Three Essays on Venture Capital Finance

A thesis presented

by

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Chapter 1: Public Policy and Venture Capital

In addition to financing, venture capitalists provide advice and expertise in management, commercialization, and development that enhance the value, success, and marketability of high-risk, high return projects. Venture capitalists also have skills in selecting projects with potentially high returns. The first chapter investigates the contracting relationship between venture capitalists and entrepreneurs in a setting where the venture capitalist and entrepreneur contribute intangible assets (advice and effort) to a project that are non-contractible and non-verifiable. In general, in the private market equilibrium, advice provided by the venture capitalist and the number of projects funded are lower than the social optimum. Government tax and investment policies may alleviate these market failures. We evaluate the impact of a capital gains tax, a tax on entrepreneur’s revenue, and an investment subsidy to venture capitalists. We also examine the impact of government project enhancing programs that complement venture capitalist advice. Finally, we analyze the effects of a government venture capital firm competing with private venture capital.

Chapter 2: Imperfect Competition and Entry Deterrence in Venture Capital Markets

The second chapter focuses on competition in venture capital markets. Venture capital markets typically have few firms competing over innovative projects and are characterized by imperfect competition. We model a three-stage game of fund raising, investment in innovative projects and input of advice and effort, where fund raising is used as an entry deterrence mechanism. We examine the impacts of taxes and subsidies on venture capital
market structure. We find that a tax on venture capitalist revenue and a tax on entrepreneur revenue increase the likelihood of entry deterrence and reduce the number of projects funded in equilibrium. A subsidy on investment reduces the likelihood of entry deterrence and increases the number of projects funded. A subsidy on fund raising decreases the likelihood of entry deterrence and has an ambiguous effect on the number of projects funded. In Canada, Labour-sponsored venture capital corporations (LSVCCs) receive a targeted tax credit, reducing the cost of raising funds. Interestingly, we find that such a targeted tax credit does not impact the likelihood of entry deterrence or the number of projects funded.

**Chapter 3: Project Selection and Venture Capital**

The third chapter examines the venture capitalist choice of investment in project selection skills and investment in managerial advice. Venture capitalists screen projects, selecting the ones with the most potential, and contribute managerial advice to projects that enhance the value and probability the project is successful. We model, separately, a private venture capitalist and a labour-sponsored venture capitalist (LSVCC) with different objectives. A LSVCC is a special type of venture capitalist fund that is sponsored by a labour union and raises its funds from individual investors. The private venture capitalist maximizes its expected profits, while the LSVCC maximizes a weighted function of expected profits and returns to labour. Consistent with empirical evidence, the quality of projects, determined by project selection skills and managerial advice, is higher for the private venture capitalist. The private venture capitalist invests in fewer projects, but invests in more skills, project selection and managerial advice, relative to the labour fund. Comparing to the social optimum, we find both private venture capitalists and LSVCC under-invest in managerial
advice while their investment in project selection skills and number of projects financed are ambiguous.
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General Introduction

The topic of this thesis is venture capital financing. Venture capital finances high-risk, high-innovation projects. These projects have difficulty securing traditional forms of financing (e.g. bank financing) due to the high-risk and low assets. In addition to providing financing, venture capitalists have two skill sets that add to innovative projects. First, venture capitalists invest in project selection and screening skills that enable it to select projects that have high expected returns. The ability to select high quality projects comes from focusing investment in a specific industry or stage where the venture capitalist is an expert. Second, the venture capitalist contributes managerial advice, such as commercialization expertise, project management skill, industry knowledge, and network connections, which enhance the value of a project and increase the probability that a project is successful. To summarize, venture capitalists invest in two types of skills: project selection and managerial advice. Both of these skills increase the venture capitalists expected returns from investing in innovative projects. Entrepreneurs seek out venture capital financing for two reasons: they have difficulty securing financing from traditional investors, and they desire the managerial advice that a venture capitalist contributes to a project.

The supply of venture capital has been raised as an issue in Canada. The argument stems from R&D being under-provided relative to the social optimum due to spillovers and incentives (for example Jaffe (1996)). The government of Canada has sought to increase the supply of venture capital using two policies: indirect tax policy (R&D and S&T tax credits, Labour-sponsored venture capital tax credits) and direct investment policy (government
financed venture capital corporations). While the R&D and S&T tax credits target innovation in general, the Labour-sponsored venture capital tax credits have specifically targeted the venture capital market. The efficiency and quality of the labour-sponsored venture capitalists created by the tax credit have been called into question (Cumming and MacIntosh (2006), Brander et al. (2008)). It is argued that government created venture capital crowds-out private venture capital. In addition to the quantity of venture capital financing, the quality, determined by ability to select projects and managerial advice, needs to be considered. This thesis considers both the quantity and quality aspects of venture capital financing and the impact that government policies have on the private market.

The thesis considers several different aspects of venture capital financing: quality of venture capitalists, determined by managerial advice and project selection skill, quantity of venture capital investments (number of investments), quality of venture capital (quality of venture capitalists, plus the entrepreneurs effort), the impact of government tax policies on venture capital, the effect of government venture capital competing with private venture capital, and how labour-sponsored funds operate. Each chapter models certain aspects of the venture capital market that shed light on how markets work, how they respond to government intervention, and how they perform relative to the social optimum.

The first chapter investigates the contracting relationship between venture capitalists and entrepreneurs and the implications of ownership of the project (property rights). The framework of the model coincides with the existing literature on venture capital where there are two stages: venture capital investment in projects and contribution of managerial advice. The chapter contributes to the literature in several ways: entrepreneurs are assumed heterogeneous; the distribution of property rights between the entrepreneur and venture
capitalist are examined; the introduction of public support mechanisms for venture capital; and government venture capital competing with private venture capital. The impact of capital gains tax, entrepreneur revenue tax, and investment subsidies are examined. Capital gains tax reduces managerial advice and the number of projects funded if the venture capitalist has market power. If the entrepreneur has the property rights, then the number of projects funded decreases, high return projects receive less advice from the venture capitalist, but low return projects are not affected. Public support programs that compliment venture capitalist advice increase the number of projects funded and the advice the venture capitalist provides. When a government venture capitalist competes with a private venture capitalist, the government venture capitalist crowds-out the private venture capitalist.

The second chapter introduces imperfect competition between venture capitalists. There is no literature on competition between venture capitalists. Given the potential effects of government policy on market structure, this chapter examines the impacts of government tax policy (tax on venture capitalist revenue, investment and fund raising subsidies, and targeted subsidies) on market structure. Considering the goal of such policies to raise venture capital investment, the quality of venture capital investment, determined by managerial advice and the effort of the entrepreneur, is also analyzed. The model adds a fund raising stage for venture capitalists – the venture capitalist raises funds from an outside investor and then invests in innovative, entrepreneurial projects. These policies are examined in a three stage model of fund raising, project investment, and managerial advice. Following the literature on entry deterrence (Spence (1977) and Dixit (1980)), fund raising can be used by the incumbent as an entry deterring mechanism. The market power implications of government policies are analyzed. A tax on venture capitalist revenue
increases the likelihood of entry deterrence and has an ambiguous effect on quality, while an investment subsidy decreases the likelihood of entry deterrence, but has no effect on quality. Finally, targeted subsidies are introduced to resemble the labour-sponsored tax credits. A targeted subsidy on fund raising of an incumbent has no impact on the likelihood of entry deterrence or project quality.

The third chapter focuses on two aspects largely ignored in the literature: project selection and labour-sponsored venture capital. Although project selection in venture capital markets is identified in empirical work (Gompers et. al (2006)), it has not been explicitly modeled. The same is true for labour-sponsored venture capital (for example, Cumming and MacIntosh (2006)). The chapter models both project selection and labour-sponsored funds. Labour-sponsored funds, influenced by union sponsorship, have different objectives than private venture capital funds. Private venture capitalists and labour-sponsored venture capitalists are modeled separately, and the quality of funds and the number of investments are compared. Labour-sponsored funds are found to have lower total expected returns and lower quality (project selection and managerial skill) than private venture capitalists, and invest in more projects. Compared to the social optimum, both private venture capitalists and labour-sponsored funds invest too little in managerial advice. However, their investments in project selection skill and number of projects are ambiguous relative to the social optimum.
Chapter 1

Public Policy and Venture Capital
1.1 Introduction

Venture capital (VC) differs from other types of financing, bank financing in particular, in a distinct way: venture capital financiers offer specific and unique knowledge and offer advice to the projects that they fund in the form of managerial, commercialization, and development knowledge. The expertise of VC can be divided into two areas: project selection and project development. First, venture capital financiers are better able to select projects due to their skills and expertise in developing and commercializing new products or services. Observations of VC markets show VC financiers tend to specialize in stage of development (early stage development or later stage established innovation) and/or industries. They are able to obtain specific knowledge pertaining to that stage and/or industry and evaluate projects more effectively. In essence, the venture capitalist is able to alleviate some of the information asymmetry that exists between the entrepreneur and the financier. Second, a VC financier provides advice or expertise in management, commercialization, and development of projects that enhance the value, probability of success and marketability of projects. Project development pertains to the advice that VC financiers can contribute to a project. While bank finance and other financiers offer only the investment capital required in order to initiate the project, VC financier offers assets that increase the value, probability of success, and marketability of the output from the project in addition to financing. It is these investments in advice that makes VC investment desirable for innovative projects where expertise of outside investors can help the project reach its full potential. Entrepreneurs that seek VC financing are likely doing so for two reasons: (1) they desire the advice aspect of VC financing; and (2) due to the high risk nature of innovative projects are unable to obtain bank or standard financing.
This paper examines the venture capital market in a setting where effort and advice are non-contractible and non-verifiable (moral hazard). The analysis characterizes the level of advice provided by venture capitalists in projects that are funded (the quality of VC financing) and the number of projects that receive funding in private market equilibrium (the size of the VC market). These results are then compared to the social optimum level of advice and number of projects funded. We find that the private market under-provides advice and funds fewer projects relative to the social optimum. Different tax instruments and government policy initiatives are analyzed in the context of the model. We find that an appropriate capital gains subsidy or a subsidy on entrepreneur’s revenue would correct both under-provision of advice and number of projects funded. A subsidy on investment corrects the under-provision of number of projects, but has no impact on the advice provided by private venture capitalists. Government programs that facilitate and encourage commercialization may have positive or negative impacts on advice and number of projects depending on whether they are complements or substitutes with private advice. We introduce a government venture capitalist that competes with private VC for projects and find that there is crowding-out of private investment and an ambiguous effect on the quality of private VC financing.

VC financing, and private equity financing in general, is an integral part of funding, creating, nurturing, supporting, and developing high-risk entrepreneurial enterprises. Beginning in the early 1980’s, developing a strong venture capital market has been a priority of policy makers in Canada. The classic argument concerns R&D being under-provided relative to the social optimum due to spillovers and incentives, and constitutes a
rationale for government intervention in the markets for innovation.¹ Kortum and Lerner (1998) find that firms with VC financing have contributed to a disproportionate share of innovation in the U.S. However, research on the impact of VC on firm’s returns suggests that Canada VC performs poorly. Venture capital databases, such as Thomson Reuters (US) and Thomson Financial (Canada), show that while U.S. VC investments out perform publicly traded stock exchanges (NASDAQ, S&P 500), Canadian VC under performs.² Ayayi (2004) finds that, in Canada, labour-sponsored venture capital underperforms comparable investments (NASDAQ composite, TSE 300 total return index, Nesbitt Burns Canadian Small Cap Index) and Duruflé (2006) presents evidence that both private and labour-sponsored funds have negative internal rates of return (IRR). The evidence suggests that Canadian VC performs relatively poorly compared to alternative investments and relative to the U.S. Baygan’s (2003) OECD study of venture capital investment finds Canada ranked 2nd behind the U.S. in VC as a percentage of GDP over the period 1998-2001. When VC is benchmarked as a percentage of BERD (Business Expenditure on R&D) Canada performs better relative to the US.³ However, unlike the U.S., the Canadian VC market has relied heavily on tax incentives (i.e. Labour-sponsored Venture Capital) and government VC companies (e.g. BDC). Graph 1 shows the number of companies invested in by government created entities versus private and institutional investors.⁴ It is clear that outside the tech-bubble (1999-2001), government VC has invested in nearly 50 percent of

¹ See, for example, Jaffe (1996).
² Source: NVCA.org for US and canadavc.com for Canada
³ Duhamel and Peter (2008) suggest that BERD is a better benchmark for size of VC because BERD relates directly to the innovative capacity of a country.
⁴ The results allow for syndication of investments. Syndication is the term used for co-investment. Co-investment can occur in the same round of financing or across financing rounds. Non-government VC includes foreign VC investors in Canadian companies.
the companies funded in the VC market in Canada. Given this large market share, the role of government in VC markets and its impact needs to be examined carefully.

Graph 1: Canada - Number of Companies Financed by VC

There are several policy instruments the government can use to stimulate the VC market. These policy instruments can be broken down into two large categories: direct and indirect policy. Direct policy methods involve the government investing directly into VC projects through debt or equity financing. The rationale behind such policies revolve around increasing the overall size of the VC market and targeting projects with positive social benefits that may not be funded in the private market. In Canada, the government uses entities such as the Business Development Bank of Canada (BDC), Canada’s Export Development Corporation, and Ontario Venture Capital Fund. Indirect policy involves tax policies to stimulate the existing private market (e.g. reduction in capital gains tax, investment subsidies such as the S&T tax credits) and target the entire VC market in general. The government can also use targeted tax measures to create new investors. In Canada this is done, for example, through retail funds dominated by Labour-sponsored venture capital corporations (LSVCC).
Cumming and MacIntosh (2006) have been critical of these targeted government subsidies, presenting empirical evidence that is consistent with crowding-out of private funds. Brander et al. (2008) consider government venture capital (GVC) as both retail and government funds and find lower returns and investment in industries with lower predicted returns (e.g. manufacturing). They too find evidence consistent with crowding-out and with lower quality of advice. Ayayi (2002) and Cumming and MacIntosh (2006) present evidence that suggests a lower quality of advice provided by retail funds. Overall, the view of public VC is poor in that evidence suggests crowding-out and lower quality of public VC. These studies indicate that the solution to suboptimal quality of advice due to adverse selection and moral hazard and possible under-provision of funds may not be a targeted subsidy but rather broad tax policy or direct government investment.

There is also evidence that suggests there is an excess amount of funds raised by venture capitalists, but not invested. This is referred to as the “overhang” (graph 2). Cumming and MacIntosh (2006) suggest that the existence of this “overhang” may be interpreted as under-investment of VC relative to the optimum.

**Graph 2: Overhang of Venture Capital in Canada**

![Overhang of Venture Capital in Canada](image)

The “overhang” of uninvested funds is defined as capital available after investments have been made. Source: Thomson Financial
There is a small literature devoted to the analysis of taxes and subsidies to correct inefficiencies from asymmetric information between VC financiers and entrepreneurs (Keuschnigg and Nielsen (2001, 2003, 2004a, 2004b) and Keuschnigg (2004)). In these models venture capitalists structure contracts such that they receive a share of the expected returns of a project in exchange for financing and advice (managerial, commercial, etc…). Venture capitalists do not provide advice unless they receive compensation for this special investment. The advice of the venture capitalist directly increases the probability of a successful project in these models.\(^5\)

Although there is discussion on indirect government policies to correct VC market inefficiencies, literature that examines the impact of direct government intervention is almost non-existent. Keuschnigg and Nielsen (2001) indicate that some governments run their own VC funds and analyze the impact of a government policy variable that increases the probability of a successful project. Secrieru and Vigneault (2004) focus on the role of a government venture capitalist. In their setting, the government offers debt contracts to entrepreneurs and contributes advice that enhances the probability of success. The government venture capitalist acts like a bank providing financing to entrepreneurs who could not receive debt financing from a conventional bank.

One of the contributions of this paper is to model the government venture capitalist as its own entity, where it operates like a venture capitalist, but has a different objective. It offers equity contracts to entrepreneurs in exchange for investment and advice that enhances the probability of success. The rationale behind this interpretation of government

\(^5\) An alternative approach is to model advice increasing the returns on a project. See de Bettignies and Brander (2007).
VC is that in addition to introducing policy to enhance the success probability of projects, the government can compete with and act as a venture capitalist.

While the literature tends to restrict projects to be homogenous in terms of probability and returns, we examine the case where returns are heterogeneous. This quality dimension of projects differentiates between good and bad projects; that is, projects with high expected returns and low expected returns. As is customary in the literature, we assume that the probability of success function is identical for all projects. Introducing a quality aspect opens the door for future research on adverse selection combined with moral hazard.

The model developed in this paper furthers the research on both tax policy and the role of a public VC financier. There are several objectives the paper attempts to address in adding to the existing literature: (1) Characterizing the efficiency of the market and the effects of different tax and subsidy policies in a setting where the quality of entrepreneurs is heterogeneous (different expected returns on projects); (2) Examining the impact of different distribution of market power between VC financiers and entrepreneurs; (3) Analyzing the impact of public VC support mechanisms (e.g. public support for commercialization); and (4) Characterizing the implications of a government VC competing with private VC.

The paper proceeds as follows: Section 2 details the specific characteristics and sequences of the model. Section 3 develops the private market equilibrium while Section 4 develops the social optimum and makes comparisons to the private market solution. Section 5 introduces government tax policy variables. In addition, this section describes different

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6 Keuschnigg and Nielsen (2007) develop a model where returns classify the type of project: good or bad.

7 An alternative way to model the quality aspect is to have heterogeneous probability functions and homogeneous returns (see, for example, de Bettignies and Brander (2007)).
outcomes based on different allocations of market power. Section 6 examines the policy implications of government programs (e.g. commercialization, seminars, and networking) that target the probability of success or returns of projects. Section 7 introduces competition between government VC and private VC. Section 8 concludes.

1.2 The Model

In our characterization of the VC market, each entrepreneur holds a single innovative project. Projects returns, when successful, are denoted by $R$ and are distributed uniformly over the interval $(\underline{R}, \overline{R})$.

This paper focuses on the advice aspect of VC investment and assumes that the project selection capabilities of the venture capitalist are such that no informational asymmetry between entrepreneurs and VC financiers exist. Returns on successful projects are known ex-ante and are perfectly observable ex-post.\(^8\)

Throughout the paper, we assume that the VC market is not competitive. Market power can be leveraged to capture positive total expected profits. This reflects observations from VC markets worldwide. The venture capitalist and entrepreneur make large sunk investments in skills relating to investing in risky projects and entering the market. These act as barriers to entry.

Venture capitalists and entrepreneurs are risk neutral. Entrepreneurs differ in their quality in terms of returns. Their quality and distribution are known to both entrepreneurs and venture capitalists.\(^9\) Projects require a capital investment. The VC financier will

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\(^8\) A possible extension would assume that the venture capitalist incurs a sunk cost ($F$) that reveals the quality of the project, and then the venture capitalist decides whether or not to fund the project. The project being examined would be randomly drawn from the sample.

