector \( j \) at time \( t \); \( e_{j,s} \) = quantity of physical capital used in sector \( j \) at time \( t \); \( e_{j,s} \) = quantity of knowledge capital used in sector \( j \) at time \( t \); \( \beta_{j,s} \) = exogenous emission intensity parameter used to allocate permit; \( \alpha \) = emission scale-back parameter (determined endogenously in the model); \( e_{j,s} = \sum_{i,j} e_{i,j,t} \) = emission factor for input \( i \) used in sector \( j \) at time \( t \), aggregated for sector \( j \); and \( \nu_{j,s} \) = rate of subsidy on R&D investment at time \( t \).

Also note that, in the model, investment in knowledge capital is synonymously used to represent R&D investment, which occurs in all but the R&D sector of the economy.
for $s = t, t+1, t+2,...$

and where, all variables are expressed in labour efficiency units

here, $U_t = $ inter-temporal utility function evaluated at time $t$; $\alpha = $ share parameter in the utility function; $C_t = $ per-capita aggregate consumption at time $t$; $L_{s,j} = $ aggregate labour supplied at time $t$; $A_t = $ financial wealth held by the representative household at time $t$; $Y_t = $ household aggregate disposable income at time $t$; $\bar{Y}_{t,s} = $ household gross non-dividend income at time $t$; $Y_{D,t} = $ household gross dividend income at time $t$; $S_{F,j} = $ foreign savings at time $t$; $S_{G,j} = $ government savings at time $t$; $T_{G,j} = $ government transfer to household at time $t$; $P_{c,j} = $ index price of unit aggregate consumption at time $t$; $L_{j,t} = $ labour supplied to sector $j$ at time $t$; $N_t = $ population at time $t$; $n = $ population growth rate; $r = $ interest rate; $w_t = $ net of (payroll) tax wage rate; $EM_t = $ total emission at time $t$; $ER_t = $ exchange rate (price of foreign currency in terms of domestic currency) at time $t$; $\beta_{H,j} = $ household share of total emission at time $t$; $\tau_y = $ income tax rate on non-dividend income; $\tau_a = $ income tax rate on dividend income; $\rho = $ rate of time preference; and $n = $ rate of population growth.

The objective function $U_t$ in equation [1] is the familiar additive separable homogenous utility function with a constant elasticity of inter-temporal substitution. This formulation assumes that the household utility at time $t$ ($= 0$) is the sum of all future discounted utilities stemming from the aggregate consumption ($C_t$) and the amount of leisure enjoyed by the household ($1-L_t$). Hence, the household exhibits labour-leisure choice in the model. The rate of time preference $\rho$ ($> 0$), on the other hand, indicates that the consumption is valued less in the future.

Given the assumption of no-ponzi-scheme as evident in equation [2-4], the inter-temporal (dynamic) budget constraint in equation [2-1] essentially implies that, in equilibrium, the sum of the net present value of consumption expenditure is equal to the sum of the net present value of disposable income. Due to the small-country assumption, the inter-temporal discount rate ($r$) is given by the world market interest rate and is used as a policy variable into the model.

Disposable income for consumption ($Y_t$) in equation [2-2] is defined as a function of gross non-dividend income ($Y_{H,t}$), dividend income ($Y_{D,t}$), and government saving ($S_{G,t}$). Since
\((Y_t - P_{c,j}C_t)\) is private saving, the inter-temporal budget constraint in equation [2-1] is well defined at the national scope such that the national income identity holds.

Equation [2-3] is the definition of household income \((Y_{H,t})\) that consists of the sum of total labour income net of the payroll tax \((\sum w_t L_{i,j})\), government transfer \((T_{G,t})\), household share of total proceeds from emission permit trading \((\beta_{H,j}P_iEM_{j})\), and foreign savings \((ER, S_{F,t})\).

Equation [2-4] presents the transversality condition for the dynamic asset accumulation equation. This condition ensures that the representative consumer is restricted to using a *ponzi-scheme* to finance her optimal consumption path. Equation [2-5] provides the initial asset level at the base-run scenario \((A_0)\).

Finally, equation [2-6] and [2-7] give the definition of aggregate labour supply and the motion equation for population growth respectively.

The Lagrangian for the optimization problem can be written as:

\[
\mathcal{L}_t = \sum_{s=t}^{\infty} (1 + \rho)^{s-t} \left[ \alpha \ln C_s + (1 - \alpha) \ln (1 - L_s) \right] + \lambda_u \left[ (1 + n) A_{s+1} - (1 + r) A_s - Y_s + P_{C,s}C_s \right]
\]

where \(\lambda_u\) is the Lagrange multiplier.

From the first order conditions (FOC) for consumptions between two adjacent periods, we can derive the inter-temporal condition for consumption. Such a condition requires that the marginal utility of consumption in period \(t\) and \(s(> t)\) satisfy:

\[
\frac{C_{s+1}}{C_t} = \left[ \frac{(1 + r) P_{C,j}}{(1 + \rho) P_{C,s+1}} \right]^{\rho} \tag{O-1}
\]

Further the first order optimality conditions between the temporal consumption and leisure yields:
Finally the FOC also yields the lifetime budget constraint, which can be equivalently written as:

\[ (1+n)A_{s+1} = (1+r)A_s + Y_s - P_{c,s}C_s \]  

with the transversality condition

\[ \lim_{t \to \infty} \left( \frac{1}{1+r} \right)^t A_{s+1} = 0 \]

Equation [O-1] is the well-known Euler equation that defines the growth rate of aggregate consumption between two adjacent periods as a function of the growth rate of index prices for composite consumption bundle \( (P_{c,s}) \), rate of time preference \( (\rho) \), and the exogenous market rate of interest \( (r) \).

Equation [O-2] signifies that the representative consumer equates the marginal utility of consumption to the marginal utility of leisure in all periods.

Equation [O-3] gives the dynamic budget constraint. Equation [O-4] is the transversality condition, which holds with equality as long as the marginal utility of consumption is positive.

Combining the Euler equation with the budget constraint and the transversality condition, the level of aggregate consumption \( (C_s) \) can be determined in each period, which takes the expression:

\[ C_s = \frac{\text{Wealth}_s (\rho - n)(1+r)^t}{P_{c,s}(1+\rho)^{t+1}} \]

where, \( \text{Wealth}_s = (1+r)A_0 + \sum_{s=1}^{\infty} \left( \frac{1+n}{1+r} \right)^{t-s} Y_s \)

In our numerical context, however, we do not need to use an explicit analytical expression for composite consumption in each period. The FOCs that signify the optimal
behaviour satisfy the requirement for simultaneously solving the full set of equations in the model.

2.3.1.2 Intra-temporal Optimization Problem

Once the optimal path of aggregate spending on composite consumption is determined inter-temporally, the representative household then intra-temporally allocates the consumption spending among different goods through a cost minimization rule. The intra-temporal optimization problem is postulated as follows:

\[
\text{Min } P_{C,s} = \sum_i P_{C,i,s} C_{i,s}, \quad \text{[4]}
\]

Subject to: \( C_s = \sum_i A C_i \left[ \prod_i^{\sigma_{s,i}} \frac{\sigma_{s,i}}{\sigma_{s,i}} \right], \) where \( \sum_i \alpha_{c,i} = 1 \) when \( C_s = f(\text{CES}) \) [5-1]

\( C_s = A \prod_i^{\sigma_{s,i}} C_{i,s}, \) where \( \sum_i \alpha_{c,i} = 1 \) when \( C_s = f(\text{C-D}) \) [5-2]

\( C_s = \min \left[ \frac{C_{i,s}}{\alpha_{c,1}}, \frac{C_{i,s}}{\alpha_{c,2}}, ..., \frac{C_{i,s}}{\alpha_{c,s}} \right], \) where \( \alpha_{c,i} > 0, \) when \( C_s = f(\text{Leontief}) \) [5-3]

for \( s = t, t+1, t+2, ... \)

where, \( C_{i,t} = \) consumption of good \( i \) at time \( t; P_{C,i,t} = \) gross of tax price of good \( i \) at time \( t; \)

\( P_{C,i,t} = \) net of tax price of good \( i \) at time \( t; \tau_c = \) indirect tax rate on consumption goods; \( \bar{P}_t = \) price of emission permit at time \( t; \xi_{H,j,t} = \) emission factor of good \( i \) used in household consumption at time \( t; \sigma_{c,i} = \) elasticity of substitution between consumption goods; \( \alpha_{c,i} = \) share parameter in the consumption aggregator function; and \( A_c = \) shift parameter in the consumption aggregator function

and where the expression of gross of tax price of good \( i, P_{C,i,t}, \) at time \( t \) is:

\[
P_{C,i,t} = P_{C,i,t} (1 + \tau_c) + \bar{P}_t, \quad \xi_{H,j,t}
\]

Within each period, the representative household allocates the optimal aggregate consumption \( (C_s) \) across different consumption commodities \( (C_{i,t}) \). Equation [4] indicates that
the aggregate consumption expenditure in each period \( (P_{c,t}C_t) \) equals the sum of consumption spending across different commodities \( (\sum_i P_{c,i,t}C_{i,t}) \) in that period. In equation [5], we specify the aggregate consumption as an aggregation of different consumption commodities where the aggregator function takes one of the following three functional forms – Constant Elasticity of Substitution (CES), Cobb-Douglas (C-D), or Leontief function.

![Figure 1: Schematic representation of household preference](image)

Figure 1 provides a schematic representation of the representative household’s choice structure. As evident in Figure 1, a system of nine nodes represents various composition of commodity use\(^{35}\) either through a CES, a C-D, or a Leontief aggregator function. These aggregator functions allow us to mimic the real life substitution behaviour exhibited by the households of the modeled economy. Further the nested structure of the model allows for substitution, on the one hand, between energy and non-energy products and, on the other hand, among various energy goods.

From the intra-temporal optimization behaviour, we derive the following first order necessary conditions:

\[
A_c \alpha_{c,i} C_{c,i}^{\sigma_{c,i} - 1} \left( \sum_i \alpha_{c,i} C_{i,t}^{\sigma_{c,i} - 1} \right)^{-1} = \frac{P_{c,t}}{\lambda_c} \quad \text{when, } C_s = f(\CES) \quad [O-5-1]
\]

\(^{35}\) In the model, the economy’s product lines are disaggregated into a total of 18 commodities as depicted in Figure 1.
\[ A \alpha_{c,i} C_{i,t}^{\alpha_{c,i}} \prod_{j \neq i} C_{j,t}^{\alpha_{c,j}} = \frac{P_{C,i}}{\lambda_c} \quad \text{when}, \ C = f \left( \text{C-D} \right) \]  

where, \( \lambda_c \) is the relevant Lagrange multiplier.

and for the Leontief function, efficiency condition dictates:

\[ C_{i,t} \frac{\alpha_{c,i}}{\alpha_{c,j}} = \frac{C_{j,t}}{\alpha_{c,j}} \]  

\[ P_{C,i} C_i = \sum_i P_{C,i,t} C_{i,t} \]  

By invoking the concept of Shephard’s Lemma, equations [O-5] and [O-6] are manipulated further to finally get the intra-temporal consumption demand for each commodity \( (C_{i,t}) \) and the aggregate price level \( (P_{C,i,t}) \) (for a CES aggregator function) as:

\[ C_{i,t} = \frac{\alpha_{c,i}^{\alpha_{c,i}} P_{C,i} C_i}{\sum_j \left[ \sigma_{c,i}^{\alpha_{c,i}} P_{C,j,t} \right]} \]  

\[ P_{C,i} = \frac{1}{A_c} \sum_i \left[ \alpha_{c,i} P_{C,i,t} \right]^{-1/\sigma_{c,i}} \]

As shown in Figure 1, in the specification of the model, a CES aggregator function is used for almost all the commodities except for natural gas, refined petroleum and motive fuel products. Whereas, for the aggregator function, a Leontief specification is used for the natural gas products, for commodities in the other two categories a C-D specification is used\(^{37}\). Along all the structural nodes of the model, however, the same cost minimization principle is used to

\( ^{36} \) Note that equation [6] can be manipulated to get \( C_{i,t} = A_c^{\alpha_{c,i}} C_i^{\alpha_{c,i}} \left[ \alpha_{c,i} P_{C,i} C_i \right]^{-\sigma_{c,i}}, \) where the definition of \( P_{C,i} \) is used from equation [7].

\( ^{37} \) For the Leontief specification the equivalent expressions for equation [6] and [7] are:

\[ C_{i,t} = \alpha_{c,i} C_i; \text{ and } P_{C,i} = \sum_i \left[ \alpha_{c,i} P_{C,i,t} \right] \]

And for the Cobb-Douglas specification they are:

\[ C_{i,t} = \frac{\alpha_{c,i} P_{C,i} C_i}{P_{C,i,t} \sum_i \alpha_{c,i}}; \text{ and } P_{C,i} = \frac{1}{A_c} \prod_i \left( \frac{P_{C,i,t}}{\alpha_{c,i}} \right)^{\alpha_{c,i}} \]
optimally distribute aggregate spending across the commodities of the economy in any given period of time.

2.3.2 Firms: Production, Investments, and Abatement Decision with OBA

The model identifies 15 production sectors. Each sector is assumed to behave as a single representative firm, producing a single homogenous commodity. Over the model horizon, each firm takes inter-temporal investment decisions that determine the availability of physical and knowledge capital stock in each period. Governed by the production technology, within the period, each firm allocates aggregate investment spending over different investment commodities, and employs a combination of knowledge capital, physical capital, labour and intermediate inputs to produce the output.

2.3.2.1 Inter-temporal Investment Optimization Problem

Figure 2 provides a schematic representation of various nesting of technologies used by the representative firm in each sector of the economy. The representative firms are modeled as forward-looking agents with perfect foresight.

To model OBA of emission permits, we adopt a technique that is similar to the authors like Dissou (2006), Fischer and Fox (2004), Fischer (2001), and Goulder et al. (1999). We model OBA through allocation of free emission permits under the standard permit trading system. Since free permits are linked to output in OBA where permits have a market value \( \tilde{P}_t \), allocation of permits through OBA affects both factor utilization decisions and the effective output price received by the producers.

\[ \text{The only exception is the R&D sector, where the representative firm does not invest in R&D to create knowledge capital in that sector. Unlike other papers in the literature, ours is the only one where we do not use an ad-hoc rule to determine economy's aggregate R&D investment. Based on Statistics Canada's calculation method, we capitalized R&D spending from other sectors and combined them to create R&D as a separate investment sector of the economy.} \]
Equation [8] provides the decision problem faced by a representative firm in sector $j$. In the presence of emission cost, and with the incentive to reduce emission through OBA of permits, the firm chooses optimal levels of labour, capital, and intermediate inputs along with the R&D investments by maximizing the net present value of the firm, subject to the standard laws of motion for the stock of physical and knowledge capital as specified in equation [9-10].

$$\text{Max } V_{j,t} = \sum_{s=t}^{\infty} (1+r)^{t-s} \left[ P_{Q,j,s} Q_{j,s} + \bar{P}_s \left( \alpha \beta_{j,s} Q_{j,s} - \sum_{i} c_{i,j,s} q_{i,j,s} \right) - \sum_{i} P_{i,s} q_{i,j,s} \right] - w_t \left( 1 - \tau_{L,s} \right) P_{H,j,s} J_{H,j,s},$$

where, $P_{Q,j,s} = P_{Q,j,s} \left( 1 - \tau_{Q,j} \right), P_{i,s} = P_{i,s} \left( 1 + \tau_{i,j} \right) + \bar{P}_s \xi_{i,s,j} \text{ and } \bar{w_t} = w_t \left( 1 - \tau_{L,j} \right)$ [8]

Subject to:

$$K_{j,s+1} = (1 - \delta_{K,j}) K_{j,s} + I_{K,j,s} \quad [9]$$

$$H_{j,s+1} = (1 - \delta_{H,j}) H_{j,s} + I_{H,j,s} \quad [10]$$

$$K_{j,0} \text{ given and } \lim_{s \to \infty} \left( \frac{1}{1+r} \right) P_{K,j,s} K_{j,s} \geq 0 \quad [11]$$

$$H_{j,0} \text{ given and } \lim_{s \to \infty} \left( \frac{1}{1+r} \right) P_{H,j,s} H_{j,s} \geq 0 \quad [12]$$

where, $J_{K,j,s} = \frac{1}{2} \left( \frac{I_{K,j,s}}{K_{j,s}} \right)$
\[ J_{H,j,s} = I_{H,j,s} \left[ 1 + \frac{\beta_{H,j}^s}{2} \begin{pmatrix} I_{H,j,s} \end{pmatrix} \right] \quad [14] \]

\[ Q_{j,s} = \gamma_j \left( H_{j,s} \right) \left[ \frac{\sigma_{Q,j}}{\sigma_{P,j}}^{\sigma_{Q,j}-1} \phi_{j} H_{j,s} + \left(1 - \phi_{j} \right) X_{j,s} \right] \quad [15] \]

\[ H_{j,s} = h(L_{j,s}) ; \quad X_{j,s} = x(L_{j,s}, K_{j,s}, H_{j,s}) \]

\[ H_{j,s+1} = H_{j,s} + \bar{\beta} \bar{T}_{H,j,s} \]

and \( s = t, t+1, t+2, \ldots \)

here, \( V_{j,t} \) = present-value of the net return in industry \( j \) at time \( t \); \( Q_{j,t} \) = output of sector \( j \) at time \( t \); \( q_{i,j,t} \) = intermediate input \( i \) used in industry \( j \) at time \( t \); \( L_{j,t} \) = amount of labour demanded by industry \( j \) at time \( t \); \( I_{K,j,t} \) = investment demand for physical capital in sector \( j \) at time \( t \); \( I_{H,j,t} \) = R&D investment demand in sector \( j \) at time \( t \); \( P_{Q,j,t} \) = gross (of production tax) price of industry \( j \)'s output at time \( t \); \( P_{Q,j,t} \) = net (of production tax) price of industry \( j \)'s output at time \( t \); \( P_{r,t} \) = price of emission permit at time \( t \); \( P_{i,t} \) = gross (of tax) price of intermediate input \( i \) at time \( t \); \( P_{\bar{r},t} \) = net (of tax) price of intermediate input used in sector \( i \) at time \( t \); \( \xi_{i,j,t} \) = emission factor of good \( i \) used as intermediate input in production at time \( t \); \( \bar{w}_t \) = wage rate net of payroll tax in period \( t \); \( P_{K,t} \) = rental rate of physical capital in period \( t \); \( P_{H,t} \) = price of knowledge capital in period \( t \); \( V_{j,t} \) = rate of subsidy on R&D investment at time \( t \); \( \delta_{K,j} \) = physical capital depreciate rate; \( \delta_{H,j} \) = knowledge capital depreciation rate; \( \beta_{j,t} \) = exogenous emission intensity parameter used to allocate permits; \( \alpha_j \) = emission scale-back parameter (determined endogenously in the model); \( e_{i,j,t} \) = emission factor for input \( i \) in sector \( j \) at time \( t \); \( \tau_{Q,j} \) = sales tax rate on goods produced in sector \( j \); \( \tau_{i,j} \) = tax rate on commodity \( i \) used as intermediate goods in sector \( j \); \( \tau_{\ell,j} \) = payroll tax rate at time \( t \); \( \gamma_j \) = shift parameter for production function in sector \( j \); and \( \varphi_{j} \) = share parameter for production function in sector \( j \); and \( \bar{H}_{j,t} \) = average knowledge stock in sector \( j \) at time \( t \) (i.e. the non-excludable knowledge stock in sector \( j \) at time \( t \)); \( \bar{T}_{H,j,t} \) = industry wide expenditure on R&D in
industry \( j \) at time \( t \) and \( \beta = \) the parameter that regulates the magnitude of technological spillovers.

Note that \( P_{K,t} \) and \( P_{H,t} \) can be interpreted as the per-unit acquisition cost of physical and knowledge capital, respectively. In equation [13], \( J_{K,j,t} \) represents the quantity of gross investment in physical capital by destination sector, which is a function of net investment \( (I_{K,j,t}) \), existing capital stock \( (K_{j,t}) \), and adjustment cost parameters \( (\beta_{K,j}) \). Similarly, in equation [14], \( J_{H,j,t} \) represents the quantity of gross investment in knowledge capital by destination sector, which is a function of R&D investment \( (I_{H,j,t}) \), existing knowledge capital stock \( (H_{j,t}) \), and adjustment cost parameters \( (\beta_{H,j}) \).

It is worth to have a look at the investment adjustment cost functions for physical and knowledge capital. Observe that, by construction, \( \frac{\partial J_{K}}{\partial I_{K}} > 0 \), \( \frac{\partial^2 J_{K}}{\partial I_{K}^2} > 0 \), \( \frac{\partial J_{K}}{\partial K} < 0 \), and \( \frac{\partial^2 J_{K}}{\partial K^2} > 0 \). This also holds for \( J_{H} \), namely \( \frac{\partial J_{H}}{\partial I_{H}} > 0 \), \( \frac{\partial^2 J_{H}}{\partial I_{H}^2} > 0 \), \( \frac{\partial J_{H}}{\partial H} < 0 \), and \( \frac{\partial^2 J_{H}}{\partial H^2} > 0 \). These properties have two implications. First, the marginal cost of investing in either form of capital formation is positive and increasing, which implies that the faster the rate at which the firm adjusts its capital stock, the higher the cost will be. Second, the larger the existing capital stock that the firms start with, the lower the cost of adjustment will be. Also note that the investment definitions in equation [13] and [14] are linear homogenous functions.

The Lagrangian of the firm’s problem is:

\[
\mathcal{L}_{j,t} = \sum_{s=1}^{\infty} (1 + r)^{-s} \left[ P_{Q,t,s} Q_{j,s} + P_s \left( \alpha_s \beta_{j,s} - \sum_i q_{i,j,s} q_{i,j,s} \right) - \sum_i P_{t,s} q_{i,j,s} - w_t L_{j,t} - P_{K,t,s} J_{K,j,s} - (1 - \nu_{j,s}) P_{H,t,s} J_{H,j,s} \right]
\]

39 For the class of linear homogenous (investment cost) functions, Hayashi (1982) shows that the Tobin’s marginal Q is equal to the average Q.
\[
\begin{align*}
+ \sum_{s,t} (1 + r)^{-t} \lambda_{K,j,s} \left[ (1 - \delta_{K,j}) K_{j,s} + I_{K,j,s} - K_{j,s+1} \right] \\
+ \sum_{s,t} (1 + r)^{-t} \lambda_{H,j,s} \left[ (1 - \delta_{H,j}) H_{j,s} + I_{H,j,s} - H_{j,s+1} \right]
\end{align*}
\]

where, \( \lambda_{K,j,s} \) and \( \lambda_{H,j,s} \) are the Lagrange multipliers (the co-state variables).

The Lagrangian yields eight necessary FOCs for the set of choice variables: \( L_{j,t}, q_{i,j,t}, I_{K,j,t}, I_{H,j,t}, K_{j,t+1}, H_{j,t+1}, \lambda_{K,j,t} \) and \( \lambda_{H,j,t} \). These are listed as equation FOC[O-7] – FOC[O-14] and are included in Appendix A for the interested readers. The interpretation of these equations is, however, provided below.

For the choice variables labour (\( L_{j,t} \)) and intermediate inputs (\( q_{i,j,t} \)), FOCs imply that the firm will hire labour and continue purchasing inputs until their marginal value products are equal to the wage rate and the price of intermediate input, respectively.

Note that, in these two optimal conditions, per unit output subsidy due to the OBA of permits are expressed as \( \alpha_i \beta_{j,t} \bar{P}_t \), where \( \beta_{j,t} \) is the exogenous emission intensity parameter that is used \textit{ex-ante} for allocation of the permits\(^{40}\). It is important to note that the assigned emissions intensity is a predetermined value; it is different from the emissions intensity actually achieved by the industry observed only \textit{ex-post}. However, given that the total number of emission permits is limited, we use the scale-back parameter \( \alpha_i \) (where, \( \alpha_i < 1 \)) that is determined endogenously in the model and serves to ensure that the allocated amounts of free permits do not exceed the total number of permits available at any given period of time.

As evident in equation [O-7] and [O-8], the permit price (\( \bar{P}_t \)) affects representative firm’s behaviour in two ways. First, it penalizes the use of the most polluting inputs and encourages firms to substitute among fossil fuels or to substitute physical capital for energy. Second, the higher the permit price, the higher the output subsidy received by the firms and the

---

\(^{40}\) Thus OBA allows firms to offset (at least partially) the negative effects of abatement efforts. It has an impact on firms' behaviour, in contrast to the grandfathering system in which permit assignment is based on past emissions.
lower the negative impact of the permit price on output. It follows that the negative impact of GHG abatement cost expenditures on firm’s output is reduced. Compared to the GFA scheme, therefore, under the OBA method firms have less incentive to reduce their output for curbing their emission intensity.

The FOCs with respect to other two choice variables, net investments in physical \((I_{K,j,t})\) capital and in R&D (i.e. the knowledge capital \((I_{H,j,t})\)), imply that the respective firm invests until its marginal cost of investment is equal to the shadow price of the added unit of capital.

With respect to two state variables, \(K_{j,t+1}\) and \(H_{j,t+1}\), the FOCs show that the required returns to investments in physical and knowledge capital, \(r\lambda_{K,j,t}\) and \(r\lambda_{H,j,t}\) respectively, are equal to their respective net marginal revenue products of the added unit of capital, \(R_{K,j,t}\) and \(R_{H,j,t}\), plus capital gains, \(\Delta\lambda_{K,j,t}\) and \(\Delta\lambda_{H,j,t}\), net of depreciation loss, \(\delta_{K,j}\lambda_{K,j,t+1}\) and \(\delta_{H,j}\lambda_{H,j,t+1}\).

Finally, FOCs with respect to the co-state variables, \(\lambda_{K,j,t}\) and \(\lambda_{H,j,t}\), respectively provide the motion equation for physical and knowledge capital.

