Competition and the Allee effect

Tzvia Iljon, Dr. Frithjof Lutscher
University of Ottawa

Background and Research Questions

❖Facilitation is defined as an increase in the density of one species as a result of the presence or increasing density of another species (Feldman).

❖We examine the possibility of species A alleviating the Allee effect of species B at low densities through some form of pollination facilitation, while competing with species B at higher densities for resources and space.

❖The system of interest consists of two plant species who share a generalist pollinator. At low densities, the presence of a second species may reduce pollen limitation as it attracts pollinators to the patch. When the densities of both species increase, resources and space may be limited resulting in competition between the two species.

- ➢Generalist pollinator: that is, a pollinator that does not distinguish between the two species.

❖Research Question: Is it possible for facilitation to overcome the detrimental effects of Allee effects so that one species is actually benefiting from the interaction of the two species?

petition

Allee effects

❖The Allee effect is a biological phenomenon characterized by a positive correlation between population density and the per capita population growth rate [Allee,1949]

❖ We observe growth of a single species with respect to its density by plotting $F(x,0)$ vs x

❖The diagonal line is given by $F(x)=x$, and intersection of the growth function with this line indicates an equilibrium point.

❖Lower equilibrium is given by F_* and the upper equilibrium with F^*.

❖When the density of the single species is below the diagonal, the species will not be able to grow and persist and the population will die out. If the density is above the diagonal, then the species is capable of growth and will persist and tend towards F^*

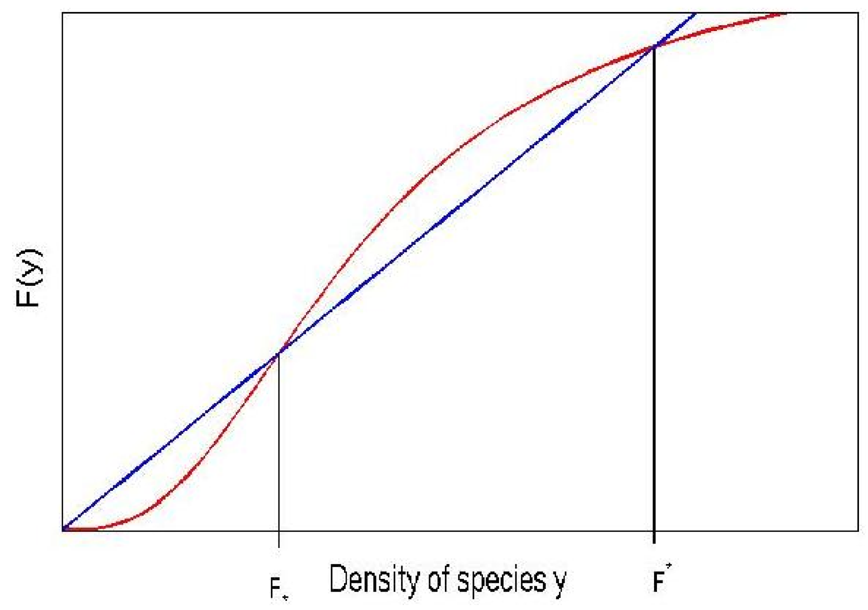

Figure 1. Allee effect and corresponding steady states for a single species

Graphical Analysis

Graphical Analysis:

❖Plots illustrate growth of each species with respect to the densities of itself and the other species.

❖Zero growth isoclines of model (1) for species *x* and *y* given by x_{t+1}-x_t=0 and y_{t+1}-y_t=0 illustrated the impact of each species on the other.

❖If population density of a species is outside of the space enclosed by the zero growth isocline then its density will decrease.

❖If the density lies inside the enclosed region, the population will grow.

❖Arrows indicate the growth of each species in the given region - horizontal movement describes the change in density of species *x* while vertical movement describes the change in density of species *y.*

❖Points of intersection with the axes represent the single species equilibria corresponding to Allee plot.

❖What does this tell us about facilitation and competition? If facilitation is possible at low densities then the presence of one species will result in increased growth of the other.