\(^9\) A possible extension would introduce an adverse selection dimension to the problem. The entrepreneur’s quality would be known to them, but unknown to venture capitalists who would only know the distribution of quality.
provide managerial advice to the project that will raise the probability of the project being successful. Entrepreneurs provide effort that is necessary for project success. The investment required is identical for all projects.

The sequence of events is standard in the VC literature. The principal offers a contract to the agent that stipulates the equity share of returns each party receives. If the venture capitalist has the market power, he offers a take-it-or-leave-it contract to the entrepreneur that extracts the entire surplus of the project. The entrepreneur is in the market for funds; she has an innovation but lacks the capital to move forward. The venture capitalist observes the entrepreneur looking for funds and makes an offer. The entrepreneur’s decision is to take the investment, and the accompanying managerial advice, or not. If the entrepreneur has the market power, she owns the property rights of the project, and approaches the venture capitalist with a contract. The venture capitalist decides whether to accept the terms of the contract and commit the capital investment to the project.

Sequence of Events:

1. The principal offers an equity contract based on share of returns (s) that defines how the revenue from the project will be allocated between itself and the agent. Projects require an investment of capital (I). The contract determines the incentives to provide advice by the venture capitalist (a) and effort by the entrepreneur (e).

2. The agent decides whether or not to enter the market based on its share of expected profits.

3. The venture capitalist and entrepreneur select the level of advice and effort for the project. Advice and effort are non-verifiable and non-contractible.

4. Returns are realized.
The number of projects that will be financed is denoted by $N$. The expected profit of the venture capitalist is denoted by $\Pi$ and the expected profit of the entrepreneur by $\pi$. Total profits for the venture capitalist are the sum of realized returns on all $N$ projects.

Defining the set of Entrepreneurs:
Entrepreneurs are ranked by quality of returns. We assume a uniform distribution. Entrepreneurs and venture capitalists can observe project quality. The distribution of returns is such that: $R \in (R, \bar{R})$. $\bar{R}$ is the return on the marginal project that enters the market. The total number of projects undertaken is equal to $\bar{R} - \bar{R}$.

Defining the Actions of the Principal and Agent:
The principal selects the contract terms it demands from the agent. The contract terms involve the share of revenue that the principal and agent will receive. In exchange for a share of revenue, the VC financier invests in and contributes advice to the project while the entrepreneur agrees to provide effort. Advice and effort increase the probability that the project will succeed. The agent chooses whether to enter into an agreement. In much of the literature, the effort of the entrepreneur is critical to the success of the project.\(^{10}\) Effort is a discrete (zero, one) variable.

\(^{10}\)See for example Keuschnigg and Nielsen (2003), Kanninen and Keuschnigg (2003, 2004), and Keuschnigg (2004) where discrete effort of the entrepreneur is critical for project success. If effort is zero, probability of success is zero.
The contract takes the form of an equity contract and revenues are allocated between the VC financier and entrepreneur. Revenues must be shared in the model to induce both the VC financier and entrepreneur to provide advice and effort respectively. The contract is structured such that if the entrepreneur does accept the terms of the contract, the entrepreneur gives full effort. This assumption implies that entrepreneurs want to participate fully in their project. There are no shirking opportunities. In fact, as will be demonstrated, given the setup of the model, the incentive constraint for the entrepreneur is everywhere interior to the participation constraint, thus if the participation constraint of the entrepreneur is satisfied then the incentive constraint of the entrepreneur is satisfied automatically and effort is given. The problem then reduces to a single choice of action for the entrepreneur in this model, that of entry. Given the distribution of returns, entrepreneurs that have negative expected profits given the contract offer, taking into account the shares and advice they expect to receive, will not enter the market. Thus projects with expected returns greater than the marginal projects return ($\bar{R}$) enter the market. In addition, the VC financier will only fund and make offers to projects with positive net expected profit.

**Structural Form of Venture Capitalist and Entrepreneurial Actions:**

The objective of the venture capitalist is to maximize its total profits. The VC financier has expected profits equal to the sum of expected profits from funded projects, where the number of projects is defined as the distance between the marginal project and the best project in the distribution.

\[
\Pi = \int_{\bar{R}}^R \left[ s_i \Pr(a_i, e)R_i - I - C(a_i) \right] dR
\]  

(2.1)
The venture capitalist incurs investment cost $I$ to fund the project and a cost of advice $C(a)$ per project.\footnote{The rate of return on capital is normalized to zero.} The probability of realizing returns, $R_i$, is $\Pr(a_i, e_i)$ where $a_i$ is the advice of the venture capitalist and $e_i$ is the effort of the entrepreneur on project $i$. The probability of success function is concave in both advice and effort and the cost function of the venture capitalist is convex:

$$\Pr_a (a, e) > 0, \Pr_{aa} (a, e) < 0$$

$$C_a (a) > 0, C_{aa} \geq 0$$  \hfill (2.2)

By assumption, if the entrepreneur provides less than full effort (i.e. does not participate) the probability of success is zero. Thus in equilibrium, effort is equal to one when there is participation and the probability function takes the form $\Pr(a_i, 1)$.

The entrepreneur has a risky project with return ($R$) and a safe job available paying wage ($w$). There is a cost of effort $v(e_i)$ for the entrepreneur.\footnote{Discrete effort: $v(e_i = 0) = 0$ and $v(e_i = 1) > 0$.} The entrepreneur must give up equity shares in the project in order to get the venture capitalist to participate and invest advice into the project. If debt was issued, there would be lower incentives for the venture capitalist to provide advice. The problem is equivalent to bank financing when there is no advice.\footnote{de Bettignies and Brander (2007), Casamatta (2003) model bank financiers using this definition. See de Meza and Webb (1987) for a characterization of financing without advice under debt and equity contracts.}

In venture capital markets, we generally observe equity contracts. With a risk neutral principal and agent, and discrete effort of the entrepreneur, an equity-debt linear contract would be optimal: the entrepreneur would bear all the risk and the venture capitalist would provide optimal effort. The venture capitalist would capture the entire
equity share of returns and transfer profits to the entrepreneur to satisfy the participation constraint. However, equity contracts are used in practice. Setting up the model with entrepreneur’s effort as discrete isolates the impacts on venture capitalist’s advice. In a model with double moral hazard, and entrepreneur’s effort modeled as a continuous variable, the distortions would be more pronounced as the decision of both the venture capitalist and the entrepreneur would be distorted. We use equity contracts specifically because it generally characterizes the venture capitalist and entrepreneur contract in practice.

The participation and incentive constraints of the entrepreneur are as follows:

$$PC_E: (1-s)Pr(a,e)R - v \geq -w$$

$$IC_E: (1-s)Pr(a,e)R - v \geq 0$$

Since $w \geq 0$, the incentive constraint is always in the interior to the participation constraint.\[14\] The sequence of events defined in the model states that the entrepreneur decides to enter before the venture capitalist selects advice. The incentive constraint of the entrepreneur ($IC_E$) is satisfied upon her choosing to participate. Thus, we substitute the advice of the venture capitalist into the participation constraint of the entrepreneur ($PC_E$). If the $PC_E$ is satisfied, then the $IC_E$ is satisfied for the entrepreneur. Effort is ensured when the entrepreneur participates.

1.3 The Market Equilibrium

The characterization of the basic model assumes the VC financier has market power and is a monopolist. Effort of the entrepreneur is discrete and advice of the venture capitalist is

\[14\] When outside wages are zero, the IC and PC constraints are identical.
continuous. Under this situation, since returns are observable, the venture capitalist extracts the entire surplus from the project.

The venture capitalist will offer a contract to an entrepreneur if expected profits of funding that project are greater than or equal to zero. The condition for the marginal project that receives an offer is:

$$\hat{\Pi} = \hat{\gamma} \Pr(\hat{a}) \hat{R} - I - C(\hat{a}) = 0$$

(3.1)

Entrepreneur $i$ will enter the market if a contract is offered and the participation constraint holds:

$$\pi_i = (1 - s_i) \Pr(a_i, e) R_i - v - w = 0$$

(3.2)

We assume that revenues are such that if a contract is offered, then entrepreneurs will enter the market. An additional assumption required is that there exists a contract with $s_i \in (0,1)$ that satisfies the entrepreneur’s constraint for all projects. The solution is interior for all projects.

Solving the Private Venture Capitalist’s Problem:

We use backward induction to solve the private venture capitalist’s problem. In the second stage, the venture capitalist maximizes its returns from advice taking shares and project quality as given:

$$\max_a \quad \Pi_i = s_i \Pr(a_i) R_i - I - C(a_i)$$

The first-order condition is:

---

15 The assumption breaks down to expected revenue always being greater than expected costs. This assumption ensures that the number of projects that enter the market is determined solely by the venture capitalist. When the assumption is relaxed, the marginal project (and thus the number of projects) is the higher (lower number of projects) of the participation constraint of entrepreneur and the project selection constraint of the venture capitalist.
\[
\frac{\partial \Pi_i}{\partial a} = s_i \Pr_a R_i - C_a = 0 \quad \forall i = 1, \ldots, R
\]

and the solution is denoted by:

\[
a_i^*(s_i, R_i) \quad (3.3)
\]

Using the implicit function theorem:

\[
F = s_i \Pr_a R_i - C_a = 0
\]

\[
\frac{\partial a}{\partial s} = -\frac{F_s}{F_a} = -\frac{\Pr_a R_i}{s_i P_{aa} - C_{aa}} > 0
\]

\[
\frac{\partial a}{\partial R} = -\frac{F_R}{F_a} = -\frac{s_i \Pr_a}{s_i P_{aa} - C_{aa}} > 0 \quad (3.4)
\]

since \( P_a > 0, R_i > 0, P_{aa} < 0 \) and \( C_{aa} > 0 \)

An increase in the profit share of the venture capitalist or an increase in the total return of a project increases advice. An increase in returns increases the benefit of providing more advice.

Substituting the optimal advice into the entrepreneur’s participation constraint yields the following condition:

\[
\pi_i = (1 - s_i) \Pr(a_i^*(s_i, R_i))R_i - v - w = 0 \quad (3.5)
\]

The expected profit of the entrepreneur is zero when the venture capitalist has market power (perfectly discriminating). The entrepreneur is just compensated for her costs. From this zero profit condition for the entrepreneur, the share of profits going to the VC financier is computed to be:

\[
s_i^*(R_i, v, w) \quad (3.6)
\]

Using the implicit function theorem we compute comparative statics:
The numerator is positive. The first term in the denominator is negative, but the second term is positive, thus the sign depends on the relative magnitude of these two effects.

The share of profits function is strictly concave over the interval \( s_i \in (0,1) \) and reaches a maximum when the term inside the bracket is zero.\(^{16}\) There are two equilibria that arise from this result. The first exists where venture capitalists share of profits is smaller and advice is smaller, and the other where venture capitalists share of profits is larger and advice is larger. The sign of comparative statics in the general model cannot be determined without an assumption being made as to whether the denominator is positive or negative.

We assume the term is positive, that is we are on the downward sloping part of the concave function and the larger venture capitalist share of profits is selected (corresponding to larger advice provided by the VC financier). Comparative static analysis shows that, when the denominator is positive, venture capitalist share of profits increase in returns and decrease in entrepreneur’s cost of effort and outside wages:

\[
\frac{ds}{dR} > 0, \frac{ds}{dv} < 0, \frac{ds}{dw} < 0
\]

(3.7)

Based on the advice selected in the second stage (equation 3.3) and the zero profit condition of the entrepreneur (equation 3.5), it is possible to calculate the marginal project that is financed by the venture capitalist. The marginal project financed has zero expected

\(^{16}\) Parameters are such that the second order condition satisfies:

\[
\frac{d^2 \pi}{ds^2} = (1 - s_i) \Pr_{\text{in}} (a_i^*(s_i, R_i)) - 2 \Pr_{\text{a}} \frac{\partial a}{\partial s} R_i < 0 \quad \forall s_i \in (0,1)
\]
profits – the venture capitalist expects zero profits on the worst project. The condition characterizing the marginal project is:

\[
\tilde{\Pi}^{\text{VC}} = s^*_j(R_j, v, w) \cdot \Pr\left(\alpha_j^*(s^*_j(R_j, v, w), R_j)\right) \cdot R_j - I - C\left(\alpha_j^*(s^*_j(R_j, v, w), R_j)\right) = 0 \quad \forall j = \tilde{R}
\]

And the marginal project and total number of projects financed are denoted by:

\[
\tilde{R}_j^*(v, w, I); \quad N^* = \tilde{R} - \tilde{R}^*
\]  

(3.8)

The marginal project is a function of the exogenous variables (cost of entrepreneurial effort, outside wages, and the investment of the VC financier in the project). The marginal project decreases in these variables. The total number of projects financed depends on the marginal project selected by the VC financier. The total expected profits of the venture capitalist are \(\sum \Pi_j^{\text{VC}} \geq 0\). At worst, the venture capitalist funds zero projects and expect zero profits. When more than a single project is financed, the VC financier’s total expected profits are strictly greater than zero.

**Proposition 1:**

(i) Venture capitalist advice and share of profits are complements. An increase in the share of profits captured by the venture capitalist increases the advice he provides to the project.

(ii) The impact of returns, cost of effort, and outside wage on venture capitalist share of profits are ambiguous. Assumptions about the equilibrium must be made in order to get definitive results concerning the comparative static results. If we assume that at the equilibrium the cost of additional shares is dominated by the indirect effect of an increase in advice on expected returns, then entrepreneurs with higher returns receive more advice but a smaller share of returns. As cost of effort or outside wages increase entrepreneurs receive less advice and a greater share of returns.
1.4 The Social Optimum

The social optimum maximizes the social surplus (the sum of VC financier and entrepreneur profits). In the optimum, projects with positive net returns are undertaken, given the advice selected.

\[
TSS = \int_{R}^{\infty} \left[ \prod_{i}^{\text{VC}} + \pi^{E} \right] dR = \int_{R}^{\infty} [\Pr(a_{i}, e)R_{i} - I - C(a_{i}) - v - w] 
\]

(4.1)

The social surplus of project \( i \) is:

\[
SS^{i} = \Pr(a_{i}, e)R_{i} - I - C(a_{i}) - v - w 
\]

(4.2)

The social optimum maximizes the social surplus on each project (equation 4.2) subject to:

IC/PC entrepreneur

\[
(1 - s_{i}) \Pr(a_{i}, e)R_{i} - v - w \geq 0 
\]

And the selection of advice in the second stage:

\[
\max_{a} \Pr(a_{i})R_{i} - I - v - w - C(a_{i}) 
\]

FOC: \( \Pr_{a}(a_{i}, e)R_{i} - C_{a}(a_{i}) = 0 \)

The first order condition yields the optimal level of advice:

\[
a_{i}^{\ast}(R_{i}) > a_{i}^{\ast}(R_{i}) 
\]

(4.3)

The social optimum level of advice is strictly larger than the advice provided by the private venture capitalist. The private venture capitalist captures only a portion of the benefits from advice, but bears the entire cost of advice because they cede a portion of returns to the entrepreneur. Socially optimum advice is independent of shares. Thus, the private venture capitalist under-provides advice relative to the social optimum. The moral hazard problem
leads to this result. Advice is increasing in returns; projects with higher returns receive relatively more advice.

The constraint on the entrepreneur may bind or be slack, since the allocation of profits between the venture capitalist and the entrepreneur is undetermined in the social optimum. The minimum shares to ensure the participation of the entrepreneur is denoted by:

\[ s_i^{**}(R_i, v, w) \quad (4.4) \]

While the private VC financier wants to maximize its profits, the social optimum maximizes the social surplus. Thus, in the optimum projects are funded up to the point where the net value of undertaking that project, equation (4.2), is equal to zero. The condition for the marginal project is independent of equity shares.

This condition differs from the private market condition (equation 3.8) through choice of advice \((a)\). The marginal project is determined by the social planner’s objective. Substituting the optimal level of advice into the social surplus expression:

\[
SS = Pr(a_i^{**}(R_i), e)R_i - I - C(a_i^{**}(R_i)) - v - w = 0
\]

\[ \tilde{R}^{**} : Pr(a^{**})R_i - I - C(a^{**}) - v - w = 0 \]

results in the following relationship:

\[ \tilde{R}_i^{**}(v, w, I) \quad (4.5) \]

Using the implicit function of the marginal project, we find that there is under-financing (credit rationing) of entrepreneurs in the private market relative to the social optimum.

Using equation (3.5) and equation (3.8), the marginal project of the private venture capitalist becomes:
\[ \tilde{R}^* : \Pr(a^*)R_i - I - C(a^*) - \nu - w = 0 \]

From equation (3.3), we know that at \( a^* \) the marginal increase in profits from an increase in advice is greater than zero and at \( a^{**} \) it reaches a maximum:

\[ \frac{\partial [\Pr(a^*)R_i - C(a^*)]}{\partial a} > 0; \frac{\partial [\Pr(a^{**})R_i - C(a^{**})]}{\partial a} = 0 \]

Hence we know there is under-financing of projects in the private market relative to the social optimum:

\[ \tilde{R}_i^*(v, I) > \tilde{R}_i^{**}(v, I) \] (4.6)

The marginal project for the private venture capitalist is higher than in the social optimum. Projects that should get funded using a positive net benefit criteria are not all funded in the private market. Why are there fewer projects funded in the private market compared to the social optimum? The source of inefficiency in the private market stems from the sub-optimal advice created by a moral hazard problem. The sub-optimality of advice is exacerbated as the share of returns captured by the venture capitalist decreases.

**Proposition 2:**

The private venture capitalist under-provides both advice and total number of projects relative to the social optimum.

In the next sections, the impacts of different policies on the level of advice and the number of projects funded are examined. The policy options considered are: (1) Capital gains tax; (2) Tax on entrepreneur’s revenue; (3) Investment subsidy for VC financiers and entrepreneur; (4) government programs designed to increase the probability of success or the returns of projects in the private market (e.g. services, workshops/seminars, expertise on
project planning and management, and commercialization); and (5) Direct government VC investment.

1.5 Tax and Subsidy Policy

We introduce the following functional forms that allow for a more intuitive analysis:

\[
\Pr(a) = a^{1/2} \\
C(a) = a
\]

There are two scenarios that we examine in the private market. (1) The venture capitalist has market power and captures the entire surplus (rents) from a project, and (2) the entrepreneur has the power and can capture the entire surplus from a project.\(^{17}\) We include policy instruments in the form of capital gains tax (\(\tau\)), tax on the revenue of entrepreneurs (\(t\)), and an investment subsidy for VC (\(\sigma\)) that is net of capital gains tax.\(^{18}\)

Case 1: Venture Capitalist has the bargaining power

The venture capitalist has the bargaining power and is able to extract the entire surplus from the entrepreneur. He maximizes total profits subject to the participation and incentive constraints of the entrepreneur and the selection of his advice in the second stage:

\[
\max \prod_{s_i, R} \int_R [(1 - \tau) s_i a_i^{1/2} R_i - (1 - \sigma) I - a_i] dR
\]

subject to

\[
\text{PC}^E: (1 - t) (1 - s_i) a_i^{1/2} R_i - v \geq w \quad \forall i = 1, \ldots, N \\
\text{IC}^\text{VC}: \max_a \prod_i = (1 - \tau) s_i a_i^{1/2} R_i - (1 - \sigma) I - a_i \quad \forall i = 1, \ldots, N
\]

\(^{17}\) We chose to examine the two extreme cases of Nash bargaining. When bargaining results in sharing of market power, the results lie somewhere in between these two extremes.