The solution of the dynamic problem is the investment sequences for physical and knowledge capital that are dependent on the tax-adjusted Tobin’s \(Q\) and the parameters of the adjustment cost functions that takes the form: 

\[
\frac{Investment\ Flow\ at\ t}{Stock\ at\ t} = \alpha + \frac{1}{\beta}Q_t.
\]

Whereas, for the physical capital, it takes the form: 

\[
\frac{I_{K,j,t}}{K_{j,t}} = \frac{1}{\beta_{K,j}} \left[ \frac{\lambda_{K,j,t}}{P_{K,j,t}} - 1 \right],
\]

for the knowledge capital it is: 

\[
\frac{I_{H,j,t}}{H_{j,t}} = \frac{1}{\beta_{H,j}} \left[ \frac{\lambda_{H,j,t}}{(1-\nu_{j,t})P_{H,j,t}} - 1 \right] \quad \text{with} \quad Q_{K,j,t} = \frac{\lambda_{K,j,t}}{P_{K,j,t}} \quad \text{and} \quad Q_{H,j,t} = \frac{\lambda_{H,j,t}}{(1-\nu_{j,t})P_{H,j,t}}.
\]

Here, both \(Q_{*,j,t}\) are the ratio of the shadow price of capital \(\lambda_{*,j,t}\) and
the replacement cost of capital \( P_{k,j,t} \), adjusted for various taxes. Moreover, by Hayashi’s identity (1982), in a perfectly competitive market setting, for the class of linear homogenous investment functions Tobin’s marginal \( Q \) is also equal to the average \( Q \). The latter is the ratio of the value of the firm to its capital stock. Hence:

\[
Q_{k,j,t} = \frac{\lambda_{k,j,t}}{P_{k,j,t}} = \frac{V_{j,t}}{P_{k,j,t}K_{j,t}} \quad \text{for physical capital,}
\]

and \( Q_{h,j,t} = \frac{\lambda_{h,j,t}}{(1-\nu_{j,t})P_{h,j,t}} = \frac{V_{j,t}}{(1-\nu_{j,t})P_{h,j,t}H_{j,t}} \) for knowledge capital.

### 2.3.2.2 Allocation of Capital Goods

A cost-minimisation process governs a representative agent’s decision about the composition of the capital goods in a given period.

As indicated below in equation [17-1], for intra-temporal investment allocation in R&D (i.e. the knowledge capital), total investment expenditure \( \left( \sum_{j=1}^{s} P_{h,j,s}J_{h,j,s} \right) \) equals the sum of the spending on different investment goods \( \left( \sum_{j=1}^{s} P_{c,j,s}I_{c,j,s} \right)^{41} \). Similar condition holds for allocation of investment funds in physical capital as shown in equation [17-2]. Equation [18-1] shows that the investments, by destination sector, for R&D \( (J_{h,j,s}) \) are specified as the C-D aggregation of different investment commodities \( (I_{h,j,s}) \). Similarly, equation [18-2] signifies that the investments, by destination sector, for physical capital \( (J_{k,j,s}) \) are specified as the Leontief aggregation of different investment commodities \( (I_{k,j,s}) \). With these specifications, we postulate the investment allocation problems as follows.

For R&D investment:

\[
\text{Min} \sum_{j=1}^{s} P_{h,j,s}J_{h,j,s} = \sum_{i=1}^{s} P_{c,i,s}I_{c,i,s} \quad [17-1]
\]

---

41 Note that, for R&D investment, the sum is over \((i-1)\) sectors since by assumption R&D sector does not invest in itself. By construction, R&D sector is the aggregation of capitalized R&D investments in rest of the sectors in the economy.
Subject to \( J_{H,j,s} = \prod_{i=1}^{n}(I_{H,i,s})^{\beta_{HINV,i}} \), where \( \sum_{i=1}^{n} \beta_{HINV,i} = 1 \) \[18-1\]

for all \( s = t, t+1, t+2, \ldots \)

that, by optimality, yields:

\[
I_{H,j,s} = \frac{\left( \sum_{j=1}^{n} P_{H,j,s} J_{H,j,s} \right)}{P_{\text{c}},j,s}
\]

\[O-15\]

And for physical capital investment:

\[
\text{Min} \sum_{j} P_{K,j,s} J_{K,j,s} = \sum_{i} \beta_{KINV,i} P_{\text{c}},j,s (1+\tau_{t,j}) I_{K,i,s}
\]

\[17-2\]

Subject to \( J_{K,j,s} = \text{Min}\left[ I_{K,1,s}, I_{K,2,s}, \ldots, I_{K,n,s} \right] \beta_{K,1}, \beta_{K,2}, \ldots, \beta_{K,n} \) \[18-2\]

for all \( s = t, t+1, t+2, \ldots \)

that, by efficiency requirement, yields:

\[
I_{K,j,s} = \beta_{KINV,i} J_{K,j,s}
\]

\[O-17\]

here, \( \beta_{HINV,i} \) = share of good \( i \) in R&D investment demand; \( \beta_{KINV,i} \) = investment demand coefficient of physical capital for good \( i \); \( I_{H,j,s} \) = R&D investment demand of good \( i \) in sector \( j \); \( I_{K,j,s} \) = physical capital investment demand of good \( i \) in sector \( j \); \( \tau_{t,j} \) = investment tax on physical capital of good \( i \); and \( P_{\text{c}},i,s \) = price of good \( i \) net of investment tax.

The optimality conditions in equations [O-15], [O-16] and [O-17], [O-18] indicate that the demand for each investment commodity is a fixed proportion of investments by destination, and that the purchased price of capital stock is the weighted average (gross) prices of investment commodities.

### 2.3.2.3 Description of the Production Nests

In the model, production is disaggregated into 11 production nests. While 8 of them are governed by the CES production technology, 1 is governed by the C-D production technique. The remaining 2 production processes are governed by the Leontief technology. Figure 2 provides a schematic representation of representative firms’ production nests at various levels.
The structure of the top production nest essentially embodies ITC and traces the idea as presented in Goulder and Schneider (1999), and in Sue Wing (2003). It is argued that the relative price effects, induced by the effluent charge, bring significant intra-sectoral substitution and inter-sectoral reallocation of knowledge inputs across different sectors of the economy. This, in turn, leads to an increase in gross input substitutability on the supply side of the economy. To capture this substitution, as shown in equation [15] below, a CES aggregation of traditional output \((X_{j,t})\) and the knowledge input \((H_{j,t})\) is used at the top node. Given the assumption of perfectly competitive markets, the zero profit condition would imply that the net revenue \((\left( P_{Q,j,t} - \bar{P}_t e_{j,t} + \alpha \beta_j \bar{P}_t \right) Q_{j,t})\) equals the sum of the R&D investment cost \((\left( 1 - \nu_{j,t} \right) P_{H,j,t} J_{H,j,t} + P_{X,j,t} X_{j,t} )\). Therefore, we postulate a cost minimization problem at the top production nest as:

\[
\begin{align*}
\text{Min} \; \left( P_{Q,j,t} - \bar{P}_t e_{j,t} + \alpha \beta_j \bar{P}_t \right) Q_{j,t} &= \left( 1 - \nu_{j,t} \right) P_{H,j,t} J_{H,j,t} + P_{X,j,t} X_{j,t} \\
\text{Subject to,} \; Q_{j,t} &= \gamma_j \left( \bar{H}_{j,t} \right) \left[ \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \right]
\end{align*}
\]

[19] [same as 15]

for all \(s = t, t + 1, t + 2,...\)

The optimality conditions are:

\[
\begin{align*}
\frac{H_{j,t}}{X_{j,t}} &= \left[ \frac{\phi_j}{(1 - \phi_j)} \left( 1 - \nu_{j,t} \right) P_{H,j,t} J_{H,j,t} \right]^{\sigma_{0,j,t}} \\
Q_{j,t} &= \gamma_j \left( \bar{H}_{j,t} \right) \left[ \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \frac{\sigma_{0,j,t}^{-1}}{\sigma_{0,j,t}^{-1}} \right]
\end{align*}
\]

[O-19] [O-20]

Equation [O-19] indicates that the optimal knowledge capital and traditional output ratio is the function of the ratio of return on capital stock to the price of output and the parameters in the CES production function.
As depicted in Figure 2, in the remaining ten production nests, the same cost minimization principle is used to allocate the cost of the composite input bundle into its component input costs. A CES aggregator function is used for almost all the nests except for the refined petroleum, natural gas, and material intermediate inputs. Whereas, for the combustible petroleum input bundle a C-D aggregator function is used, for the natural gas and material intermediate inputs a Leontief specification is used.  

2.3.3 Trade

Domestic economy is linked to the world economy through trade and capital markets. By the assumption of a small-open economy, a domestic market faces perfect elastic world supply and demand curves. On the demand side, for any given commodity, domestic agents (firms, households, and government) face a composite of locally produced and imported goods. On the supply side, for any given domestic output, each firm can either sell its output domestically or export it to foreign markets. Therefore, there are two optimization problems that govern the allocation of commodities demanded and supplied between the modeled economy and the rest of the world – the optimal combination of composite goods, and the optimal allocation of domestic output. For capital market, the small-open economy balances its saving-investment gap (or the external balance) through external borrowing at a given world rate of interest.

2.3.3.1 Allocation of Domestic Output

Following Armington (1969), the composite output \( X_{jt} \) that is produced by the representative firm in each industry is allocated between domestic sales \( X^D_{jt} \) and export \( X^E_{jt} \). As

\[ \begin{align*}
  x_{jt} &= A_x^\alpha_i X_{jt} \left[ \frac{\alpha_i P_{X_{jt}}}{P_{jt}} \right]^\sigma_i \quad \text{and} \quad P_{X_{jt}} = A_x \sum_j \left[ \alpha_i P_{X_{jt}} \right]^\delta_i \\
  x_{jt} &= \alpha_i X_{jt} \quad \text{and} \quad P_{X_{jt}} = \sum_j \left[ \alpha_i P_{X_{jt}} \right] \\
  x_{jt} &= \frac{\alpha_i P_{X_{jt}} X_{jt}}{P_{jt} \sum j} \quad \text{and} \quad P_{X_{jt}} = \frac{1}{A_x} \prod_j \left( \frac{P_{X_{jt}}}{\alpha_i} \right)^{\sigma_i}
\end{align*} \]

42 For CES specification, a further manipulation of equation [O-19], [O-20], and [19] yields the equivalent expressions for quantity and index price:
expressed in equation [22], we use a Constant Elasticity of Transformation (CET) aggregator function for this purpose. The CET specification implies imperfect transformability between these two destinations and allows for cross-hauling behaviour as is empirically observed in the international trade data. In equation [21], total revenue \( (P_{X,j,s} X_{j,s}) \) is comprised of the value of domestic sales \( (P_{X,j,s} X_{j,s}) \) and the value of exports \( (P_{X,j,s} X_{j,s}) \). We postulate the optimal behaviour as a revenue maximization problem as:

\[
\text{Max } P_{X,j,s} X_{j,s} = P_{X,j,s}^D X_{j,s}^D + P_{X,j,s}^E X_{j,s}^E
\]

Subject to,

\[
X_{j,s} = A_{X,j} \left[ \eta_j \left( X_{j,s}^D \right)^{1+\sigma_{s,j}} + \left( 1 - \eta_j \right) \left( X_{j,s}^E \right)^{1+\sigma_{s,j}} \right]^{\frac{\sigma_{s,j}}{1+\sigma_{s,j}}}
\]

for all \( s = t, t+1, t+2, \ldots \)

This yields the optimality conditions:

\[
\frac{X_{j,s}^D}{X_{j,s}^E} = \left[ \frac{1-\eta_j}{\eta_j} \right]^{\frac{\sigma_{s,j}}{1+\sigma_{s,j}}} \text{ [O-21-1]}
\]

\[
X_{j,s} = A_{X,j} \left[ \eta_j \left( X_{j,s}^D \right)^{1+\sigma_{s,j}} + \left( 1 - \eta_j \right) \left( X_{j,s}^E \right)^{1+\sigma_{s,j}} \right]^{\frac{\sigma_{s,j}}{1+\sigma_{s,j}}} \text{ [O-22]}
\]

here, \( \sigma_{s,j} \) = elasticity of substitution for export; \( \eta_j \) = share parameter in the aggregator function; and \( A_{X,j} \) = shift parameter in the aggregator function.

Equation [O-21-1] implies that the optimal ratio of exports to domestic sale is a function of the ratio of export price to the domestic market price and the parameters in CET function.

A further manipulation of equation [O-21-1], [O-22] and [20] yields equivalent expressions for optimality conditions:

\[
P_{X,j,s} = \frac{1}{A_{X,j}} \left[ \left( \eta_j \right)^{-\sigma_{s,j}} \left( P_{X,j,s}^D \right)^{1+\sigma_{s,j}} + \left( 1 - \eta_j \right)^{-\sigma_{s,j}} \left( P_{X,j,s}^E \right)^{1+\sigma_{s,j}} \right]^{\frac{1}{1+\sigma_{s,j}}} \text{ [O-21-2]}
\]
In addition, since domestic firms are price-takers, export price of each commodity in local currency units (\(P^E_{X_{j,t}}\)) is the product of exchange rate (\(ER_t\)) and its world market price in foreign currency units (\(P^W_{X_{j,t}}\)):

\[P^E_{X_{j,t}} = ER_t P^W_{X_{j,t}}\]  \[22\]

### 2.3.3.2 The Optimal Combination of Composite Goods

For any commodity \(i\) that are available in the domestic markets, the composite commodity \((C_{i,t})\) is the combination of locally produced \((C^D_{i,t})\) quantity and imported \((C^I_{i,t})\) quantity. The behaviour of domestic agents (firms, households, and government) in choosing their optimal combination is governed by the Armington assumption as indicated in equation [24], which implies imperfect substitution between imported and local goods.

Let \(P_{C_{i,t}}\), \(P^D_{C_{i,t}}\), and \(P^I_{C_{i,t}}\) be the index price of composite goods, the price of locally produced quantity, and the price of imported quantity in local currency units, respectively. The budget constraint in equation [23] requires the total expenditure of domestic agents on composite goods (net of sales tax) be equal to the sum of spending on locally produced quantity and spending on imported quantity. The optimization problem of domestic agents can be depicted as an expenditure minimization problem as:

\[
\text{Min} \ P_{C_{i,s}} C_{i,s} = P^D_{C_{i,s}} C^D_{i,s} + P^I_{C_{i,s}} C^I_{i,s} \]

\[23\]

Subject to, \(C_{i,s} = A_{m,i} \left[ \kappa_i \left( C^D_{i,s} \right)^{\sigma_{m,i}^{-1}} \right] + (1 - \kappa_i) \left( C^I_{i,s} \right)^{\sigma_{m,i}^{-1}} \]

\[24\]

for all \(s = t, t + 1, t + 2,\ldots\)
This yields the optimality conditions:

\[
\frac{C_{i,t}^D}{C_{i,t}^{BM}} = \frac{\kappa_i}{(1 - \kappa_i)} \frac{P_{C_i,t}^{BM}}{P_{C_i,t}^D} \begin{pmatrix} \sigma_{m,i} \\ \sigma_{m,i}^{-1} \end{pmatrix} \quad [O-23-1]
\]

\[
C_{i,t} = A_{m,i} \left( \kappa_i \left( \frac{C_{i,t}^D}{C_{i,t}^{BM}} \right)^{\sigma_{m,i}^{-1}} + (1 - \kappa_i) \left( \frac{C_{i,t}^{BM}}{C_{i,t}^D} \right)^{\sigma_{m,i}^{-1}} \right) \quad [O-24]
\]

Here, \( \sigma_{m,i} \) = elasticity of substitution for import; \( \kappa_i \) = share parameter in the aggregator function; and \( A_{m,i} \) = shift parameter in the aggregator function.

Equation [O-23-1] implies that the optimal ratio of imported to locally produced quantities is a function of the ratio of their relative prices.

A further manipulation of equation [O-23-1], [O-24] and [23] yields equivalent expressions for optimality conditions:

\[
P_{C_i,t}^D = \frac{1}{A_{m,i}} \left[ \left( \kappa_i \right)^{\sigma_{m,i}} \left( P_{C_i,t}^D \right)^{1-\sigma_{m,i}} \right] \left( \left( \frac{P_{C_i,t}^{BM}}{P_{C_i,t}^D} \right)^{-\sigma_{m,i}} \right)^{1/(1-\sigma_{m,i})} \quad [O-23-2]
\]

\[
C_{i,t}^D = \left( A_{m,i} \right)^{\sigma_{m,i}^{-1}} C_{i,t} \left( \frac{\kappa_i P_{C_i,t}^{BM}}{P_{C_i,t}^D} \right)^{\sigma_{m,i}} \quad [O-23-3]
\]

\[
C_{i,t}^{BM} = \left( A_{m,i} \right)^{\sigma_{m,i}^{-1}} C_{i,t} \left( \frac{(1 - \kappa_i) P_{C_i,t}^{BM}}{P_{C_i,t}^D} \right)^{\sigma_{m,i}} \quad [O-23-4]
\]

In addition, since import tariff is imposed on the value of imports, the import price in local currency units (\( P_{C_i,t}^{BM} \)) is defined gross of tax as:

\[
P_{C_i,t}^{BM} = E R_i \left( 1 + \tau_{BM,i} \right) P_{C_i,t}^W \quad [25]
\]

2.3.3.3 Capital Market

Current account position (\( E R S_{F,i} \)) is defined in terms of domestic currency units. It is the sum of spending on imports (\( \sum_{i=1} I P_{C_i,t}^{BM} \)), interest rate payments on external debt (\( rB_r \)), and
government transfer to the rest of the world ($T_{G,t}^W$) minus the sum of export incomes ($\sum_{j-1} P_{X,j,t}^EX X_{j,t}^EX$). With this specification, a positive value of $S_{F,t}$ implies current account deficit or, in other words, domestic savings insufficiently met the required optimal investment:

$$ER_i S_{F,t} = \sum_{i-1} P_{C,i,t}^IM C_{i,t}^{IM} + rB_i + T_{G,t}^W - \sum_{j-1} P_{X,j,t}^EX X_{j,t}^EX$$ [26]

$$(1+n)B_{t+1} = (1+r)B_t + ER_i S_{F,t}$$ [27]

Domestic economy balances its current account ($ER_i S_{F,t}$) through borrowing or lending to the world capital market at a given world rate of interest ($r$). The current account position affects the external debt accumulation process in equation [27]. This specification implies that the domestic economy can access the world capital market without any constraint, and as such the external borrowing is a capital market clearing variable.

### 2.3.4 Government

Government behaviour is kept simple in the model. It earns revenues via various forms of taxation and spends its income on government consumption ($\sum_i P_{C,i,t} C_{i,t}$), transfers to the households ($T_{G,t}$) and the rest of the world ($T_{W,t}^W$), and on interest payments on foreign borrowing ($rER_i B_t$). Equation [28] identifies the components of tax revenues ($Y_{G,t}$) – direct tax, value-added tax, indirect tax on activities, import tax and sales tax. Tax and tariff rates are assumed to be exogenous and fixed throughout the model horizon. Equation [29] defines government saving as the sum of tax income minus the sum of government consumption expenditure, government transfers to households and to the rest of the world, and interest payment on foreign borrowing.

$$Y_{G,t} = \sum_f \tau_Q P_{Q,f,t} Q_{f,t} + \sum_f \sum_j \tau_{i,j} P_{C,j,t} q_{i,j,t} + \sum_i \tau_{C,i} P_{C,i,t} C_{i,t} + \tau_{L,t} W_j L_{j,t} + \tau_{I,t} I_{j,t} + \sum_{i-1} ER_i \tau_{IM,i} P_{C,i,t}^{IM} C_{i,t}^{IM} + \tau_{Y,t} Y_{H,t} + \tau_{D} Y_{D,t}$$ [28]

Note that the spending on R&D resources is not included in this equation since the spending is capitalized under R&D investment.
2.3.5 Within Period Equilibrium Conditions

For the economy to reach equilibrium, in each period, the following equilibrium conditions much be satisfied.

2.3.5.1 Commodity Market Equilibrium

In each period, aggregate demand for each commodity is composed of consumption demand \((C_{i,t})\), investment demands \((I_{H,i,t} + I_{K,i,t})\), government consumption demand \((G_{i,t})\), and intermediate inputs demand \(\sum_{j} q_{i,j,t}\). Commodity market equilibrium condition in equation [30] requires that the aggregate demand for each commodity be equal to its composite supply \((X_{j,t})\).

\[
X_{j,t} = C_{i,t} + I_{H,i,t} + I_{K,i,t} + G_{i,t} + \sum_{j} q_{i,j,t}
\]

Equation [31] also requires that, in equilibrium, demand for locally produced commodities \((C_{i,t}^D)\) be equal to the quantity of domestic output supplied to the domestic market \((X_{j,t}^D)\).

\[
C_{i,t}^D = X_{j,t}^D
\]

2.3.5.2 Labour Market Equilibrium

Equation [32] provides the equilibrium condition for labour market. The equilibrium requires that the sum of the labour demand across production sectors \((L_{D,j} = \sum_{j} L_{j,t})\) in each period be equal to the labour supply \((L_{S,j})\).

\[
L_{D,j} = L_{S,j}
\]
2.3.5.3 Internal-External Balance

Equation [33] provides economy-wide value of the firms \( (Z_t) \) that depends on the sum of the value of total physical \( \left( \sum_{j} \lambda_{k,j,t} K_{j,t} \right) \) and knowledge \( \left( \sum_{j=1}^{t} \lambda_{h,j,t} H_{j,t} \right) \) capital held by the economy in each period. Equilibrium in internal-external balance requires that, in each period, the sum of the assets held by the representative household \( (A_t) \) and the net foreign borrowing \( (B_t) \) must equate with the total value of the firms. This is given in equation [34].

\[
Z_t = \sum_{j} \lambda_{k,j,t} K_{j,t} + \sum_{j=1}^{t} \lambda_{h,j,t} H_{j,t} \tag{33}
\]


t = B_t + A_t \tag{34}

2.3.6 Steady-State Conditions

The model is calibrated to the base year data collected from 2004 Social Accounting Matrix of the Canadian economy and to the parameter values as specified in Table 2. The economy is assumed to be at the steady-state equilibrium in the base period. Steady-state conditions are also imposed at the terminal period. This ensures consistency with the economic theory and enables the model to generate a steady-state equilibrium solution with values that are consistent with the benchmark data.

2.3.6.1 Steady-State Household Budget Constraint

Equation [35] is the budget constraint from \( t = ss \) and onward. Given that, in the steady-state, asset holding is constant \( (A_{ss}) \), household’s total income is the sum of the total return (net of population growth) to asset and the disposable income. It follows that in equation [35], the steady-state value of consumption \( (P_{ss} C_{ss}) \) equals the steady-state total income.

\[
(r - n) A_{ss} + Y_{ss} = P_{ss} C_{ss} \tag{35}
\]

2.3.6.2 Steady-State Investments

For investment in R&D:

\[
I_{h,j,ss} = \left( n + \delta_{h,j} \right) H_{j,ss} \tag{36}
\]

For investment in physical capital:

\[
I_{k,j,ss} = \left( n + \delta_{k,j} \right) K_{j,ss} \tag{37}
\]
Equation [36] and [37] imply that steady-state requires each sectoral investment in R&D and physical capital to be just enough to recuperate the portion of capital stock lost due to depreciation, plus the amount of investment needed to keep the per-capita stock of knowledge and physical capital constant such that the stocks do not change over time. Therefore, in the steady-state, the value of capital stock of each production sector remains constant.

2.3.6.3 Steady-State Shadow Value of Knowledge and Physical Capital

In steady-state, shadow value of capital stops changing. Therefore, the steady-state condition in equation [38] and [39] are the market equilibrium conditions that require the returns on investments be equal to the market rate of interest plus the rate of respective depreciations.

\[
(\delta_{K,j} + r) \lambda_{K,j,ss} = \sum_{j=1} (P_{Q,j,ss} - \overline{P}_{ss} e_{j,ss} + \alpha_{ss} \beta_{j,ss} \overline{P}_{ss}) RR_{H,j,ss} + \sum_{j=1} (1 - v_{j,ss}) P_{H,ss} J_{H,j,ss} \tag{38}
\]

\[
(\delta_{H,j} + r) \lambda_{H,j,ss} = \sum_{j=1} P_{KET,j,ss} RR_{K,j,ss} + \sum_{j=1} P_{K,ss} J_{K,j,ss} \tag{39}
\]

where, \(RR_{K,j,ss}\) and \(RR_{H,j,ss}\) are as defined in equation [O-11] and [O-12], respectively.

2.3.6.4 Steady-State Condition for Internal-External Balance

Equation [40] provides the condition for balancing current account with the external debt accumulation process in the steady-state. As indicated in the equation below, given that the amount of debt is constant in the steady-state, the balance is restored by ensuring that foreign savings in terms of domestic currency (\(ER_{ss} S_{F,ss}\)) is just enough to keep per-capita domestic debt (\(nB_{ss}\)), net of interest payment (\(rB_{ss}\)), constant.

\[
(n - r) B_{ss} = ER_{ss} S_{F,ss} \tag{40}
\]

With the steady-state terminal conditions and the consistent steady-state values for the identified variables, we solve the model as a social planner’s problem for intra- and inter-temporal solutions from a reference run over a horizon of 150 periods. Since the solution in the reference run represents the steady-state equilibrium of the Canadian economy, advent of policy shocks (such as the OBA as oppose to the GFA of emission permits with and without an R&D subsidy) traces an alternative time path that deviates from the steady-state equilibrium of the
In the results section, we trace out these deviations for some variables of interest in terms of percentage change relative to their base-values.

