

Investigating Traveling Spots Using Three-Component Differential Equations and the Euler's Approximation Method

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Introduction

In May 1997, C.P. Schenk, M. Or-Guil, M. Bode, and H.-G. Purwins published an article called *Interacting Pulses in Three-Component Reaction-Diffusion Systems on Two Dimensional Domains*. In this article they describe a situation in which they are given an initial condition on the XY-plane, and while using a numerical method to approximate the graph at the next level of the z-axis, a traveling spot would occur. The objective of our project is to recreate this experiment using Three-Component Systems and the Euler's Approximation Method on MATLAB to try to find out why these traveling spots occur.

Background

Differential equations are equations involving one or more derivatives of the unknown function. A three-component differential equation is a set of three equations u , v , and w that depends on three different variables x , y , and t . The three variables represent the three axis of the three-dimensional space, and each equation determines how the variables change depending on the values of the other two variables.

The Euler's Approximation Method is one of the most basic methods of solving differential equations numerically. For any differential equations, if given an initial condition, one can approximate the next value by using the formula $y_{n+1} = y_n + hf(x_n, y_n)$ where y_n is the preceding value, h is the chosen step size, and $f(x_n, y_n)$ is the function. The Euler's Method can be applied to the three-component system if the initial values that are given span the XY-plane. Each approximation will then result in a new set of values for the XY-plane while moving a step size higher on the Z-axis.

Specifications

The three-component system for my project has the form:

$$\begin{aligned} u_t &= D_u (u_{xx} + u_{yy}) + f(u) - v - \kappa_3 w + \kappa_1 \\ v_t &= (D_v (v_{xx} + v_{yy}) + u - v) / \tau \\ w_t &= (D_w (w_{xx} + w_{yy}) + u - w) / \theta \end{aligned}$$

Following the specifications of the article, we get the system:

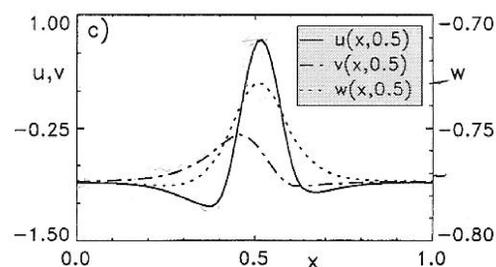
$$\begin{aligned} u_t &= 0.001 (u_{xx} + u_{yy}) + (2u - u^3) - v - 8.5w - 6.92 \\ v_t &= (0.00125 (v_{xx} + v_{yy}) + u - v) / 48 \\ w_t &= (0.064 (w_{xx} + w_{yy}) + u - w) \end{aligned}$$

MATLAB Program

The first step to recreating the traveling spot experiment involved adapting an existing code for a two-component system to the specific three-component system. A new component was added to the program, and set values and equations were modified to match the specifications of the article. This program allowed us to compute and plot the Euler's Approximations using a chosen step size. Examining the figures for each individual step size, one should see a spot that moves with each step size increment, thus a traveling spot.

Finding Initial Conditions – Method 1

Since the article did not specify the initial conditions that are needed in order to use the Euler's Method to create the traveling spot, we must try to find the initial conditions using the results of the article. Our first attempt was the to find the equations to the typical shape of a traveling three-component spot.

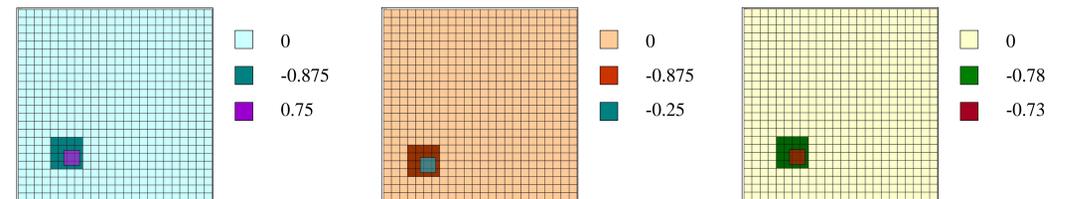


Typical shape of the traveling three-component spot in *Interacting Pulses in Three-Component Reaction-Diffusion Systems on Two-Dimensional Domains*.