\(^{18}\) We do not include loss offset in the model. If projects differ in risk and the tax code allows for loss offset, with taxes and risk sharing, the venture capitalists (and entrepreneurs) may take on more risk.
The constraints bind for each project financed. Since returns are observable, there is no adverse selection problem. The venture capitalist extracts the entire surplus and funds projects until profit is equal to zero.

Solving the first order condition of the incentive constraint problem yields:

\[ \text{IC}^{VC}: \quad a^*_i = \left( \frac{(1 - \tau)s_iR_i}{2} \right)^2 \quad (5.2) \]

Using this result, we can compute the comparative static results for advice:

\[ \frac{da}{d\tau} < 0 \quad (5.3) \]

For a given share and return, venture capitalist advice decreases in the capital gains tax.\(^{20}\)

Substituting the selection of advice into the participation constraint for the entrepreneur:

\[ s_i(1 - s_i) = \frac{2(w + v)}{(1 - t)(1 - \tau)R_i^2} \quad (5.4) \]

There are two possible equilibria. In one equilibrium, the venture capitalist keeps a larger share of profits and puts more advice into the project. In the other, the venture capitalist cedes a larger share of profits to entrepreneurs and contributes less advice.\(^ {21}\) The VC financier selects the first of these equilibria since it captures a larger share of profits and the probability of project success is higher. The venture capitalists profit is maximized in selecting the highest share of profit possible. This condition defines the shares that satisfy

\(^ {19}\) The Second Order Condition to equation (5.2) ensures that the advice selected maximizes the function.

\(^ {20}\) Keuschnigg and Nielsen (2003, 2004a, 2004b) also find that a capital gains tax decreases advice.

\(^ {21}\) Kanniainen and Keuschnigg (2003, 2004) and Keuschnigg and Nielsen (2007) present arguments as to why maximum shares are selected. The venture capitalist selects the larger shares and advice that maximize expected profits. The second order conditions verify the concavity of the share of returns function. See appendix A(1).
the participation and incentive constraints of the entrepreneur (equation 5.1). Defining the shares function as an implicit function, comparative statics are computed:\textsuperscript{22}

\[ \frac{ds}{d\tau} < 0, \frac{ds}{dt} < 0, \frac{ds}{d\sigma} = 0 \] \hspace{1cm} (5.5)

An increase in the capital gains tax or entrepreneur revenue tax reduces the equity shares of the venture capitalist.\textsuperscript{23} The equity shares of the venture capitalist remain constant with respect to an investment subsidy.

The number of projects is determined by a zero profit condition on the marginal project the venture capitalist finances:

\[ \prod_{j, \sigma} = (1 - \tau)s_{j}^* a_{j}^{1/2} R_j - (1 - \sigma)I - a_{j}^* = 0 \quad \forall j = \tilde{R} \] \hspace{1cm} (5.6)

Using the implicit function theorem, this yields the following comparative static results (see Appendix A(3) for derivation):

\[ \frac{d\tilde{R}}{d\tau} > 0, \frac{d\tilde{R}}{dt} > 0, \frac{d\tilde{R}}{d\sigma} < 0 \] \hspace{1cm} (5.7)

A capital gains tax decreases the marginal benefits of an additional project, thus the number of projects falls with the capital gains tax. An investment subsidy will increase the number of projects that are funded.\textsuperscript{24} The reduced cost lowers the marginal project threshold.

Using the equations (5.2), (5.6), and the participation constraint of the entrepreneur, we derive a condition for the marginal project (see Appendix A(4) for derivation):

\textsuperscript{22} See appendix A(2) for computation of comparative static results.

\textsuperscript{23} The comparative static for capital gains tax and revenue tax on entrepreneurs assume that $t \neq \tau$, i.e., the tax on entrepreneurs and venture capitalists are not identical. The results do not change if I allow these rates to be identical.

\textsuperscript{24} Keuschnigg (2004) and Keuschnigg and Neilsen (2001, 2003, 2004a, 2004b) also find that a capital gains tax reduces, and an investment subsidy increases, the number of projects funded.
\[(\tilde{R}^*)^2 = \frac{4(1-\sigma)I + v + w}{s(1-\tau)(1-t)} \] (5.8)

The social optimum problem results in a level of advice that is not distorted by the moral hazard problem or by the distortions from taxation. The socially optimal level of advice is:

\[a_{i^*}^* = \left(\frac{R_s}{2}\right)^2 > a_i = \left(\frac{(1-\tau)s_iR_{i^*}}{2}\right)^2 \] (5.9)

We find the socially optimum share of returns required to ensure the participation of the entrepreneur.\(^{25}\) The participation constraint of the entrepreneur in the social optimum takes the form:

\[(1 - s_i) \frac{R_i^2}{2} - v - w = 0 \] (5.10)

Substituting equation (5.10) into the social optimum objective yields the marginal project funded in the social optimum:

\[\tilde{R}^2 - I - \tilde{R}^2 4 - v - w = 0 \] \[(\tilde{R}^*)^2 = 4(I + v + w) \] (5.11)

There are 4 situations that need to be considered when evaluating the number of projects funded by the private venture capitalist relative to the social optimum.

(i) If \(\tau = 0, t = 0,\) and \(\sigma = 0,\) then there is under financing of projects relative to the social optimum. Starting where there is no public policy intervention, it is clear that there are too few projects funded by the private VC financier relative to the social optimum, as shown in section 4.

\(^{25}\) The surplus can be redistributed ex-post using lump-sum transfers.
(ii) If $\tau > 0$ (or $t > 0$) and $\sigma = 0$, then there is under financing of projects relative to the social optimum. The capital gains tax (tax on entrepreneur revenue) further distorts both the advice provided to each project and the number of projects funded by the private VC financier.

(iii) If $\tau = 0$, $t = 0$ and $\sigma > 0$, then there may be under financing, optimal financing or over financing of projects relative to the social optimum. The introduction of a subsidy on investment decreases the cost to the venture capitalist and induces the funding of more projects. It increases the break-even condition for the VC financier on the marginal project.

(iv) If $\tau > 0$ (and/or $t > 0$) and $\sigma > 0$, then there may under financing, optimal financing or over financing of projects relative to the social optimum. There are two effects moving in opposite directions. The first is that the capital gains tax (tax on entrepreneur revenue) is reducing incentives to advise projects and thus reduces the expected returns and number of projects. The second effect is that the subsidy encourages funding of more projects. Hence the overall effect is ambiguous.

**Proposition 3:**

For $s > 1/2$, (i) An increase (decrease) in the capital gains tax or entrepreneur revenue tax leads to a decrease (increase) in venture capitalist advice and share of returns in a project and a decrease (increase) in the number of projects funded. (ii) An increase (decrease) in a subsidy on investment has no impact on venture capitalist advice or shares, but leads to an increase (decrease) in the number of projects funded. (iii) An increase (decrease) in the capital gains tax or entrepreneur revenue tax leads away from (leads towards) the social...
optimal advice and number of projects. (iv) An increase (decrease) in an investment subsidy leads towards (leads away from) the socially optimal number of projects.

Case 2: Entrepreneur has the bargaining power

When the entrepreneur has the bargaining power, she compensates the venture capitalist to induce entry and advice. In this characterization, the entrepreneur is the principal and the venture capitalist is the agent. The entrepreneur has the innovative idea and the property rights. The value (return) of a project is observable and the entrepreneur seeks financing to develop and commercialize the innovation. The entrepreneur’s problem takes the following form:

\[
\max_{s} \quad \pi = (1 - t)(1 - s_i) a_i^{1/2} R_i - v - w \quad \quad (5.12)
\]

subject to

\[
PC_{VC}: \quad (1 - \tau)s_i a_i^{1/2} R_i - (1 - \sigma)I - a_i \geq 0
\]

\[
IC_{VC}: \quad \max_a \quad \prod_i = (1 - \tau)s_i a_i^{1/2} R_i - (1 - \sigma)I - a_i
\]

The incentive constraint for the venture capitalist determines the advice dedicated to the project. The participation constraint determines if the venture capitalist finds it profitable to enter. Equation (5.2) defines the advice provided by the venture capitalist. The PC\textsuperscript{VC} is the binding constraint that determines the shares function:

\[ s_i = \frac{\gamma}{2} \text{ when } PC_{VC} > 0 \]

and
\[ s_i = \left( \frac{4(1-\sigma)I}{R_i^2(1-\tau)^2} \right)^{1/2} > \frac{1}{2} \text{ when } PC_{\text{VC}} = 0 \]  

(5.13)

An interior solution exists with share of revenue between zero and one.\(^{26}\)

The entrepreneur maximizes equation (5.12). If the entrepreneur maximizes expected profits with respect to shares without considering the participation constraint of the venture capitalist, her expected profits are maximized when \(s=1/2\). If the venture capitalist will still participate at this level of shares, then this is the solution. However, if the venture capitalist will not participate when \(s=1/2\), then the entrepreneur must now consider the participation constraint. In this case, the share of returns to the venture capitalist needed to induce participation is greater than 50 percent.\(^{27}\) The entrepreneur desires to give only \(s=1/2\) to the venture capitalist, but if the venture capitalist is unwilling to participate at \(s=1/2\), the entrepreneur cedes a larger share of revenue to the venture capitalist (\(s>1/2\)) until the participation constraint is satisfied.

This implies that for a set of projects with sufficiently high returns, the entrepreneur requires only a 50 percent share of profits. Entrepreneurs with sufficiently high returns, trade shares for additional advice and maximize profits by yielding more shares than are necessary to meet the VC financier’s participation constraint.\(^{28}\) For projects with lower returns, entrepreneurs must offer shares greater than \(1/2\) to the venture capitalist in order to satisfy the venture capitalists participation constraint.

\(^{26}\)The condition \(4(1-\sigma)I < R_i^2(1-\tau)^2\) ensures that \(s < 1\) and is assumed satisfied. Note that \(s > 0\) is already imposed by \(R \geq 0, I \geq 0, t \geq 0\) and \(\tau \geq 0\). If the taxes were assumed to be subsidies, the results still hold.

\(^{27}\)The result is analogous to de Bettignies and Brander (2007).

\(^{28}\)For a more general proof of the result see Appendix A(5).
As returns increase, the entrepreneur can capture a larger share of profits while satisfying the VC financier’s participation constraint. At \( s > \frac{1}{2} \) \((1-s < \frac{1}{2})\), the entrepreneur extracts the entire surplus from the venture capitalist and the participation constraint binds. When \( s = \frac{1}{2} \), the participation constraint of the venture capitalist is slack and the entrepreneur cedes share of returns to the venture capitalist in exchange for more advice and thus a higher probability of success. To summarize, (1) high quality entrepreneurs give a share of profits to the venture capitalist equal to \( \frac{1}{2} \) and (2) low quality entrepreneurs offer the venture capitalist a share of profits greater than \( \frac{1}{2} \). The relation is shown in graph 3.

**Advice:**

(1) High Quality Entrepreneurs

When the entrepreneur extracts only a portion of the entire surplus, leaving surplus to the venture capitalist, advice increases in returns and decreases in capital gains tax. Advice is unaffected by investment costs and subsidies.

\[
\frac{da}{d\tau} < 0, \quad \frac{da}{d\sigma} = \frac{da}{dl} = 0 \quad (5.14)
\]
(2) Low Quality Entrepreneurs

When the entrepreneur extracts the entire surplus, advice increases in returns and shares, and decreases in capital gains tax. The results are identical to those when the venture capitalist has the market power (equation 5.3).

Substituting equation (5.13) into equation (5.2), we find optimal venture capital advice in terms of exogenous variables:

\[ a^* = (1 - \sigma)I \quad (5.15) \]

There is a positive relationship between the share of profits of the venture capitalist (s) and the advice (a) he provides. An increase in returns increases advice, but decreases shares. Thus, there is a direct effect of returns on advice and an indirect effect through selection of shares. Similarly, an increase in the capital gains tax decreases advice, but increases the venture capitalists share of profits. The direct and indirect effects cancel each other out. As a result, the overall impact of an increase in returns or capital gains tax on advice is zero.

The direct effect (first term) is offset by the indirect effect (second term):

\[ \frac{da}{dR} = \frac{\partial a}{\partial R} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial R} = 0 \quad \text{and} \quad \frac{da}{d\tau} = \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} = 0 \quad (5.16) \]

An increase in investment cost results in an increase in advice, while an increase in investment subsidy results in a decrease in advice. The indirect effect of investment cost and subsidy impacts the advice provided by the venture capitalist.

\[ \frac{da}{dl} > 0, \frac{da}{d\sigma} < 0 \quad (5.17) \]
Shares and the marginal project:

(1) For the high quality projects, changes in policy have no affect on the share of revenue offered to the venture capitalist. However, changes in policy or exogenous variables affect the threshold at which the participation constraint of the venture capitalist binds.

\[
\frac{ds}{dR} = \frac{ds}{dT} = \frac{ds}{dI} = \frac{ds}{d\sigma} = 0 \quad (5.18)
\]

Suppose all projects are “high quality” such that the participation constraint of the venture capitalist is not binding. The marginal project increases in a capital gains tax and a revenue tax on entrepreneurs. These are identical to the effects when the venture capitalist has the market power:

\[
\frac{d\tilde{R}}{dT} > 0, \frac{d\tilde{R}}{dt} > 0, \frac{d\tilde{R}}{dI} = 0, \frac{d\tilde{R}}{d\sigma} = 0 \quad (5.19)
\]

(2) For low quality projects, changes in exogenous variables affect the share of profits offered to the venture capitalist as follows:

\[
\frac{ds}{dR} < 0, \frac{ds}{dT} > 0, \frac{ds}{dI} > 0, \frac{ds}{d\sigma} < 0 \quad (5.20)
\]

The comparative static results for the marginal project are the following:

\[
\frac{d\tilde{R}}{dT} > 0, \frac{d\tilde{R}}{dt} > 0, \frac{d\tilde{R}}{dI} > 0, \frac{d\tilde{R}}{d\sigma} = 0 \quad (5.21)
\]

For a binding participation constraint, the venture capitalist share of profits decreases in returns. The share of profits offered to the venture capitalist increases with the capital gains tax and increases the threshold project for which the participation constraint binds. An increase in the investment cost increases the share of profits required and the threshold project for which the participation constraint binds. A subsidy on VC investment
decreases the share of profits required and decreases the threshold for which the participation constraint binds.

**Proposition 4:**

When the entrepreneur has market power and the project is “low quality”, a change in the capital gains tax has zero net impact on venture capitalist advice. An increase (decrease) in the capital gains tax increases (decreases) venture capitalist share of profits and decreases (increases) the advice provided by the venture capitalist. These impact the selection of advice in opposite directions and cancel each other out.

**Summary of Results: Advice**

The comparative static analysis for advice and number of projects are considered for both high quality and low quality projects. For high quality projects, an increase in returns or decrease in capital gains tax increases advice. When the project is low quality, capital gains tax has no effect on advice. An increase in the investment cost or a decrease in an investment subsidy increases advice.

For a complete summary of advice, shares, and marginal project comparisons between high quality and low quality projects see the table in Appendix B.

**Proposition 5:**

For high quality projects: (i) The capital gains tax has no impact on the distribution of shares of profits; (ii) Advice decreases with capital gains tax; (iii) The number of projects funded decreases with the tax; and (iv) Investment subsidies have no impact on advice, shares, or number of projects.
For low quality projects (i) The number of projects funded decreases with the capital gains tax, (ii) An investment subsidy does not impact advice, decreases venture capitalist share of profits, and has an ambiguous impact on number of projects.

1.6 Government Support for Commercialization

Keuschnigg and Nielsen (2001) suggest and discuss how governments may introduce programs that increase the probability of successful projects. Further, they discuss and demonstrate that advice and government programs can be complements or substitutes. In practice, the government uses a myriad of programs, services, workshops and seminars, networking, expertise on project planning and management, and commercialization, which enhance the probability of a successful project and the value of projects. The target of government policy, in this context, is to stimulate and increase innovation. These programs increase the expected returns of venture capital projects and may succeed in increasing the supply and the quality of venture capital financing.

In this section, we briefly analyze the impact of a government support program on the level of advice provided by the venture capitalist, the number of projects funded and the shares of returns captured by the venture capitalist and the entrepreneur. To do so, we assume that the probability function takes the following form:29

$$\Pr(a, g, e) = \frac{ea^\alpha g^\beta}{\alpha \beta}$$  \hspace{1cm} (6.1)

where $g$ denotes government support and $\beta < \alpha$, capturing the assumption that the marginal benefit of additional government support is less than the marginal benefit of VC financier advice. Venture capitalist hands-on advice should contribute more to the probability of a

---

29 If government support affects project return, $R$, rather than the probability of success, it does not change the results.
successful project than indirect government programs designed to better equip managers and entrepreneurs. The probability function satisfies the following properties:

\[ \Pr_g > 0, \Pr_{gg} < 0, \Pr_a > 0, \Pr_{aa} < 0, \Pr_{ag} = \Pr_{ga} > 0 \]  \hspace{1cm} (6.2)

Hence, it is assumed that government support and VC advice are complements.\(^{30}\)

The venture capitalist chooses advice, \(a\), to maximize expected return:

\[
\max_a \quad s \frac{a^\alpha g^\beta}{\alpha \beta} R - a
\]

FOC: \( s \frac{a^{\alpha - 1} g^\beta}{\beta} R - 1 = 0 \)

\[ a(g) = \left( \frac{sg^\beta R}{\beta} \right)^{\frac{1}{1-\alpha}} \]  \hspace{1cm} (6.3)

Equation (6.3) is the venture capitalist’s best-response function. It defines the optimal level of advice in the private market, given \(g\). Advice increases with government support at a decreasing rate. Government policy therefore improves the under-provision of advice in the private market. An increase in the return to government support (\(\beta\)) also increases the advice provided by the venture capitalist. Since the government program parameter and advice are complements, government policy does not crowd-out private advice. Thus, we have:

\[ \frac{da}{dg} > 0, \frac{da}{d\beta} > 0, \frac{da}{d\alpha} > 0 \]  \hspace{1cm} (6.4)

The impact of government support on the share of profits captured by the venture capitalist and the number of projects are positive. The VC financier captures the entire profits.