2.4 Data, Calibration and Numerical Solution Strategy:

The model is calibrated on a horizon of 150 periods (years) using data sourced from 2004 social accounting matrix (SAM) developed by Statistics Canada. Detailed sectoral emissions data by fuel type are built using sectoral emissions data supplied by Statistics Canada and emissions forecasts contained in the Canadian Emissions Outlook produced by Natural Resources Canada\textsuperscript{44}. Following the general convention, the observed total labour supply, including technological progress, is normalized to one.

The rate of population growth is held at 2\%\textsuperscript{45}. To disentangle the dynamics resulting from the exogenous growth of the population from the dynamics induced by policy shocks, all real variables are expressed in labour efficiency units. Throughout the model horizon, these variables are held constant for the reference period in order to trace out the percentage change in these variables relative to the base period of 2004, where the assumption of the steady state is used. Tables 1a and 1b present the structure of the SAM for 2004 for the Canadian economy.

\textsuperscript{44} For further details consult the Analysis and Modelling Group (AMG) Report (1999).

\textsuperscript{45} This value is the implicit growth rate of GDP compatible with the forecasts of GHG emissions in 2010 that was developed in the AMG Report (1999).
Table 1a: Some characteristics of the social accounting matrix for Canada in the base period (percent shares)

<table>
<thead>
<tr>
<th>Industries</th>
<th>GDP at factor cost</th>
<th>GDP at market price</th>
<th>Household consumption</th>
<th>Exports</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2.1</td>
<td>0.9</td>
<td>1.2</td>
<td>2.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Mining</td>
<td>1.0</td>
<td>0.5</td>
<td>0.0</td>
<td>1.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Coal</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Crude Oil and Natural Gas</td>
<td>6.2</td>
<td>3.1</td>
<td>0.6</td>
<td>10.8</td>
<td>4.7</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>1.5</td>
<td>1.4</td>
<td>2.5</td>
<td>6.4</td>
<td>3.7</td>
</tr>
<tr>
<td>Cement</td>
<td>0.5</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>1.1</td>
<td>0.6</td>
<td>0.0</td>
<td>5.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Chemicals</td>
<td>2.2</td>
<td>0.3</td>
<td>1.8</td>
<td>7.9</td>
<td>11.0</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>10.4</td>
<td>12.5</td>
<td>17.9</td>
<td>45.3</td>
<td>56.6</td>
</tr>
<tr>
<td>Refineries</td>
<td>0.4</td>
<td>1.6</td>
<td>0.0</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Gas Pipelines</td>
<td>0.7</td>
<td>0.5</td>
<td>0.4</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>Electricity</td>
<td>2.1</td>
<td>1.1</td>
<td>1.9</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Transport Industry</td>
<td>3.8</td>
<td>2.8</td>
<td>2.8</td>
<td>4.8</td>
<td>2.1</td>
</tr>
<tr>
<td>Services</td>
<td>67.2</td>
<td>73.4</td>
<td>70.5</td>
<td>13.5</td>
<td>13.3</td>
</tr>
<tr>
<td>R&amp;D Industry</td>
<td>0.8</td>
<td>1.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

Source: Statistics Canada, data from various sources and author’s calculations
Table 1b: Some characteristics of the social accounting matrix for Canada in the base period (percent shares)

<table>
<thead>
<tr>
<th>Industries</th>
<th>Exports as % of output</th>
<th>Domestic sales as % of output</th>
<th>Imports as % of total domestic demand</th>
<th>Domestic goods as % of total domestic demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>17.4</td>
<td>82.6</td>
<td>14.0</td>
<td>86.0</td>
</tr>
<tr>
<td>Mining</td>
<td>51.4</td>
<td>48.6</td>
<td>27.8</td>
<td>72.2</td>
</tr>
<tr>
<td>Coal</td>
<td>60.4</td>
<td>39.6</td>
<td>68.2</td>
<td>31.8</td>
</tr>
<tr>
<td>Crude Oil</td>
<td>47.3</td>
<td>52.7</td>
<td>25.9</td>
<td>74.1</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>57.3</td>
<td>42.7</td>
<td>34.4</td>
<td>65.6</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>47.7</td>
<td>52.3</td>
<td>31.3</td>
<td>68.7</td>
</tr>
<tr>
<td>Cement</td>
<td>21.5</td>
<td>78.5</td>
<td>30.1</td>
<td>69.9</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>53.8</td>
<td>46.2</td>
<td>46.1</td>
<td>53.9</td>
</tr>
<tr>
<td>Chemicals</td>
<td>48.9</td>
<td>51.1</td>
<td>54.5</td>
<td>45.5</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>58.3</td>
<td>41.7</td>
<td>61.0</td>
<td>39.0</td>
</tr>
<tr>
<td>Gasoline</td>
<td>15.0</td>
<td>85.0</td>
<td>8.8</td>
<td>91.2</td>
</tr>
<tr>
<td>Diesel</td>
<td>21.8</td>
<td>78.2</td>
<td>13.2</td>
<td>86.8</td>
</tr>
<tr>
<td>Liquid petroleum products</td>
<td>52.7</td>
<td>47.3</td>
<td>37.8</td>
<td>62.2</td>
</tr>
<tr>
<td>Other refined petroleum products</td>
<td>23.7</td>
<td>76.3</td>
<td>14.5</td>
<td>85.5</td>
</tr>
<tr>
<td>Gas Pipelines</td>
<td>29.6</td>
<td>70.4</td>
<td>4.8</td>
<td>95.2</td>
</tr>
<tr>
<td>Electricity</td>
<td>6.1</td>
<td>93.9</td>
<td>3.6</td>
<td>96.4</td>
</tr>
<tr>
<td>Transport Industry</td>
<td>26.1</td>
<td>73.9</td>
<td>12.2</td>
<td>87.8</td>
</tr>
<tr>
<td>Services</td>
<td>4.9</td>
<td>95.1</td>
<td>4.3</td>
<td>95.7</td>
</tr>
<tr>
<td>R&amp;D Industry</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Source: Statistics Canada, from other sources and author’s calculations

Using data from SAM and the emissions table, we calculate emission factors for different fossil fuels and the emission intensities by industry. Table 2 presents the values used for various behavioural parameters. These values, which are borrowed from previous studies on Canada, such as Ab Iorwerth et al. (2000) and Wigle (2001), are not very different from the values used in many other general equilibrium models of Canada or the United States. Table 3 presents the sectoral distribution of emissions in 2010.
Table 2: Values for behavioural parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution elasticity</td>
<td></td>
</tr>
<tr>
<td>between knowledge capital and aggregate input</td>
<td>1.5</td>
</tr>
<tr>
<td>between value added-energy and intermediate inputs</td>
<td>0.2-0.7</td>
</tr>
<tr>
<td>between labour &amp; capital-energy</td>
<td>1.0</td>
</tr>
<tr>
<td>between capital &amp; energy</td>
<td>0.25-0.8</td>
</tr>
<tr>
<td>between electricity &amp; fossil energy</td>
<td>0.5-0.7</td>
</tr>
<tr>
<td>among stationary fossil fuels</td>
<td>0.4-0.7</td>
</tr>
<tr>
<td>between other intermediate inputs &amp; mobile fossil fuels</td>
<td>0.2-0.8</td>
</tr>
<tr>
<td>among mobile fossil fuels</td>
<td>1.0</td>
</tr>
<tr>
<td>between imports and domestic goods</td>
<td>0.75-2.5</td>
</tr>
<tr>
<td>between exports and domestic goods</td>
<td>2.0</td>
</tr>
<tr>
<td>among same industry products*</td>
<td>2.0</td>
</tr>
<tr>
<td>Other parameters</td>
<td></td>
</tr>
<tr>
<td>Physical capital adjustment cost parameter</td>
<td>3.0</td>
</tr>
<tr>
<td>Knowledge capital adjustment cost parameter</td>
<td>2.0</td>
</tr>
<tr>
<td>Depreciation rate for physical capital</td>
<td>0.06</td>
</tr>
<tr>
<td>Depreciation rate for knowledge capital</td>
<td>0.10</td>
</tr>
<tr>
<td>Population growth rate** (%)</td>
<td>2.0</td>
</tr>
<tr>
<td>World interest rate (%)</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Sources: Various studies
* For multi-products industries
** With Harrod-neutral technological progress

The calibration of the model involves using the SAM, exogenous parameters, first-order conditions, and the steady state conditions to recover other parameters in the behavioural functions and the values of non-observed variables to reproduce the reference situation. To this end, we use the calibration procedures frequently employed in static and dynamic general equilibrium models as explained in Dissou (2002), Keuschnigg and Kohler (1994), and Mansur and Whalley (1984).
Table 3: Sectoral emission shares (%) in the base period

<table>
<thead>
<tr>
<th>Industries</th>
<th>Shares in total emissions (%)</th>
<th>Shares in total industrial emissions (%)</th>
<th>Industry group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2.5</td>
<td>3.0</td>
<td>Non-LFE</td>
</tr>
<tr>
<td>Mining</td>
<td>1.0</td>
<td>1.3</td>
<td>LFE</td>
</tr>
<tr>
<td>Coal</td>
<td>0.2</td>
<td>0.2</td>
<td>LFE</td>
</tr>
<tr>
<td>Crude Oil and Natural Gas</td>
<td>13.2</td>
<td>16.2</td>
<td>LFE</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>1.7</td>
<td>2.1</td>
<td>LFE</td>
</tr>
<tr>
<td>Cement</td>
<td>2.9</td>
<td>3.5</td>
<td>LFE</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>4.1</td>
<td>5.0</td>
<td>LFE</td>
</tr>
<tr>
<td>Chemicals</td>
<td>3.1</td>
<td>3.8</td>
<td>LFE</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>2.3</td>
<td>2.8</td>
<td>LFE</td>
</tr>
<tr>
<td>Refineries</td>
<td>4.7</td>
<td>5.8</td>
<td>LFE</td>
</tr>
<tr>
<td>Gas Pipelines</td>
<td>1.6</td>
<td>1.9</td>
<td>LFE</td>
</tr>
<tr>
<td>Electricity</td>
<td>20.2</td>
<td>24.8</td>
<td>LFE</td>
</tr>
<tr>
<td>Transport Industry**</td>
<td>8.5</td>
<td>10.4</td>
<td>Non-LFE</td>
</tr>
<tr>
<td>Services</td>
<td>15.4</td>
<td>18.9</td>
<td>Non-LFE</td>
</tr>
<tr>
<td>R&amp;D Industry</td>
<td>0.0</td>
<td>0.0</td>
<td>Non-LFE</td>
</tr>
<tr>
<td>Household</td>
<td>18.7</td>
<td>-</td>
<td>Non-LFE</td>
</tr>
<tr>
<td>Total</td>
<td>100.0</td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>

Source: Statistics Canada, Natural Resources Canada, Domestic Emissions Trading Working Group and author’s calculations

*Industrial emissions do not include those related to transportation activities, i.e., from mobile sources, like gasoline, diesel.

** Emissions in the transport industry are not identical to emissions related to transportation activities that are carried out in several other industries. Transport industry has transportation as its main activity.

The model is solved numerically by treating it as a “two-point boundary problem” in which the initial conditions are set for the state variables, and the terminal conditions are imposed on the jumping variables to avoid the Bang-Bang problem. We use the “Extended Path” method suggested by Gagnon (1990) to solve the non-linear system that contains difference equations.

2.5 Simulation:

2.5.1 Description of the Simulations

Four policy experiments are considered: the impacts of using a pure output-based allocation (OBA) of emission permits are compared with those of three other systems: (i) the
grandfathered allocation of emission permits (GFA), (ii) the auctioning of permits with the proceeds being used to reduce labour (i.e. payroll) tax (RPT), and (iii) a combination of OBA and R&D investment subsidy (O-R&D). In GFA, permits are freely distributed to firms according to their historical emissions. No public revenue is raised in this tradeable permit system. As the allocation method has no behavioural impact on firms, it is equivalent to a wealth transfer to firm owners, who are assumed to be the households. In the O-R&D simulation, a portion of the permit proceeds$^{46}$ corresponding to the auctioning of Canada’s total emission rights are used to pay for the subsidy provided to the R&D investors. The remainder of the permit proceeds is distributed among the firms in the form of emission permits using OBA policy. In the trading system with OBA, emission credits corresponding to the firm’s share in Canada’s total emission rents are redistributed to them according to their output and their assigned emissions intensity$^{47}$. All domestic economic agents are required to hold permits for emissions sourced from them.

In all simulations, a constant permit price policy is considered for the entire model horizon. Simulations are run keeping the permit price constant at $40 per ton of CO$_2$ while letting the level of emissions to be determined endogenously.

For a good understanding of the differences among the simulations, the following discussions are focused on the basic mechanisms at play in the simulation with GFA (the most commonly cited allocation scheme in the literature), and on how results change for the other three policy simulations.

2.5.2 Results

Unless otherwise mentioned, all the results presented in this paper are expressed as a percentage deviation from the reference situation.

---

$^{46}$ For the O-R&D policy simulation, we hold total R&D subsidy to be equal to 5% of the total permit proceeds. The choice of 5% subsidy was motivated by the desire to keep the overall R&D subsidy amount to be meaningful relative to the size of the capitalized R&D investment sector of the economy.

$^{47}$ Since households also emit GHG related to their direct consumption of energy products, firms do not receive the total amount of Canada’s emissions revenues. They only receive a portion corresponding to their share in total BAU emissions as indicated in Table 3.
2.5.2.1 Reducing GHG Emissions with Grandfathered Allocation of Emission Permits (GFA)

The results of this simulation are contained in Tables 4-9. The permit price affects the prices of polluting goods and, thus, all relative prices in the economy. It has both direct and indirect effects, characterized by changes in production costs, composition of aggregate demand, and household welfare.

2.5.2.1.1 Aggregate Impacts

Referring to Table 4, household welfare decreases as a result of the carbon abatement policy with GFA even though shareholders receive emission rents. This result is in line with Dissou (2006), and Fullerton and Metcalf (2001). The estimated measure of welfare loss over their entire lifetime is –0.46% 48. Following the implementation of GFA policy, in the 20th period real GDP declines by 1.95%. Total level of employment falls by 0.66%, and the real exchange rate depreciates by 0.5%. Household consumption in real terms falls by 1.42% leading to the loss in overall welfare. The biggest fall is registered in real investment in physical capital by 4.27%. R&D investment, on the other hand, falls marginally by 0.08%. Real imports observe a decline by 2.67%, while real exports fall by 3.24%.

48 Note that the index of welfare change does not take into account the environmental improvement due to the reduction in GHG emissions. Moreover, it is not directly comparable with the one reported in static models. The measure is an indication of the percentage change in the household consumption stream in the BAU situation that would yield the same lifetime utility level as the one with the policy change. See Dissou (2002) and Goulder and Eichengreen (1992) for details on the computation of this welfare measure.
Table 4: Impacts on selected aggregate variables in the 20th period from various simulations (percentage deviation from BAU unless otherwise mentioned)

<table>
<thead>
<tr>
<th></th>
<th>GFA</th>
<th>OBA</th>
<th>RPT</th>
<th>O-R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare change</td>
<td>-0.46</td>
<td>-0.29</td>
<td>-0.40</td>
<td>-0.28</td>
</tr>
<tr>
<td>GDP at market prices</td>
<td>-1.95</td>
<td>-0.66</td>
<td>-1.15</td>
<td>-0.51</td>
</tr>
<tr>
<td>Employment</td>
<td>-0.66</td>
<td>-0.10</td>
<td>0.14</td>
<td>-0.09</td>
</tr>
<tr>
<td>Household aggregate real consumption</td>
<td>-1.42</td>
<td>-0.59</td>
<td>-0.55</td>
<td>-0.55</td>
</tr>
<tr>
<td>Real investment (physical capital)</td>
<td>-4.27</td>
<td>-1.33</td>
<td>-3.34</td>
<td>-1.38</td>
</tr>
<tr>
<td>Real investment (R&amp;D)</td>
<td>-0.08</td>
<td>0.21</td>
<td>0.88</td>
<td>8.91</td>
</tr>
<tr>
<td>Total real imports</td>
<td>-2.67</td>
<td>-1.17</td>
<td>-1.84</td>
<td>-1.24</td>
</tr>
<tr>
<td>Total real exports</td>
<td>-3.24</td>
<td>-1.28</td>
<td>-2.18</td>
<td>-1.31</td>
</tr>
<tr>
<td>Real exchange rate*</td>
<td>0.52</td>
<td>0.12</td>
<td>0.57</td>
<td>0.10</td>
</tr>
<tr>
<td>Aggregate emissions</td>
<td>-35.83</td>
<td>-31.84</td>
<td>-35.27</td>
<td>-31.91</td>
</tr>
<tr>
<td>Permit price</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Source: Simulation results
* A positive number means depreciation
GFA: Grandfathered allocation of permits
OBA: Output-based allocation of permits
RPT: Payroll tax reduction with permit proceeds
O-R&D: OBA with subsidy in R&D investments

Pronounced negative impact on household consumption and on physical investment compared to other GDP components is not surprising. Since households and firms have forward-looking behaviour, consumption and investment do not depend solely on contemporary variables; they are also affected by the state of the economy in future periods. The requirement to hold costly emission permits puts an upward pressure on the cost of production. Under the circumstances, absent any incentive on production, firms respond by reducing the overall level of production resulting in a low demand for labour, and investment in physical capital. Since knowledge capital facilitates the development of carbon-saving technological innovations49, and thus serves as an imperfect substitute to the traditional production process, investment in R&D, however, does not fall drastically. Low production also translates into low level of exports and low household aggregate consumption leading to a decrease in the overall welfare.

49 In this context, innovative carbon-saving technological process refers to new production processes that are able to produce same level of output with less amount of CO₂ emissions.
It is interesting to note that quantitatively these aggregate results differ substantially from those found in other studies on carbon abatement policies in Canada like Dissou (2006), and Dissou and Robichaud (2003). This occurs primarily due to our modeling framework that, unlike these other models, allows for the endogenous technological change to occur through investments in R&D (i.e. investments that build the knowledge capital). Even without OBA and/or R&D investment incentives (e.g. R&D subsidy), accumulation of knowledge capital paves the way to innovative new carbon-saving technologies. This, in turn, assists firms to lower their cost of production and ultimately improves the overall welfare. Consequently, we find improved welfare implications for all the policy scenarios.\textsuperscript{50}

These aggregate results, which summarize the impact of the policy change on Canadian economy as a whole, however do not provide information on the variety of sectoral adjustments. The sectoral impacts that shed light on the equity aspects are discussed below, where intuitive explanations are provided and the main transmission mechanisms are highlighted.

\textbf{2.5.2.1.2 Sectoral Impacts}

Since not all industries are affected in similar ways, a distinction between different categories of industries is made. Further, we distinguish energy- or carbon-intensive industries as well as differentiate fossil energy-producers from the others.\textsuperscript{51}

Tables 5 and 6 present the sectoral impacts of the abatement policies on sectoral output, employment and investment.

\textsuperscript{50} For example, the GDP cost of the carbon abatement policy under GFA in Dissou (2006) is 2.9\% in 2010 as oppose to 0.46\% in our model.

\textsuperscript{51} The last column in Table 9 presents the GDP share of each category in the baseline situation.
Table 5a: Sectoral impacts on value-added and employment in the 20th period from various simulations (percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Industries</th>
<th>Value added</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GFA</td>
<td>OBA</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.43</td>
<td>0.14</td>
</tr>
<tr>
<td>Mining</td>
<td>-6.22</td>
<td>1.01</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>-4.81</td>
<td>-1.05</td>
</tr>
<tr>
<td>Cement</td>
<td>-5.78</td>
<td>0.32</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>-7.94</td>
<td>0.25</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-5.81</td>
<td>-1.53</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>1.51</td>
<td>-0.03</td>
</tr>
<tr>
<td>Refineries</td>
<td>-17.09</td>
<td>-12.44</td>
</tr>
<tr>
<td>Electricity</td>
<td>-9.21</td>
<td>0.14</td>
</tr>
<tr>
<td>Transport Industry</td>
<td>-2.53</td>
<td>0.03</td>
</tr>
<tr>
<td>Services</td>
<td>-0.96</td>
<td>-0.30</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.33</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Source: Simulation results

Figures in Table 5a suggest, in the 20th period following the implementation of GFA policy, the impact on sectoral GDP at factor cost varies between -40% for the Coal industry, and +1.5% for the “Other Manufacturing” industry. These results are qualitatively similar to the results found in Dissou (2006). In general, the energy-producing large carbon emitting industries are affected the most, followed by the non-energy producing emitting industries. Within the latter group of industries, pulp and paper is the least affected one. Interestingly, sectoral GDP in the “Other Manufacturing” industry increases as a consequence of the change in the demand composition. This is because users shift away from polluting and energy-intensive commodities to less polluting goods.

Table 7 provides a snapshot of the emissions reduction efforts across the industries. It is important to note that, under a constant permit price (held at $40 per ton of CO2) policy, emissions at the individual industry level vary depending on the mode of emission permit
allocation policy in the form of equivalent amount of recycled emission rents. As GFA does not have any behavioural implication for the producers, with increasing production cost, not surprisingly producers respond by cutting down the production levels. Consequently emissions also fall. Evidently, among all the carbon emitting industries (CEI), energy producing industries (i.e. the energy producing CEI industries) generally suffer the most even though the highest reduction is registered for Cement industry with an emissions reduction of 61%.

Table 5b: Sectoral impacts on investments in the 20th period from various simulations
(percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Industries</th>
<th>Physical Capital Investment</th>
<th>R&amp;D Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GFA</td>
<td>OBA</td>
</tr>
<tr>
<td>Agriculture</td>
<td>-0.01</td>
<td>0.25</td>
</tr>
<tr>
<td>Mining</td>
<td>-7.23</td>
<td>1.69</td>
</tr>
<tr>
<td>Coal</td>
<td>-45.94</td>
<td>-36.67</td>
</tr>
<tr>
<td>Crude Oil and Natural Gas</td>
<td>-17.45</td>
<td>-8.99</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>-4.29</td>
<td>-0.20</td>
</tr>
<tr>
<td>Cement</td>
<td>-5.07</td>
<td>1.80</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>-7.02</td>
<td>2.32</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-5.16</td>
<td>-0.43</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>0.95</td>
<td>0.04</td>
</tr>
<tr>
<td>Refineries</td>
<td>-11.82</td>
<td>-7.22</td>
</tr>
<tr>
<td>Electricity</td>
<td>-5.81</td>
<td>4.55</td>
</tr>
<tr>
<td>Transport Industry</td>
<td>-2.95</td>
<td>0.32</td>
</tr>
<tr>
<td>Services</td>
<td>-1.81</td>
<td>-0.38</td>
</tr>
</tbody>
</table>

Source: Simulation results

The decline in output observed in some industries is the consequence of firms’ abatement efforts. *Ceteris paribus*, the requirement to hold costly permits increases the cost of using fossil fuel inputs and thus the output prices. For example, the user price of fossil energy products increases as shown in Table 6c, where coal experiences the largest increase (639.6%) followed by refined petroleum products (67.2%). Changes in relative prices induce some substitution effects among inputs to minimize production cost. In the case of fossil fuel inputs, firms substitute to the least polluting input for the others. Moreover, the decrease in the use of

---

52 The CEI entails large final emitters (LFE) that include all industries except Agriculture, Transport and Services.
53 The energy producing CEI is a subset of industries that includes the following industry sectors: Coal, Crude Oil and Natural Gas, Refineries, Gas Pipelines and Electricity.
fossil fuels in production process lowers the marginal product of labour and puts a downward pressure on real wage. As a consequence, labour supply falls.