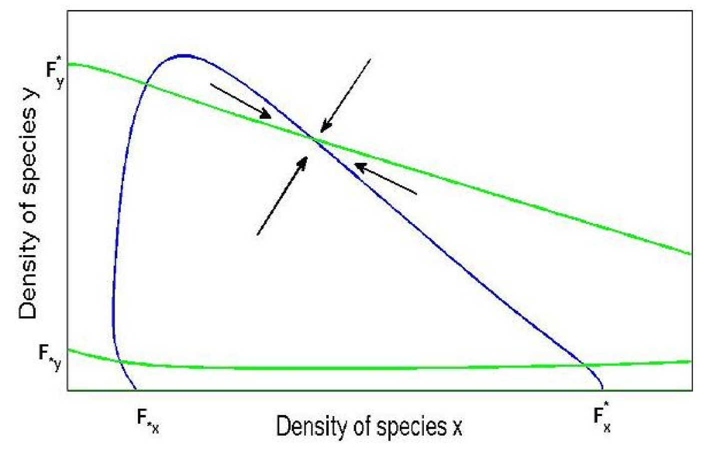

Figure 2. Example figure to explain graphical analysis.

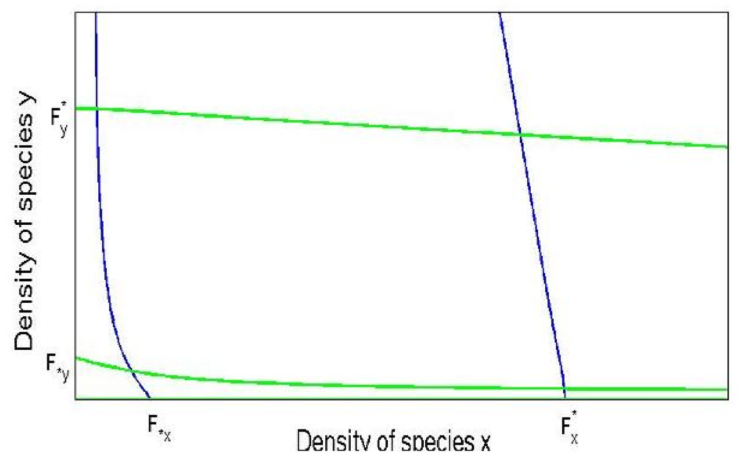

Figure 3. Demonstration of facilitation at low densities and competition at high densities. Parameter values: a_1=0.2,a_2=0.3,κ=0.87,β=4,α_1=1.4, α_2=1.3,s_1=4,s_2=4

Interpretation:

•Negative slope at F_* indicates lowering of Allee threshold for species *y* in the presence of species *x.*

•Negative slope at F* indicates a decrease in carrying capacity in the presence of species *x*, which would result from competition for resources and space at higher densities

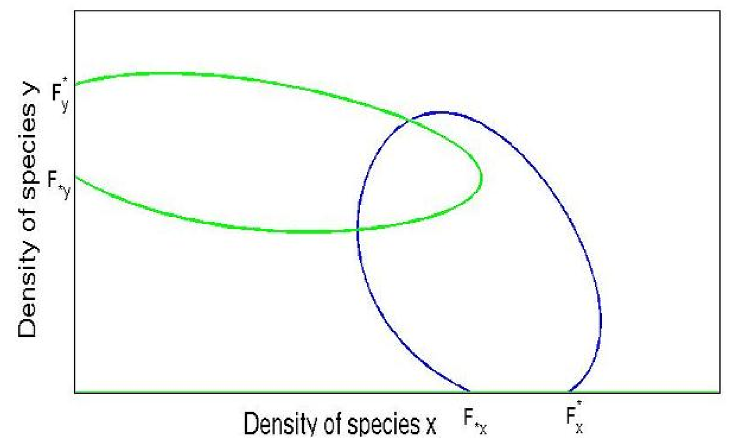

Figure 4. Demonstration of facilitation at low densities and facilitation at high densities. Parameter values: a_1=0.1,a_2=0.1,κ=8.4,β=6,α_1=1.4, α_2=1.2,s_1=4,s_2=4

Interpretation:

•Negative slope at F_* indicates lowering of Allee threshold for species *y* in the presence of species *x.*

• Positive slope at F* indicates increase in carrying capacity of species *y* in the presence of species *x*

• Biologically speaking: At both high and low densities one species is benefiting from the presence of the other