\(^{30}\) If government support and VC advice are substitutes, there is crowding-out of venture capitalist advice. The VC financier reduces the amount of advice it provides in favor of using government support; the venture capitalist reduces its costs by reducing its advice.
surplus from a project and as the probability of success increases, is able to capture a larger share of project returns. The same reasoning holds for the marginal project. Projects with lower returns become more profitable at no cost to the venture capitalist. The returns required on the marginal project falls and the number of projects funded increases. We obtain:

$$\frac{ds}{dg} > 0, \frac{d\hat{R}}{dg} > 0$$

(6.5)

The government chooses support, $g$, to maximize the social surplus of the project:

$$\max_g \frac{a^\alpha g^\beta}{\alpha \beta} R - a - g - I - v - w$$

FOC: $$\frac{a^\alpha g^{\beta-1}}{\alpha} R - 1 = 0$$

$$g(a) = \left(\frac{a^\alpha R}{\alpha}\right)^{\frac{1}{1-\beta}}$$

(6.6)

Equation (6.6) defines the best-response of the government programs to the selection of advice by the venture capitalist. Using equations (6.3) and (6.6), we solve for the optimal level of government support and venture capitalist advice as functions of exogenous parameters:

$$g^* = \left(\frac{s R^{\frac{\alpha}{1-\alpha}}}{R^{\frac{\beta}{1-\alpha}}}\right)$$

$$a^* = \left(\frac{s R^{\frac{\beta}{1-\beta} (1-\alpha)}}{R^{\frac{1-\beta}{1-\beta} (1-\alpha)}}\right)$$

(6.7)

The results of this section are listed in the following proposition.
Proposition 6:

Complementary government programs increase the probability of success of a project (i) decrease the under-provision of advice by the VC financier, (ii) increase the number of projects funded in equilibrium, and (iii) increase the share of profits captured by the venture capitalist.

Note that, when venture capitalists have market power and are able to extract the entire surplus from a project, the entrepreneur does not gain from receiving government support for its project. The terms of the financing contract adjust, implying that the entire additional return is captured by the venture capitalists.

1.7 Competition between Government Venture Capital and Private Venture Capital

In this section, we examine the impact of introducing a government venture capitalist that competes with the private venture capitalist. In order to do so, we assume that, as the government venture capitalist finances more projects, the costs of the government venture capitalist and of the private venture capitalist increase. This could be seen as reflecting search costs: the more projects are funded, the more difficult it is to find promising projects. The simplest way to introduce this in the model is to assume that the cost function, for both the government and private venture capitalists, is as follows:

\[
C(a, N) = a \cdot N = a \cdot [N^p + N^G] = a \cdot [(\overline{R} - \tilde{R}) + N^G]
\]  

(7.1)

where \( N = N^p + N^G \), and \( N^p \) and \( N^G \) are the number of projects funded by the private and public venture capitalists, respectively.

The venture capitalist maximizes its expected profit taking into account the effect additional advice has on total cost of advice rather than just the cost of advice for that project. The selection of advice in the second stage follows:
\[
\max \ a \ E \prod_{i}^{VC} = s_i a_i^{1/2} R_i - a_i N \quad \forall i = 1, ..., N
\]

\[
a_i = \left( \frac{s_i R_i}{2(N^P + N^G)} \right)^2
\]  

(7.2)

As the number of projects increases, the amount of advice provided per project decreases. In choosing how many projects to finance, the VC financier will balance the benefits of additional projects against a decrease in advice per project. Since the venture capitalist has perfect information and practices first degree discrimination, capturing the entire surplus of a project, he funds projects until expected profit equals zero.

**Competition from the government venture capitalist:**

We assume that the government selects only project that would not be financed by the venture capitalist, but are still socially desirable. Therefore there is market segmentation. The government is assumed to have the objective of maximizing the social surplus of the project it funds.

Since projects are selected outside the set of private projects, the number of private projects can be defined as the set of projects from the highest return to the marginal project.

The shares function defined by the participation constraint on the entrepreneur becomes:

\[
s_i (1 - s_i) = \frac{2(v + w)[(\overline{R} - \tilde{R}) + N^G]}{R_i^2}
\]  

(7.3)

Define as an implicit function (F):

\[
F = s_i (1 - s_i) - \frac{2(v + w)[(\overline{R} - \tilde{R}) + N^G]}{R_i^2} = 0
\]

Totally differentiating with respect to the parameters of interest \((s, N^G)\):
\[
[(1 - s_i) - s_i]ds - \left\{ \frac{2(v + w)((\bar{R} - \tilde{R}) + N^G)}{R^2_i} \right\}dN^G = 0
\]

\[
\frac{ds}{dN^G} < 0 \quad (7.4)
\]

The first term is negative \((s > 1/2)\) and the second term inside the bracket is positive, as the number of government projects increases, the share of profits captured by the venture capitalist decreases. This is intuitive since the advice provided to the project decreases, the probability of success falls and the share of profits captured by the venture capitalist decreases.

The introduction of a government venture capitalist has an additional impact: the marginal project of the private venture capitalist will change. The venture capitalist captures the entire surplus from a project, but an increase in government projects increases costs. The marginal project that satisfies the zero profit condition of the venture capitalist has changed. Substituting the advice of the venture capitalist into the profit of the marginal project we obtain:

\[
\Pi(\tilde{R}) = s\frac{s\tilde{R}}{2[(\bar{R} - \tilde{R}) + N^G]} - I - \left( \frac{s\tilde{R}}{2[(\bar{R} - \tilde{R}) + N^G]} \right)^2 \left[(\bar{R} - \tilde{R}) + N^G \right] = 0 \quad (7.5)
\]

Remember that \(s\) is a function of number of projects, wages, and cost of entrepreneur effort. We find that an increase in the number of government projects increases the return required on the marginal project. The number of venture capitalist projects is reduced and crowded-out by government projects. That is,

\[
\frac{d\tilde{R}}{dN^G} > 0 \quad (7.6)
\]
Proposition 7:

(i) A government venture capitalist that selects low return, but socially desirable projects, crowds-out private venture capital projects with better returns. (ii) The overall impact on advice and shares is ambiguous and depends on the extent of the crowding-out.

1.8 Conclusion

In addition to financing, venture capitalists contribute managerial expertise and advice that increases the probability of a successful venture and are able to better select and screen potential projects than other types of financiers such as banks. In exchange for financing and managerial advice the venture capitalist requires an equity share in the project. There is a moral hazard problem that sees venture capitalists under-provide advice relative to the social optimum.

The government can implement tax and investment policies to stimulate the venture capital market. In Canada, the R&D tax credits act as an investment subsidy while capital gains tax has been criticized as a major impediment for investments in innovation. Programs that encourage and support entrepreneurial activity are continually being advocated to develop an innovative character in Canada. These programs target increasing knowledge, experience, and the supply of investment funds. Government venture capital corporations have been established to increase investment.

Our model provides several interesting insights on the impact of government policy on venture capital markets. We examine the effect of a capital gains tax, a tax on entrepreneur revenue, and an investment subsidy on the private market equilibrium. The benchmark is where the venture capitalist has market power. The venture capitalist under-provides advice and the number of projects it funds relative to the social optimum. A capital
gains tax or tax on entrepreneur revenues decreases the advice provided and the number of projects funded. An investment subsidy increases the number of projects funded. If the entrepreneur has the market power, she may maximize her returns by ceding a portion of the surplus to the venture capitalist. High quality projects will cede rent to the venture capitalist while low quality projects will extract the entire surplus. A capital gains tax has no impact on venture capitalist advice provided to low quality projects but decreases advice provided to high quality projects. An investment subsidy lowers advice on low quality projects. Irrespective of project quality, a capital gains tax unambiguously decreases the number of projects funded.

Government initiatives such as commercialization programs, seminars on successful business practices, and networking conferences increase the probability of a successful project or increase the value of a project. Government support programs that complement venture capitalist advice increase venture capitalist advice and the number of projects funded.

In Canada, there are several government venture capitalists (e.g., BDC) that directly invest and advise innovative projects. We find that when there is competition between government venture capitalist and private venture capitalist, the government venture capitalist crowds-out the private venture capitalist.

There are several possibilities for future research concerning the impact of government venture capital. First, including an adverse selection problem in the evaluation on government and private VC competition would enhance understanding. Second, a more rigorous model of competition with strategic investment would help to shed light on the “overhang” and relatively low returns to venture capital observed in Canada.
Chapter 2

Imperfect Competition and Entry Deterrence in Venture Capital Markets
2.1 Introduction

Venture capitalists finance highly innovative, high risk projects that have difficulty securing financing from traditional financiers (e.g. banks) due to high risk and low assets of the firm. In addition to providing investment in the project, the venture capitalist contributes advice, such as commercialization knowledge, that enhances the value of the project. The venture capitalist is also skilled in selecting the best projects. Its knowledge and skills are usually specific to certain industries or stages of development (early stage startup or later stage established firm innovation). Thus, the venture capitalist tends to focus where its expertise and advice receive the maximum returns.

The relatively small supply of venture capital in Canada has been raised as an issue and has attracted considerable policy interest, as the following suggests:

“Because of the general perception that there has been an insufficient supply of private venture capital, both federal and provincial governments have sought to redress this market failure and assist SMEs in overcoming the barriers that inhibit their capital supply.”
Osborne and Sandler (1998), pp.536

Government policy, taxes, and programs have focused on increasing the supply of venture capital in Canada. The main tax instruments to increase the supply of venture capital are investment subsidies (R&D and S&T tax credits) and subsidies designed to increase the funds raised (Labour fund tax credits). In addition, the size of corporate taxes has been identified as a barrier to venture capital funding and innovative activity. It is argued that a reduction in capital gains taxes would increase the amount of innovation (Cumming (2007)). The effect of these subsidies and taxes, not only on the overall supply of venture capital, but also on the venture capital market structure, needs to be considered and examined. That is the main purpose of this paper.
The venture capital market is characterized by few venture capital financiers competing in specific industries (biotechnology, software, IT, etc…) or stage of financing (early stage development or later stage marketing). Venture capitalists, in Canada, tend to specialize in concentrated industry or stage focus. They possess knowledge that is specific to the type of industry and stage they specialize in.

Venture capitalists raise funds from external investors to finance entrepreneurial projects. There are different types of venture capital funds that operate in Canada. The largest types of venture capital financiers are private and Labour-sponsored venture capital corporations (LSVCCs).³¹ Private venture capitalists raise funds from a few sources while LSVCCs raise funds from many individual investors each contributing small amounts. LSVCCs raise funds through a tax credit system that rewards individual investors for making contributions.³² The labour funds are approved by the government and sponsored by a union. The provincial government approves of a labour fund, before it receives the benefits of the tax credit. The federal government provides a matching tax credit. Recently, the efficiency and implications of labour funds has been questioned.³³ It is argued that labour funds are of lower quality and evidence suggests that LSVCCs crowd-out private venture capital. Venture capitalist fund raising, especially when subsidized by governments, may be used as a strategic investment in capacity. Fund raising could act as a potential barrier to entry and give market power.

There are several tax instruments that the government uses to influence the venture capital market. A revenue tax on venture capitalists and a tax on entrepreneur’s profits are

³¹ Foreign investors, mostly from the United States of America, are also very large contributors of venture capital finance in Canada.
³² The tax credits are provided by the provincial government and a matching credit from the Federal government. There is a maximum claim that can be made by the investor.
two taxes that venture capital firms and entrepreneurs face. These taxes may distort the advice and effort provided by the venture capitalist and entrepreneur and the number of projects funded. In addition, these taxes may affect the incentives of an incumbent venture capital firm to exercise market power and deter entry. Subsidies on investment, such as the R&D tax credit, may also affect the number of projects financed by venture capitalists and the likelihood of entry deterrence. In reality, governments provide subsidies that target a specific type of venture capitalist: LSVCCs. A targeted subsidy on fund raising, consistent with the LSVCCs tax credit, may also impact the number of projects that receive financing or the likelihood of entry deterrence.

This paper examines the impact of these government policies in a venture capital market that is highly concentrated and in which an incumbent firm can choose to raise funds strategically in order to deter entry. We develop a model that describes when the venture capitalist may find it profit maximizing and credible to deter entry. Various tax and government policy initiatives (tax on revenues, tax on entrepreneurs revenue, investment subsidy, subsidy on funds raised) are examined and the impacts on the decision of the incumbent regarding number of investments or projects funded and entry deterrence. The effect on the total number of projects financed in the entire market is also examined. The paper contributes to understanding how venture capitalists compete and the impact of taxes and subsidies on venture capital market structure.

Spence (1977) and Dixit (1980) are the seminal papers on strategic investment in capacity and entry deterrence. The game structure, in both, is a two stage game. In the first stage, the incumbent makes a strategic investment in capacity. The entrant observes the investment and makes its decision on whether to enter the market. In the second stage, the
incumbent and entrant compete under a specified game structure (Stackelberg, Cournot) if entry occurs. If entry is deterred, the incumbent is alone in the market and produces either the monopoly output or the output that just, and profitably, deters entry (limit output). The incumbent firm uses its first mover advantage to make a strategic, irreversible, investment in capacity. It is able to commit itself to an action based on its investment. The decision of the incumbent must be credible and profit maximizing. In Dixit (1980), capacity can be expanded in the second stage product competition. Although raising high capacity may be used to deter entry, holding excess (idle) capacity is never optimal. The incumbent uses all the capacity it has built in the previous stage. The incumbent has a first mover advantage and commits to a strategy, entry deterrence or accommodation, which is credible and profit maximizing. Ware (1984) questions the sequence of events in the Dixit model, arguing that it is more consistent to model capacity as a three stage game where the incumbent sets capacity, the entrant sets capacity, and then firms compete in Cournot. The set of equilibrium possibilities is reduced, but the general conclusions of the Dixit model still hold. Using the Dixit framework, Dixit and Kyle (1985) conduct an analysis of taxation in an international market. The optimal tax responses of governments are examined and conditions for entry deterrence.

Keuschnigg and Neilson (2001, 2003, 2004a, 2004b) and Keuschnigg (2004) use a principal-agent model to examine how taxes and an investment subsidy impact the advice decision of the venture capitalist and the number of projects financed. More advice from the venture capitalist increases the probability of a successful project. They find that a capital gains tax reduces the number of projects funded and dilutes the advice provided by the

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34 Bulow et al. (1985) show demand conditions that lead to output being strategic complements, at least over a range of output, and holding excess capacity may be optimal. Maskin (1999) shows that under demand uncertainty it may be optimal to hold excess capacity.
venture capitalist. To our knowledge, there is no research concerning the impact of taxes and subsidies on the venture capital market structure and on strategic fund raising decisions. Other models of venture capital include Bernile et al. (2008) and de Bettignies and Brander (2007). In both of these papers the focus is on the selection of inputs by the entrepreneur and the venture capitalists that increase the revenue of an entrepreneurial project.

The paper proceeds as follows: Section 2 describes the model. Section 3 derives the different equilibrium possibilities and entry deterrence conditions. Section 4 examines the impact of a venture capitalist revenue tax and an entrepreneur revenue tax on the market conditions. Section 5 introduces a subsidy on investment and a subsidy on raising funds and examines the impact on market conditions. Section 6 introduces targeted subsidies, for the entrant or incumbent, consistent with what is observed in reality. The results are compared to generally, or universally, applied subsidies. Section 7 briefly discusses the effect of government programs and subsidies that target the fixed costs of establishing a venture capital fund. Section 8 concludes.

2.2 The Model

There are three players in the game: investors, venture capitalists, and entrepreneurs. There is initially an incumbent venture capitalist in the market and one potential entrant. The model is a three stage game. In the first stage, the incumbent venture capitalist raises funds from the investors. In the second stage, if the potential entrant enters the market it raises funds. The venture capitalist(s) then invest in entrepreneurial projects and determine the terms of the equity contracts with the entrepreneurs that stipulate how profits are shared. Finally, in the third stage, the venture capitalist(s) and entrepreneurs, respectively, provide advice and effort to the projects that are financed.
Sequence of Events:

1st Stage: Incumbent venture capitalist raises funds from investors.

2nd Stage: The entrant decides whether or not to enter. If he enters, he then raises funds. The venture capitalist(s) select the number of projects (entrepreneurs) and equity shares of revenue.

3rd Stage: The entrepreneur and venture capitalist(s) contribute, respectively, effort and advice to the projects that are financed.

The venture capital market is characterized by imperfect competition, which is captured by assuming that there are only two venture capital firms: an incumbent and a potential entrant. The incumbent venture capitalist firm has a first mover advantage, raising its funds in the first stage. This first mover advantage may be used to deter entry. If entry is deterred, the incumbent venture capitalist is alone in the market and invests in either the monopoly number of investments or the “limit” investments (i.e., the number of investments obtaining under monopoly or the limit that deters entry). Its choice, to deter or accommodate entry, must be both credible and profit maximizing.

The incumbent venture capitalist’s fund raising capacity is selected in the first stage. The incumbent is assumed not to raise funds in the second stage.35 In the real world, venture capital fund raising, in limited partnerships, is typically done over a fixed period of time. Once the funds have been raised, then the investment stage begins.36 Funds are raised and intended for venture capital investment. If the funds were used elsewhere, it would

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35 If this assumption is relaxed, the outcome is the same. The incumbent that finds it profitable to increase its capacity will do so in the first stage since it gains a first-mover advantage that is lost if it raises capacity in the second period.

36 Cressy (2006) describes the establishment of a venture capital fund as follows: “Investors are invited to participate in a fund… with an obvious sector or stage focus… The fund size is normally fixed. Once the sum has been raised, investments will be made…” (Cressy, 2006, pp. 361)
“violate” the contract with the investors. LSVCCs are required by statute to invest in venture capital projects. Funds raised for venture capital investments are a capital expenditure that is sunk. Fund raising is also assumed to be irreversible. This implies that the incumbent can commit to a capacity level and use fund raising as a strategic investment. If the funds are not invested in venture capital, it remains uninvested.

It is assumed that investors are willing to lend funds to venture capitalists at an exogenously determined interest rate, \( r \). The venture capitalist borrows enough funds to have a given capacity. In return for financing the venture capitalist, an investor receives its investment plus the interest. The venture capitalist demands an equity share, \( s \), of profits in exchange for financing and contributing advice to the project. The required capital investment per project is \( I \). Shares of profits are determined endogenously to satisfy the incentive constraints of the venture capitalist and entrepreneur. The participation constraint of entrepreneurs is always satisfied. Since the entrepreneur faces zero outside wage possibilities, as long as he gets positive returns he will participate.

Venture capital financiers contribute value-enhancing advice, \( a \), and the entrepreneur supplies effort, \( e \), to the project. Both are continuous variables. The probability of a successful project, \( p \), is exogenous. Entrepreneurs are risk neutral.\(^{37}\) There are good and bad projects. Total returns on successful good projects are \( R \) and zero on bad ones. Only good projects will receive an investment. Venture capitalists incur search costs to perfectly discern good projects from bad projects.