Table 6a: Sectoral impacts on supply in various markets in the 20th period from various simulations (percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Products</th>
<th>Total supply</th>
<th></th>
<th></th>
<th></th>
<th>Exports</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GFA</td>
<td>OBA</td>
<td>RPT</td>
<td>O-R&amp;D</td>
<td>GFA</td>
<td>OBA</td>
<td>RPT</td>
<td>O-R&amp;D</td>
</tr>
<tr>
<td>Agriculture</td>
<td>-0.40</td>
<td>-0.29</td>
<td>0.67</td>
<td>-0.41</td>
<td>-0.88</td>
<td>-0.70</td>
<td>0.26</td>
<td>-0.87</td>
</tr>
<tr>
<td>Mining</td>
<td>-6.44</td>
<td>0.92</td>
<td>-5.41</td>
<td>0.64</td>
<td>-6.86</td>
<td>0.76</td>
<td>-5.82</td>
<td>0.44</td>
</tr>
<tr>
<td>Coal</td>
<td>-41.34</td>
<td>-33.16</td>
<td>-40.74</td>
<td>-33.33</td>
<td>-41.20</td>
<td>-32.13</td>
<td>-40.59</td>
<td>-32.32</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>-4.48</td>
<td>-0.64</td>
<td>-3.54</td>
<td>-0.65</td>
<td>-5.63</td>
<td>-0.78</td>
<td>-4.65</td>
<td>-0.79</td>
</tr>
<tr>
<td>Cement</td>
<td>-5.13</td>
<td>1.49</td>
<td>-4.26</td>
<td>1.38</td>
<td>-9.26</td>
<td>3.46</td>
<td>-8.35</td>
<td>3.27</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>-7.42</td>
<td>1.17</td>
<td>-6.36</td>
<td>0.95</td>
<td>-10.04</td>
<td>1.56</td>
<td>-8.97</td>
<td>1.32</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-5.39</td>
<td>-1.02</td>
<td>-4.40</td>
<td>-0.95</td>
<td>-6.56</td>
<td>-1.22</td>
<td>-5.55</td>
<td>-1.09</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>1.21</td>
<td>0.03</td>
<td>2.36</td>
<td>0.05</td>
<td>2.09</td>
<td>0.16</td>
<td>3.31</td>
<td>0.21</td>
</tr>
<tr>
<td>Other refined petroleum</td>
<td>-18.04</td>
<td>-12.64</td>
<td>-17.30</td>
<td>-12.70</td>
<td>-18.98</td>
<td>-10.15</td>
<td>-18.22</td>
<td>-10.22</td>
</tr>
<tr>
<td>Pipelines</td>
<td>-14.96</td>
<td>-11.31</td>
<td>-14.21</td>
<td>-11.38</td>
<td>-17.08</td>
<td>-11.32</td>
<td>-16.30</td>
<td>-11.45</td>
</tr>
<tr>
<td>Electricity</td>
<td>-8.40</td>
<td>1.11</td>
<td>-7.58</td>
<td>1.06</td>
<td>-20.44</td>
<td>2.43</td>
<td>-19.67</td>
<td>2.32</td>
</tr>
<tr>
<td>Transport</td>
<td>-3.44</td>
<td>-0.49</td>
<td>-2.56</td>
<td>-0.52</td>
<td>-5.59</td>
<td>-1.00</td>
<td>-4.65</td>
<td>-1.10</td>
</tr>
<tr>
<td>Services</td>
<td>-1.22</td>
<td>-0.36</td>
<td>-0.51</td>
<td>-0.34</td>
<td>1.00</td>
<td>-0.09</td>
<td>1.84</td>
<td>-0.12</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>-0.08</td>
<td>0.21</td>
<td>0.88</td>
<td>8.91</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 6b: Sectoral impacts on total demand, imports, and domestic demand in the 20\textsuperscript{th} period from various simulations (percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Products</th>
<th>Total demand</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GFA</td>
<td>OBA</td>
</tr>
<tr>
<td>Agriculture</td>
<td>-0.25</td>
<td>-0.16</td>
</tr>
<tr>
<td>Mining</td>
<td>-5.75</td>
<td>1.19</td>
</tr>
<tr>
<td>Coal</td>
<td>-41.67</td>
<td>-35.55</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>-2.59</td>
<td>-0.41</td>
</tr>
<tr>
<td>Cement</td>
<td>-2.31</td>
<td>0.21</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>-2.13</td>
<td>0.39</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-2.99</td>
<td>-0.63</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>-1.36</td>
<td>-0.38</td>
</tr>
<tr>
<td>Gasoline</td>
<td>-11.53</td>
<td>-8.82</td>
</tr>
<tr>
<td>Diesel</td>
<td>-11.44</td>
<td>-7.84</td>
</tr>
<tr>
<td>Liquid petroleum products</td>
<td>-17.94</td>
<td>-14.43</td>
</tr>
<tr>
<td>Electricity</td>
<td>-7.15</td>
<td>0.97</td>
</tr>
<tr>
<td>Transport</td>
<td>-2.55</td>
<td>-0.27</td>
</tr>
<tr>
<td>Services</td>
<td>-1.38</td>
<td>-0.38</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>-0.08</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 6c: Sectoral impacts on consumption, and consumer prices in the 20\textsuperscript{th} period from various simulations (percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Products</th>
<th>Consumption GFA</th>
<th>Consumption OBA</th>
<th>Consumption RPT</th>
<th>Consumption O-R&amp;D</th>
<th>Consumer Prices GFA</th>
<th>Consumer Prices OBA</th>
<th>Consumer Prices RPT</th>
<th>Consumer Prices O-R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-2.39</td>
<td>-0.65</td>
<td>-1.54</td>
<td>-0.63</td>
<td>0.25</td>
<td>0.21</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>Mining</td>
<td>-2.51</td>
<td>-0.51</td>
<td>-1.70</td>
<td>-0.50</td>
<td>0.34</td>
<td>0.12</td>
<td>0.32</td>
<td>0.15</td>
</tr>
<tr>
<td>Coal</td>
<td>-90.38</td>
<td>-90.51</td>
<td>-90.30</td>
<td>-90.50</td>
<td>639.61</td>
<td>639.08</td>
<td>639.60</td>
<td>639.10</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>-15.23</td>
<td>-15.06</td>
<td>-14.50</td>
<td>-15.05</td>
<td>25.33</td>
<td>23.83</td>
<td>25.30</td>
<td>23.86</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>-3.17</td>
<td>-0.47</td>
<td>-2.33</td>
<td>-0.41</td>
<td>0.79</td>
<td>0.09</td>
<td>0.75</td>
<td>0.09</td>
</tr>
<tr>
<td>Cement</td>
<td>-4.86</td>
<td>0.96</td>
<td>-4.03</td>
<td>0.97</td>
<td>1.98</td>
<td>-0.85</td>
<td>1.94</td>
<td>-0.83</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>-4.38</td>
<td>0.00</td>
<td>-3.58</td>
<td>0.04</td>
<td>1.64</td>
<td>-0.23</td>
<td>1.62</td>
<td>-0.21</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-2.83</td>
<td>-0.46</td>
<td>-2.01</td>
<td>-0.37</td>
<td>0.55</td>
<td>0.09</td>
<td>0.53</td>
<td>0.06</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>-1.42</td>
<td>-0.24</td>
<td>-0.57</td>
<td>-0.16</td>
<td>-0.41</td>
<td>-0.06</td>
<td>-0.44</td>
<td>-0.08</td>
</tr>
<tr>
<td>Gasoline</td>
<td>-13.07</td>
<td>-10.26</td>
<td>-12.31</td>
<td>-10.24</td>
<td>17.86</td>
<td>14.74</td>
<td>17.83</td>
<td>14.79</td>
</tr>
<tr>
<td>Diesel</td>
<td>-3.99</td>
<td>-1.22</td>
<td>-3.16</td>
<td>-1.18</td>
<td>6.72</td>
<td>4.25</td>
<td>6.70</td>
<td>4.27</td>
</tr>
<tr>
<td>Other refined petroleum</td>
<td>-42.78</td>
<td>-42.69</td>
<td>-42.30</td>
<td>-42.66</td>
<td>67.19</td>
<td>64.97</td>
<td>67.17</td>
<td>64.98</td>
</tr>
<tr>
<td>Pipelines</td>
<td>-15.23</td>
<td>-15.06</td>
<td>-14.50</td>
<td>-15.05</td>
<td>1.71</td>
<td>0.01</td>
<td>1.67</td>
<td>0.05</td>
</tr>
<tr>
<td>Electricity</td>
<td>-5.11</td>
<td>3.45</td>
<td>-4.27</td>
<td>3.48</td>
<td>7.44</td>
<td>-0.66</td>
<td>7.40</td>
<td>-0.64</td>
</tr>
<tr>
<td>Transport</td>
<td>-3.96</td>
<td>-0.79</td>
<td>-3.10</td>
<td>-0.80</td>
<td>1.34</td>
<td>0.31</td>
<td>1.29</td>
<td>0.35</td>
</tr>
<tr>
<td>Services</td>
<td>-0.36</td>
<td>-0.13</td>
<td>0.54</td>
<td>-0.10</td>
<td>-1.12</td>
<td>-0.14</td>
<td>-1.17</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

Source: Simulation results
Table 7: Sectoral impacts on emission in the 20th period from various simulations (percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Industries</th>
<th>GFA</th>
<th>OBA</th>
<th>RPT</th>
<th>O-R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-11.17</td>
<td>-9.99</td>
<td>-10.21</td>
<td>-10.11</td>
</tr>
<tr>
<td>Coal</td>
<td>-56.65</td>
<td>-49.61</td>
<td>-56.21</td>
<td>-49.75</td>
</tr>
<tr>
<td>Crude Oil and Natural Gas</td>
<td>-51.39</td>
<td>-46.56</td>
<td>-50.94</td>
<td>-46.65</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>-23.57</td>
<td>-19.87</td>
<td>-22.82</td>
<td>-19.96</td>
</tr>
<tr>
<td>Cement</td>
<td>-61.31</td>
<td>-58.24</td>
<td>-60.96</td>
<td>-58.30</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>-50.87</td>
<td>-45.81</td>
<td>-50.30</td>
<td>-45.96</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-12.64</td>
<td>-7.43</td>
<td>-11.73</td>
<td>-7.57</td>
</tr>
<tr>
<td>Other manufact. products</td>
<td>-9.53</td>
<td>-9.64</td>
<td>-8.51</td>
<td>-9.77</td>
</tr>
<tr>
<td>Refineries</td>
<td>-28.39</td>
<td>-23.92</td>
<td>-27.77</td>
<td>-23.96</td>
</tr>
<tr>
<td>Gas Pipelines</td>
<td>-41.13</td>
<td>-37.76</td>
<td>-40.61</td>
<td>-37.81</td>
</tr>
<tr>
<td>Electricity</td>
<td>-53.91</td>
<td>-48.60</td>
<td>-53.51</td>
<td>-48.65</td>
</tr>
<tr>
<td>Transport Industry</td>
<td>-29.75</td>
<td>-26.45</td>
<td>-29.12</td>
<td>-26.46</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 8: Impacts on consumer prices of selected fossil energy products in the 20th period from various simulations (percentage deviation from the BAU)

<table>
<thead>
<tr>
<th>Energy products</th>
<th>GFA</th>
<th>OBA</th>
<th>RPT</th>
<th>O-R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>639.6</td>
<td>639.1</td>
<td>639.6</td>
<td>639.1</td>
</tr>
<tr>
<td>Natural gas</td>
<td>25.3</td>
<td>23.8</td>
<td>25.3</td>
<td>23.9</td>
</tr>
<tr>
<td>Gasoline</td>
<td>17.9</td>
<td>14.7</td>
<td>17.8</td>
<td>14.8</td>
</tr>
<tr>
<td>Diesel</td>
<td>6.7</td>
<td>4.2</td>
<td>6.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Liquid petroleum products</td>
<td>12.4</td>
<td>11.0</td>
<td>12.4</td>
<td>11.0</td>
</tr>
<tr>
<td>Other refined petrol.</td>
<td>67.2</td>
<td>65.0</td>
<td>67.2</td>
<td>65.0</td>
</tr>
</tbody>
</table>

Source: Simulation results

Table 9: Sectoral GDP share at market price in the 20th period from various simulations

<table>
<thead>
<tr>
<th>Industry categories</th>
<th>GFA</th>
<th>OBA</th>
<th>RPT</th>
<th>O-R&amp;D</th>
<th>Initial GDP shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Carbon Emitting Industries (CEI)</td>
<td>23.3</td>
<td>23.7</td>
<td>23.4</td>
<td>23.7</td>
<td>23.2</td>
</tr>
<tr>
<td>CEI Energy Producers</td>
<td>5.6</td>
<td>6.1</td>
<td>5.6</td>
<td>6.1</td>
<td>6.2</td>
</tr>
<tr>
<td>CEI Non-Energy Producers</td>
<td>17.7</td>
<td>17.6</td>
<td>17.7</td>
<td>17.6</td>
<td>15.2</td>
</tr>
<tr>
<td>Non-CEI industries</td>
<td>76.7</td>
<td>76.3</td>
<td>76.6</td>
<td>76.3</td>
<td>78.6</td>
</tr>
</tbody>
</table>

Source: Simulation results
With regard to the change in sectoral distribution of output according to industry categories, implementation of GFA policy leads to a slight increase in the GDP share of all CEI industries by 0.1%, while non-CEI industries\textsuperscript{54} bear the burden and registers a decline of 1.9% (Table 9). However, real beneficiaries turn out to be the non-energy producing CEI industries\textsuperscript{55} that, relative to the base period, experience an increase of 2.5% in their GDP share at the expense of energy producing CEI industries. Together these lead to an overall decline in GDP at market price of the amount 1.95% (Table 4).

The same pattern of sectoral distribution of the burden of carbon abatement with GFA, in which energy producers bear a significant share, is also observed when variables other than sectoral GDP are considered. For example, as shown in Tables 5a, 5b, 6a, 6b, and 6c employment, investments in both physical capital and in R&D, exports and domestic sales decrease more for energy-producing carbon emitting firms than for the other sectors.

\subsection*{2.5.2.2 Reducing GHG Emissions with Output-Based Allocation of Emission Permits}

In comparison to the GFA simulation, under the output-based allocation of emission permits (OBA), all industries are provided a share of the emission rents according to their output and their (\textit{ex-ante}) assigned emissions intensity ($\beta_{jt}$). The latter parameter is multiplied by an endogenously determined common scale-back factor ($\alpha_t$) in order to equate the amount of emission rents with industries’ share in Canada’s assigned emission rights. It is important to note that in this OBA simulation, households receive a lump-sum transfer (scarcity rents from the value of the permits given to them). These represent the value of their shares in Canada’s assigned emission rights since not all emissions are generated by the production sectors alone. Households do emit due to their direct consumption of energy products (i.e. mobile energy products). Hence, Canada’s total rights are not attributed to firms alone.

As in the simulation with GFA, the endogenous permit price in OBA is also held at $40 per kilo ton of CO$_2$. With OBA, real GDP at market prices declines by 0.66%. The measure of

\textsuperscript{54} For the list of CEI and Non-CEI industries, see footnote 52 on page 115.
\textsuperscript{55} The non-energy producing CEI is a subset of CEI industries that includes the following sectors: Mining, Pulp and Paper, Cement, Iron and Steel, Chemicals and Other manufact. products.
household welfare change is recorded at –0.29% (in comparison to –0.46% in the previous simulation) registering a welfare improvement of 36%.

The differences in the aggregate impacts of the two simulation scenarios are mainly explained by the differences in the sectoral impacts fetched through the disparity in the permit allocation schemes. Relative to GFA, OBA policy has a behavioural impact on firms through the implicit output subsidy that is provided to the output producers. While imposing the requirement to hold costly permits put an upward pressure on firms’ cost of production thereby putting a downward pressure on the level of outputs, the implicit output subsidy renders incentives to firms not to reduce their output. The larger the intensity of emission for an industry under the BAU scenario, the greater the benefit that accrues to that sector under the OBA policy.

Akin to the previous simulation, the increase in production cost from the permit trading system induces an upward shift of the supply curve. This upward shift is partially offset by a downward movement of the supply curve that is generated by the subsidy. Under the OBA, however, a higher permit price also implies a higher subsidy. With preexisting distortions in resource allocation in the supply side (e.g. production and consumption taxes) of the economy, the implicit subsidy provided by OBA could be seen as a revenue-recycling scheme to reduce these distortions.

The subsidy reduces the price of consumer goods, which mitigates the fall in real net wage (as is the case with GFA) (Table 6c). In addition, the implicit output subsidy benefits all production factors, including labour, which experience a significant rise in total demand (it falls from 0.66% under GFA to 0.10% with OBA, Table 4). This occurs due to the increased investments in both R&D and physical capital. This leads to an improvement in the marginal productivity of labour that consequently induces an increase in the total labour supply through an increase in real wage. However, it is worth noting that this recycling method introduces additional distortions since the subsidy rate depends on the assigned emissions intensities that vary across different sectors of the economy.
The welfare results in the two simulations support the hypothesis that non-revenue-raising market-based instruments induce more welfare loss than the instruments that bring in revenues. In the simulation with OBA, a portion\textsuperscript{56} of the permit proceeds is used to reduce pre-existing distortions in resource allocation on the supply side of the economy. As shown in Dissou (2006), Parry et al. (2002) and Goulder et al. (1999), non-revenue-raising tradeable permit system (like GFA) could be a very costly way to achieve environmental objectives in the presence of such pre-existing distortions.

The third column in Table 5a shows the impacts on sectoral GDP at factor prices following the implementation of the OBA policy. Evidently OBA leads to an increase in GDP of more industry sectors compared to that with GFA. The policy helps large emitters (i.e. the energy producing CEI industries\textsuperscript{57}) mitigate against the rising cost of production. This is evident in their sectoral GDP shares, which stands at 6.1% with OBA as oppose to 5.6% under GFA (Table 9). It is, however, interesting to note that the non-emitting industries (i.e., Non-CEI industries\textsuperscript{58}) experience a further decrease in their GDP shares that fall from 76.7% under GFA to 76.3% with OBA (Table 9). The interesting revelation, however, lies in further decomposition of GDP at factor cost for the CEI – between energy-producing and non-energy-producing industry groups. OBA policy leads to a substantial improvement in GDP of the former group at a marginal expense to the latter (Table 5a). Hence, not only the adverse GDP impacts are reduced under OBA, but also the sectoral distribution of burden between the energy-producing and non-energy producing CEI are minimized. The same story is reflected in Table 7 for emissions reduction efforts across the industries. Relative to GFA, with OBA, energy-producing CEI are better off significantly. This, therefore, reduces the likelihood of potential political resistance to the implementation of a permit trading system for emissions control.

Overall, in comparison to GFA, the OBA scheme helps dampen, at least for energy-intensive industries as a whole, the negative impacts of the increase in production cost induced by the requirement to hold costly emission permits.

\textsuperscript{56} Households receive part of the permit proceeds, in proportion to their direct emissions.  
\textsuperscript{57} For the list of energy producing CEI industries, see footnote 53 on page 115. 
\textsuperscript{58} For the list of Non-CEI industries, see footnote 52 on page 115.
Table 10: Long-run impacts (30th period) on some aggregate variables from various simulations (percentage deviation from BAU)

<table>
<thead>
<tr>
<th></th>
<th>GFA</th>
<th>OBA</th>
<th>RPT</th>
<th>O-R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare change</td>
<td>-0.46</td>
<td>-0.29</td>
<td>-0.40</td>
<td>-1.30</td>
</tr>
<tr>
<td>GDP at market prices</td>
<td>-2.02</td>
<td>-0.68</td>
<td>-1.22</td>
<td>-0.52</td>
</tr>
<tr>
<td>Employment</td>
<td>-0.67</td>
<td>-0.11</td>
<td>0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>Household aggregate real consumption</td>
<td>-1.42</td>
<td>-0.57</td>
<td>-0.56</td>
<td>-0.54</td>
</tr>
<tr>
<td>Real investment (physical capital)</td>
<td>-3.94</td>
<td>-1.23</td>
<td>-3.10</td>
<td>-1.28</td>
</tr>
<tr>
<td>Real investment (R&amp;D)</td>
<td>-0.04</td>
<td>0.21</td>
<td>0.85</td>
<td>8.81</td>
</tr>
<tr>
<td>Total real imports</td>
<td>-2.65</td>
<td>-1.14</td>
<td>-1.82</td>
<td>-1.19</td>
</tr>
<tr>
<td>Total real exports</td>
<td>-3.58</td>
<td>-1.38</td>
<td>-2.47</td>
<td>-1.36</td>
</tr>
<tr>
<td>Real exchange rate*</td>
<td>0.47</td>
<td>0.11</td>
<td>0.53</td>
<td>0.09</td>
</tr>
<tr>
<td>Aggregate emissions</td>
<td>-35.56</td>
<td>-31.45</td>
<td>-34.98</td>
<td>-31.51</td>
</tr>
<tr>
<td>Permit price**</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Source: Simulation results

* A positive number means depreciation

** In Canadian dollars

2.5.2.3 Reducing GHG Emissions with Auctioning of Permits and Reduction in Payroll Taxes

The main difference in this simulation with the two previous ones is the revenue recycling method – permit proceeds are used to reduce payroll taxes. Adjustments in labour tax rate are applied equally to all industries.

Following the implementation of this policy (i.e. the RPT policy), the measure of household welfare change is registered at -0.40% (against -0.46% and -0.29% respectively with OBA and GFA). This is an interesting result since it runs contrary to the received wisdom in the literature that suggests that RPT yields higher welfare compared to GFA or OBA (Dissou, 2006).

We attribute this seeming anomaly to our modeling framework. In particular, in our model this occurs due to the incorporation of endogenous technological change into the modeling structure. While RPT provides incentive to the households to supply more labour thus enabling the producers to substitute more labour for other inputs, it does not provide incentive to invest in R&D activities. As OBA, on the other hand, provides implicit output subsidy and hence the incentive to produce more, producers have the impetus to invest more in
R&D leading to a greater substitutability on the supply side of the economy (Table 6b). Consequently, OBA leads to a higher welfare ranking compared to RPT as an emission permit allocation policy.

Table 4 also shows that real GDP at market prices declines by 1.15% with RPT (in comparison to -1.95% and -0.66% respectively with GFA and OBA). Since reduction in payroll tax essentially acts as a labour subsidy, not surprisingly household labour supply increases by 0.14% (in comparison to a decrease of 0.10% with OBA and 0.66% with GFA). The differences between the aggregate impacts of this simulation and the others stem from this positive impact on labour supply that is provided through the reduction in the payroll tax rate.

As evident in Table 7, with RPT industry sectors undergo more reductions in emissions levels relative to OBA. The pattern of reduction efforts, however, mimics that of with other two policies – that is, energy producing CEI, in general, have to reduce more emissions compared to the non-energy producing industry groups.

Households benefit more from the rise in real wage with RPT in comparison to OBA, as their real consumption decrease less (-0.55% vs. -0.59%). The main reason explaining this result is that the user prices of carbon-intensive products increase less with RPT than with OBA (Table 6c). This occurs due to the wage subsidy that induces increased labour supply and enables producers to substitute more labour for expensive energy inputs thereby putting a downward pressure on the producer prices.

The negative impact on the carbon emitting industries (CEI) with RPT is much higher than that in the simulation with OBA. This is because recycling of permit revenue is not primarily targeted to the industries that are the most affected by the abatement policy. Akin to simulation with GFA and OBA, while GDP share of, CEI industries rise by 0.2%, for the Non-CEI industries it falls by 2% (Table 9). Within the former group of industries, GDP share of non-energy producing CEI industries increases by 2.5% at the expense of energy producing CEI industries.

---

59 Sue Wing (2003) refers to this as the substitution effect.
60 For the list of CEI and Non-CEI industries, see footnote 52 on page 115.
industries\textsuperscript{61} that registers a fall of 0.6\% (compared to a decrease of 0.1\% under the OBA, Table 9). This result is not surprising since in the simulation with RPT, the reduction in labour tax rate benefits all industries, and especially those that are more labour intensive. In contrast, OBA is relatively more beneficial to energy producing CEI industries.

These results suggest that the distributional impact of RPT policy is clearly less in favour of the energy producing industries than in the simulation with OBA. As compared to OBA, generally received perception of RPT as welfare improving policy tool seems to hold only in the absence of ETC into the modeling structure. Simulation results in Dissou (2006), therefore, can be attributed to its modeling framework that does not allow for ETC. With ETC, results from our model unequivocally indicate the superiority of OBA as a policy tool over RPT. This result also suggests that when accounting for heterogeneity, OBA performs better as a policy tool to address unevenness of the sectoral distributional outcome of GHG abatement policy.

2.5.2.4 Reducing GHG Emissions with OBA and R&D Subsidy

As expected, under this policy welfare goes up relative to the remaining three other policy options. As compared to RPT’s welfare metric of -0.40, this policy (i.e. the O-R&D policy) registers a welfare measure of -0.28 leading to an improvement of 29\%. Relative to GFA, however, O-R&D leads to an overall improvement of 38\%. Welfare measurement stands close between OBA and O-R&D and critically depends on the amount of subsidies provided to the R&D sector.

O-R&D also leads to a better performance in overall GDP at market prices (which falls by 0.28\%, Table 4) that translates into an increase in the GDP share by 0.5\% under this policy relative to GFA and RPT policies that register a rise by 0.1\% and 0.2\% respectively (Table 9). Akin to other scenarios while, quantitatively, the carbon emitting industries (CEI) benefit more than the Non-CEI industries\textsuperscript{62}, and even within the CEI industry group non-energy producers benefit at the expense of energy producing CEI industries\textsuperscript{63}, there is a qualitative difference (Table 9). As evident in Table 5a, performance of the Non-CEI industries improves under O-

\textsuperscript{61} For the list of energy producing and non-energy producing CEI industries, see footnote 53 and 55 on page 115 and 120 respectively.
\textsuperscript{62} For the list of CEI and Non-CEI industries, see footnote 52 on page 115.
\textsuperscript{63} For the list of energy producing and non-energy producing CEI industries, see footnote 53 and 55 on page 115 and 120 respectively.
R&D as compared to GFA and RPT scenarios. But, interestingly, this does not come at the cost of increased burden sharing by the energy-producing CEI industries. Indeed, due to the hybrid nature of O-R&D policy (i.e. the part-OBA scheme), we retain similar results under O-R&D as in pure OBA. Thus, the policy mitigates against the burden cost due to the sectoral unevenness.

Not surprisingly the biggest qualitative impact of O-R&D policy is manifested in the energy-producing CEI industries. Due to the R&D subsidy, investments in R&D rise in all sectors. However, this is particularly significant in the energy-producing sectors (Table 5b). Indeed it is interesting to observe that relatively non-carbon intensive energy sector, like Electricity, also undertakes significant investment in R&D activities (a rise in investment by 24% under O-R&D as oppose to 14%, 5% and 4% with OBA, RPT and GFA respectively). Higher R&D investments lead to increased productivity for both labour and physical capital. Consequently, employment and investment in physical capital also rises.

As shown in Table 7, emissions reduction activities with O-R&D policy closely reflects that of the OBA. The policy effectively reduces the uneven burden cost across the industries. The added benefit, however, comes from the productivity gains for all production factors due to the substantial increase in R&D investments that also leads to increased production in physical capital.

Higher investments subsequently translate into accumulation of more knowledge and physical capital that, in turn, raises substitution in the supply side of the economy thereby enabling the economy to produce more. Furthermore, with the increase in real wage due to increased labour demand, households are also able to consume more. This leads to improved welfare performance as a whole.

2.5.3 Sensitivity Analysis

One of the general criticisms of CGE modeling is that it relies on the elasticity of substitution (EOS) parameters that are exogenously provided. To address this concern and to test for the robustness of the results, we perform sensitivity analysis.
Qualitative implications derived from the model results, by and large, hold up. Table 11 provides information on the welfare measure under various policy scenarios where EOS for substitutability between knowledge capital and the traditional production process is altered. As indicated, O-R&D turns out to be the best policy option followed by OBA.