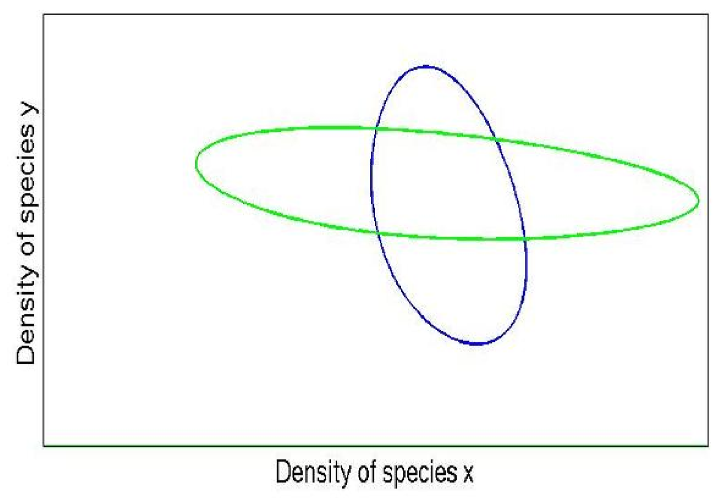

Figure 5. Extreme case of mutualism. Parameter values: a_1=0.1, a_2=0.1, κ=0.75, β=6, α_1=2.1, α_2=2, s_1=4, s_2=4

Interpretation:

• Zero isoclines do not intersect axes for both species, meaning that neither species has a single-species steady state

• Stable existence of each species is dependent on the presence of the other species.

Pollination model

Parameterization

B	Bees carrying no pollen
P_1	Bees carrying pollen of species 1
P_2	Bees carrying pollen of species 2
F_1	Flower species 1
F_2	Flower species 2
N	Total population of bees
a_i	Visitation rate of generalist pollinator to plants of given species
β	Loss of pollen to the environment
γ	Attraction of pollinator to the patch

Pollination sub-model

System A:

$$\frac{\partial B}{dt} = -\alpha_1 F_{1,0} B - \alpha_2 F_{2,0} B + \beta P_1 + \beta P_2$$

$$\frac{\partial P_1}{\partial t} = \alpha_1 F_{1,0} B - \alpha_2 F_{2,0} P_1 + \alpha_1 F_{1,0} P_2 - \beta P_1$$

$$\frac{\partial P_2}{\partial t} = \alpha_2 F_{2,0} B + \alpha_2 F_{2,0} P_1 + \alpha_1 F_{1,0} P_2 - \beta P_2$$

System B:

$$\frac{\partial F_1}{\partial t} = \alpha_1 P_1 (F_{1,0} - F_1)$$

$$\frac{\partial F_2}{\partial t} = \alpha_2 P_2 (F_{2,0} - F_2)$$

$$B + P_1 + P_2 = N = \gamma (F_1 + F_2)$$

Equilibria for System A are given by the following:

$$P_1^* = \frac{\alpha_1 F_{1,0} N}{\alpha_1 F_{1,0} + \alpha_2 F_{2,0} + \beta}$$

$$P_2^* = \frac{\alpha_2 F_{2,0} N}{\alpha_1 F_{1,0} + \alpha_2 F_{2,0} + \beta}$$

$$B^* = \frac{\beta N}{\alpha_1 F_{1,0} + \alpha_2 F_{2,0} + \beta}$$

Using the equilibria above, we solve System B describing the probability of pollination of flowers of each species and yield the following equations.

$$\psi_1(x,y) = 1 - \exp\left(-\kappa \frac{\alpha_1 x (x+y)}{\alpha_1 x + \alpha_2 y + \beta}\right)$$

$$\psi_2(x,y) = 1 - \exp\left(-\kappa \frac{\alpha_2 y (x+y)}{\alpha_1 x + \alpha_2 y + \beta}\right)$$

Conclusions

❖In response to the primary research question, it is clear that it is possible to have the presence of a second plant species facilitate the growth of a single species at low densities while also competing for resources and space at higher densities.

❖In particular, the potential for facilitation at low densities is important to conservation biology as it demonstrates that removing a competitor from the environment may not be beneficial to an endangered species.

❖Through variation of parameters, we observe a wide range of interaction between the two plant species, giving rise to many biologically interesting situations.

❖Further research may aim to address the question of whether the parameter values required for such interactions are biologically feasible, and if so, do these conditions actually allow for such interactions in real environments.

References

Feldman, Tracy S., Morris, William F. and Wilson, William G. (2004). When can two plant species facilitate each other's pollination?. OIKOS, 105, 197-207

Allee, W.C., (1949). Principles of Animal Ecology. Philadelphia: Saunders Co.

Beverton, R. J. H.; Holt, S. J. (1957), On the Dynamics of Exploited Fish Populations. London: Chapman and Hall