Following Bernile et al. (2007) and de Bettignies and Brander (2008) we model advice and effort as simultaneous choices, capturing a double moral hazard problem. We

\(^{37}\) For tractability reasons there is no outside option available to the entrepreneur.
assume advice and effort are complements. The value-enhancing function is additively separable. The total (expected) return, TR, on any given project is

\[ TR = (\varepsilon e + \alpha a) pR \]  

(2.1)

Equation (2.1) states the relation between advice and effort and their impact on the expected revenue of a project. The relative weight of value-enhancing contributions for the venture capitalist and entrepreneur are \( \alpha \) and \( \varepsilon \) respectively. The importance of the entrepreneur in generating a successful project is assumed to be greater than the venture capitalist, thus \( \varepsilon > \alpha \).

The cost of venture capitalist advice on each project, \( C(a) \), and the cost of entrepreneurial effort, \( v(e) \), increase at an increasing rate:

\[ C(a) = \frac{a^2}{2}, v(e) = \frac{e^2}{2} \]  

(2.2)

\[ C_a > 0, C_{aa} > 0, v_e > 0, v_{ee} > 0 \]  

(2.3)

The total cost of advice for the venture capitalist is:

\[ \frac{a^2}{2} n_i \]  

(2.4)

The cost of advising projects increases linearly in the number of projects financed, \( n_i \).

The venture capitalist faces fixed costs, \( F \), that relate to creating a venture network or investing in the skills necessary to advise projects. We assume the fixed cost is the same for both incumbent and potential entrant.
There are search costs, $S(n)$, associated with investing in a project. Before actually investing, venture capital firms need to search in order to identify good projects.\textsuperscript{38} Search costs are increasing in the number of own projects financed and the number of projects the competitor finances.

$$S(n) = (n_i + n_j)n_i \quad \forall i \neq j$$  \hspace{1cm} (2.5)

where $i, j$ denote the incumbent and entrant.

with the properties:

$$\frac{\partial S_i}{\partial n_i} > 0, \frac{\partial^2 S_i}{\partial n_i^2} > 0, \frac{\partial S_j}{\partial n_j} > 0, \frac{\partial^2 S_j}{\partial n_j^2} = 0$$  \hspace{1cm} (2.6)

In what follows, the incumbent venture capital firm is denoted by subscript 1 and the potential entrant by subscript 2.

**Venture Capital Fund Raising:**

The incumbent raises enough funds for, $k_1$, projects in the first period. The total funds raised are equal to $Ik_1$. The venture capitalist pays the interest rate, $r$, on the funds raised. The total cost of raising funds is therefore equal to $(1 + r)Ik_1$. The cost of investment in $n_1$ projects is $In_1$. Equation (2.7) gives the net cost of investment for the incumbent.

$$rIk_1 + In_1$$  \hspace{1cm} (2.7)

The incumbent is capacity constrained. We assume that once funds are raised, capacity cannot be increased; the round of raising finance has ended, thus $k_1 \geq n_1$.

\textsuperscript{38} The search process leads to perfect information. Venture capitalists search out and fund only good projects. Entrepreneurs, with returns greater than zero, are funded while projects with zero returns are eliminated as possible investments.
The entrant faces the cost of raising funds and investing in the same period. The interest rate on funds borrowed in the second period is \( r \). The entrant raises funds for and invests in \( n_2 \) projects. Equation (2.8) gives the total cost of investment for the entrant.

\[
rlk_2 + In_2
\]  

Equation (2.8)

The entrant will not raise more funds than it will use, thus \( k_2 = n_2 \).

The incumbent raises funds prior to the entrant which leads to the incumbent having a cost advantage in the second period. The incumbent uses the business strategy of raising funds prior to potential entry to reduce the marginal cost of investment it faces in the second period. Strategically selecting capacity allows the incumbent to act as a Stackelberg leader.

### 2.3 Entry Deterrence

As explained above, there is an incumbent venture capitalist, a potential entrant venture capitalist, and many entrepreneurs who are seeking financing from venture capital for their innovative project. The venture capitalist(s) offers an equity contract, as a take-it-or-leave-it offer, to the entrepreneur. In exchange for a share of profits the venture capitalist provides the capital required to fund the project and advice that enhances the potential revenue of the project. Equity contracts are consistent with real world venture capital markets as the dominant type of contract (Keuschnigg (2004)). There is a revenue tax, \( \tau \), imposed on the venture capitalist and a revenue tax, \( t \), imposed on the entrepreneur.

The equilibrium is derived using backward induction, starting with stage three.
3rd Stage: The venture capitalist and entrepreneur select advice and effort.

The expected profit of the venture capitalist and entrepreneur are given by, $\Pi$ and $\pi$, respectively. Since all entrepreneurs that receive investment are identical, the analysis is conducted for a representative entrepreneur. The problem of the venture capitalist is:

$$\max_a \quad \Pi_i = (1 - \tau)s_i(\varepsilon \alpha + \alpha a)pRn_i - \frac{a^2}{2}n_i - rIk_i - In_i - (n_i + n_j)n_j - F$$

The solution to this problem is:

$$a^* = (1 - \tau)s_i\alpha pR \quad \text{(3.1)}$$

Similarly, the problem of the entrepreneur is:

$$\max_e \quad \pi = (1 - t)(1 - s_i)(\varepsilon \alpha + \alpha a_i)pR - \frac{\varepsilon^2}{2}$$

And the solution is:

$$e^* = (1 - t)(1 - s_i)e pR \quad \text{(3.2)}$$

The advice provided by the venture capitalist is independent of the number of projects in which it invests and not directly affected by the entrepreneurial revenue tax. An increase in the revenue tax on venture capitalists reduces the advice provided by the venture capitalist. Likewise, the effort of the entrepreneur is independent of the number of projects and not directly affected by the venture capitalist revenue tax. The entrepreneur lowers its effort as the revenue tax increases. Both advice and effort increase in the probability of a successful project and the returns. Substituting the solution into the objective, we find entrepreneur’s profit:

$$\pi = \left[\frac{(1 - t)(1 - s_i)pR^F}{2}\right] + [(1 - t)(1 - \tau)(1 - s_i)e pR]^2 \geq 0 \quad \text{(3.3)}$$
Equation (3.3) satisfies the participation constraint of the entrepreneur since its profits are necessarily equal or greater than zero.\(^{39}\)

2\(^{nd}\) Stage: Selection of number of investments and shares of profits.

The venture capitalist(s) select the equity share of returns it extracts and the number of projects it will fund that maximize its expected returns. The venture capitalist knows the market conditions it faces. If there is entry, the venture capitalists compete over investment in projects. The funds for investment have already been raised by the incumbent and are sunk when it makes its investment and contract decisions. The entrant must raise funds and invest in the same stage if entry occurs.

The incumbent’s maximization problem is:

$$\max_{n_1,s_1} \Pi_i = (1-\tau)s_1(\alpha e + \alpha a)pRn_1 - \frac{a^2}{2}n_1 - rlk_1 - ln_1 - (n_1 + n_2)n_1 - F$$  \hspace{1cm} (3.4)

Subject to: Equations (3.1), (3.2), (3.3), and \(n_1 \leq k_1\).

The entrant’s maximization problem is:

$$\max_{n_2,s_2} \Pi_2 = (1-\tau)s_2(\alpha e + \alpha a)pRn_2 - \frac{a^2}{2}n_2 - rlk_2 - ln_2 - (n_1 + n_2)n_2 - F$$  \hspace{1cm} (3.5)

Subject to: Equations (3.1), (3.2), and (3.3).

There are four potential stage 2 equilibria that are considered in determining whether entry is deterred or accommodated: Cournot, Monopoly, Limit, and Stackelberg.

a. Cournot Equilibrium:

The incumbent solves (3.4) with respect to its equity share of returns and the number of projects it will finance.

---

\(^{39}\) The participation constraint of the entrepreneur is satisfied for any \(s \in (0,1)\), \(\tau < 1\), and \(t < 1\).
The equilibrium share of profits is constrained to $s \in (0,1)$.\(^{40}\) It can be easily verified that the second order conditions for a maximum are satisfied.

Equation (3.6) defines the optimal equity share of returns captured by the venture capitalist. The share of revenue is independent of the probability of success, returns and number of projects. As the relative importance of venture capitalist advice increases (decreases) the share of revenue captured by the venture capitalist increases (decreases). As the relative importance of entrepreneur effort increases (decreases) the share of revenue captured by the venture capitalist decreases (increases).

**Proposition 1:**

The optimal equity contract and equilibrium advice and effort are such that (i) An increase (decrease) in the entrepreneur revenue tax increases (decreases) the share of profits captured by the venture capitalist, and (ii) An increase (decrease) in the tax on venture capitalist revenue decreases (increases) the venture capitalists share of profits.

Equation (3.7) represents the best-response function of the incumbent, given that sufficient funds ($n_1 \leq k_1$) have been raised in the first stage. An increase in the entrepreneur’s equity share of profit decreases the number of projects funded. When the returns to innovation or probability of a successful project are high, the incumbent invests in more projects.

\[^{40}\text{For } s \in (0,1), (1-t)e^2 > (1-\tau)\alpha^2 \text{ must be satisfied.}
\]

\[
s_1^* = \frac{(1-t)(1-\tau)e^2}{2(1-t)(1-\tau)e^2 - (1-\tau)^2 \alpha^2}
\]

\[
n_1 = \frac{(1-t)(1-\tau)s(1-s)e^2 p^2 R^2 + (\frac{1}{2})(1-\tau)^2 s^2 \alpha^2 p^2 R^2 - 2I - n_2}{2}
\]

\[
\frac{\partial^2 \Pi}{\partial s^2} < 0 \text{ and } \frac{\partial^2 \Pi}{\partial n^2} < 0 \text{ for all } s \text{ in the relevant range. The profit function is strictly concave.}
\]
The entrant solves (3.5) with respect to its equity share of revenue and the number of projects it will invest in. The share of revenue equation is identical for both the incumbent and entrant venture capitalist; equation (3.6) is identical for both the incumbent and the entrant. The equity share selected by the venture capitalist maximizes its profits. Shares induce contribution of both advice and effort. If the venture capitalist offers a higher equity share to the entrepreneur, it reduces its profits since it decreases the number of projects it funds and the equity share it receives. The profit maximizing share of profit is independent of the number of investments. The best-response function of the entrant is given by:

\[ n_2 = \frac{(1-t)(1-\tau)s(1-s)e^2p^2R^2 + (\frac{1}{3})(1-\tau)^2s^2\alpha^2 p^2R^2 - 2I(1+r) - n_1}{2} \] (3.8)

Differentiating equations (3.4) and (3.5) with respect to the number of investments, we define investment in projects as substitutes and strategic substitutes.\(^{41}\)

\[ \frac{\partial \Pi_i}{\partial n_j} < 0, \quad \frac{\partial^2 \Pi_i}{\partial n_j \partial n_i} < 0 \quad \forall i \neq j, \quad i, j = 1,2 \]

An increase in the number of competitors investments reduces own profits and reduces marginal profits. This is driven by the search cost rather than the usual demand function explanation. Strategic substitutes ensure that all the funds raised by the incumbent are invested; there will be no idle capacity (Bulow et al., 1985b).

Solving the best-response functions, equations (3.7) and (3.8), of the incumbent and entrant venture capitalists respectively we find the Cournot number of investments.

\[ n^c_i = \frac{(1-t)(1-\tau)s(1-s)e^2p^2R^2 + (\frac{1}{3})(1-\tau)^2s^2\alpha^2 p^2R^2 - 2I + I(1+r)}{3} \] (3.9)

\(^{41}\) Bulow et al. (1985a) provide the distinction between substitutes that reduce profit and strategic substitutes that reduce marginal profit.
An increase in the cost of investment reduces the total number of projects funded. A decrease in the share of revenue captured by the venture capitalist decreases the number of projects funded. Equations (3.9), (3.10), and (3.11) are functions of only exogenous variables since $s = s^*$ ($\tau$, $t$, $\epsilon$, $\alpha$).

Some interesting characteristics of the Cournot equilibrium are listed in propositions 2 and 3.

**Proposition 2:**
An increase in the cost of raising funds, $r$, increases the number of projects funded by the incumbent and reduces both the number of projects funded by the entrant and the total projects funded.

As the cost of raising funds increases, the first-mover advantage, through sunk costs in the first stage, is larger and the incumbent enjoys a more dominant position.

**Proposition 3:**
(i) The number of investments made by the incumbent and entrant decrease as the venture capitalist revenue tax rises. (ii) The number of investments made by the incumbent and entrant decrease as the entrepreneur revenue tax rises. (iii) The revenue of the venture capitalist per project decreases with both taxes while the total cost of investment in a project remains constant.

The profits of the incumbent in the Cournot equilibrium are equal to:\footnote{Profit is reported at the equilibrium.}

\[ n^c_2 = \frac{(1-t)(1-\tau)s(1-s)\epsilon^2 p^2 R^2 + (\frac{1}{2})(1-\tau)^2 s^2 \alpha^2 p^2 R^2 + I - 2I(1+r)}{3} \]  
\[ N^c = \frac{2(1-t)(1-\tau)s(1-s)\epsilon^2 p^2 R^2 + (1-\tau)^2 s^2 \alpha^2 p^2 R^2 - I - I(1+r)}{3} \]
\[
\Pi_i^c = n_1^c \cdot n_2^c - F \quad (3.12)
\]

This will be useful in the analysis below.

b. Monopoly number of projects:

If we suppose there is no threat of entry, the incumbent acts as a monopolist, the optimal number of investments includes the cost of raising funds. The incumbent venture capitalist maximizes equation (3.4) with respect to equity share of revenue and number of investments where \( k_1 = n_1 \) and \( n_2 = 0 \). A monopolist selects a number of investments equal to:

\[
n_1^M = \frac{(1 - \tau)(1 - \tau)s(1 - s)e^2p^2R^2 + \left(\frac{1}{s}\right)(1 - \tau)^2s^2\alpha^2p^2R^2 - I(1 + r)}{2} \quad (3.13)
\]

An increase in the probability of a successful project or returns of a project increases the number of projects funded. An increase in the investment cost decreases the number of projects funded. In contrast to the Cournot case, an increase in the interest rate lowers the number of projects funded by the incumbent since the first-mover advantage plays no role in the monopoly case. The profits of a monopolist are equal to:

\[
\Pi_i^M = (n_1^M)^2 - F \quad (3.14)
\]

c. The “Limit” Number of Investments:

The limit number of investments is the number of projects that the incumbent would need to fund in order to drive entrant profits to zero.

Solving \( \Pi_2 = (n_1^L, n_2^L) - F = 0 \), where \( n_2 \) is given by equation (3.8), yields:

\[
n_1^L = (1 - \tau)(1 - \tau)s(1 - s)e^2p^2R^2 + \left(\frac{1}{s}\right)(1 - \tau)^2s^2\alpha^2p^2R^2 - I(1 + r) - 2\sqrt{F} \quad (3.15)
\]

Comparing equations (3.13) and (3.15) we find that:
If \( \frac{(1-t)(1-\tau)s(1-s)e^2 p^2 R^2 + \frac{1}{2}(1-\tau)^2 s^2 \alpha^2 p^2 R^2 - I(1+r)}{2} < 2\sqrt{F} \) then \( n^L_i < n^M_i \) and entry will be deterred by the incumbent. The monopoly level of output is sufficient to deter entry by driving entrant’s profits to zero, is credible, and maximizes the profits of the incumbent venture capitalist.

The limit number of investments for the entrant makes him indifferent between entering or not. If the potential entrant enters the market facing the limit investment, he will invest in \( \sqrt{F} \) or zero projects (in either case its profits are zero). We assume that when faced with zero profits, the potential entrant does not enter the market. The profit of the incumbent at the limit number of projects is:

\[
\Pi^L_i = (n^L_i)^2 - F
\]  

(3.16)

d. The Stackelberg Outcome:

The incumbent enjoys a first mover advantage since it selects investment capacity in the first stage. In selecting capacity, the incumbent may act as a Stackelberg leader. The incumbent may use its first-mover advantage to induce the Stackelberg outcome by raising funds equal to the Stackelberg number of projects (i.e. act as if it were a Stackelberg leader by committing to the Stackelberg number of projects). The entrant then selects its number of projects. In this case, the incumbent maximizes equation (3.4) subject to the best-response function of the entrant, equation (3.8). The solution is:

\[
n^S_i = \frac{(1-t)(1-\tau)s(1-s)e^2 p^2 R^2 + \frac{1}{2}(1-\tau)^2 s^2 \alpha^2 p^2 R^2 - 2I + I(1+r)}{2} \]  

(3.17)

\[
n^S_2 = \frac{(1-t)(1-\tau)s(1-s)e^2 p^2 R^2 + \frac{1}{2}(1-\tau)^2 s^2 \alpha^2 p^2 R^2 + 2I - 5I(1+r)}{4} \]  

(3.18)
Comparing equation (3.17) to equation (3.13), we see that the incumbent Stackelberg number of projects is strictly greater than the monopoly number of projects. The profit of the incumbent in a Stackelberg outcome is:

\[ \Pi_i^s = n_i^s \cdot P_i^s - F \]  

1st Stage: The Selection of Capacity.

The incumbent selects capacity equal to the equilibrium number of projects in the second stage. The venture capitalist has incentives to fully use any capacity it raises – it will not hold excess (idle) capacity.\(^{43}\) In equilibrium, \(k_i = n_i^*\), where \(n_i^*\) is the sub-game perfect Nash equilibrium (SPNE) investment selection of the incumbent. What is the equilibrium number of projects for the incumbent?

In determining the optimal capacity the incumbent considers whether or not to deter entry and this depends on two factors: Whether it is credible and profit-maximizing. In order for entry deterrence to be credible, the incumbent must be able to commit to his action and not have an incentive to deviate. If entry deterrence is credible then it must also be profit maximizing for it to be the equilibrium.

The conditions for entry deterrence or accommodation are similar to those derived in Dixit (1980), in the context of the current model. The different cases are outlined below.

Case (1): Accommodate Entry

\[ n_1^M < n_i^c < n_1^L, n_i^s \]

\(^{43}\)This result follows the intuition and results of Dixit (1980) that no idle capacity is raised. “…marginal revenue is decreasing in the other’s output” (Bulow et. al (1985b), pp. 178). In this venture capital capacity model, marginal cost is increasing in the other VC firm’s investments. The result is there is no incentive to raise funds that will not be used.
The equilibrium is the Cournot number of projects. The incumbent cannot commit to either the limit or Stackelberg number of projects since if entry did occur, the incumbent would find it profit maximizing to invest in the Cournot number of projects. The monopoly level does not deter entry and the incumbent is best to accommodate entry and raise and invest in $k_i = n_i^c$ projects. The profits of the incumbent are $\Pi_i^c$.