Table 11: Welfare indicators for various sensitivity scenarios (SS)

<table>
<thead>
<tr>
<th></th>
<th>Base Scenario</th>
<th>SS 1</th>
<th>SS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFA</td>
<td>-0.458</td>
<td>-0.458</td>
<td>-0.458</td>
</tr>
<tr>
<td>OBA</td>
<td>-0.292</td>
<td>-0.292</td>
<td>-0.292</td>
</tr>
<tr>
<td>RPT</td>
<td>-0.403</td>
<td>-0.403</td>
<td>-0.403</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 5%</td>
<td>-0.284</td>
<td>-0.283</td>
<td>-0.283</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 10%</td>
<td>-0.286</td>
<td>-0.282</td>
<td>-0.283</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 20%</td>
<td>-0.333</td>
<td>-0.311</td>
<td>-0.318</td>
</tr>
</tbody>
</table>

Source: Simulation results
Elasticity of substitution (EOS) for the base scenario is 1.5
EOS for Sensitivity Scenario 1 (SS1) is 1.2
EOS for SS2 is 1.3

Table 12 summarizes information that measures percentage improvements for various policy options relative to their base scenario with the EOS value of 1.5. Results indicate that welfare tends to go up for each of the policy scenarios with larger substitution between knowledge capital and traditional production process. In other words, larger substitution possibilities generally prove to be welfare improving when we allow for technological change to occur endogenously.

Table 12: Welfare improvements (%) in various scenarios relative to the Base Scenario (BS)

<table>
<thead>
<tr>
<th></th>
<th>BS to SS 1</th>
<th>BS to SS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFA</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>OBA</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>RPT</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 5%</td>
<td>0.43</td>
<td>0.29</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 10%</td>
<td>1.58</td>
<td>1.06</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 20%</td>
<td>6.58</td>
<td>4.45</td>
</tr>
</tbody>
</table>

Source: Simulation results
Note: A positive value implies welfare improvement relative to the base case

Finally Table 13 shows information on welfare improvements that occur due to increased amount of subsidy accorded to R&D investments. We track this by comparing...
percentage change in welfare indicator for varying degrees of subsidy with O-R&D policy relative to the pure OBA policy\textsuperscript{64}.

Table 13: Welfare improvements (%) due to R&D Subsidy with various O-R&D scenarios over pure OBA

<table>
<thead>
<tr>
<th></th>
<th>SS 1</th>
<th>SS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-R&amp;D with Subsidy at 5%</td>
<td>3.10</td>
<td>2.97</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 10%</td>
<td>3.60</td>
<td>3.11</td>
</tr>
<tr>
<td>O-R&amp;D with Subsidy at 20%</td>
<td>-6.55</td>
<td>-8.97</td>
</tr>
</tbody>
</table>

Source: Simulation results
Note: A positive value implies welfare improvement relative to the base case

Results derived from this sensitivity analysis provide interesting revelation – as the level of subsidy increases, welfare tends to increase. However, beyond a critical level of R&D subsidy, further increase in the level of support reduces welfare. Further more, the critical level of optimal subsidy tends to decrease as EOS between knowledge capital and traditional production process increases.

These results, however, are not surprising. Unlike models in the existing literature, since accumulation process of knowledge stock undergoes a convex adjustment process in our model, with larger amount R&D subsidies, as firms invest further in R&D, resources are increasingly taken away from the OBA. This implies increasing unevenness due to less amount of implicit output subsidy. As a consequence welfare tends to dampen. With increased substitutability between the knowledge capital and the traditional production process, on the other hand, increasing R&D subsidy tends to reinforce the process since firms have stronger incentive to invest in R&D. As a result, the dampening effect on welfare sets out faster implying a reduction in the critical level of R&D subsidy.

2.6 Concluding Observations:

Addressing the uneven sectoral distributional outcomes of GHG abatement policies with market-based instruments may be a prerequisite for these policies to get the required political support. So far climate policy models that attempt to provide insights on available policy

\textsuperscript{64} Intuitively, one may consider pure OBA policy to be a hybrid O-R&D policy with R&D subsidy level of 0%.
options from this perspective have largely treated technological change as exogenous. However, recent advancements in the endogenous technological change (ETC) modeling literature indicate that ETC has the potential to greatly ameliorate the painful burden cost on hydrocarbon-intensive sectors of the economy. Combining ETC into assessing distributional outcome, we provide welfare ranking of four policy options – GFA, OBA, RPT, and O-R&D – where technological change is treated endogenously into the production process. The novelty in our approach lies in introducing two accumulable factors – physical and knowledge capital – where, unlike existing models in the literature, our treatment to the adjustment in the latter factor is not based on an *ad-hoc* approach.

Evidently permit trading systems with OBA are gaining popularity among policymakers due to their ability to mitigate the abatement cost burden in most energy intensive industries. As emissions allowances are linked to firms’ current output under OBA, it provides an implicit output subsidy, thus correcting for distributional equity, that helps the firms lower their prices. The existing literature suggests that the remedy for distributional issues generally comes at the expense of efficiency. However, with ETC incorporated into the framework, OBA and particularly O-R&D seem to perform better on both of these aspects. More specifically, O-R&D provides the best welfare implication as it takes care of both the uneven distributional burden cost and the market imperfection in the knowledge accumulation process.

In this study, we use an inter-temporal, multi-sector general equilibrium model with ETC to examine the trade-off between efficiency and uneven sectoral distributional effects of OBA in a second-best setting with pre-existing distortions. We compare the welfare and distributional impacts of OBA with three other systems: (i) an emissions trading with GFA, (ii) an emissions trading with auction permits where permit revenue is recycled to lower payroll taxes (RPT), and (iii) auction permits with a hybrid of OBA and R&D subsidy. The simulation results indicate that, with endogenous technology, each policy yields an improved welfare metric vis-à-vis the results derived from the models with exogenous technological change (such as Dissou, 2006). More interestingly, the results also indicate that welfare implication of the double dividend hypothesis does not hold true when technological change is endogenized into the modeling framework. Whereas without ETC, RPT policy yields the highest level of welfare, with ETC welfare levels improve by 27.5% for OBA and by 30% for O-R&D as compared to
the RPT policy. The GFA policy, however, yields the lowest welfare both in models with and without the ETC.

These results are largely in accordance with the received wisdom as indicated in the existing literature. The results support the hypothesis that nonrevenue-raising market-based instruments (i.e. GFA) induce more welfare loss than instruments that raise revenues (Goulder 2001, Parry et. al., 1999). Welfare cost of the RPT, OBA and O-R&D policy is lower than the GFA policy because the latter does not raise any revenue that could be recycled in order to generate efficiency gains. The implicit output subsidy provided through the OBA and O-R&D policy further facilitate to reduce the negative effects induced by the high cost of emission permits when technological change is endogenized into the model.

Using the relative contribution of different categories of industries to the overall carbon mitigation effort as the indicator for sectoral distributional effect, we find that OBA and O-R&D improves the uneven distributional outcome for energy-intensive industries considered as whole. Further, unlike Dissou (2006), we find that, in terms of welfare metric, relative to RPT, OBA unequivocally improves the sectoral distributional outcome of GHG abatement policies with emissions trading among varying energy-intensive industries. The welfare comparison between O-R&D and OBA, however, relies on the optimal level of R&D subsidy that critically depends on parameters of knowledge capital accumulation adjustment costs and the elasticity of substitution. Regardless of the extent of R&D subsidy, however, the beneficiaries of OBA or O-R&D policy are the energy-producing emitting industries. Contrary to the previous results, we find that these industries are able to benefit more from the output incentives provided through these allocation mechanisms when ETC is embedded into the modeling structure. The ETC also, in general, leads to improvements in welfare (a more important measure) for all emission permit trading policy schemes.
Appendices to Chapter Two

**Appendix A: First order conditions from the representative firm’s optimization problem**

Following is the list of eight necessary FOCs derived from the representative firm’s optimization problem:

\[
L_{j,t} : \left( P_{q,j,t} + \alpha_j \beta_j \bar{P}_t \right) \left[ \frac{\partial Q_{j,t}}{\partial H_{j,t}} \frac{\partial H_{j,t}}{\partial L_{j,t}} + \frac{\partial Q_{j,t}}{\partial X_{j,t}} \frac{\partial X_{j,t}}{\partial L_{j,t}} \right] = \bar{w}_t \quad [O-7]
\]

\[
q_{i,j,t} : \left( P_{q,j,t} - \bar{P}_t e_{i,j,t} + \alpha_j \beta_j \bar{P}_t \right) \frac{\partial Q_{j,t}}{\partial q_{i,j,t}} = \bar{p}_{i,t} \quad [O-8]
\]

\[
I_{K,j,t} : P_{k,j,t} \frac{\partial J_{K,j,t}}{\partial I_{K,j,t}} = \lambda_{K,j,t}, \quad \text{where} \quad \frac{\partial J_{K,j,t}}{\partial I_{K,j,t}} = 1 + \beta_{K,j} \left( \frac{I_{K,j,t}}{K_{j,t}} \right) \quad [O-9]
\]

\[
I_{H,j,t} : \left( 1 - v_{j,t} \right) P_{h,j,t} \frac{\partial J_{H,j,t}}{\partial I_{H,j,t}} = \lambda_{H,j,t}, \quad \text{where} \quad \frac{\partial J_{H,j,t}}{\partial I_{H,j,t}} = 1 + \beta_{H,j} \left( \frac{I_{H,j,t}}{H_{j,t}} \right) \quad [O-10]
\]

\[
K_{j,t+1} : \left( 1 - \delta_{K,j} \right) \lambda_{K,j,t+1} = \left( 1 + r \right) \lambda_{K,j,t}
\]

\[
- \left( P_{q,j,t+1} - \bar{P}_{t+1} e_{j,t+1} + \alpha_{t+1} \beta_{j+1,t+1} \bar{P}_{t+1} \right) \left[ \frac{\partial Q_{j,t+1}}{\partial X_{j,t+1}} \frac{\partial X_{j,t+1}}{\partial K_{j,t+1}} \right] \\
+ P_{K,j,t+1} \frac{\partial J_{K,j,t+1}}{\partial K_{j,t+1}}, \quad \text{where} \quad \frac{\partial J_{K,j,t+1}}{\partial K_{j,t+1}} = - \frac{\beta_{K,j}}{2} \left( \frac{I_{K,j,t+1}}{K_{j,t+1}} \right)^2 \Rightarrow r \lambda_{K,j,t} = R_{K,j,t+1} + \Delta \lambda_{K,j,t} - \delta_{K,j} \lambda_{K,j,t+1},
\]

where, \( R_{K,j,t+1} = \left( P_{q,j,t+1} + \bar{P}_t e_{j,t+1} + \alpha_{t+1} \beta_{j+1,t+1} \bar{P}_{t+1} \right) \left[ \frac{\partial Q_{j,t+1}}{\partial X_{j,t+1}} \frac{\partial X_{j,t+1}}{\partial K_{j,t+1}} \right] + \frac{1}{2} P_{K,j,t+1} \beta_{K,j} \left( \frac{I_{K,j,t+1}}{K_{j,t+1}} \right)^2 \quad [O-11]

\]

and \( e_{j,t} = \sum_i e_{i,j,t} \)

\[
H_{j,t+1} : \left( 1 - \delta_{H,j} \right) \lambda_{H,j,t+1} = \left( 1 + r \right) \lambda_{H,j,t}
\]

\[
- \left( P_{q,j,t+1} - \bar{P}_{t+1} e_{j,t+1} + \alpha_{t+1} \beta_{j+1,t+1} \bar{P}_{t+1} \right) \left[ \frac{\partial Q_{j,t+1}}{\partial H_{j,t+1}} \frac{\partial H_{j,t+1}}{\partial X_{j,t+1}} + \frac{\partial Q_{j,t+1}}{\partial X_{j,t+1}} \frac{\partial X_{j,t+1}}{\partial H_{j,t+1}} \right]
\]
\begin{align*}
&+(1-v_{j,t}) P_{H,j,t+1} \frac{\partial J_{H,j,t+1}}{\partial H_{j,t+1}}, \quad \text{where} \quad \frac{\partial J_{H,j,t+1}}{\partial H_{j,t+1}} = -\frac{\beta_{H,j}}{2} \left( \frac{I_{H,j,t+1}}{H_{j,t+1}} \right)^2

\Rightarrow r_{H,j,t} &= R_{H,j,t+1} + \Delta \lambda_{H,j,t} - \delta_{H,j} \lambda_{H,j,t+1},

\text{where,} \quad R_{H,j,t+1} = \left( P_{\omega,j,t+1} + \bar{P}_t e_{j,t+1} + \alpha_{t,j} \beta_{j,t+1} \right) \left[ \frac{\partial Q_{j,t+1}}{\partial H_{j,t+1}} + \frac{\partial Q_{j,t+1}}{\partial X_{j,t+1}} \frac{\partial X_{j,t+1}}{\partial H_{j,t+1}} \right] + \frac{1}{2} \beta_{H,j} (1-v_{j,t}) P_{H,j,t+1} \left( \frac{I_{H,j,t+1}}{H_{j,t+1}} \right)^2 [O-12]

\text{and} \quad e_{j,t} = \sum_i e_{i,j,t}

\lambda_{K,j,t} : \quad (1+n) k_{j,t+1} = (1-\delta_{K,j}) k_{j,t} + i_{K,j,t} \quad [O-13]

\text{where variables are expressed in intensive form such that:}

\frac{K_{j,t+1}}{L_{j,t}} = (1+n) k_{j,t+1}, \quad \frac{K_{j,t}}{L_{j,t}} = k_{j,t} \quad \text{and} \quad \frac{I_{j,t}}{L_{j,t}} = i_{K,j,t}

\lambda_{H,j,t} : \quad h_{j,t+1} = (1-\delta_{H,j}) h_{j,t} + i_{H,j,t} \quad [O-14]

\text{where corresponding variables are expressed in intensive form such that:}

\frac{H_{j,t+1}}{L_{j,t}} = (1+n) h_{j,t+1}, \quad \frac{H_{j,t}}{L_{j,t}} = h_{j,t} \quad \text{and} \quad \frac{I_{j,t}}{L_{j,t}} = i_{K,j,t}
\end{align*}
Appendix B: List of equations

Household:

\[
\frac{C_{t+1}}{C_t} = \left[ \frac{(1+r)P_{C,t}}{(1+\rho)P_{C,t+1}} \right]
\]

\[
\alpha w_i (1-L_i) = (1-\alpha)P_{C,t} C_t
\]

\[
(1+n)A_{t+1} = (1+r)A_t + Y_t - P_{C,t} C_t
\]

\[
Y_t = (1-\tau_{Y,t}) Y_{H,t} + (1-\tau_{D,t}) Y_{D,t} + S_{G,t}
\]

\[
Y_{H,t} = \sum_i w_i L_{i,t} + T_{G,t} + ER_i S_{F,t} + \beta_{H,t} \bar{P}_t EM_t
\]

\[
Y_{D,t} = \left( \sum_j P_{\text{HET},j,t} RR_{K,j,t} K_{j,t} - \sum_j P_{\text{Ket},j,t} J_{K,j,t} \right)
+ \left( \sum_{j=1} \left( P_{Q,j,t} - \bar{P}_t e_{j,t} + \alpha \beta_{j,t} \bar{P}_t \right) RR_{H,j,t} H_{j,t} - \sum_{j=1} \left( 1-\nu_{j,t} \right) P_{H,t} J_{H,j,t} \right)
\]

\[
P_{C,t} = \frac{1}{A_C} \left[ (\theta_C)^{\sigma_C} (P_{\text{HET},t})^{1-\sigma_C} + (1-\theta_C)^{\sigma_C} (P_{\text{HOG},t})^{1-\sigma_C} \right]^{-\frac{1}{1-\sigma_C}}
\]

\[
HET_t = \left( A_C \right)^{\sigma_C} C_t \left[ \frac{\theta_C P_{C,t}}{P_{\text{HET},t}} \right]^{-\sigma_C}
\]

\[
HOG_t = \left( A_C \right)^{\sigma_C} C_t \left[ \frac{(1-\theta_C) P_{C,t}}{P_{\text{HOG},t}} \right]^{-\sigma_C}
\]

\[
P_{\text{HET},t} = \frac{1}{A_{\text{HET}}} \left[ (\theta_{\text{HET}})^{\sigma_{\text{HET}}} (P_{\text{HET},t})^{1-\sigma_{\text{HET}}} + (1-\theta_{\text{HET}})^{\sigma_{\text{HET}}} (P_{\text{ELEC},t})^{1-\sigma_{\text{HET}}} \right]^{-\frac{1}{1-\sigma_{\text{HET}}}}
\]

\[
HEF_t = \left( A_{\text{HET}} \right)^{\sigma_{\text{HET}}} HET_t \left[ \frac{\theta_{\text{HET}} P_{\text{HET},t}}{P_{\text{HET},t}} \right]^{\sigma_{\text{HET}}}
\]

\[
ELEC_t = \left( A_{\text{HET}} \right)^{\sigma_{\text{HET}}} HET_t \left[ \frac{(1-\theta_{\text{HET}}) P_{\text{HET},t}}{P_{\text{ELEC},t}} \right]^{\sigma_{\text{HET}}}
\]

\[
p_{\text{HET},t} = \frac{1}{A_{\text{HET}}} \left[ (\theta_{\text{HET},1})^{\sigma_{\text{HET}}} (P_{\text{HRP},t})^{1-\sigma_{\text{HET}}} + (\theta_{\text{HET},2})^{\sigma_{\text{HET}}} (P_{\text{HTGAS},t})^{1-\sigma_{\text{HET}}} + (1-\theta_{\text{HET},1}-\theta_{\text{HET},2})^{\sigma_{\text{HET}}} (P_{\text{COAL},t})^{1-\sigma_{\text{HET}}} \right]^{-\frac{1}{1-\sigma_{\text{HET}}}}
\]

\[
HRP_t = \left( A_{\text{HET}} \right)^{\sigma_{\text{HET}}} HEF_t \left[ \frac{\theta_{\text{HET},1} P_{\text{HET},t}}{P_{\text{HRP},t}} \right]^{\sigma_{\text{HET}}}
\]

\[
HTGAS_t = \left( A_{\text{HET}} \right)^{\sigma_{\text{HET}}} HEF_t \left[ \frac{\theta_{\text{HET},2} P_{\text{HET},t}}{P_{\text{HTGAS},t}} \right]^{\sigma_{\text{HET}}}
\]
\[ HCOAL_t = (A_{HEF})^{\sigma_{HEF}-1}_{HEF} F_t \left[ \frac{\left(1 - \theta_{HEF,1} - \theta_{HEF,2}\right) P_{HEF,t}}{P_{HCOAL,t}} \right]^{\sigma_{HEF}} \]

\[ P_{HRP,t} = \frac{1}{A_{HRP}} \prod_{i=1}^{n} \left( \frac{P_{HRP,i,t}}{\theta_{HRP,i}} \right)^{\theta_{HRP,i}} \]

\[ HRP_{i,t} = P_{HRP,i,t} \theta_{HRP,i} \]

\[ P_{HTGAS,t} = \sum_{i \in GAS} \gamma_{H,i} P_{GAS,i,t} \]

\[ HGAS_{i,t} = \gamma_{H,i} HTGAS_{i,t} \]

\[ P_{HOG,t} = \frac{1}{A_{HOG}} \left[ \left( \theta_{HOG} \right) \left( P_{HMFS,t} \right)^{1-\sigma_{HOF}} + (1-\theta_{HOG}) \left( P_{HMFS,t} \right)^{1-\sigma_{HOF}} \right]^{1-\sigma_{HOF}} \]

\[ HMF_t = (A_{HOG})^{\sigma_{HOF}-1}_{HOG} \left[ \left( \theta_{HOG} P_{HOG,t} \right) P_{HMFS,t} \right]^{\sigma_{HOF}} \]

\[ HMAT_t = (A_{HOF})^{\sigma_{HOF}-1}_{HOF} \left[ \left( 1 - \theta_{HOF} P_{HOG,t} \right) P_{HMFS,t} \right]^{\sigma_{HOF}} \]

\[ P_{HMFS,t} = \frac{1}{A_{HMFS}} \prod_{i=1}^{n} \left( \frac{P_{HMFS,i,t}}{\theta_{HMFS,i}} \right)^{\theta_{HMFS,i}} \]

\[ HMF_{i,t} = P_{HMFS,i,t} \theta_{HMFS,i} \]

\[ P_{HMAT,t} = \frac{1}{A_{HMAT}} \left[ \sum_{i \in HMAT} \left( \theta_{HMAT,i} \right) P_{HMAT,i,t} \right]^{1-\sigma_{HMAT}} \]

\[ HMAT_{i,t} = (A_{HMAT})^{\sigma_{HMAT}-1}_{HMAT} \left[ \theta_{HMAT,i} P_{HMAT,i,t} \right]^{\sigma_{HMAT}} \]

**Firm:**

\[ Q_{j,t} = A_{Q,j} \left[ \phi_j \left( H_{j,t} \right)^{\sigma_{Q,j}-1}_{Q,j} + (1-\phi_j) \left( X_{j,t} \right)^{\sigma_{Q,j}-1}_{Q,j} \right]^{\sigma_{Q,j}} \]

where \( A_{Q,j} = \gamma_j \left( H_{j,t} \right) \) and \( \overline{H}_{j,t} = H_{j,t} + \beta \overline{I}_{H,j,t} \)

\[ X_{j,t} = (A_{Q,j})^{\sigma_{Q,j}-1}_{Q,j} Q_{j,t} \left[ \left( 1-\phi_j \right) \left( P_{Q,j,t} - \overline{P}_t e_{j,t} + \alpha_j \beta_j \overline{P}_t \right) \right]^{\sigma_{Q,j}}, \text{ where } e_{j,t} = \sum_i e_{i,j,t} \]
\[
(1 - \delta_{H, j}) \lambda_{H, j+1} = (1 + r) \lambda_{H, j} - \left( P_{Q, j+1} - \overline{P}_{t+1} e_{j, t+1} + \alpha_{t+1} \beta_{j, t+1} \overline{P}_{t+1} \right) RR_{H, j+1} + (1 - \nu_{j, t}) P_{H, j+1} \frac{\partial J_{H, j+1}}{\partial H_{j+1}}
\]

\[
RR_{H, j+1} = \left[ \frac{\partial Q_{j, t+1}}{\partial H_{j, t+1}} + \frac{\partial Q_{j, t+1}}{\partial X_{j, t+1}} \frac{\partial X_{j, t+1}}{\partial H_{j, t+1}} \right]
\]

\[
\frac{\partial J_{H, j+1}}{\partial H_{j, t+1}} = - \frac{\beta_{H, j}}{2} \left( \frac{I_{H, j+1}}{H_{j, t+1}} \right)^2
\]

\[
(1 + n) H_{j, t+1} = (1 - \delta_{H, j}) H_{j, t} + I_{H, j, t}
\]

\[
(1 - \nu_{j, t}) P_{H, t} \frac{\partial J_{H, j, t}}{\partial I_{H, j, t}} = \lambda_{H, j, t}
\]

\[
\frac{\partial J_{H, j, t}}{\partial I_{H, j, t}} = 1 + \beta_{H, j} \left( \frac{I_{H, j, t}}{H_{j, t}} \right)
\]

\[
P_{x, j, t} = \frac{1}{A_{x, j}} \left[ (\varphi_{x, j})^{\sigma_{x, j}} \left( P_{INT, j, t} \right)^{1 - \sigma_{x, j}} + (1 - \varphi_{x, j})^{\sigma_{x, j}} \left( P_{VAE, j, t} \right)^{1 - \sigma_{x, j}} \right]^{\frac{1}{1 - \sigma_{x, j}}}
\]

\[
INT_{j, t} = \left( A_{x, j} \right)^{\sigma_{x, j}} X_{j, t} \left[ \frac{\varphi_{x, j} P_{x, j, t}}{P_{INT, j, t}} \right]^{\sigma_{x, j}}
\]

\[
VAE_{j, t} = \left( A_{x, j} \right)^{\sigma_{x, j}} X_{j, t} \left[ \frac{(1 - \varphi_{x, j}) P_{x, j, t}}{P_{VAE, j, t}} \right]^{\sigma_{x, j}}
\]

\[
P_{VAE, j, t} = \frac{1}{A_{VAE, j}} \left[ (\varphi_{VAE, j})^{\sigma_{VAE, j}} \left( P_{KET, j, t} \right)^{1 - \sigma_{VAE, j}} + (1 - \varphi_{VAE, j})^{\sigma_{VAE, j}} \left( w_{t} \right)^{1 - \sigma_{VAE, j}} \right]^{\frac{1}{1 - \sigma_{VAE, j}}}
\]

\[
\overline{w_{t}} = w_{t} \left( 1 - \tau_{L} \right)
\]

\[
KET_{j, t} = \left( A_{VAE, j} \right)^{\sigma_{VAE, j}} VAE_{j, t} \left[ \frac{\varphi_{VAE, j} P_{VAE, j, t}}{P_{KET, j, t}} \right]^{\sigma_{VAE, j}}
\]

\[
L_{j, t} = \left( A_{VAE, j} \right)^{\sigma_{VAE, j}} VAE_{j, t} \left[ \frac{(1 - \varphi_{VAE, j}) P_{VAE, j, t}}{w_{t}} \right]^{\sigma_{VAE, j}}
\]

\[
KET_{j, t} = A_{KE, j} \left[ \varphi_{KET, j} \left( K_{j, t} \right)^{\sigma_{KET, j}} + (1 - \varphi_{KET, j}) \left( E_{T, j} \right)^{\sigma_{KET, j}} \right]^{\sigma_{KET, j}}
\]