We were able to find equations that matched the shape of the graphs using exponential, rational, and arctan functions, but scaling the equations to match the axis on the figure was too complicated, since there were too many coefficients that will transform the shape of the graphs. We proceeded to attempt another method.

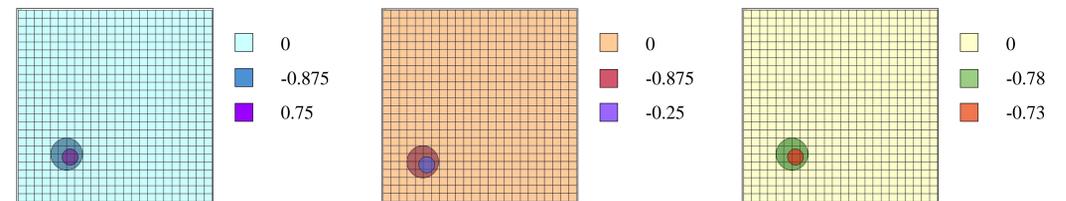
Finding Initial Conditions – Method 2

Our second attempt was to make a grid on the XY-plane and assign specific values to certain parts of the grid. The first trial is shown below with the outer squares having the values of the horizontal asymptotes of the typical shape of a traveling spot, and the inner squares having the values of the maximum points.



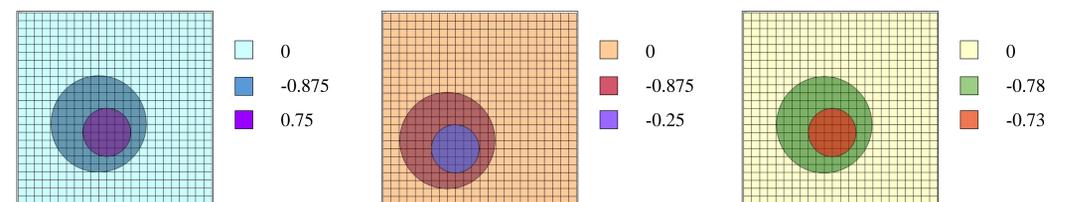
Square grids for the first estimates of the initial conditions using two squares.

However, these initial conditions did not yield the expected shape of a small hill, and eventually encountered errors as the values approached infinity. To solve the shape of the graphs, we changed the squares to circles.

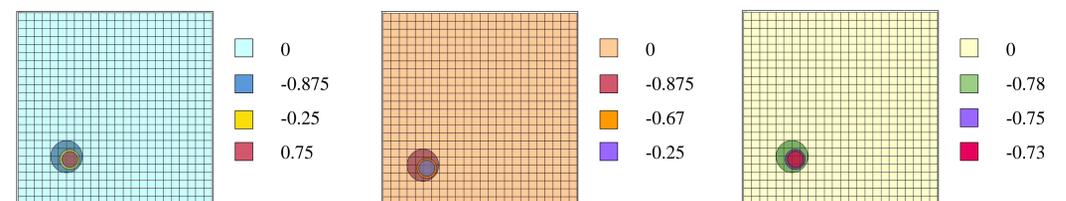


Square grids for the second estimates of the initial conditions using two circles.

This set of initial conditions gave the correct shape but did not fix the errors resulting from infinity values. We decided to try to use bigger circles, more levels of circles, as well as making gradual transition of assigned values from the smaller circle to the larger circle.



Square grids for the fourth estimates of the initial conditions using two larger circles.



Square grids for the third estimates of the initial conditions using three circles.

Unfortunately, none of these methods were able to solved the infinity problem.

Next Steps

Since we were unable to find the initial conditions by creating a grid for the XY-plane, the next option is to give another attempt to finding the equations that model the typical shape of a traveling three-component spot. After finding the initial conditions, we can run the MATLAB program to create a series of figures that will reveal a traveling spot. The results can then be analyzed to determine why the traveling spot occurred.

Bibliography

Boyce, William E., DiPrima, Richard C. (1997). *Elementary Differential Equations and Boundary Value Problems Sixth Edition*. United States: John Wiley & Sons Inc.

Schenk, C.P. et al. (1997). Interacting pulses in three-component reaction-diffusion systems on two-dimensional domains. *Physical Review Letters*, 78(19), 3781-3784.