Case (2): Deter or Accommodate Entry

$$n_i^M < n_i^S < n_i^L < n_i^c$$

The equilibrium is either $n_i^L$ or $n_i^S$ depending on where profits are maximized. The incumbent can credibly commit to the limit number of projects since it is less than the Cournot equilibrium. It can also commit to the Stackelberg number of projects since it is less than the Cournot equilibrium. If the Stackelberg number of projects maximizes profits, then entry occurs. However, if the limit number of projects maximizes profits, then entry is deterred. The equilibrium requirements on profits are $\Pi_i^L, \Pi_i^S \geq \Pi_i^c$.  

Case (3): Deter Entry

(i)  

$$n_i^M < n_i^S < n_i^L < n_i^c$$

The equilibrium is at $n_i^L$. The incumbent can credibly commit to the limit number of projects since it is less than the Cournot equilibrium. The equilibrium requirement on profit is $\Pi_i^L > \Pi_i^S$. The incumbent always deters entry if the Stackelberg number of projects is greater than the limit number of projects.

---

44 The Stackelberg profit must be at least as large as the Cournot profit. The incumbent venture capitalist is selecting a point on the best-response function of the entrant. The incumbent can select any point. If the profit at the Cournot equilibrium were greater than the Stackelberg selection, the incumbent would be better off selecting the Cournot outcome.

45 The sufficient condition for $\Pi_i^L > \Pi_i^S$ is $n_i^L < n_i^S$ (Dixit (1980))
\[(ii) \quad n_1^L < n_1^M < n_1^S < n_1^c\]

The limit is below the monopoly output in this case. Entry is deterred and the incumbent invests in the monopoly number of projects. The case holds since \(\Pi_1^M > \Pi_1^S\). This case is known as blockaded monopoly.

**Socially Optimum Investment, Advice and Effort**

The social optimum maximizes the social surplus (the sum of venture capitalist and entrepreneur profits). The social surplus of venture capital projects is:

\[
TSS = (\epsilon e + \alpha a) pRN - \frac{a^2}{2} N - I(1 + r)N - N^2 - F - \frac{e^2}{2} N \quad (3.21)
\]

Total social surplus is independent of equity share and taxes (venture capitalist revenue tax and entrepreneur revenue tax). The social optimum advice, effort, and number of projects are:

\[
a^{**} = \alpha p R \quad (3.22)
\]

\[
e^{**} = \epsilon p R \quad (3.23)
\]

\[
N^{**} = \frac{\epsilon^2 p^2 R^2 + \alpha^2 p^2 R^2 - 2I(1 + r)}{4} \quad (3.24)
\]

Comparing equations (3.22) and (3.23) to equations (3.1) and (3.2) we find that advice and effort are underprovided relative to the social optimum. Comparing the social optimum number of projects (equation 3.24) with the private market outcomes in equations (3.11), (3.13), (3.15), and (3.19) we find that there may be underinvestment, overinvestment, or optimum investment relative to the social optimum. The result is
consistent with findings in the literature that welfare effects, in a strategic investment model, are often ambiguous.\footnote{See, for example, Shapiro (1989), Church and Ware (2000).}

There are several distortionary effects at work in the model: (1) Equity contracting between venture capitalist(s) and entrepreneurs, (2) Strategic investment by the incumbent and (3) search cost.

(1) As discussed above, equity share of revenue leads to the under provision of advice and effort relative to the social optimum. The lower total quality of projects in the private market leads to underinvestment in projects, ceteris paribus.

(2) Strategic investment may lead to overinvestment, underinvestment, or the social optimum level of investment. Whether the outcome is Cournot, Stackelberg, Monopoly or Limit, the incumbent venture capitalist has market power. The incumbent uses strategic investment to alter the market outcome. The incumbent’s selection depends on credibility and maximizing its own profits. Strategic investment raises the incumbent’s profit, but decreases the entrant’s profit.

(3) There is a spillover (search cost) that the venture capitalist(s) do not take into consideration. The venture capitalist maximizes its own profits, not taking into consideration the impact its selection of investments has on the search costs of the competitor. This leads the venture capitalist to overinvest relative to social optimum.

These different effects can lead to overinvestment, underinvestment or the socially optimum level of investment. The equity contract leads to underinvestment, search costs to overinvestment, and the strategic investment has an ambiguous effect. When we consider the welfare effect of venture capitalist revenue tax and entrepreneur revenue tax the results remain ambiguous.
2.4 Venture Capitalist Revenue Tax and Entrepreneur Revenue Tax

We examine how taxation affects the equilibrium outcome and entry deterrence decision of the incumbent venture capitalist. The comparative static analysis for the venture capitalist revenue tax and the entrepreneur revenue tax follow.

The likelihood of entry deterrence depends on the two criteria for equilibrium: credibility and profit maximizing. As the different cases of entry accommodation and deterrence above outline, credibility and profit maximization move in the same direction. If an investment decision is credible, it is profitable to commit to a strategic outcome (Stackelberg, limit, or monopoly) and not the Cournot. The incumbent’s strategic choice of investment capacity, if credible, creates a tactical advantage that it uses in the investment stage. Cases 3(i) or 3(ii), entry deterrence, are credible and profit maximizing, when the limit number of investments is below both the Stackelberg and Cournot number of investments. These relations are used to evaluate the likelihood of entry deterrence.

i. Venture Capitalist Revenue Tax:

Differentiating equations (3.9), (3.13), (3.15) and (3.17) with respect to the tax on venture capitalist revenue we find:

\[
\frac{\partial n^L_t}{\partial \tau}, \frac{\partial n^M_t}{\partial \tau}, \frac{\partial n^S_t}{\partial \tau}, \frac{\partial n^C_t}{\partial \tau} < 0
\]  

(4.1)

An increase in the tax on venture capitalist revenue reduces the number of projects funded by the incumbent in equilibrium. The marginal effect of an increase in the tax on venture capitalist revenue is larger for the limit number of projects than for monopoly, Stackelberg, or Cournot number of projects. That is, an increase in the tax on venture capitalist revenue reduces the limit number of projects more than it reduces the Cournot
number of projects. This implies that the limit investment is more likely the SPNE equilibrium. We can show that:

\[
\left| \frac{\partial n_i^L}{\partial \tau} \right| > \left| \frac{\partial n_i^S}{\partial \tau} \right| > \left| \frac{\partial n_i^M}{\partial \tau} \right| > \left| \frac{\partial n_i^c}{\partial \tau} \right| \tag{4.2}
\]

**Proposition 4:**

An increase in the venture capitalist revenue tax makes it more likely that entry deterrence occurs. An increase in the tax on venture capitalist revenue reduces the number of projects in the limit case at a greater rate than the reduction for the Cournot case. It therefore makes it more likely that Case (2) or (3) is the equilibrium and \( n_i^L, n_i^S < n_i^c \). The total number of projects financed decrease (increase) as the tax on venture capitalist revenue increases (decreases). The tax on venture capitalist revenue decreases revenue for both the incumbent and the entrant. In equilibrium, the total number of projects is lower.

The interesting insight we find is, in addition to reducing the number of projects, a venture capitalist revenue tax increases the likelihood of entry deterrence. Therefore, there are market power implications of a tax on venture capitalist revenue.

**ii. Entrepreneur Revenue Tax:**

Differentiating equations (3.9), (3.13), (3.15) and (3.17) with respect to the entrepreneur revenue tax we find:

\[
\frac{\partial n_i^L}{\partial \tau}, \frac{\partial n_i^M}{\partial t}, \frac{\partial n_i^S}{\partial t}, \frac{\partial n_i^c}{\partial t} < 0 \tag{4.3}
\]

The number of projects funded in equilibrium decreases as the entrepreneur revenue tax increases. The limit number of projects falls more quickly than the number of projects for monopoly, Stackelberg, or Cournot as the entrepreneur revenue tax increases. We can also derive the following:
Proposition 5:

An increase in the entrepreneur revenue tax makes it more likely that entry deterrence occurs. The number of projects in the limit case decrease at a greater rate than the reduction for the Cournot (and the Stackelberg) case, making it more likely that Case (2) or (3) is the equilibrium. As the entrepreneur revenue tax increases (decreases) the total number of projects financed decrease (increase). The revenue for both the incumbent and the entrant decrease in the entrepreneur revenue tax.

Similar to the tax on venture capitalist revenue case, in addition to reducing the number of projects, an entrepreneur revenue tax increases the likelihood of entry deterrence.

iii. Quality of Projects

The quality of projects is an important aspect of venture capital finance. Substituting equation (3.6) into equations (3.1) and (3.2) we can find the marginal effects of an increase in venture capitalist revenue tax and entrepreneur revenue tax on the equilibrium advice and effort:

\[
\frac{de^*}{d\tau} > 0, \quad \frac{da^*}{d\tau} < 0, \quad \frac{de^*}{dt} < 0, \quad \frac{da^*}{dt} > 0
\]  

(4.5)

Proposition 6:

(i) An increase (decrease) in venture capitalist revenue tax increases (decreases) the effort provided by the entrepreneur and decreases (increases) the advice provided by the venture capitalist. The equity share decreases in the tax on venture capitalist revenue, further reducing the advice provided by the venture capitalist. The entrepreneur’s equity share
increases in the venture capitalist revenue tax, increasing the effort he provides. (ii) The impact of a venture capitalist revenue tax on total quality (equation 2.1), is ambiguous. (iii) An increase (decrease) in the entrepreneur revenue tax decreases (increases) the effort provided by the entrepreneur and increases (decreases) the advice provided by the venture capitalist. (iv) An increase (decrease) in the entrepreneur revenue tax negatively (positively) affects project quality.

Both the venture capitalist revenue tax and entrepreneur revenue tax increase the likelihood of entry deterrence and reduce the number of projects funded in equilibrium. An entrepreneurial revenue tax reduces the quality of projects while a venture capitalist revenue tax has an ambiguous effect on quality.

2.5 Subsidies on Investment and Fund Raising

There is a perceived lack of supply of venture capital in Canada. As such, government policy (e.g. R&D tax credits, S&T subsidies) focuses on increasing the overall supply of venture capital investments. We examine how subsidy on investment, $\sigma$, and a subsidy on fund raising, $\gamma$, affect the equilibrium choice of the incumbent venture capitalist. We re-characterize the model to evaluate the impact of these subsidies on the equilibrium. The capital commitment to a project is reduced to $1 - \sigma$ per project. The raising of funds by the venture capitalist takes into account the reduced capital commitment. Both the investment and fund raising subsidies are assumed to be identical for the incumbent and the entrant. This assumption is relaxed in Section 6.

3rd Stage: The venture capitalist and entrepreneur select advice and effort.

The venture capitalist and entrepreneur contribute value-enhancing advice and effort to the project. The problem of the venture capitalist is:
The optimal venture capitalist advice and entrepreneur effort are independent of both an investment subsidy and a subsidy on raising funds.

2nd Stage: The venture capitalist(s) select their equity share and number of investments.

The venture capitalist selects the terms of the contract, share of revenue, and the number of projects it funds. Both subsidies reduce the effective cost of investment the venture capitalist faces. The problem becomes:

\[
\max_{n_1,r_1} \Pi_1 = s_i(ee + \alpha a) p R n_1 - \frac{a^2}{2} n_1 - I(1 - \sigma) n_1 - I(1 - \sigma) [r(1 - \gamma)] k_1 - (n_1 + n_2) n_1 - F
\]

Subject to: Equations (5.1), (5.2), \(\pi^E > 0\), and \(n_1 \leq k_1\). (5.3)

The entrant’s problem is:

\[
\max_{n_2,r_2} \Pi_1 = (1 - \tau) s_i(ee + \alpha a) p R n_2 - \frac{a^2}{2} n_2 - I(1 - \sigma) n_2 - I(1 - \sigma) [r(1 - \gamma)] k_2 - (n_1 + n_2) n_2 - F
\]

Subject to: Equations (5.1), (5.2), and \(\pi^E > 0\). (5.4)

The first order conditions for problems (5.3) and (5.4) yield the optimal shares of revenue and the best-response functions for the incumbent and entrant venture capitalists:
Equation (5.5) is the optimal equity share demanded by the venture capitalist. The share of revenues is independent of the number of projects, the investment subsidy, and the subsidy on fund raising. The quality of venture capital projects is independent of subsidies that target the cost of investment. Equations (5.6) and (5.7) are the best-response functions of the incumbent and entrant, respectively. An increase in the investment subsidy shifts the best-response function of the incumbent and entrant out—the total number of projects increases. The cost for a venture capitalist to invest in a project has decreased. A subsidy on fund raising shifts the best-response of the entrant out. The entrant now faces a lower cost of investment.

The optimal number of projects under Cournot, monopoly, limit and Stackelberg cases follow the derivations of section 3.

a. Cournot:
Solving the best-response functions of the incumbent and entrant yields the equilibrium number of investments:

\[ n_1^c = \frac{s(1-s)s^2 p^2 R^2 + \frac{1}{2} s^2 \alpha^2 p^2 R^2 - 2I(1-\sigma) + I(1-\sigma)[1 + r(1-\gamma)]}{3} \]  

\[ n_2^c = \frac{s(1-s)s^2 p^2 R^2 + \frac{1}{2} s^2 \alpha^2 p^2 R^2 + I(1-\sigma) - 2I(1-\sigma)[1 + r(1-\gamma)]}{3} \]  

\[ s^* = \frac{\epsilon^2}{2\epsilon^2 - \alpha^2} \]  

\[ n_1 = \frac{s(1-s)s^2 p^2 R^2 + \frac{1}{2} s^2 \alpha^2 p^2 R^2 - I(1-\sigma) - n_2}{2} \]  

\[ n_2 = \frac{s(1-s)s^2 p^2 R^2 + \frac{1}{2} s^2 \alpha^2 p^2 R^2 - I(1-\sigma)[1 + r(1-\gamma)] - n_1}{2} \]

\[ 47 \text{ The optimal share of profits is such that } s \in (0,1) \text{ as long as } \epsilon > \alpha , \text{ as assumed.} \]
\[ N^c = \frac{2s(1-s)e^2p^2R^2 + (\frac{1}{2})s^2\alpha^2 p^2R^2 - I(1-\sigma) - I(1-\sigma)[1+r(1-\gamma)]}{3} \] (5.10)

b. Monopoly:

The incumbent maximizes its profit and takes into consideration the total cost of investment (including the cost of raising funds):

\[ n^M = \frac{s(1-s)e^2p^2R^2 + (\frac{1}{2})s^2\alpha^2 p^2R^2 - I(1-\sigma)[1+r(1-\gamma)]}{2} \] (5.11)

c. Limit: The number of projects the incumbent must finance in order to drive entrant profits to zero:

\[ n^L = s(1-s)e^2p^2R^2 + (\frac{1}{2})s^2\alpha^2 p^2R^2 - I(1-\sigma)[1+r(1-\gamma)] - 2\sqrt{F} \] (5.12)

d. Stackelberg: The incumbent acts as a first-mover and takes the best-response of the entrant as given, leading to:

\[ n^S_1 = \frac{s(1-s)e^2p^2R^2 + (\frac{1}{2})s^2\alpha^2 p^2R^2 - 2I(1-\sigma) + I(1-\sigma)[1+r(1-\gamma)]}{2} \] (5.13)

\[ n^S_2 = \frac{s(1-s)e^2p^2R^2 + (\frac{1}{2})s^2\alpha^2 p^2R^2 + 2I(1-\sigma) - 3I(1-\sigma)[1+r(1-\gamma)]}{4} \] (5.14)

\[ N^S = \frac{3s(1-s)e^2p^2R^2 + (\frac{1}{2})s^2\alpha^2 p^2R^2 - 2I(1-\sigma) - I(1-\sigma)[1+r(1-\gamma)]}{4} \] (5.15)

From equations (5.10), (5.11), (5.12), and (5.15) we determine the effects of the investment subsidy and fund raising subsidy on entry deterrence and the total number of projects financed.

i. Investment Subsidy

An investment subsidy affects all possible equilibrium outcomes. The subsidy reduces the cost of investment in the second stage. Both the incumbent’s and the entrant’s variable cost of investment is reduced. We find the following:
\[
\frac{\partial n^I_c}{\partial \sigma}, \frac{\partial n^M_i}{\partial \sigma} > 0 
\]
\[
\frac{\partial N^S}{\partial \sigma}, \frac{\partial N^C}{\partial \sigma} > 0 
\]
\[
\frac{\partial n^S_i}{\partial \sigma}, \frac{\partial n^C_i}{\partial \sigma} > 0 \text{ if } r(1 - \gamma) < 1 \text{ and } \frac{\partial n^S_i}{\partial \sigma}, \frac{\partial n^C_i}{\partial \sigma} \leq 0 \text{ otherwise.} 
\]

An increase in the subsidy on investment increases the number of projects financed (equation 5.16 and 5.17) under all circumstances. The Cournot and Stackelberg number of incumbent projects may increase or decrease depending on the cost of fund raising (equation 5.18). The incumbent’s advantage in raising funds is determined by the interest rate. As the interest rate increases, and the cost of fund raising increases, its advantage is diminished and this affects its decision to invest.

**Proposition 7:**

(i) An increase in the investment subsidy decreases the likelihood of entry deterrence. The limit number of investments is more likely to be beyond the Cournot and Stackelberg number of investments as the investment subsidy increases, and thus non-credible. (ii) The total number of projects financed increases with an investment subsidy. (iii) If the cost of raising funds is sufficiently low, the number of projects financed by the incumbent increases with an investment subsidy, independently of whether entry is deterred or accommodated. (iv) If the cost of raising funds is high, an investment subsidy decreases the number of projects financed by the incumbent under entry accommodation and increases the number of projects financed under entry deterrence.
ii. A subsidy on Fund Raising

Subsidizing fund raising reduces the cost for both the incumbent and the entrant. The impact on the incumbent is less pronounced since its cost is sunk in the first stage. The entrant faces a smaller cost disadvantage in the second stage. We can show that:

\[
\frac{\partial n^L}{\partial \gamma}, \frac{\partial n^M}{\partial \gamma} > 0 \tag{5.19}
\]

\[
\frac{\partial N^S}{\partial \gamma}, \frac{\partial N^C}{\partial \gamma} > 0 \tag{5.20}
\]

\[
\frac{\partial n^S}{\partial \gamma}, \frac{\partial n^C}{\partial \gamma} < 0 \tag{5.21}
\]

An increase in a subsidy on raising funds increases the number of projects of the incumbent under monopoly or limit, but decreases the number of projects of the incumbent under Cournot and Stackelberg. The incumbent is less likely to be able to credibly commit to entry deterrence. The strategic advantage, in Cournot and Stackelberg, of the incumbent is diminished by a subsidy on raising funds. The entrant faces less of a cost disadvantage in the second period and competes on more equal terms. The subsidy on fund raising shifts the best-response function of the entrant to the right. The result is an increase in the total number of projects funded under both the Cournot and Stackelberg cases.