135
\((1 - \delta_{K,j}) \lambda_{K,j,t+1} = (1 + r) \lambda_{K,j,t} - \left(P_{Q,j,t+1} - \bar{P}_{j,t+1} \ e_{j,t+1} + \alpha_{t+1} \beta_{j,t+1} \bar{P}_{j,t+1} \right) R K_{K,j,t+1} + P_{K,j,t+1} \frac{\partial J_{K,j,t+1}}{\partial K_{j,t+1}}\)

\[RR_{K,j,t+1} = \left[ \frac{\partial Q_{j,t+1}}{\partial X_{j,t+1}} \right] \]

\[\frac{\partial J_{K,j,t+1}}{\partial K_{j,t+1}} = - \beta_{K,j} \left( \frac{I_{K,j,t+1}}{K_{j,t+1}} \right)^2\]

\[(1 + n) K_{j,t+1} = (1 - \delta_{K,j}) K_{j,t} + I_{K,j,t}\]

\[P_{K,j,t} \frac{\partial J_{K,j,t}}{\partial I_{K,j,t}} = \lambda_{K,j,t}\]

\[\frac{\partial J_{K,j,t}}{\partial I_{K,j,t}} = 1 + \beta_{K,j} \left( \frac{I_{K,j,t}}{K_{j,t}} \right)\]

\[P_{ET,j,t} = \left( \frac{1}{A_{ET,j,t}} \right) \sigma_{ET,j,t}^{-1} \sigma_{ET,j} \left( 1 - \varphi_{ET,j,t} \right) P_{ET,j,t} \left( \frac{ET_{j,t}}{KET_{j,t}} \right) \]

\[P_{ET,j,t} = \frac{1}{A_{ET,j,t}} \left[ \left( \varphi_{ET,j} \right)^{\sigma_{ET,j}} \left( P_{EF,j,t} \right)^{1 - \sigma_{ET,j}} + \left( 1 - \varphi_{ET,j} \right) \left( P_{ELEC,j,t} \right)^{1 - \sigma_{ET,j}} \right]^{-1}
\]

\[EF_{j,t} = \left( A_{ET,j,t} \right)^{\sigma_{ET,j,t}^{-1} \sigma_{ET,j}} \left( \varphi_{ET,j} \frac{P_{ET,j,t}}{P_{EF,j,t}} \right)^{\sigma_{ET,j}} \]

\[ELEC_{j,t} = \left( A_{ET,j,t} \right)^{\sigma_{ET,j,t}^{-1} \sigma_{ET,j}} \left( \frac{1 - \varphi_{ET,j}}{P_{ELEC,j,t}} \right)^{\sigma_{ET,j}} \]

\[P_{EF,j,t} = \frac{1}{A_{EF,j,t}} \left[ \left( \varphi_{EF,j} \right)^{\sigma_{EF,j}} \left( P_{RP,j,t} \right)^{1 - \sigma_{EF,j}} + \left( \varphi_{EF,2,j} \right)^{\sigma_{EF,j}} \left( P_{TGAS,j,t} \right)^{1 - \sigma_{EF,j}} + \left( 1 - \varphi_{EF,3,j} - \varphi_{EF,2,j} \right) \left( P_{COAL,j,t} \right)^{1 - \sigma_{EF,j}} \right]^{-1}
\]

\[RP_{j,t} = \left( A_{EF,j,t} \right)^{\sigma_{EF,j,t}^{-1} \sigma_{EF,j}} \left( \varphi_{EF,j} \frac{P_{EF,j,t}}{P_{RP,j,t}} \right)^{\sigma_{EF,j}} \]

\[TGAS_{j,t} = \left( A_{EF,j,t} \right)^{\sigma_{EF,j,t}^{-1} \sigma_{EF,j}} \left( \varphi_{EF,2,j} \frac{P_{EF,j,t}}{P_{TGAS,j,t}} \right)^{\sigma_{EF,j}} \]

\[COAL_{j,t} = \left( A_{EF,j,t} \right)^{\sigma_{EF,j,t}^{-1} \sigma_{EF,j}} \left( \frac{1 - \varphi_{EF,3,j} - \varphi_{EF,2,j}}{P_{COAL,j,t}} P_{EF,j,t} \right)^{\sigma_{EF,j}} \]
\[
P_{RP,j,t} = \frac{1}{A_{RP,j,t}} \prod_{i=1}^{n} \left( \frac{P_{RP,i,j,t}}{\varphi_{RP,i,j}} \right)^{\sigma_{RP,j}} \\
RP_{i,j} \quad P_{RP,i,j,t} = \varphi_{RP,i,j} \quad RP_{j,t} \quad P_{RP,j,t} \\
P_{GAS,j,t} = \sum_{i \in GAS,i,j} Y_{GAS,i,j} P_{GAS,i,j,t} \\
GAS_{i,j,t} = Y_{GAS,i,j} \quad TGAS_{j,t} \\
\]

\[
P_{INT,j,t} = \frac{1}{A_{INT,j}} \left[ \left( \varphi_{INT,j} \right)^{\sigma_{INT,j}} \left( P_{MF,j,t} \right)^{1-\sigma_{INT,j}} + \left( 1 - \varphi_{INT,j} \right)^{\sigma_{INT,j}} \left( P_{MAT,j,t} \right)^{1-\sigma_{INT,j}} \right]^{\frac{1}{1-\sigma_{INT,j}}} \\
MF_{j,t} = \left( A_{INT,j} \right)^{\sigma_{INT,j}-1} \quad INT_{j,t} \left[ \frac{\varphi_{INT,j} P_{INT,j,t}}{P_{MF,j,t}} \right]^{\sigma_{INT,j}} \\
MAT_{j,t} = \left( A_{INT,j} \right)^{\sigma_{INT,j}-1} \quad INT_{j,t} \left[ \frac{(1 - \varphi_{INT,j}) P_{INT,j,t}}{P_{MAT,j,t}} \right]^{\sigma_{INT,j}} \\
\]

\[
P_{MF,j,t} = \frac{1}{A_{MF,j}} \sum_{i \in MF,i,j} \left( \varphi_{MF,i,j} \right)^{\sigma_{MF,j}} \left( P_{MF,i,j,t} \right)^{1-\sigma_{MF,j}} \left[ \frac{\varphi_{MF,i,j} P_{MF,i,j,t}}{P_{MF,j,t}} \right]^{\sigma_{MF,j}} \\
MF_{i,j,t} = \left( A_{MF,j} \right)^{\sigma_{MF,j}-1} \quad MF_{j,t} \left[ \frac{\varphi_{MF,i,j} P_{MF,i,j,t}}{P_{MF,j,t}} \right]^{\sigma_{MF,j}} \\
P_{MAT,j,t} = \sum_{i \in MAT,i,j} Y_{MAT,i,j} P_{MAT,i,j,t} \\
MAT_{i,j,t} = Y_{MAT,i,j} \quad MAT_{j,t} \\
\]

**Government:**

\[
Y_{G,t} = \sum_{j} \tau_{Q,j} P_{Q,j,t} Q_{j,t} + \sum_{i} \tau_{E,i} P_{E,i,j,t} q_{i,j,t} + \sum_{i} \tau_{C,i} P_{C,i,j,t} C_{i,t} + \tau_{L,w} L_{j,t} + \sum \tau_{K,j} P_{K,j,t} K_{j,t} + \sum \tau_{ER} \tau_{IM,t} P_{c,t}^{W} C_{j,t}^{IM} + \tau_{Y} Y_{H,t} + \tau_{D} Y_{D,t} \\
S_{G,t} = Y_{G,t} - \sum_{i} P_{c,i,j} C_{G,i,t} - T_{G,t} - T_{W} G_{t} - \tau_{ER} B_{i} \\
\]

**Trade:**

\[
P_{X,j,t} = \frac{1}{A_{X,j}} \left[ \left( \eta_{j} \right)^{\sigma_{X,j}} \left( P_{D,X,j,t} \right)^{1-\sigma_{X}} + \left( 1 - \eta_{j} \right)^{\sigma_{X}} \left( P_{E,X,j,t} \right)^{1-\sigma_{X,j}} \right]^{\frac{1}{1-\sigma_{X,j}}} \\
\]

137
\[ X_{j,t}^D = (A_{X_{j}})^{-1-\sigma_{x,j}} X_{j,t} \left( \frac{P_{X,j,t}^D}{\eta_j P_{X,j,t}} \right)^{\sigma_{x,j}} \]

\[ X_{j,t}^{EX} = (A_{X_{j}})^{-1-\sigma_{x,j}} X_{j,t} \left( \frac{P_{X,j,t}^{EX}}{(1-\eta_j) P_{X,j,t}} \right)^{\sigma_{x,j}} \]

\[ P_{C,j,t}^D = \frac{1}{A_{m,t}} \left[ (\kappa_i)^{\sigma_{m,j}} \left( P_{C,j,t}^D \right)^{1-\sigma_{m,j}} + (1 - \kappa_i)^{\sigma_{m,j}} \left( P_{C,j,t}^{IM} \right)^{1-\sigma_{m,j}} \right] \]

\[ C_{C,j,t}^D = \left( A_{m,t} \right)^{\sigma_{m,j} - 1} C_{C,t} \left( \frac{\kappa_i P_{C,j,t}^D}{P_{C,j,t}^{D}} \right)^{\sigma_{m,j}} \]

\[ C_{C,j,t}^{IM} = \left( A_{m,t} \right)^{\sigma_{m,j} - 1} C_{C,t} \left( \frac{(1 - \kappa_i) P_{C,j,t}^{IM}}{P_{C,j,t}^{IM}} \right)^{\sigma_{m,j}} \]

Capital account (balance of payments):

\[ ER_S F_{t} = \sum_{i=1}^{n} P_{C_{i,t}}^{IM} C_{t,i}^{IM} + rB_{t} + T_{G,t}^{W} - \sum_{j=1}^{n} P_{X,j,t}^{EX} X_{j,t}^{EX} \]

\[(1 + n) B_{t+1} = (1 + r) B_{t} + ER_S F_{t} \]

Demand for investment commodities and total demand:

\[ J_{H,j,t} = I_{H,j,t} \left[ 1 + \frac{\beta_{H,j}}{2} \left( \frac{I_{H,j,t}}{H_{j,t}} \right) \right] \]

\[ I_{H,j,t} = \beta_{HINV,j} \frac{P_{C,j,t}^D}{P_{C,j,t}^{D}} \]

\[ J_{K,j,t} = I_{K,j,t} \left[ 1 + \frac{\beta_{K,j}}{2} \left( \frac{I_{K,j,t}}{K_{j,t}} \right) \right] \]

\[ I_{K,j,t} = \beta_{KINV,j} \sum_{j} J_{K,j,t} \]

Prices:

\[ P_{X,j,t}^{EX} = ER P_{X,j,t}^{W} \]

\[ P_{C,j,t}^{IM} = ER \left( 1 + \tau_{IM,j} \right) P_{C,j,t}^{W} \]

\[ P_{Q,j,t}^{EX} = P_{Q,j,t}^{W} \left( 1 - \tau_{Q,j} \right) \]
\begin{align*}
P_{C,j,t} &= P_{C,j,t} (1 + \tau_{C,j,t}) + \xi_{H,j,t}
\xi_{P,j,t} &= P_{C,j,t} (1 + \tau_{C,j,t}) + \xi_{Q,j,t}
\xi_{P,k,t} &= \sum_i \beta_{KINV,j} P_{C,i,t} (1 + \tau_{C,i,t})
P_{H,t} &= P_{C,j,t} (1 + \tau_{C,j,t}) + \xi_{H,j,t}
\end{align*}

Equilibrium conditions:
\begin{align*}
X_{j,t} &= C_{i,t} + I_{H,i,t} + I_{K,j,t} + C_{G,i,t} + \sum_j q_{i,j,t}
C_{i,t} &= X_{j,t}
L_{D,t} &= \sum_j L_{j,t}
L_{D,t} &= L_{S,t}
Z_t &= \sum_j \lambda_{K,j,t} K_{j,t} + \sum_{j-1} \lambda_{H,j,t} H_{j,t}
Z_t &= B_t + A_t
\end{align*}

Steady-state conditions:
\begin{align*}
(r - n) A_{ss} + Y_{ss} &= P_{C,ss} C_{ss}
I_{H,j,ss} &= (n + \delta_{H,j}) H_{j,ss}
I_{K,j,ss} &= (n + \delta_{K,j}) K_{j,ss}
(\delta_{K,j} + r) \lambda_{K,j,ss} &= \sum_j (P_{Q,j,ss} - \bar{P}_{ss} e_{j,ss} + \alpha_{\beta,j,ss} \bar{P}_{ss}) RR_{H,j,ss} + \sum_j (1 - v_{j,ss}) P_{H,ss} J_{H,j,ss}
(\delta_{H,j} + r) \lambda_{H,j,ss} &= \sum_j P_{KE,ss} RR_{K,j,ss} + \sum_j P_{K,ss} J_{K,j,ss}
(n - r) B_{ss} &= ER_{ss} S_{F,ss}
\end{align*}

Emissions:
\begin{align*}
E_{t} &= E_{H,t} + \sum_j E_{Q,j,t}
E_{H,t} &= \sum_j (\xi_{H,j,t} C_{i,j})
E_{Q,j,t} &= \sum_i (\xi_{Q,i,j} q_{i,j,t})
\alpha_t \sum_j \beta_{j,t} Q_{j,t} &= E_{Q,j,t}
\end{align*}
Appendix C: List of variables (subscript \( t \) in any variable denotes the time period \( t \ ))

\( C_t \): per-capita aggregate consumption at time \( t \)
\( P_{C,t} \): index price of unit aggregate consumption at time \( t \)
\( w_i \): gross rate of return to labour (i.e. wage rate gross of (payroll) tax) in period \( t \)
\( \bar{w}_t \): wage rate net of payroll tax in period \( t \)
\( A_t \): financial wealth held by the representative household at time \( t \)
\( Y_t \): aggregate disposable income of the representative household at time \( t \)
\( Y_{H,t} \): aggregate gross income of household at time \( t \)
\( Y_{D,t} \): household gross dividend income at time \( t \)
\( T_{G,t} \): government transfer to household at time \( t \)
\( T_{G,W}^t \): government transfer to rest of the world at time \( t \)
\( S_{F,t} \): foreign savings at time \( t \)
\( S_{G,t} \): government savings at time \( t \)
\( \bar{P} \): price of emission permit at time \( t \)
\( EM_t \): total emission at time \( t \)
\( L_{S,t} \): aggregate labour supplied at time \( t \)
\( N_t \): population at time \( t \)
\( \beta_{H,t} \): household share of total emission at time \( t \)
\( \nu_{j,t} \): rate of subsidy on R&D investment at time \( t \)
\( \tau_{L,t} \): payroll tax rate at time \( t \)

\( P_{KET,j,t} \): index price of capital-energy factor used in industry \( j \) at time \( t \)
\( RR_{K,j,t} \): marginal product of physical capital in sector \( j \) at time \( t \)
\( K_{j,t} \): quantity of physical capital demanded by sector \( j \) at time \( t \)
\( P_{K,t} \): unit cost of physical capital investment at time \( t \)
\( J_{K,j,t} \): gross investment in physical capital in sector \( j \) at time \( t \)

\( RR_{H,j,t} \): marginal product of knowledge capital in sector \( j \) at time \( t \)
\( H_{j,t} \): quantity of knowledge capital demanded by sector \( j \) at time \( t \)
\( P_{H,t} \): unit cost of knowledge capital investment at time \( t \)

\( P_{Q,j,t} \): gross (of production tax) price of industry \( j \)'s output at time \( t \)
\( P_{Q,j,t} \): net (of production tax) price of industry \( j \)'s output at time \( t \)
\( P_{i,t} \): gross (of tax) price of intermediate input \( i \) at time \( t \)
\( P_{\xi,j,t} \): net (of tax) price of intermediate input \( i \) at time \( t \)

\( \alpha \): emission scale-back parameter (determined endogenously in the model)

\( J_{H,j,t} \): gross investment in knowledge capital in sector \( j \) at time \( t \)

\( Q_{j,t} \): quantity of composite output produced by firm \( j \) at time \( t \)

\( q_{i,j,t} \): intermediate input \( i \) demanded by industry \( j \) at time \( t \)

\( I_{K,j,t} \): investment demand for physical capital in sector \( j \) at time \( t \)

\( I_{H,j,t} \): investment demand for R&D (i.e. knowledge capital) in sector \( j \) at time \( t \)

\( H_{ET,t} \): composite of (non-mobile) energy goods consumed by the household at time \( t \)

\( P_{HET,t} \): index price of composite (non-mobile) energy goods at time \( t \)

\( HOG_{j} \): composite of materials-mobile energy commodities (i.e. other household goods) at time \( t \)

\( P_{HOG,j} \): index price of composite materials-mobile energy commodities at time \( t \)

\( HEF_{t} \): composite of combustible energy goods consumed by the household at time \( t \)

\( P_{HEF,t} \): index price of combustible energy goods at time \( t \)

\( ELEC_{t} \): composite of non-combustible commodities at time \( t \)

\( P_{ELEC,t} \): index price of non-combustible energy commodities at time \( t \)

\( HRP_{t} \): composite of refined petroleum goods consumed by the household at time \( t \)

\( P_{HRP,t} \): index price of refined petroleum goods at time \( t \)

\( HTGAS_{t} \): composite of natural gas consumed by the household at time \( t \)

\( P_{HTGAS,t} \): index price of natural gas at time \( t \)

\( HCOAL_{t} \): quantity of coal consumed by the household at time \( t \)

\( P_{HCOAL,t} \): price of coal at time \( t \)

\( HMF_{t} \): composite of mobile energy goods consumed by the household at time \( t \)

\( P_{HMF,t} \): index price of mobile energy goods at time \( t \)

\( HMAT_{t} \): composite of material goods consumed by the household at time \( t \)

\( P_{HMAT,t} \): index price of material goods at time \( t \)

\( X_{j,t} \): quantity of output produced by firm \( j \) at time \( t \)

\( P_{x,j,t} \): index price of output produced in industry \( j \) at time \( t \)

\( \lambda_{H,j,t} \): shadow price of knowledge capital in industry \( j \) at time \( t \)
\( \text{INT}_{j,t} \): composite of intermediate inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{INT},j,t} \): index price of intermediate goods demanded by industry \( j \) at time \( t \)
\( \text{VAE}_{j,t} \): composite of value-added–energy inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{VAE},j,t} \): index price of value-added–energy composite inputs demanded by industry \( j \) at time \( t \)
\( \lambda_{K,j,t} \): shadow price of physical capital in industry \( j \) at time \( t \)
\( \text{KET}_{j,t} \): composite of physical capital and energy inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{KET},j,t} \): index price for the composite of physical capital and energy inputs demanded by industry \( j \) at time \( t \)
\( L_{j,t} \): quantity of labour input demanded by industry \( j \) at time \( t \)
\( L_{D,t} \): aggregate labour input demand at time \( t \)
\( \text{ET}_{j,t} \): composite of energy inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{ET},j,t} \): index price of energy inputs demanded by industry \( j \) at time \( t \)
\( \text{EF}_{j,t} \): composite of combustible energy inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{EF},j,t} \): index price of combustible energy inputs demanded by industry \( j \) at time \( t \)
\( \text{ELEC}_{j,t} \): non-combustible energy input demanded by industry \( j \) at time \( t \)
\( P_{\text{ELEC},j,t} \): price of non-combustible energy input demanded by industry \( j \) at time \( t \)
\( \text{RP}_{j,t} \): composite of refined petroleum inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{RP},j,t} \): index price of refined petroleum inputs demanded by industry \( j \) at time \( t \)
\( \text{TGAS}_{j,t} \): composite of natural gas input demanded by industry \( j \) at time \( t \)
\( P_{\text{TGAS},j,t} \): index price of natural gas input demanded by industry \( j \) at time \( t \)
\( \text{COAL}_{j,t} \): quantity of coal demanded by industry \( j \) at time \( t \)
\( P_{\text{COAL},j,t} \): price of coal that is demanded by industry \( j \) at time \( t \)
\( \text{MF}_{j,t} \): composite of mobile energy inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{MF},j,t} \): index price of mobile energy inputs demanded by industry \( j \) at time \( t \)
\( \text{MAT}_{j,t} \): composite of material inputs demanded by industry \( j \) at time \( t \)
\( P_{\text{MAT},j,t} \): index price of material inputs that is demanded by industry \( j \) at time \( t \)
\( Y_{G,t} \): government income at time \( t \)
\( I_{K,i,j,t} \): investment demand for physical capital of good \( i \) at time \( t \)
\[ I_{Hi,t} \] : investment demand for knowledge capital of good \( i \) at time \( t \)

\[ P_{Hi,t} \] : unit cost of knowledge capital investment of good \( i \) at time \( t \)

\[ C_{Gi,t} \] : government consumption of good \( i \) in period \( t \)

\[ P_{Ci,t} \] : gross price of good \( i \) at time \( t \)

\[ C_{i,t} \] : quantity of composite of good \( i \) at time \( t \)

\[ B_t \] : foreign borrowing at time \( t \) in foreign currency unit

\[ X_{j,t}^D \] : quantity supplied to domestic market from industry \( j \) at time \( t \)

\[ P_{X_{j,t}}^D \] : price of good supplied to domestic market from industry \( j \) at time \( t \)

\[ X_{j,t}^E \] : quantity supplied to export markets from industry \( j \) at time \( t \)

\[ P_{X_{j,t}}^E \] : export price of good supplied from industry \( j \) at time \( t \)

\[ C_{i,t}^D \] : quantity consumed of good \( i \) sourced from domestic market at time \( t \)

\[ P_{C_{i,t}}^D \] : index price of good \( i \) sourced from domestic market at time \( t \)

\[ C_{i,t}^{IM} \] : quantity of import of good \( i \) at time \( t \)

\[ P_{C_{i,t}}^{IM} \] : price of import of good \( i \) at time \( t \)

\[ Z_t \] : economy-wide value of the firms at time \( t \)

* Variables with time subscript \( \tau \) represent their steady-state values

\[ E_t \] : total emissions at time \( t \)

\[ E_{Hi,t} \] : household emissions at time \( t \)

\[ E_{Q_{j,t}} \] : industrial (process) emissions produced from good \( j \) at time \( t \)
Appendix D: List of parameters (subscript $t$ in any variable denotes the time period $t$)

$ER_t$: exchange rate (price of foreign currency in terms of domestic currency); model numéraire

$\alpha$: share parameter in the utility function

$\beta_{j,t}$: exogenous emission intensity parameter used to allocate permit

$e_{j,t} \left( = \sum_i e_{i,j,t} \right)$: emission factor for input $i$ used in sector $j$ at time $t$

$n$: population growth rate

$r$: interest rate

$\rho$: rate of time preference

$\delta_{K,j}$: physical capital depreciate rate

$\delta_{H,j}$: knowledge capital depreciation rate

$\tau_y$: income tax rate on non-dividend income

$\tau_d$: income tax rate on dividend income

$\tau_{c,i}$: indirect tax rate on consumption good $i$

$\tau_{Q,j}$: producer sales tax rate on good produced in sector $j$

$\tau_{i,j}$: tax rate on commodity $i$ used as intermediate goods in sector $j$

$\xi_{H,i,t}$: emission factor of good $i$ used in household consumption at time $t$

$\xi_{Q,i,t} \left( = \xi_{Q,i,j,t} \right)$: emission factor of good $i$ used as intermediate (in industry $j$) input in production at time $t$

$\theta_c$: share parameter for consumption aggregator function

$A_c$: shift parameter for consumption aggregator function

$\sigma_c$: elasticity of substitution between energy and non-energy consumption goods

$A_{HEF}$: shift parameter for household energy aggregator function

$\theta_{HEF}$: share parameter for household energy aggregator function

$\sigma_{HEF}$: elasticity of substitution between combustible and non-combustible energy goods

$A_{HEF}$: shift parameter for household combustible energy aggregator function

$\theta_{HEF}$: share parameter for household combustible energy aggregator function

$\sigma_{HEF}$: elasticity of substitution among combustible energy commodities

$A_{HRP}$: shift parameter for household refined petroleum aggregator function
\( \theta_{HRP,j} \): share parameter for household refined petroleum aggregator function

\( \gamma_{H,j} \): Leontief co-efficient for natural gas used in the household

\( A_{HOG} \): shift parameter for household non-energy commodities aggregator function
\( \theta_{HOG} \): share parameter for household non-energy commodities aggregator function
\( \sigma_{HOG} \): elasticity of substitution between mobile energy and material goods

\( A_{HMF} \): shift parameter for household mobile energy aggregator function
\( \theta_{HMF,j} \): share parameter for household mobile energy aggregator function

\( A_{HMAT} \): shift parameter for household material goods aggregator function
\( \theta_{HMAT,j} \): share parameter for household material goods aggregator function
\( \sigma_{HMAT} \): elasticity of substitution among material goods

\( \gamma_j \): shift parameter for composite production function in sector \( j \)
\( \phi_j \): share parameter for composite production function in sector \( j \)
\( \bar{H}_{j,t} \): average knowledge stock in sector \( j \) at time \( t \)
\( \bar{T}_{H,j,t} \): industry wide expenditure on R&D in industry \( j \) at time \( t \)
\( \bar{\beta} \): the parameter that regulates the magnitude of technological spillovers
\( \sigma_{Q,j} \): elasticity of substitution between knowledge capital and composite output in sector \( j \)
\( \alpha_{Q,j} \): composite shift parameter for production function in sector \( c \)

\( \beta_{K,j} \): adjustment cost parameter for physical capital investment in sector \( j \)
\( \beta_{H,j} \): adjustment cost parameter for knowledge capital investment in sector \( j \)

\( A_{x,j} \): shift parameter for production function in industry \( j \)
\( \phi_{x,j} \): share parameter for production function in industry \( j \)
\( \sigma_{x,j} \): elasticity of substitution between composite of intermediate and value-added–energy goods

\( A_{VAE,j} \): shift parameter for value-added–energy aggregator function in industry \( j \)
\( \phi_{VAE,j} \): share parameter for value-added–energy aggregator function in industry \( j \)
\( \sigma_{VAE,j} \): elasticity of substitution between capital-energy composite and labour used in industry \( j \)

\( A_{KET,j} \): shift parameter for physical capital–energy aggregator function in industry \( j \)
\( \phi_{KET,j} \): share parameter for physical capital–energy aggregator function in industry \( j \)
\( \sigma_{KET,j} \): elasticity of substitution between physical capital and energy composite inputs used in industry \( j \)

\( A_{ET,j} \): shift parameter for energy aggregator function in industry \( j \)

\( \varphi_{ET,j} \): share parameter for energy aggregator function in industry \( j \)

\( \sigma_{ET,j} \): elasticity of substitution between combustible and non-combustible composite energy inputs used in industry \( j \)

\( A_{EF,j} \): shift parameter for combustible energy aggregator function in industry \( j \)

\( \varphi_{EF,j} \): share parameter for combustible energy aggregator function in industry \( j \)

\( \sigma_{EF,j} \): elasticity of substitution among combustible energy inputs used in industry \( j \)

\( A_{RP,j} \): shift parameter for refined petroleum aggregator function in industry \( j \)

\( \varphi_{RP,j} \): share parameter for refined petroleum aggregator function in industry \( j \)

\( \varphi_{GAS,j} \): Leontief co-efficient for natural gas used as input in industry \( j \)

\( A_{INT,j} \): shift parameter for intermediate inputs aggregator function in industry \( j \)

\( \varphi_{INT,j} \): share parameter for intermediate inputs aggregator function in industry \( j \)

\( \sigma_{INT,j} \): elasticity of substitution between mobile energy and material inputs in industry \( j \)

\( A_{MF,j} \): shift parameter for mobile energy aggregator function in industry \( j \)

\( \varphi_{MF,j} \): share parameter for mobile energy aggregator function in industry \( j \)

\( \sigma_{MF,j} \): elasticity of substitution among mobile energy and material goods

\( \varphi_{MAT,j} \): Leontief co-efficient for materials used as inputs in industry \( j \)

\( P_{i,t} \): net of tariff price of imported good \( i \) at time \( t \) in foreign currency units

\( P_{j,t} \): price of exported good \( j \) in world market at time \( t \) in foreign currency units

\( \tau_{IM,j} \): import tariff rate on good \( i \)

\( \tau_{I,j} \): tax rate on physical capital investment

\( A_{X,j} \): shift parameter in export aggregator function from firm \( j \)

\( \eta_{j} \): share parameter in export aggregator function from firm \( j \)

\( \sigma_{X,j} \): elasticity of substitution between domestic and export supplies from firm \( j \)
\( A_{m,i} \): shift parameter in import aggregator function for good \( i \)

\( \kappa_i \): share parameter in import aggregator function for good \( i \)

\( \sigma_{m,i} \): elasticity of substitution between domestic and import demands for good \( i \)

\( \beta_{HINV,i} \): share of good \( i \) in R&D investment demand

\( \beta_{KINV,i} \): investment demand coefficient of physical capital for good \( i \)
CHAPTER THREE
Emission Permit Banking and Induced Technological Change

3.1 Introduction:

Economic literature on climate change mitigation policy has established a number of important properties regarding the efficiency of marketable emission permits that can be used to attain a predetermined level of emissions. The most important of all is the property that was initially established by Montgomery (1972). It states that for any given emissions standard or a targeted level of aggregate emissions, a system of marketable permits can achieve the target at the least cost.