**Proposition 8:**

A subsidy on fund raising decreases the likelihood of entry deterrence. The number of projects in the limit and monopoly increase while it decreases in the Cournot and Stackelberg cases. If entry occurs, the Cournot and Stackelberg investments by the incumbent are reduced. A subsidy on fund raising reduces the strategic advantage that raising capacity offers the incumbent venture capitalist.
2.6 Targeted Subsidies on Investment and Fund Raising

In this section the assumption in section 5 that subsidies are applied equally to both the incumbent and entrant venture capitalist is relaxed. Instead, it is assumed that the government uses targeted subsidies. These subsidies target the incumbent or the entrant rather than being universally applied. The Government of Canada offers targeted subsidies to certain venture capital financiers. These targeted subsidies are intended to expand the supply of venture capital funds in Canada. For example, programs such as LSVCCs receive special tax breaks in raising funds. Those that invest in LSVCC receive tax credits, while those that invest in private venture capital corporations receive no such benefit. These tax credits can be viewed as a targeted subsidy on fund raising – it reduces the burden that the LSVCC pays back to its investors. Another possible targeted tax policy is reducing the cost of investment for entrants, designed to increase the size of the venture capital market. We examine the impacts of targeted subsidies on the market equilibrium and the likelihood of entry deterrence.

i. Targeted Subsidy on Fund Raising

We start with a subsidy on fundraising for the incumbent (which could be seen here as an LSVCC). With a targeted subsidy on fund raising for the incumbent, $\gamma_1$, the best-response functions of the incumbent and entrant as given in equations (5.6) and (5.7) become:

$$n_1 = \frac{s(1 - s)e^2 p^2 R^2 + \frac{(1 - \tau)}{2} s^2 \alpha^2 p^2 R^2 - I - n_2}{2}$$  \hspace{1cm} (6.1)$$

$$n_2 = \frac{s(1 - s)e^2 p^2 R^2 + \frac{(1 - \tau)}{2}(1 - \tau) s^2 \alpha^2 p^2 R^2 - I(1 + r) - n_1}{2}$$  \hspace{1cm} (6.2)$$

The best-response functions of the incumbent and entrant are independent of the fund raising subsidy. The incumbent and entrant do not alter their optimal selections of number
of projects to invest in. The limit and Stackelberg number of projects are not affected by a subsidy on the incumbent’s fund raising.

The monopoly number of projects is larger with the targeted subsidy.

\[ n^M_1 = \frac{s(1-s)\epsilon^2 p^2 R^2 + (\frac{1}{2})s^2 \alpha^2 p^2 R^2 - I[1 + r(1 - \gamma)]}{2} \]  

\textbf{Proposition 9:}

(i) A targeted subsidy on fund raising for the incumbent has no impact on the number of investments made by the incumbent or the total number of projects funded if entry occurs. The incumbent’s cost of raising funds is sunk in the project selection stage. Therefore, a subsidy on the incumbent’s fund raising does not affect the incumbent’s best-response function. (ii) The likelihood of entry deterrence is not affected by the targeted subsidy. (iii) An increase in a targeted subsidy on fund raising increases the likelihood that \( n^L_1 < n^M_1 \) and when entry is profitably and credibly deterred, the incumbent selects the monopolist number of investments.

\textbf{ii. Targeted Subsidy on Investment}

A targeted subsidy on investment, \( \sigma_2 \), for the entrant only could be used to stimulate venture capital investment. The Cournot equilibrium number of projects selected by the incumbent and entrant become:

\[ n_1 = \frac{s(1-s)\epsilon^2 p^2 R^2 + (\frac{1}{2})s^2 \alpha^2 p^2 R^2 - 2I + I(1 - \sigma_2)(1 + r)}{3} \]  

\( n_2 = \frac{s(1-s)\epsilon^2 p^2 R^2 + (\frac{1}{2})(1 - \tau)^2 s^2 \alpha^2 p^2 R^2 - 2I(1 - \sigma_2)(1 + r) + I}{3} \]  

The comparative statics for the Cournot case are given in equation (6.6), for Stackelberg in equation (6.7) and for the limit and monopoly number of investments in equation (6.8).
\[
\frac{\partial n^c_1}{\partial \sigma^c_2} < 0, \frac{\partial n^c_2}{\partial \sigma^c_2} > 0, \frac{\partial N^c}{\partial \sigma^c_2} < 0 \quad (6.6)
\]

\[
\frac{\partial n^S_1}{\partial \sigma^S_2} < 0, \frac{\partial n^S_2}{\partial \sigma^S_2} > 0, \frac{\partial N^S}{\partial \sigma^S_2} > 0 \quad (6.7)
\]

\[
\frac{\partial n^L_1}{\partial \sigma^L_2} > 0, \frac{\partial n^M}{\partial \sigma^M_2} = 0 \quad (6.8)
\]

The targeted investment subsidy reduces the Cournot total number of projects financed. The Stackelberg total number of projects financed increases in the targeted subsidy. The sunk cost advantage of the incumbent is reduced by the targeted subsidy and results in the reduction of incumbent number of projects in the Cournot and Stackelberg outcomes.

**Proposition 10:**

(i) A targeted investment subsidy makes entry deterrence less likely. (ii) A targeted subsidy that reduces the cost of investment for the entrant may reduce or increase the total number of projects funded if entry occurs. If the entry equilibrium is Cournot, the total number of projects financed decreases, however if the entry equilibrium is Stackelberg, the total number of projects financed increases. (iii) If entry is deterred, the number of projects funded is at least as many as without the subsidy.

### 2.7 Policy that Reduces Fixed Costs

Another policy instrument observed in reality is a subsidy or government run programs that reduce fixed costs. The fixed costs component of a venture capital firm includes networking and industry connections and experience. Policy designed to reduce these costs are implemented by the government in networking and training seminars designed to connect venture capitalists with the knowledge and connections needed to raise funds and develop a
rapport with the industry where it will invest. The market is more contestable when the fixed cost of entry is reduced. The entrant faces a lower fixed cost of entry and is more likely to enter the market. From equation (3.15), the limit number of projects depends on the fixed cost of entry. Differentiating the limit number of projects, when there is a subsidy on the fixed cost of investment for the entrant, we find:

$$\frac{\partial n^L_i}{\partial \delta^*_2} = \sqrt{\frac{F}{1 - \delta^*_2}} > 0$$  \hfill (7.1)

The policy, $\delta$, reduces the fixed costs of operating in the venture capital market. There are three cases: (1) The policy applies to both incumbent and entrant; (2) the policy applies only to the incumbent; and (3) the policy applies only to the entrant. In case (1) and (3), the “limit” number of investments increases as the policy increases, as indicated by equation (7.1). Entry is more difficult to deter as the entrant’s fixed cost of entry, a barrier to entry, decreases. The likelihood of entry deterrence decreases as the fixed cost of the entrant decreases. None of the other potential outcomes (Monopoly, Cournot, or Stackelberg) are affected by a reduction in fixed costs. In case (2), the reduction in incumbents fixed cost increases the profits of the incumbent, but does not affect the outcomes. The likelihood of entry deterrence and the quantity of projects in equilibrium are unaffected.

**Proposition 11:**

A subsidy or government program that reduces the fixed costs of a venture capitalist:

(i) Reduces the likelihood of entry deterrence when it reduces the fixed costs of the entrant, but (ii) has no impact when it reduces the fixed costs of the incumbent.
2.8 Conclusion

The venture capital market is typically characterized by few firms competing over projects. Economies of scale exist in venture capital markets and firms are seen as making strategic sunk investments in funding capacity. These market conditions create imperfect competition. Government tax and subsidy policy has been used to increase venture capitalist financing. These policies need to take into consideration the impact on the market structure and not just the overall supply of venture capital funds. In addition, there are consequences on the quality of venture capital financing that also need to be taken into account. This paper analyzed these issues.

We find that an increase in tax on venture capitalist revenue or tax on entrepreneur revenue increases the likelihood of entry deterrence and reduces the number of projects funded in equilibrium. The reduction in the number of projects is more pronounced if entry is successfully deterred by the incumbent venture capitalist. An investment subsidy decreases the likelihood of entry deterrence and increases the number of projects funded in equilibrium. A subsidy on fund raising decreases the likelihood of entry deterrence. If entry is accommodated, the number of projects financed is reduced. If entry is deterred, the number of projects financed increases. In practice, the government uses targeted subsidies (e.g. LSVCCs). A targeted fund raising subsidy on the incumbent does not affect the likelihood of entry deterrence or the number of projects funded in equilibrium. Targeted subsidies on investment are less efficient (have a smaller impact) than a general, universally applied, subsidy. Government policy that reduces the cost of investment decreases the likelihood of entry deterrence.
Strategic investment capacity is an important aspect of venture capital markets. The incentive for an incumbent venture capitalist to raise funds and act as a leader is applicable to the real world. There are other strategic instruments in which a venture capitalist can invest, such as expertise, establishing reputation, and project selection capabilities. An investigation into how these strategic instruments are used in venture capital markets would lead to a better understanding of how venture capitalists compete. The paper does not explain or address the excess (idle) investment capacity that is observed. The “overhang” of funds raised by venture capitalists remains an issue that has not been fully examined. The excess capacity is probably due to several factors including the uncertainty of project quality and returns. Given Canada’s large “overhang” of uninvested venture capital funds, future research to examine the reasons for holding excess investment capacity in venture capital markets would be valuable.
Chapter 3

Project Selection and Venture Capital
3.1 Introduction

Venture capitalists invest in high-risk, high-return projects that have difficulty securing other types of financing. In addition to financing innovative projects, venture capitalists apply two types of intangible assets to projects: (1) project selection skills and (2) project management skills. A venture capitalist is better able to identify good quality innovative projects than other types of financiers (e.g. banks). The ability of the venture capitalist to better identify project quality stems from specific investment in project evaluation skills. The venture capitalist invests in these skills by hiring experts and concentrating investment in a specific industry or stage of project development. Hiring experts in technology or project evaluation is costly. The expert’s knowledge comes from education and experience. There is also learn-by-doing for the venture capitalist. Becoming more familiar with a particular industry allows the venture capitalist to build a knowledge base that will assist in selecting better projects in the future.

Once a project has been selected for investment, the venture capitalist contributes managerial advice to the project. The advice of the venture capitalist increases the probability of a successful project and the value of the project. The venture capitalist invests in managerial advice; it builds a team of project leaders that have expertise in commercialization, firm management structure, and networking. Project management and commercialization knowledge are a different set of skills than project selection ability. Project evaluation involves intimate knowledge of the industry and the technological details of the project. For example, engineers or scientists are better able to evaluate the feasibility of a project. However, business experts (e.g. MBA) are better able to evaluate how to successfully market and commercialize a project.
We develop a model where, in addition to financing innovative projects, a venture capitalist can invest in project selection skills and managerial skills. In our analysis we, separately, model a private venture capitalist and a labour-sponsored venture capital corporation (LSVCC). We model these two types of venture capital differently. The private venture capitalist is modeled as a profit-maximizing firm. The LSVCC, however, is assumed to have a labour objective in addition to a profit-maximizing objective. The labour objective, modeled as a total wage bill consideration, is derived from the influence of labour unions in these funds. The venture capitalist allocates funds between project evaluation skills and the number of project it invests in. Managerial advice is contributed after investments are made and project quality becomes known. We find that labour funds invest less in project selection and managerial skills than a private venture capitalist. The total average gross return on a project is lower for the labour fund because its quality is lower. The labour fund invests in more projects than the private venture capitalist. Both the private venture capitalist and LSVCC under-invest in project management (quality of advice) relative to the social optimum. The investments of the private and LSVCC in projects and project selection skill relative to the social optimum are ambiguous; there may be optimal, under-, or over-investment. There is an inverse relationship between number of projects and project selection skills; if there is over-investment in number of projects, there is under-investment in project selection skills, relative to the social optimum.

In Canada, LSVCCs are venture capital funds that raise funds through a tax-credit system. These funds must invest in innovative projects and be sponsored by a labour union. In effect, the union exercises some control over the fund. Labour funds tend to invest more in traditional sectors (see graph 1), consistent with union sponsorship, where returns to
innovative projects are lower, but tend to be more labour intensive.\textsuperscript{48} The industry statistics, presented in graph 1, clearly show that LSVCC invest substantially more in traditional sectors than private venture capital funds. The union sponsorship explains this observation, at least in part. In conjunction with a labour union oriented objective, it is observed that some LSVCCs may “… pursue objectives other than profit maximization.” (Cumming and MacIntosh (2006), pp.583). Evidence suggests a negative impact in the venture capital market as a result of LSVCCs and government venture capital funds (Cumming and MacIntosh (2006) and Brander et. al. (2008)). Returns are low, and mostly negative, for LSVCC (Cumming (2007)) and there may be crowding-out of private funds. In addition to investing in more traditional sectors where returns are lower, there is also evidence that low returns may be caused by poor skills of LSVCC managers.\textsuperscript{49} Cumming and MacIntosh (2006) and Ayayi (2002) suggest that LSVCCs have lower fund management skills, than private venture capitalists, and are not good at identifying good quality projects.

Graph 1: Proportion of Canadian Venture Capital Investments in Traditional Sectors

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{graph1.png}
\caption{Proportion of Canadian Venture Capital Investments in Traditional Sectors}
\end{figure}

\textsuperscript{48} Traditional sectors: consumer and business services, consumer products, manufacturing, and retailing. (www.canadavc.com)

\textsuperscript{49} There is also a social cost of LSVCC funds that decreases the returns further. The tax credit to keep the in operation means the returns even lower.
There is substantial literature that examines the probability-enhancing and value-added contribution of venture capital investment (Keuschnigg and Neilson (2001, 2003, 2004a, 2004b), Keuschnigg (2004), Bernile et al. (2008), and de Bettignies and Brander (2007)). However, although the importance of project selection skills of the venture capitalists is identified in the venture capital literature, it is not explicitly modeled. Gompers et al (2006) find that venture capitalists contribute project screening (selection) and value-added to the venture capital market. However, their empirical analysis does not disentangle the two; venture capitalists have high (better) success rates due to some combination of these skills. Brander et. al. (2002) evaluates project selection and value-enhancing as reasons for syndication (joint investment) of venture capitalists. Ueda (2004) models the venture capitalist as having better project selection ability than banks. Keuschnigg and Nielsen (2007) allow the venture capitalist a buyout option if the project is observed to be of low quality in a setting where the venture capitalist, through investment, learns the quality of projects.

The paper proceeds as follows: Section 2 describes the model. Section 3 presents the private venture capitalist market equilibrium. Section 4 introduces the LSVCC equilibrium and the results are compared to the private venture capitalist. The social optimum is derived in Section 5. The results of the private venture capitalist and LSVCC equilibria are compared to the social optimum. Section 6 concludes.

3.2 The Model

There are three players in the model: venture capitalist, entrepreneurs, and workers. Each project requires labour (workers). The labour contributes to the project in the form of product assembly, services related to the project, etc… that are necessary to develop an
innovative project. Entrepreneurs have innovative projects, but lack the funds required to initiate the project. Venture capitalists provide financing to the project and cover the cost of labour. The venture capitalist and entrepreneur provide managerial expertise (advice) and effort, respectively, for the project.

There are two types of projects: good (high quality) and bad (low quality). High quality projects have high returns, $R_H$, and low quality projects have returns, $R_L$, when successful.\(^{50}\) Project returns are determined exogenously. The venture capitalist and entrepreneur can enhance the probability a project is successful through contributions of expertise and effort in the project.

The venture capitalist invests in $n$ innovative projects and pays the labour costs associated with the projects. The wage paid to labour, $w$, is exogenous. Each project also requires an investment, $I$, to get the project underway. The start-up cost is identical for all entrepreneurs. The cost of financing $n$ projects is therefore:

$$ F = nw + nI \quad (2.1) $$

Before financing projects, the venture capitalists can invest in project selection skills, $x$, that enables it to better screen projects ($x \in [0,1]$).\(^{51}\) The project selection skill of the venture capitalist determines the probability of funding projects of good quality. The expected number of good quality projects is $nx$ and the expected number of bad quality projects is $n(1-x)$. Through this investment in industry specific knowledge and expertise, the venture capitalist can better discern between good projects, with high expected returns, and

\(^{50}\) Alternatively, the quality of projects could be determined by differences in the probability of a successful project (Keuschnigg and Nielsen, 2007).

\(^{51}\) Project selection ability, $x$, is constrained to be between zero and one. At best, the venture capitalist selects only high quality projects and at worst, it selects only bad projects.
bad projects, with low expected returns. The cost of investing in project selection skill, \( c[x] \), is increasing at an increasing rate.

\[
c_1[x] > 0, c_{xx}[x] > 0
\]  

(2.2)

The venture capitalist can also invest managerial advice, \( a \), to a project. Managerial advice increases the probability of a successful project. Entrepreneurs contribute effort, \( e \), to the project. The probability of a project being successful is:

\[
\Pr = \beta(a_i + e_i) \quad \forall i = H, L
\]  

(2.3)

where \( i \) denotes the quality of the project and \( \beta \) is a return on advice and effort parameter such that the probability is less than one and greater than zero: \( \Pr(a, e; \beta) \in [0,1] \). Managerial advice and entrepreneur effort are non-contractible and non-verifiable. The probability function is assumed identical for low and high quality projects. The venture capitalist invests in managerial advice after the quality of the project is revealed.

The cost of advising a project, \( c[a] \), is increasing at an increasing rate. As the number of projects increases, the cost of advice increases at a constant rate for high quality projects. The venture capitalist does not advise low quality projects in our model. We assume that the venture capitalist can only secure funds for advising high quality projects. The entrepreneur faces an increasing, at an increasing rate, cost of effort, \( v[e] \). The cost of effort is the same for both high and low quality projects. In particular, we assume the following cost functions:

\[
c[a_H] = \frac{a_H^2}{2}, C[a_H, n_H] = c[a_H]n_H = n_H\frac{a_H^2}{2}
\]  

(2.4)

This assumption does not significantly change the results and simplifies the analysis. The results do not change substantially if we allow the venture capitalist to contribute advice in both high quality and low quality projects. The venture capitalist finds it optimal to invest more advice in high quality projects, than low quality projects.
with,
\[ v[e_i] = \frac{e_i^2}{2} \] (2.5)

and,
\[
\begin{align*}
&c_a[a_H] > 0, c_{aa}[a_H] > 0 \\
&C_n[a_H, n_H] > 0, C_{mn}[a_H, n_H] = 0 \\
v_e[e_i] > 0, v_{ee}[e_i] > 0
\end{align*}
\] (2.6)

where \( n_H \) is the number of high quality projects. There is a double moral hazard problem. The venture capitalist and entrepreneur only consider their own costs when selecting probability-enhancing managerial advice and effort.

In exchange for financing the project and contributing advice, the venture capitalist demands an equity share of revenue, \( s \), from the entrepreneur. The venture capitalist offers a take-it-or-leave-it contract to the entrepreneurs. Equity contracts are used in the real world and are the dominant form of contracts in venture capital markets (Keuschnigg (2004)).