In general, research on marketable permits has identified three sources of potential cost savings (Rubin 1996). Two of these sources are intra-temporal: emissions trading among firms and emissions averaging within the sources (e.g. enforcing the principle of equi-marginal abatement costs across different plants of a firm would ensure minimization of overall costs for that firm). The other cost-saving property of marketable permits is of inter-temporal in nature, namely, emissions trading across time. The idea behind it is simple – facilitated by banking and borrowing of permits, when permits are traded across time, it allows firms to lower their cost of compliance under emission regulation (e.g. aggregate emissions, capped at a targeted level) by allowing them to adjust their emissions stream more flexibly across time (i.e. over the horizon of an emission mitigation program). In this instance, the efficiency gains arise from two fronts. First, firms are allowed to move permits among sources within the firm not only intra-temporally but also across time. Secondly, inter-temporal permit trading allows firms to trade permits among themselves thereby not only assisting the environmental regulator (i.e. the social planner) to attain the aggregate emissions regulation target at a lower cost, but also allowing the firms to benefit from the potential profit earning opportunity that arises from trading surplus permits across time.

Permit banking and borrowing, in fact, has been already put into practice by a few regulatory authorities. Most notably, in the U.S. the 1990 Clean Air Act Amendments allowed firms to bank (but not borrow) SO$_2$ permits. Another example is the Low-Emission Vehicle Program in California that allowed both banking and borrowing of hydrocarbon emissions.
One of the crucial properties of bankable permits\textsuperscript{65}, as identified in the literature, is that when firms are allowed to bank and borrow permits, the price path of permits becomes dependent on the amount of available permits at the disposal of private firms. This makes the permit price path depended across time. This occurs since without banking and borrowing the regulator essentially controls the permit price path through its authority over the number of permits it issues in each period (i.e. the authority to control the supply of permits commensurate to the overall targeted level of emissions). When banking and borrowing is allowed, the regulator partially loses its power to influence the trajectory of permit price in that the price of permits also becomes a function of permits available at the disposal of private firms. Hence, even though the overall number of permits remains constant over the horizon of a given emissions regulation program, with banking and borrowing the trajectory of permit price becomes more volatile than without\textsuperscript{66}.

This has other important regulatory implications, which has been explored in a separate, albeit related stream of the literature that focuses on the role of endogenous technological change into the emissions mitigation policy. Technological change, in this branch of the literature, is typically introduced through what is known as the inducement process. The induced technological change (ITC) hypothesis, originally credited to Hicks (1932), holds that when technology enters the scenario, the terms of trade-off between the marginal cost of pollution control and its marginal social benefit alters (Popp et al., 2009). This provides the incentive to promote innovations in order to save the use of relatively expensive polluting inputs\textsuperscript{67} into the production process. Goulder and Schneider (G&S, 1999) and Sue Wing (2003) provide modeling framework with ITC embedded into the modeling structure. Both of these papers essentially build on the approach that parallels the endogenous-growth literature by including a stock of “knowledge capital” that is generated through investments in R&D activities.

\textsuperscript{65} Through out the paper, we use ‘banking and borrowing emission permits’ and ‘bankable permits’ interchangeably.

\textsuperscript{66} In part, the environmental regulator can tackle this by issuing the correct amount of total emission rights in addition to specifying the correct inter-temporal permit trading ratio for banking and borrowing within a regulatory horizon. For further exposition on this, see Leiby and Rubin (2001).

\textsuperscript{67} Polluting inputs become relatively more expensive due to the regulatory requirement to hold emission permits or alternatively due to the imposition of emission tax.
Given the ITC process, and given its link to the change in the relative price of polluting inputs that, in turn, depends on the price of emission permits under a cap-and-trade (CAT) framework, it appears that the rate of technological change and emission mitigation policy are connected. To the extent that public policies affect the prices of emissions (e.g. permit price or, alternatively, taxes on carbon-based fuels), they affect incentives to invest in research and development (R&D) aimed at technological innovation (e.g. to develop new emissions-benign production methods). That is, through impacts on patterns of R&D spending, climate policy can alter the path of knowledge acquisition.

What does this connection imply for the design of emission mitigation policy when emission permits are bankable? In particular, how do the optimal timing and the extent of emissions abatement as well as the optimal time path of emission permit price change when we recognize the potential of ITC in the presence of bankable permits?

This question bears relevance on two grounds. First, since permit prices are not independent across time when permits are bankable, the price of permits will be affected under the ITC scenario as opposed to a scenario that does not consider ITC (the NITC scenario). To date, no research has attempted to unravel this link. Secondly, the question sheds light on the conflicting views in the literature regarding the justification for regulatory support for innovation (e.g. R&D subsidies).

To address these issues, the current paper develops a model that explicitly considers ITC in a framework that allows banking and borrowing of emission permits within the regulatory horizon. The model builds on the paper by Leiby and Rubin (2001) on marketable permits and extends the work of Goulder and Mathai (2000), who examine similar aspects of ITC within the emission control regulatory framework.

In particular, our paper extends the work of Leiby and Rubin and Goulder and Mathai by providing a more general framework that links permit trading to banking and borrowing as well as to ITC. Leiby and Rubin examine the impact of permit banking and borrowing on the time path of emissions. Goulder and Mathai, on the other hand, examine the impact of policy-induced technological change on the optimal emissions abatement path. Neither of these
papers, however, examines this and other associated issues by considering permit banking and technological change together in a unified framework. Given that emission permit price is not independent across time when banking and borrowing is allowed, it has implications for the process through which ITC functions and, consequently bears implications for the time profile of optimal emission abatement. We explore this issue in detail and address the gap in the literature by linking the two relevant streams into a unified framework in order to provide a comprehensive assessment of the emissions control mechanism. In doing so, we derive analytical expressions characterizing the optimal path of emissions and permit price path under the ITC scenario.

The present investigation thus differs from existing relevant studies, and hence contributes to the literature in two important ways. First, it derives analytical results revealing the impact of ITC on optimal time profiles of permit price and emission abatement when banking and borrowing of emission permits is allowed. Second, and more importantly, it unifies two relevant frameworks of climate policy research: it qualitatively evaluates the role of banking and borrowing of permits and the role that ITC plays in it to mitigate harmful effluents.

The highlight from the current model indicates that the presence of ITC implies a lower optimal emissions path over time. We find that the impact of ITC on the optimal permit price path varies and critically depends on the extent of the role that innovation plays in reducing the marginal cost of emissions. Our conclusion qualitatively differs from other studies (notably Goulder and Mathai 2000) that suggest that while the impact of ITC on the optimal abatement path varies, its presence lowers the path of optimal carbon taxes over time. The difference can be attributed to the fact that the framework in Goulder and Mathai does not allow for banking and borrowing of emission permits.

The rest of the paper is organized as follows. Section 2 provides an overview of the modeling framework and defines the elements that lead to the formation of the social planner’s problem. Section 3 formulates the benevolent social planner’s problem and characterizes the solution. Section 4 presents and interprets results from the decentralized model. Section 5 identifies the implication of ITC on the optimal path of emissions. Section 6 characterizes the optimal path of output and emissions. Section 7 traces out the implication of ITC on the
optimal permit price path. The final section offers some concluding remarks and suggests future research directions.

3.2 An Overview of the Modeling Framework:

In this section we provide an overview of the modeling framework. The section offers definitions of various components of the model that will help developing the problem encountered by the social planner and by a decentralized firm as identified in section 3 and 4 respectively.

The social planner (i.e. an environmental regulator) typically faces the objective to maximize consumer and producer surplus less social damages from the production of good, $Y(t) = \sum_{i=1}^{n} y_i(t)$, which causes instantaneous emission flows, $E(t) = \sum_{i=1}^{n} E_i(t)$, and a cumulative emission stock $S(t)$; along with the objective to achieve a targeted level of atmospheric emission concentration, $\bar{S}$, by a specified future date $T$. Once achieved, the regulator maintains $\bar{S}$ beyond period $T$. We assume that producers of good $y_i$ face perfect competition in both output and input markets.

At any point in time $t$, letting $S(t) = \sum_{i=1}^{n} S_i(t)$ be the total stock of emissions from all firms, the stock increases whenever the sum of all firms’ emissions is greater than the nature’s assimilative capacity. In other words, the motion equation for emission stock follows $\frac{dS(t)}{dt} = S(t) = \sum_{i=1}^{n} E_i(t) - \gamma S(t)$ where $\gamma$ represents the rate of natural decay of emissions.

Let $B(Y(t))$ be the function that captures benefits from the consumption of output $y_i$. We assume $C_i(y_i, E_i, H(t))$ to be the cost function of a perfectly competitive representative firm $i$, where $y_i(t)$ and $E_i(t)$ respectively represent its level of output and emissions at time $t$, and $H(t)$ signifies the stock of knowledge characterizing the state-of-the-art technology at period $t$. In other words, given the knowledge stock, $H(t)$, firm $i$’s minimum total cost of producing
output $y_i(t)$ and emission level $E_i(t)$ is $C_i(y_i, E_i, H_i, t)$. We further hold that the representative firm’s production behaviour satisfies the following regularity assumptions: $\frac{\partial C_i(t)}{\partial y_i} = C_y > 0$ and $\frac{\partial^2 C_i(t)}{\partial y_i^2} = C_{yy} > 0$, that is, $C_i$ is strongly convex in $y_i$; $C_E < 0$ and $C_{EE} > 0$, that is, $C_i$ is strongly convex in $E_i$. This implies that the representative firm’s total production cost decreases at an increasing rate when the regulator allows additional level of emissions (without a penalty). Furthermore, $\frac{\partial^2 C_i(t)}{\partial y_i \partial E_i} = C_{ye} < 0$, which implies that the marginal cost of production declines with additional levels of emissions.

Following Montgomery (1972), Färe et al. (1989 and 2005) and Pasurka (2005) we assume separability of production and emission abatement technology\(^{68}\), where both the production of output (a good) and the combustible emissions (a bad that is generated as a by-product in the process) are assumed to be disposable\(^{69}\) and where asymmetric treatment is accorded to the emission abatement technology\(^{70}\). This implies that $\frac{\partial^2 C_i(t)}{\partial y_i \partial H_i} = C_{yh} = 0$, that is, the effect of technological change (induced through incremental knowledge stock) is purely related to the emission abatement technology such that $\frac{dy_i(t)}{dH} = y_h = 0$. In other words, the separability between production and abatement technologies ensures that the technological change that affects the abatement technology does not influence the production process and hence does not alter the level of output $y_i$ \textit{per se}. The assumption is consistent with the end-of-pipe abatement technologies\(^{71}\) such as carbon capture and storage, carbon sequestration and carbon scrubbers.

\(^{68}\) While the modeling framework presented in Montgomery (1972) can handle both separability and non-separability issues, it is essentially based on the assumption of technological separability. For further discussion on this see Schwabe (1999).

\(^{69}\) The disposability assumption refers to the models in Färe et al. (1989 and 2005) and Pasurka (2005) where the representative firm’s production entails multiple outputs (i.e. production generates both a good and a bad output). In this context, disposability essentially implies that getting rid off the bad output (i.e. emissions) could be costly.

\(^{70}\) Following Färe (1989) and Pasurka (2005), in particular while we assume the good output to be strongly or weakly disposable, the bad output (i.e. emissions) is assumed to be only weakly disposable. This ensures that abatement technology used by the firm to get rid off emissions is a costly endeavour.

\(^{71}\) The end-of-pipe technology refers to an emission abatement technology that is separable from the production technology in that it does not alter the input mix but only is added on at the end of the production process with the purpose of reducing the (by-product) emissions.
We further assume \( \frac{\partial^2 C_i(t)}{\partial E \partial H} = C_{EH} > 0 \), that is, the cost saving from marginal emissions increases with incremental knowledge stock. This implies that the advancement of emission saving technology accentuates cost savings on the part of the firms by allowing them to reduce emissions, and thus comply with the regulation, for the same level of production.

Note that the last assumption underscores the positive impact of technological advancement. It underlies the benefits of ITC\(^{72}\) in reducing firms’ costs when the regulator makes pollution costly either by imposing a pollution tax or by compelling firms to hold pollution permits. Also note that, given our notation, \(-C_e\) represents the marginal cost of abatement. Thus, the assumption \( C_e < 0 \), or equivalently \( -C_e > 0 \), represents the marginal cost of abatement. Given this interpretation, it is easy to see that the assumption \( C_{EH} > 0 \), or equivalently \( -C_{EH} < 0 \), signifies the reduction in the marginal cost of abatement due to the positive impact of ITC (that comes through incremental knowledge stock).

Following Goulder and Mathai (2000), we model the process of technological change through an R&D investment approach, where the evolution of the knowledge stock is represented as: \( \frac{dH(t)}{dt} = H(t) = \alpha H(t) + k \Psi \left( \sum_{i=1}^{n} I_i(t), H(t) \right) \). This equation describes the process of technological change that occurs through investments in R&D, \( I_i(t) \). The R&D investments serve to increase the stock of knowledge via the knowledge accumulation function \( \Psi(\cdot) \), where we impose the regularity assumptions \( \Psi(\cdot) > 0; \; \Psi_i(\cdot) > 0; \; \Psi_{ii}(\cdot) < 0; \; \Psi_{HH}(\cdot) > 0 \) and \( \Psi_{HHH}(\cdot) > 0 \). These assumptions suggest that knowledge increases with R&D investment. However, note that R&D investments effectively contributes to the knowledge accumulation process only when the parameter \( k > 0 \). Hence, the case of \( k > 0 \) represents the induced technological change (the ITC scenario) and \( k = 0 \) would signify an instance where ITC is absent (the NITC scenario) (Goulder and Mathai, 2000). In addition to ITC, we also allow for autonomous technological change at the rate \( \alpha \) represented by the first term of the \( H(t) \) function. The presence of \( \alpha \) simply captures all non-climate policy related reasons for technological advancement.

\(^{72}\) Thus induces firms to invest in R&D activities that are assumed to bring technological advancements.
Emissions, in our model, are assumed to harm social welfare as described by the convex damage function, \( D(E(t), S(t)) \), where \( D_E > 0 \), \( D_{SE} > 0 \); \( D_S > 0 \), \( D_{ES} > 0 \); and \( D_{SE} > 0 \). These imply that not only social damage increases at an increasing rate with the level of both flow, \( E_i(t) \), and stock pollution, \( S(t) \), the damage from stock pollution intensifies with the incremental flow pollution. Note that, following Leiby and Rubin (2001), we postulate the damage function \( D(\cdot) \) to incorporate both flow and stock pollution. The rationale for including both types of pollution is based on the differing types of damages that these pollutants cause. When the stock pollution is considered, damages do not stop at the end of the regulatory program in period \( T \). Rather the damage continues until the stock pollution decays to a benign level. As identified in Leiby and Rubin (2001), it is important to consider both types of pollution for two reasons. First, the interaction between the flow and stock pollution corresponds to most types of greenhouse gas emissions where flow pollution adds to the atmospheric concentration of the stock pollution. Secondly, given the link, for the social planner’s problem it is important to take into account the damages that stock pollution continues to cause until it reaches a benign level. Consequently, we capture this through the term \( F(S(T)) \) in the regulator’s problem, where the term signifies the value of damages for all periods \( t \geq T \rightarrow \infty \) evaluated in period \( T \) dollars.

### 3.3 Inter-temporal Emission Allocation with ITC - The Social Planner’s Solution:

Given the specifications identified in section 2, we are now equipped to define the social planner’s problem. With the (social) discount rate \( \rho \), the environmental regulator encounters the following objective functional:

\[
J^* \equiv \max_{\{y_i,E_i,I_i\}} \int_0^T e^{-\rho t} \left( B \left( \sum_{i=1}^n y_i(t) \right) - \sum_{i=1}^n C_i(y_i,E_i,H_i(t)) - D \left( \sum_{i=1}^n E_i(t), S(t) \right) - \sum_{i=1}^n P_i I_i(t) \right) dt - e^{-\rho T} F(S(T)) \tag{1}
\]

Subject to:

The motion equation for emission stock: \( \dot{S}(t) = \sum_{i=1}^n E_i(t) - \gamma S(t) \) \tag{2-1}

The motion equation for knowledge stock: \( \dot{H}(t) = \alpha H(t) + k \Psi \left( \sum_{i=1}^n I_i(t), H(t) \right) \) \tag{2-2}

The regulatory objective: \( S(t) \leq \bar{S} \text{ for } t \geq T \) \tag{2-3}
with \( S(0) \) and \( H(0) \) given

and where \( P_i \) is the unit price of R&D investment in real terms\(^{73}\).

The current-value Hamiltonian, \( \mathcal{H}(t) \), associated with the optimization problem for \( t<T \) is:

\[
\mathcal{H}(t) = B \left( \sum_{i=1}^{n} y_i(t) \right) - \sum_{i=1}^{n} C_i(y_i,E_i,H_i) - D(E,S,t) - \sum_{i=1}^{n} P_i I_i(t)
+ \Lambda(t) \left[ \sum_{i=1}^{n} E_i(t) - \gamma S(t) \right] + \lambda(t) \left[ \alpha H(t) + k \Psi \left( \sum_{i=1}^{n} I_i(t), H(t) \right) \right]
\]

[3-1]

where \( \Lambda(t) \) is the shadow cost of \( S(t) \) and \( \lambda(t) \) is the shadow benefit of \( H(t) \). For \( t \geq T \), however, we must form the following Lagrangian:

\[
\mathcal{L}(t) = \mathcal{H}(t) + \eta(t) \left( \bar{S} - S(t) \right)
\]

[3-2]

Assuming an interior solution, from the Pontryagin principle we derive the following first-order conditions (FOCs)\(^{74}\):

**Control variables**

\[
\mathcal{H}_y(t) = 0 \Rightarrow B_y = C_y
\]

[O-1]

\[
\mathcal{H}_E(t) = 0 \Rightarrow -(C_E + D_E) + \Lambda = 0 \Rightarrow -C_E = D_E + (-\Lambda)
\]

[O-2]

\[
\mathcal{H}_I(t) = 0 \Rightarrow -P_i + \lambda k \Psi_i = 0 \Rightarrow P_i = \lambda k \Psi_i
\]

[O-3]

**State variables**

For \( t<T \): \( -\mathcal{H}_S(t) = \dot{\Lambda} - \rho \Lambda \)

\[
\Rightarrow -(D_S - \Lambda \gamma) = \dot{\Lambda} - \rho \Lambda \Rightarrow \dot{\Lambda} = D_S + (\gamma + \rho) \Lambda
\]

[O-4-1]

and for \( t \geq T \): \( -\mathcal{H}_S(t) = \dot{\Lambda} - \rho \Lambda - \eta \)

\[
\Rightarrow -(D_S - \Lambda \gamma) = \dot{\Lambda} - \rho \Lambda - \eta \Rightarrow \dot{\Lambda} = D_S + \eta + (\gamma + \rho) \Lambda
\]

[O-4-2]

---

\(^{73}\) Following Goulder and Schneider (1999), we hold \( P_i \left( I(t) \right) \) to be non-decreasing in \( I(t) \); that is, the average cost of R&D investment increases with the level of R&D. This captures the idea that the opportunity cost (to other sectors of the economy) of employing R&D resources for climate policy related technological advancement increases as more and more resources are employed.

\(^{74}\) To reduce clutter, we suppress the subscript \( i \) for firm \( i \).
\[-H_H(t) = \dot{\lambda} - \rho \lambda \Rightarrow -(-C_H + \lambda(\alpha + k\Psi_H)) = \dot{\lambda} - \rho \lambda \Rightarrow \dot{\lambda} = C_H - \lambda(\alpha + k\Psi_H - \rho) \quad [O-5]\]

**Co-state variables**

\[H_{\lambda}(t) = \dot{S} \Rightarrow \dot{S}(t) = \sum_{i=1}^{n} E_i(t) - \gamma S(t) \quad [O-6]\]

\[H_{\lambda}(t) = \dot{H} \Rightarrow \dot{H}(t) = \alpha H(t) + k\Psi\left(\sum_{i=1}^{n} I_i(t), H(t)\right) \quad [O-7]\]

Finally the terminal condition yields:

\[\Lambda(T) = -\frac{\partial F(S(T))}{\partial S(T)} \quad [O-8]\]

### 3.3.1 Economic Interpretation of the Necessary Conditions for Optimization of the Social Planner's Problem

The FOC [O-1] says that the social planner should equate each firm’s marginal production costs to the marginal benefit derived from the output in every period. FOC [O-2] indicates that, over the planning horizon, the regulator should choose an emission path, \(E_i(t)\), for firm \(i\) such that the marginal abatement cost is equal to the marginal (flow) damages net of the shadow benefit from saving a unit pollution plus the current shadow cost of another unit of pollution. FOC [O-3], on the other hand, shows that, when knowledge is valuable (i.e. \(k > 0\)), the social optimal level of R&D investment should be such that the unit price of investment is equal to the marginal value product of investment in every period, measured at the shadow price.

Equation [O-4-1], [O-4-2] and [O-5] provide the necessary first-order conditions with respect to the two state variables: pollution stock, \(S(t)\); and knowledge stock, \(H(t)\). Equation [O-4-1] indicates that, up to the regulatory period \(T\), the optimal path of current shadow cost of pollution stock is dictated by the damages due to the marginal increase in the pollution stock, \(D_S\); the social discount rate, \(\rho\); and the rate of nature’s emission assimilative capacity, \(\gamma\). Equation [O-4-2] indicates that for periods \(t \geq T\), the change in the shadow cost of pollution further depends on shadow cost of regulatory restriction \(\eta\). Equation [O-5], on the other hand,
shows that the optimal path of current shadow benefit of incremental knowledge stock is governed by the extent of cost saving due to the incremental knowledge stock, $C_H$ ($< 0$ by assumption); the autonomous knowledge growth rate, $\alpha$; and the social discount factor, $\rho$.

Finally equation [O-6] and [O-7] simply restate the state equations while equation [O-8] indicates that the shadow cost of stock pollution at the end of the planning horizon should be equal to the discounted marginal damages from the stock pollution measured in period $T$ dollars (until the pollution stock reaches a benign level).

The FOC [O-5] has an interesting implication. It signifies the value of incremental stock of knowledge as perceived by the society and establishes a link between the social discount rate $\rho$ and the value of incremental knowledge stock. In other words, equation [O-5] identifies the condition under which the knowledge accumulation is deemed valuable by the society\textsuperscript{75}. This is important to consider since it provides the rationale for the ITC process to be included into the framework. This condition is summarised in Proposition 1 below:

**Proposition 1:** The value of the incremental (i.e. marginal) knowledge (and the innovation derived from it) grows beyond a critical social discount rate $\rho^*$. 

FOC [O-5] yields:

$$- H_n(t) = \dot{\lambda} - \rho \lambda$$

$$\Rightarrow -(C_H + \lambda(\alpha + k\Psi_H)) = \dot{\lambda} - \rho \lambda$$

$$\Rightarrow \dot{\lambda} - \rho \lambda = C_H - \lambda(\alpha + k\Psi_H)$$

$$\Rightarrow \dot{\lambda} = C_H - \lambda(\alpha + k\Psi_H) + \lambda \rho$$

where $\lambda$ signifies the marginal value of innovation that emanates from the incremental knowledge stock (which, in turn, is derived through the investment in R&amp;D activities) and is positive. This reflects the fact that if incremental knowledge from R&amp;D activities yields emission-saving innovation, abatement cost would be lower and social return would be higher. This cost saving is denoted by $C_H$ and hence is assumed to be negative. Together these

\textsuperscript{75}The condition also underscores that society will increasingly value investments in R&amp;D the higher the discount rate it places on the future.
specifications imply that the first two terms in \([O-5]\)' are negative. Hence, \(\lambda > 0\) iff 
\[
\lambda \rho > \left[ -C_H + \lambda(\alpha + k\Psi_H) \right] \text{ or, } \rho > \rho' \text{ where, } \rho' = \left[ \frac{-C_H}{\lambda} + \alpha + k\Psi_H \right] 
\]
Q.E.D.

Proposition 1 shows that the critical value of the discount rate depends on the extent of normalized cost saving from innovation, \(-\frac{C_H}{\lambda}\); the autonomous knowledge growth rate, \(\alpha\), and the positive effect of incremental knowledge on the overall knowledge accumulation process, \(\Psi_H\). It also signifies that the importance of the role of incorporating ITC into the framework increases as the society discounts the future periods sufficiently.

3.4 Permit Banking with ITC - The Decentralized Solution:

In this section we develop the decentralized solution. To do so, however, we need to introduce two modifications to the regulator’s problem that we have seen in section 3. First we need to introduce permit banking/borrowing into the framework, which enters into the representative firm’s optimization problem in order for it to comply with the environmental regulation. Second, unlike the social planner, in the decentralized solution a representative firm only cares about maximizing the net present value of its profit stream. Consequently, the social damage function should be removed from the firm’s objective functional.

To carry out these changes, let us begin by defining \(P_y\) as the price of output \(y\), determined in a perfectly competitive output market. We assume that the regulator controls emissions by issuing flow permits, which are allocated to firms as endowments \(\omega(t)\). Given the endowments and the demand (that emanates from the optimal production decision), firms trade emission rights in a competitive emission permit market. Permit trading, thus, determines the price of the permits, \(P_x(\chi(t))\), through the forces of demand and supply in the permit market that sustain until the end of the regulatory program at the end of period \(T\). Following the convention, we assume that, in any period, production of a unit output causes a unit of emission for which the firm has to hold one permit.
Once permits are acquired, firms are allowed to hold on to them (i.e. banking \( B_i(t) \)) for use in the later periods or to loan them to others (i.e. lending; which is equivalent to borrowing for the permit recipients) in which case the borrowing firms need to pay \( r \) units for every unit borrowed. Hence, \( r \) represents the inter-temporal permit trading ratio.

Given these specifications, we can write down the equation of motion for permit banking as: 

\[
\dot{B}_i(t) = \omega_i(t) + x_i(t) + rB_i(t) - E_i(t) \]  

This indicates that over time the change in the amount of banked permits \( B_i(t) \) depends on the balance of two opposing forces: the sum of the permit endowment \( \omega_i(t) \), the amount of permit purchased by the firm \( x_i(t) \) and the return (in terms of the units of permits) from exiting (loaned or borrowed) permits \( rB_i(t) \) minus the use of permit \( E_i(t) \) by the firm in order for it to comply with the environmental regulation.

We are now ready to write down the objective functional of a representative firm \( i \), which takes the following expression:

\[
J^{**} = \max_{(0, x, \omega, I)} \int_0^T e^{-\nu t} \left( P_i y_i(t) - C_i(y_i, E_i, H_i, t) - P_i I_i(t) - P_i x_i(t) \right) dt \]  

Subject to:

\[
\dot{S}_i(t) = E_i(t) - \gamma S_i(t) \quad [5-1]
\]

\[
\dot{H}_i(t) = \alpha H_i(t) + k \Psi_i(I_i(t), H_i(t)) \quad [5-2]
\]

\[
\dot{B}_i(t) = \omega_i(t) + x_i(t) + rB_i(t) - E_i(t) \quad [5-3]
\]

The regulatory objective: \( \sum_i S_i(t) \leq \bar{S} \text{ for } t \geq T \) \quad [5-4]

with \( S_i(0), H_i(0) \) and \( B_i(0) \) given

and where \( \nu \) is the private discount rate, which may or may not be equal to the social discount rate \( \rho \). \( ^{78} \)

\( ^{76} \) A negative value of \( x_i(t) \) signifies sale of permits.

\( ^{77} \) While a positive value of \( B_i(t) \) signifies permit lending, a negative value signifies permit borrowing.

\( ^{78} \) For further discussion on this, see footnote 80 on page 164.
The current-value Hamiltonian, $\mathcal{H}_i(t)$, associated with the optimization problem for $t < T$ is:

$$\mathcal{H}_i(t) = P_y(t) - C_y(t) - C_e(t) - P_x(t) + \Lambda_i(t)(E_i(t) - \gamma S_i(t))$$

$$+ \lambda_i(t)(\alpha H_i(t) + k \Psi_i(t)) + \mu_i(t)(\alpha E_i(t) + P_i(t) + r B_i(t) - E_i(t)) \quad [6-1]$$

where $\Lambda_i(t)$, $\mu_i(t)$ and $\lambda_i(t)$ represent the shadow cost (benefit) of relaxing the respective constraints. For $t \geq T$, however, we must form the following Lagrangian:

$$\mathcal{L}_i(t) = \mathcal{H}_i(t) + \eta_i(t) \left( S - \sum_i S_i(t) \right) \quad [6-2]$$

Assuming an interior solution, from the Pontryagin principle we derive the following first-order conditions (FOCs):

Control variables

$$\mathcal{H}_y(t) = 0 \Rightarrow P_y = C_y \quad [O-9]$$

$$\mathcal{H}_E(t) = 0 \Rightarrow -C_E + \Lambda - \mu = 0 \Rightarrow -C_E = \mu + \Lambda \quad [O-10]$$

$$\mathcal{H}_i(t) = 0 \Rightarrow -P_i + \lambda_i \Psi_i = 0 \Rightarrow P_i = \lambda_i \Psi_i \quad [O-11]$$

$$\mathcal{H}_x(t) = 0 \Rightarrow -P_x + \mu = 0 \Rightarrow \mu = P_x \quad [O-12]$$

Further manipulation of equation [O-10] and [O-12] yield: $P_x = \Lambda + (-C_e)$ \quad [7]

State variables

For $t < T$: $-\mathcal{H}_s(t) = \dot{\Lambda} - \nu \Lambda \Rightarrow -(-\Lambda \gamma) = \dot{\Lambda} - \nu \Lambda \Rightarrow (\nu + \gamma) \Lambda = \dot{\Lambda} \Rightarrow \frac{\dot{\Lambda}}{\Lambda} = (\nu + \gamma) \quad [O-13-1]$ \quad [13]

and for $t \geq T$: $-\mathcal{H}_s(t) = \dot{\Lambda} - \nu \Lambda - \eta \Rightarrow -(-\Lambda \gamma) = \dot{\Lambda} - \nu \Lambda - \eta \Rightarrow \frac{\dot{\Lambda}}{\Lambda} = (\gamma + \nu) + \frac{\eta}{\Lambda} \quad [O-13-2]$ \quad [13]

$$-\mathcal{H}_H(t) = \dot{\lambda} - \nu \lambda \Rightarrow -(C_H + \lambda (\alpha + k \Psi_H)) = \dot{\lambda} - \nu \lambda \Rightarrow \dot{\lambda} = C_H - \lambda (\alpha + k \Psi_H - \nu) \quad [O-14]$$

---

79 To reduce clutter, we suppress the subscript $i$ that represents firm $i$.  

162
- $\mathcal{H}_B(t) = \dot{\mathcal{H}} = \gamma S(t) - \gamma S(t)$

Equation [O-12] and [O-15] together imply: $\frac{\dot{\mathcal{H}}}{P} = (v - r)$

Co-state variables

$\mathcal{H}_A(t) = \dot{S} \Rightarrow \dot{S}(t) = E(t) - \gamma S(t)$ [O-16]

$\mathcal{H}_A(t) = \dot{H} \Rightarrow \dot{H}(t) = \alpha H(t) + k \Psi(I(t), H(t))$ [O-17]

$\mathcal{H}_A(t) = B \Rightarrow \dot{B}(t) = \omega(t) + x(t) + rB(t) - E(t)$ [O-18]

3.4.1 Economic Interpretation of the Necessary Conditions for the Decentralized Optimization

Equation [O-16], [O-17] and [O-18] simply restate the state equations that represent the equation of motion for emission stock, for knowledge stock and for the stock of bankable permits respectively. FOC [O-9] states that, at each point in time, firms pick the level of output so as to equate the marginal cost of production with the given market price. The interpretation of FOC [O-5] carries over to the equation [O-10], where $\mu_i$ represents the shadow cost of an additional permit. Through a slight manipulation of FOC [O-10], it is also easy to see that, for the representative firm, the optimality condition dictates that the optimal emission path be such that the shadow cost of permit, $\mu_i$, be equal to the sum of the shadow cost of emission, $\Lambda_i$, and the marginal abatement cost, $-C_E$, in every period. Equation [O-11] is essentially identical to equation [O-3] and has the same economic interpretation. Introduction of the permit banking/borrowing generates FOC [O-12], which dictates the condition that determines the optimal level of permit purchase by a representative firm over the planning horizon. FOC [O-12] states that, at every period, a representative firm would buy permits until the unit price of permits equals its shadow cost. In other words, FOC [O-10] and [O-12] together imply that a representative firm’s permit price path be such that in every period the unit price of permit, $P_i$, be equal to the marginal abatement cost, $-C_E$, plus the shadow cost of pollution, $\Lambda_i$. 
FOC [O-13-1] indicates that under the decentralized solution, up to the regulatory period $T$, the optimal path of current shadow cost of stock pollutant is only dictated by the private discount factor, $\nu$, and the rate of nature’s emission assimilative capacity, $\gamma$. Unlike the social planner’s solution, private solution does not take the marginal damages, $D_s$, into account. The equation also shows that the growth rate of the current shadow cost of pollution is equal to the sum of the private discount rate and the rate of natural decay of emission. Equation [O-13-2] indicates that for periods $t \geq T$, the change in the shadow cost of pollution further depends on the normalized shadow cost of regulatory restriction $\eta_i$. Equation [O-14], which determines the optimal path of current shadow benefit of knowledge stock, is essentially similar to the one derived from the social planner’s problem except for one difference – the decentralized solution differs from the social planner’s solution only by the extent of the difference between the social and private discount rates\textsuperscript{80}. As long as the difference between the two discount rates is taken into account the qualitative result derived in Proposition 1 would carry through. FOC [O-15] determines the emission permit price path for banking an additional permit by the representative firm $i$. Equation [O-12] and [O-15] together imply that when firms are allowed to freely borrow and/or bank permits across time, the permit price will rise at the rate of discount net of the inter-temporal permit trading ratio, $r$. This is essentially a permit arbitrage condition.

### 3.5 Implication of ITC on the Optimal Level of Emissions:

We now examine the effect of ITC on the optimal level of emission. Following Goulder and Mathai (2000), we do this by considering the significance of a change in the parameter $k$. As mentioned above, the case of $k = 0$ corresponds to a scenario with no induced technological change (the NITC scenario), while positive values of $k$ imply the presence of induced technological change (the ITC scenario). Our analysis will focus on incremental increases in $k$ from the point $k = 0$.

\textsuperscript{80} The difference between the social and private discount rate may exist due to knowledge spillover, which creates a wedge between the social and public return from investment in knowledge (i.e. the return from R&D investment). While we acknowledge the existence of knowledge spillover, explicit modeling of spillover effects remains beyond the scope of the current paper.
If (as is assumed) $C_n < 0$ and $C_{nn} > 0$, then additional knowledge is valuable (i.e. the multiplier $\lambda(t)$ is positive). When $k = 0$, all of the growth in knowledge is due to the autonomous term, and the knowledge grows at the rate $\alpha$. In contrast, assuming an interior solution, for strictly positive values of $k$, the planner will find it optimal for society to accumulate at least some additional knowledge. This additional knowledge causes a decrease in optimized costs to a degree dictated by $\lambda(t)$. Thus, the introduction of the ITC option has a negative (or at least non-positive) effect on optimal cost of achieving the given concentration target.

**Proposition 2**: The presence of ITC unambiguously reduces emission along the optimal emissions path.

We examine the impact of introducing ITC on the optimal time profiles of emission by totally differentiating equation [O-2] with respect to $k$, which yields:

$$-\left(C_{EE} \frac{dE_i}{dk} + C_{EH} \frac{dH}{dk} + C_{EH} \frac{dS}{dk}\right) = \left(D_{EE} \frac{dE_i}{dk} + D_{ES} \frac{dS}{dk}\right)$$

$$\Rightarrow -C_{EE} \frac{dE_i}{dk} - C_{EH} \frac{dH}{dk} = D_{EE} \frac{dE_i}{dk} + D_{SE} \frac{dS}{dk}$$

where we have used the fact that $\frac{dy_i}{dk} = 0$ and $D_{ES} = D_{SE}$.

Note that as long as equation [2-1] governs the relationship between the stock and flow pollutants, the last term of the above equation $\frac{dS}{dk}$ can be decomposed as $\frac{\partial S}{\partial E_i} \frac{dE_i}{dk}$ with $\frac{\partial S}{\partial E_i} > 0$.

Hence, we have:

$$-C_{EE} \frac{dE_i}{dk} - C_{EH} \frac{dH}{dk} = \left(D_{EE} + D_{ES} \frac{\partial S}{\partial E_i}\right) \frac{dE_i}{dk}$$

Collecting the term $\frac{dE_i}{dk}$ and rearranging, we get the following:

$$\frac{dE_i}{dk} = \frac{-C_{EH} \frac{dH}{dk}}{\left(C_{EE} + D_{EE} + D_{SE} \frac{S_E}{k}\right)}$$
where, as identified above, all terms in the denominator and the first term in the numerator are positive. Hence, \( \text{sign} \left( \frac{dE}{dk} \right) = \text{sign} \left( -\frac{dH}{dk} \right) \). To the extent that knowledge increases as a result of ITC (i.e. \( \frac{dH}{dk} > 0 \)), we derive \( \frac{dE(t)}{dk} < 0 \). Q.E.D.

3.6 Characterizing the Optimal Time Path of Output and Emissions:

Exiting literature on emissions banking and borrowing has established important properties regarding the optimal time path of output and emissions. However, these characterizations almost exclusively have been conducted without considering the role of ITC into the emission banking and borrowing framework. Given that recent climate policy modeling literature, which incorporates ITC into the framework, indicates to a potential favourable impact of ITC in mitigating emissions abatement, in this section we characterize the optimal output and emissions path in the presence of both the ITC and the banking and borrowing of emission permits.

To do so, we begin by totally differentiating FOC [O-9] and [O-10] with respect to time \( t \).

For FOC [O-9] we derive the following:

\[
\frac{d}{dt} \left\{ P_y \left( y(t) \right) \right\} = \frac{d}{dt} \left\{ C_y \left( y(t), E(t), H(t) \right) \right\}
\]

\[
\Rightarrow \frac{\partial P_y}{\partial y} \frac{dy(t)}{dt} = C_{yy} \frac{dy(t)}{dt} + C_{ye} \frac{dE(t)}{dt} + C_{yh} \frac{dH(t)}{dt}
\]

\[
\Rightarrow \frac{\partial P_y}{\partial y} \cdot y = C_{yy} y + C_{ye} E + C_{yh} H
\]

\[
\Rightarrow \left( \frac{\partial P_y}{\partial y} - C_{yy} \right) \dot{y} = C_{ye} \dot{E} + C_{yh} \dot{H}
\]

\[ [9-1] \]

Similarly FOC [O-10] yields:

\[
\frac{d}{dt} \left\{ -C_E \left( y(t), E(t), H(t) \right) \right\} = \frac{d}{dt} \left\{ \mu(t) - \Lambda(t) \right\}
\]
By simultaneously solving equation [9-1] and [9-2], we finally arrive at:

$$\Rightarrow -C_{yE} \frac{dy(t)}{dt} - C_{EE} \frac{dE(t)}{dt} - C_{EH} \frac{dH(t)}{dt} = \frac{d\mu(t)}{dt} - \frac{d\Lambda(t)}{dt}$$

$$\Rightarrow -C_{yE} \dot{y} - C_{EE} \dot{E} - C_{EH} \dot{H} = \mu - \Lambda$$

$$\Rightarrow C_{yE} \dot{y} + C_{EE} \dot{E} = \Lambda - \mu - C_{EH} \dot{H} \quad [9-2]$$

By simultaneously solving equation [9-1] and [9-2], we finally arrive at:

$$\dot{y} = \frac{-C_{yE} \left( \gamma A + vC_E - C_{EE} \dot{E} + C_{EE} C_{yH} \dot{H} \right)}{\left[ C_{yy} - \frac{dP_s}{dy} \right] C_{EE} - C_{yE}^2}$$

$$\quad [10-1]$$

and

$$\dot{E} = \frac{\left[ C_{yy} - \frac{dP_s}{dy} \right] \left( \gamma A + vC_E - C_{EE} \dot{E} + \frac{C_{yE}}{C_{yy}} \frac{dP_s}{dy} C_{yH} \dot{H} \right)}{\left[ C_{yy} - \frac{dP_s}{dy} \right] C_{EE} - C_{yE}^2}$$

$$\quad [10-2]$$

where we have made use of FOC [O-10] and [O-13].

Equations [10-1] and [10-2] lead to the following proposition:

**Proposition 3:** With perfectly competitive output market and with technological change purely related to the emissions abatement, the optimal time path of emissions and output move in the same direction.

As long as ITC is purely related to emission abatement technology, $C_{yH} = 0$, and the last term on the numerator of equation [10-1] and [10-2] drops out. Also, given the assumption of perfect competition in the output market, $\frac{dP_s}{dy} = 0$; that is, the polluting firms under consideration are relatively small suppliers of the output, $y$, so that output price is constant with respect to changes in the polluting industries output. With these specifications, above equations evolve to the following:
\[
\dot{y} = \frac{-C_{yE} \left\{ \gamma A + \nu C_E - C_{EH} \dot{H} \right\}}{C_{yE}^2 - C_{yE}^2 - C_{yE}^2}
\]  \[11-1\]

and

\[
\dot{E} = \frac{-C_{yE} \left\{ \gamma A + \nu C_E - C_{EH} \dot{H} \right\}}{C_{yE}^2 - C_{yE}^2 - C_{yE}^2}
\]  \[11-2\]

Note that by appealing to the strong convexity of the cost function, we see that the denominator of each expression is positive. Also, the second term on the numerator for both equations is identical. Further, by assumption \( C_{yE} < 0 \) and \( C_{yE} > 0 \). Hence, \( \text{sign} \left( \dot{E} \right) = \text{sign} \left( \dot{y} \right) \); i.e. both \( \dot{y} \) and \( \dot{E} \) move in the same direction.

Q.E.D.

Proposition 3 signifies that characterization of the optimal time path of emissions is qualitatively equivalent to characterizing the time path of output. We now, therefore, characterize the optimal time path of emissions in the presence of ITC, which leads us to the following proposition:

**Proposition 4:** If the rate of change of marginal cost saving from emissions due to incremental knowledge (i.e. due to the effect of ITC) is higher than the net marginal cost of emissions, over time emissions will fall, triggering a fall in the time path of output.

To observe this, note that \( \dot{E} < 0 \) (and hence \( \dot{y} < 0 \)) iff \( \left( \gamma A + \nu C_E - C_{EH} \dot{H} \right) < 0 \). This implies:

\( \dot{E} < 0 \) iff \( C_{EH} \dot{H} > (\gamma A + \nu C_E) \)  \[12\]

where \( C_{EH} > 0 \), \( C_E < 0 \), and the parameters \( \gamma, \Lambda, \nu \) are all positive.

Note that under the ITC scenario (i.e. when \( k > 0 \)), \( \dot{H} > 0 \). This implies that the left side of equation [12] represents the rate of change of marginal cost saving from emissions due to the incremental knowledge. First term on the right side, on the other hand, represents the
(augmented) marginal cost of emissions\(^{81}\) and the second term represents the cost saving due to marginal emission. Hence, right hand side of equation [12] represents the net marginal cost of emissions. This suffices the proof for Proposition 4. Q.E.D.

Equation [12] signifies that when the effect of ITC on marginal cost saving from emissions (e.g. due to the improvement in emissions-saving technology) is sufficiently strong, emissions tends to fall over time.

3.7 Implication of ITC on the Optimal Path of Permit Price:

We now examine the effect of ITC on the optimal permit price path. To do this, we hold the parameter \( k > 0 \), that is, we consider the ITC scenario (which contrast to the NITC scenario where \( k = 0 \)).

When \( C_{ii} < 0 \) and \( C_{in} > 0 \), the additional knowledge is valuable (i.e. the multiplier \( \lambda(t) \) is positive). As well equation [7] signifies that, at the optimum, the shadow cost of permit is equal to the price of permit. In other words, under effective environmental regulation, permits bear a positive value which, according to equation [7], is equal to the shadow cost of emission plus the marginal cost of abatement. This is the key equation that leads us to the following proposition.

**Proposition 5:** The effect of ITC on the optimal emission price path is ambiguous.

To observe this, we totally differentiate equation [7] with respect to \( k \) and obtain the following:

\[
\frac{dP_i}{dk} = \frac{d\Lambda}{dk} - \left( C_{jE} \frac{dy_i}{dk} + C_{EE} \frac{dE_i}{dk} + C_{EH} \frac{dH}{dk} \right)
\]

\[
\Rightarrow \frac{dP_i}{dk} = -C_{EE} \frac{dE_i}{dk} - C_{EH} \frac{dH}{dk}
\]

where we have used the fact that \( \frac{dy_i}{dk} = 0 \) and \( \frac{d\Lambda}{dk} = 0 \)

---

\(^{81}\) Augmented by the natural decay rate of emissions, \( \gamma \).
These yield:

$$\frac{dP_k}{dk} = \left(-C_{EE}\frac{dE}{dk}\right) + \left(-C_{EH}\frac{dH}{dk}\right)$$  \[13\]

Note that the terms in equation [13] have the following signs: $C_{EE} > 0$ and $C_{EH} > 0$ (by assumption) and $\frac{dE}{dk} < 0$ (by Proposition 1). Hence, first term on the right hand side is unambiguously positive. As for the second term, to the extent that knowledge increases due of ITC, $\frac{dH}{dk} > 0$ and consequently second term would be negative. Thus, the sign of $\frac{dP_k}{dk}$ is unknown. Q.E.D.

The economic intuition underlying the result is straightforward: to the extent that knowledge increases as a result of ITC (i.e. $\frac{dH}{dk} > 0$) and contributes toward reduction of emission over time (i.e. $\frac{dE}{dk} < 0$ implying that emission path declines), optimal price path of emission permit tends to increase (decrease) as long as the change in the rate of cost saving from marginal emission (reflected in $-C_{EE}\frac{dE}{dk}$) is greater than the change in the rate of cost savings due to ITC (i.e. $C_{EH}\frac{dH}{dk}$). That is, when the presence of ITC brings a greater (smaller) relative reduction in overall emissions, firms find it profitable to buy emission permits (invest in R&D) resulting in an increased (decreased) demand for permits and thus leading to a rise (fall) in the price of permit.

3.8 Conclusion:

This paper employs an analytical model to examine the implications of induced technological change for the optimal design of emission mitigation policy. We obtain optimal time profiles for output, emissions and emission permit price when permits are bankable.

The primary contribution of the paper lies in providing a unified framework where both the inter-related concepts of permit banking and borrowing and induced technological change
(ITC) in achieving a given emission mitigation target are investigated together. While the exiting literature offers such investigation in the presence of frameworks that allow for either permit banking and borrowing or the ITC process, we provide the first attempt to combine the two.

Compared to the results from existing literature, our analytical model yields some interesting findings. With a modeling framework that does not allow for permit banking and borrowing, Goulder and Mathai (G&M 2000) show that the presence of ITC justifies a rising abatement profile (i.e. declining emissions) over time when technological change occurs through R&D investments. Our result indicates that this implication of ITC carries over to a framework that allows for permit banking and borrowing.

In terms of the impact of ITC on the optimal path of emission charges, our result contradicts with that of G&M’s. While G&M finds an unambiguously declining time profile of optimal emission charges, we find that the impact of ITC on the optimal path of permit price is analytically ambiguous. The difference in the result can be attributed to the differences in the modeling structure. Unlike G&M, the presence of permit banking and borrowing in our model reveals an explicit trade-off between cost-savings that arise from marginal emissions and the marginal technological innovations (due to ITC). The relative strength of these two forces determines the demand for emission permits, which in turn determines the optimal path of permit price.

Our work abstracts from some important issues. One such issue is uncertainty. We have assumed both that knowledge-accumulation is a deterministic process and that the cost and damage functions are perfectly known. In doing so, we have avoided difficult issues of abatement timing relating to irreversibilities and the associated need to trade off the “sunk costs and sunk benefits” of abatement (see Pyndick, 1993 and Ulph and Ulph, 1997).

Another important issue is the allocation method of permit endowment. Existing theoretical and empirical literature provide a variety of permit endowment methods, each with its implication on the optimal path of emissions and the permit price. In the current framework, while we have accounted for permit endowment, we did so in the simplest manner thus avoiding
the contentious issue of equity considerations (see Rose, 1992). We leave these important issues for future research.
REFERENCES


Pindyck, R. S., 1993. “Sunk Costs and Sunk Benefits in Environmental Policy.” Working paper, MIT.