The venture capitalist faces a budget constraint when investing in project selection skills and choosing the number of projects it finances. We assume the funds raised, \( M \), are exogenous. The venture capitalist’s pool of funds is fixed until investment in projects takes place. The venture capitalist faces the budget constraint:
\[ n(I + w) + c[x] \leq M \] (2.7)

After the investment in entrepreneurial projects is made, uncertainty regarding project quality is resolved. The venture capitalist can now raise additional funds to invest in managerial advice in projects to increase the probability of success. To simplify the analysis, we assume the venture capitalist is only able to raise funds to advise good quality projects.
Sequence of events:

1st Stage: The venture capitalist makes an investment in project selection skills, \( x \), that determines its ability to discern the high-quality projects.

2nd Stage: The venture capitalist then selects the number of projects to finance, \( n \), and the equity share of revenues, \( s \), offered to entrepreneurs.

3rd Stage: The uncertainty is resolved. The venture capitalist observes the quality of projects they finance, raises additional funds, and contributes managerial advice, \( a \). The entrepreneur selects the effort, \( e \), it will contribute to the project.

We separately model a private venture capitalist and a labour-sponsored venture capital corporation, with the objective of comparing how their decisions to invest in project-selection skills and managerial advice will differ. We compute, separately, the market equilibrium for a private venture capital firm and a LSVCC. We then compare these results to the social optimum investments in project selection skills, managerial advice, and number of projects.

3.3 The Private Venture Capitalist Market Equilibrium

The venture capitalist, subject to its budget constraint, selects the investment in project selection skills, the number of projects to finance, and the equity share of returns it will demand. The venture capitalist makes a take-it-or-leave-it offer to the entrepreneur. The expected profit of the venture capitalist and entrepreneur are given by, \( \Pi \) and \( \pi \), respectively. Expected profits depend on the expected number of good quality projects, \( nx \), and bad quality projects, \( n(1-x) \).

In the third stage, uncertainty of project quality is resolved. Since it is assumed the venture capitalist can only raise additional funds for high quality projects, the private
venture capitalist chooses its advice for high quality projects, but invests no advice in low quality projects. Both high and low quality entrepreneurs contribute effort. The problem of the venture capitalist in the third stage is:

$$\max_{a_H} \Pi_H = s\beta(a_H + e_H)R_Hn_H - \frac{a_H^2}{2}n_H$$

which gives the following solution:

$$a_H^* = s\beta R_H$$  \hspace{1cm} (3.1)

$$a_L^* = 0$$  \hspace{1cm} (3.2)

The problem of the entrepreneur is:

$$\max_{e_i} \pi_i = (1-s)\beta(a_i + e_i)R_i - \frac{e_i^2}{2}$$

which gives the following solution:

$$e_i^* = (1-s)\beta R_i, \quad \forall \quad i = H, L$$  \hspace{1cm} (3.3)

The profit of the entrepreneur is greater than or equal to zero for any equity share or revenue, $s \in (0,1)$. The entrepreneur always takes the contract offer from the venture capitalist. Both high and low quality projects participate in the market. Our analysis focuses on a “pooling equilibrium”. We assume that the venture capitalist cannot use project selection skills or equity share of revenues to drive the bad quality projects out of the market. In equilibrium, the venture capitalists portfolio will include both good and bad quality projects.

**Proposition 1:**

(i) The venture capitalists advice to high quality projects increases in its equity share of revenues, return on advice, and project returns. The effort of the entrepreneur decreases in
the venture capitalists equity share of revenue, and increases in return on effort and project
returns. (ii) Managerial advice and effort do not depend on the number of projects financed.

The venture capitalist’s managerial advice and entrepreneur’s effort depend on the
equity share of revenues they receive. If the venture capitalist demands more (fewer)
shares, the entrepreneur responds by decreasing (increasing) its effort to the project. As the
return on advice and effort parameter increases (decreases), managerial advice and effort
increases (decreases).

In the second stage, given project selection skill, $x$, chosen in the previous stage, the
venture capitalist selects the number of projects it will finance and its equity share of
revenue subject to solve:

$$
\max_{s, x} \Pi = s \beta (a_H^s + e_H^s) R_H nx + s \beta (e_I^s) R_I n (1 - x) - \frac{a_H^{s^2}}{2} nx - In - wn - c(x)
$$

(3.4)

s.t. equations (2.7), (3.1), (3.2), and (3.3)

The budget constraint represents the funds raised by the venture capitalist that are used to
build project selection ability and fund innovative projects. The funds that are owed to the
investors is the amount of funds raised, $M$. The interest rate on funds raised is assumed to
be zero.\(^{53}\) The funds raised are determined exogenously. The venture capitalist requests an
investor’s financial support and receives funds, independent of his ability and investment
decisions.

At the second stage, the number of projects that will receive funding is already
determined by the budget constraint. At this stage, the venture capitalist has made its
investment in project selection skills and the remainder is spent financing projects.

\(^{53}\) Assuming the interest rate to be zero simplifies the analysis, and has no impact on the results of the paper. If
the interest rate were modeled as a positive parameter, $r$, the results remain essentially the same.
\[ n^* = \frac{M - c[x]}{1 + w} \]  

(3.5)

The first order condition of equation 3.4, with respect to share of profits is:

\[
\frac{\partial \Pi^{vc}}{\partial s} = \beta^2 R_H^2 nx + \beta^2 (1 - s) R_L^2 n(1 - x) \\
- s \beta^2 R_H^2 nx - s \beta^2 R_L^2 n(1 - x) = 0
\]

(3.6)

and yields the venture capitalist’s equity share equation:

\[
s^* = \frac{R_H^2 x + R_L^2 (1 - x)}{R_H^2 x + 2 R_L^2 (1 - x)}
\]

(3.7)

The equity share is such that: \(s^* \in (0,1)\).⁵⁴ The venture capitalist’s share of revenue is independent of the number of projects it invests in. It depends on the returns of good and bad quality projects and on the project selection ability of the venture capitalist.

The venture capitalist selects share of revenues such that the marginal increase in profits, from capturing a larger share, equals the marginal decrease in profits from reduced effort of the entrepreneurs with low quality projects and the increase in the cost of providing more advice to high quality projects.

**Proposition 2:**

(i) The equity share of revenue captured by the venture capitalist is increasing in project selection ability. The better the venture capitalist can discern between projects, the more equity shares it demands.

An increase in project selection skills increases the probability of selecting high return projects. The marginal increase in returns, from an increase in equity share of revenue, is greater than the marginal increase in cost of advice and the loss of entrepreneur

---

⁵⁴ The condition is satisfied for \(R_H^2 x > R_L^2 (1 - x)\). The second order conditions for concavity are satisfied for \(R_H^2 x > 2 R_L^2 (1 - x)\).
effort as project selection skills increase. The venture capitalist extracts a larger share of profits, but also contributes proportionally more managerial advice to the project. The expected returns for high quality projects are assumed sufficiently larger than returns for low quality projects to drive this result.\(^{55}\) Thus, the venture capitalist finds it profitable to increase its share of equity as it invests more in project selection skills. Venture capital project selection skills and management advice are closely related. An increase in project selection skills increases the managerial advice the venture capitalist contributes to a project. The quality of the venture capitalist is determined by these two investments: managerial advice and project selection skills.

In the first stage, the venture capitalist invests in project selection skills. These skills determine the expected proportion of high quality projects it selects. Maximizing equation (3.4) with respect to \(x\) and subject to equations (2.7), (3.1), (3.2), (3.3), (3.5) and (3.6), we find the equation that determines the optimal investment in project selection skills:

\[
\frac{\partial \Pi_{VC}}{\partial x} = s^* R_H^2 + \frac{\partial s^*}{\partial x} R_H^2 n^* x - \frac{\partial n^*}{\partial x} s^* R_H^2 x + \frac{\partial s^*}{\partial x} R_L^2 n^* (1-x) - s^* R_L^2 n^*
\]

\[
+ \frac{\partial n^*}{\partial x} s^* R_L^2 (1-x) - 2s \frac{\partial s^*}{\partial x} R_L^2 n^* (1-x) + s^2 R_L^2 n^* (1-x)
\]

\[
- \frac{\partial n^*}{\partial x} s^2 R_L^2 (1-x) - \frac{\partial s^*}{\partial x} s^* R_H^2 n^* x - \frac{\partial n^*}{\partial x} s^2 R_H^2 \frac{x}{2} = 0
\]

The solution takes the form: \(x^* (R_H, R_L, w, I, M)\) and is a function of the returns to high and low quality projects, the wage paid to labour and the investment required for each project, and the size of initial funds raised.

The venture capitalist takes into consideration several benefits and costs when selecting its project selection skills. In summary, the venture capitalist balances the quality

---

\(^{55}\) The condition for concavity is satisfied.
of its portfolio and share of revenue against the number of investments, entrepreneur’s effort, and costs (investment, advice, and project selection).

An increase in project selection skills decreases the number of projects it finances. The venture capitalist’s budget constraint creates this direct tradeoff between project selection skills and number of investments. As proposition 2 states, an increase in project selection skills increases the share of revenue captured by the venture capitalist. An increase in share of revenue leads to an increase managerial advice, but a decrease in effort provided by the entrepreneur. The costs associated with managerial advice and effort increase and decrease, respectively, in project selection skills.

We present equation (3.8) to make the comparison to the LSVCC and social optimum easier; however, it is useful to write it in general terms to breakdown the different components that determine the private venture capitalists investment in project selection skills (see Appendix A). Equation (3.8) includes the substitutions made for the optimal advice, effort, and number of projects. A general equation, without these substitutions, makes it easier to interpret the marginal effects of a change in project selection skills on expected profit. The following equation equates the marginal benefit (LHS) to the marginal cost (RHS) of project selection skill investment:

\[
\frac{\partial s^*}{\partial x} \beta(a_{h}^* + e_{h}^*)R_{h}n^* x + s^* \beta(a_{h}^* + e_{h}^*)R_{h}n^* + \frac{\partial s^*}{\partial x} \beta(e_{l}^*)R_{l}n^*(1-x) - \frac{\partial n^*}{\partial x} (a_{h}^*)^2 x = \\
- \frac{\partial n^*}{\partial x} s^* \beta(a_{h}^* + e_{h}^*)R_{h}x - \frac{\partial e_{l}^*}{\partial x} s^* \beta(e_{l}^*)R_{l}n^*(1-x) - \frac{\partial n^*}{\partial x} s^* \beta(e_{l}^*)R_{l}(1-x) \\
+ s^* \beta(e_{l}^*)R_{l}n^* + (a_{h}^*)^2 n^* + \frac{\partial a_{h}^*}{\partial x} n^* x
\]

(3.9)

The benefits of better project selection skills are: extracting a larger share of revenue from projects and a higher quality portfolio of projects. There is also a reduction in cost of advice
from funding fewer projects. The costs of better project selection skills are: fewer projects, a decrease in the effort provided by the entrepreneur, and an increase in the cost of advising good projects. The investment by the private venture capitalist in project selection skills balances these marginal costs and benefits.

The venture capitalist's portfolio (number of investments) and skills (project selection and management) are inversely related. An increase in project selection skills increases managerial advice (according to proposition 2) and decreases the number of investments (equation 3.5).

We now compare the private venture capitalist's investment in project selection and number of projects, its equity share of revenue, and its investment in managerial advice to a labour fund and to the social optimum.

### 3.4 Labour-Sponsored Venture Capitalist

We incorporate the idea that LSVCCs have other objectives in our analysis by assuming that the labour fund maximizes a weighted function of expected profits and the wage bill. The total wages paid to labour are taken to be the union’s control of the fund.\(^56\) The idea of maximizing the wage bill is an older idea in the labour literature, first introduced by Dunlop (1944), but appropriate in the context of our model.

There is substantial literature that investigates the objective and economic implications of a union (labour cooperative or labour managed) as the firm.\(^57\) These labour-maximizing (LM) firms have a different objective than profit maximizing firms. It is argued that LM firms maximize the profit per worker or some other labour objective (e.g. benefits,

---

\(^56\) In addition to the benefits the union may derive from its influence on the labour fund, a union also receives a fee for renting its name (Cumming and MacIntosh, 2006).

\(^57\) See, for example, Oswald (1982), Oswald (1987) and Ben-Ner and Estrin (1991) on trade unions. Stewart (1991) and Zhang (1993) examine the impact on competition with labour managed firms.
This literature focuses on the union as the firm. However, LSVCCs are not run by unions. LSVCCs require union sponsorship and receive investment from the union, among other sources of funding, but operate independently from the union. The investment decisions of LSVCCs in entrepreneurial projects are independent, although not completely, from the union.

As discussed above, the labour-sponsored venture capitalist has a different objective than a private venture capitalist. Receiving its authority to raise funds from a union sponsor, the LSVCC, or labour fund (LSF), will be influenced away from pure profit maximizing objective.\(^ {58}\) We capture this different objective of the LSF by including both profit and a wage bill. The parameter, \(\alpha\), represents the relative weight the LSF places on profit maximization. The LSF places weight, \(1-\alpha\), on maximizing the wage bill (\(wn\)). The relative importance of profits and wage bill is such that \(\alpha \in (0,1)\); if \(\alpha = 1\), the LSF acts as a purely profit-maximizing venture capitalist. The labour fund’s objective is:

\[
\Pi^{LF} = \alpha \left( s \beta (a^{*}_{H} + e^{*}_{H}) R_{H} nx + s \beta (e^{*}_{L} R_{L} n (1-x) - \frac{a^{**}_{H}}{2} n x - L n - wn - c [x] \right) + (1 - \alpha) wn
\]

(4.1)

As in the private venture capitalists case, advice and effort are determined in the third stage and number of investments and equity share of revenue in the second stage. Equations (3.1), (3.2), and (3.3) determine the optimal advice and effort of the LSF venture capitalist and entrepreneur, respectively. Equations (3.5) and (3.6) determine the optimal number of investments and the LSF’s equity share of revenue.

\(^ {58}\) A LSVCC receives its authority from both union sponsorship and government approval of creating the fund.
The equations that determine managerial advice, entrepreneur’s effort, share of revenues, and number of investments are identical for the private venture capitalist and the labour fund. However, since these equations are all functions of project selection skills, the values may differ depending on the investment made in these skills by the LSF.

In the first stage, the LSF invests in project selection skills. The LSF maximizes equation (4.1) with respect to project selection ability, subject to equations (3.1), (3.2), (3.3), (3.5), and (3.6). The optimal investment in project selection skills is determined by:\^59

\[
\frac{\partial \Pi^{LF}}{\partial x} = \frac{\partial \Pi^{VC}}{\partial x} - \frac{(1-\alpha)w}{\alpha(1+w)\beta^2} \frac{\partial c[x]}{\partial x} = 0
\]  

Equation (4.2) states that the optimal investment in project selection skills by the LSF is equal to the equation determining the optimal investment of the private venture capitalist (equation 3.7) minus an additional term. The additional term is determined by the impact of the wage bill and the relative weight the LSF places on profit and the wage bill, respectively. The additional term is always positive and thus reduces the LSF’s project selection skills relative to the private venture capitalist. For very small weights on the wage bill, the LSF’s investment in project selection skills, number of projects, and managerial advice are very close to the private venture capitalists investments. As the objective of the LSF and private venture capitalist diverge, the difference in project selection investment, number of projects, and managerial advice, increases.

The investment in project evaluation skill depends on the relative weight the LSF places on its “union” and profit-maximizing components. An increase in the importance of

\^59 In terms of a general equation, the following holds: \( \frac{\partial \Pi^{LF}}{\partial x} = \frac{\partial \Pi^{VC}}{\partial x} - \frac{(1-\alpha)w}{\alpha} \frac{\partial n^*}{\partial x} = 0 \). For the sake of simplifying the comparison between private venture capitalist and LSVCC, we substitute equilibrium values to get at the underlying differences.
profit-maximization relative to wage bill leads to an increased investment in project selection skill. As the wage rate increases, the investment in project selection skills falls relative to the private venture capitalist's project selection skills. The LSF pays workers $w$ per project. The total return on labour is $(1-2\alpha)wn$ and this is strictly less than the “cost” of labour the private venture capitalist faces. The benefit the LSF derives from the wage bill increases as the wage paid to labour increases. The union component of the LSF objective function is the union’s influence over the LSF’s project selection skills. As the influence of the union increases, project selection skills becomes less important and the number of projects selected becomes more important. The investment in project selection falls relative to the private venture capitalist as the marginal cost of project selection increases. The relative benefit of project selection falls relative to wage bill objectives, and this leads to lower investment in project selection by the LSF. As the marginal cost of investing in project selection increases, the relative cost of investing in more projects has fallen. Finally, as the cost of investment to initiate the project increases, the LSF reduces its project selection skill relative to the private venture capitalist. The cost of investment determines the relative expense of projects versus project selection skill. As the cost of investment decreases, it becomes relatively cheaper to invest in more projects and the benefits of investing in more projects are greater than the returns to investing in project selection skills.

**Proposition 3:**

The labour funds investment in project selection ability relative to the private venture capitalist (i) decreases in the wage rate and the marginal cost of project selection skills and (ii) increases in the relative importance of profit-maximization in its objective and the cost of investment to initiate projects.
Comparing the optimal investment in skills of the LSF, characterized by equation (4.2), to the optimal skills of the private venture capitalist, characterized by equation (3.7), we find that the LSF invests less in project selection ability.

\[ x^{LF} < x^{VC} \]  \hspace{1cm} (4.3)

As a result, the LSF invests in more projects than a private venture capitalist. The venture capitalist allocates its budget between project selection skills and projects. The emphasis the LSF puts on job creation, in the form of the wage bill, induces it to invest in more projects and less in project selection skills. The result is consistent with the observations of the venture capital market in Canada. Labour funds tend to invest in more projects than private venture capital firms, and invest less per project on average.\textsuperscript{60}

There are additional implications from the LSF investing less in project selection skills. The advice the LSF provides is less than the private venture capitalist. This stems from a lower share of profits captured by the LSF relative to the private venture capitalist. The entrepreneur exerts more effort when financed by a LSF since it captures a larger share of profits. Thus, the quality of entrepreneurs, the effort they contribute to projects, is higher when financed by LSFs. However, the probability of a successful high quality project is identical for the labour and private venture capitalist. The reduction in managerial advice, due to lower project selection skills of the LSF and its lower share of revenue, is exactly offset by the increase in entrepreneur’s effort.

To evaluate the overall quality of the venture capitalist’s portfolio, we define average expected gross return of a project as the advice and effort, contributed to high quality projects and low quality projects, and the selection of projects. Quality depends on

\textsuperscript{60} Duhamel and Peter (2008).
two components: (i) project selection and (ii) advice and effort. The average gross expected
return on a project is equal to the expected return on a high quality project multiplied by the
probability of selecting a high quality project, plus the expected return on a low quality
project multiplied by the probability of selecting a low quality project. The equation that
determines average expected gross return is given by:

\[ E[R] = \beta\{R_H^2 x + R_L^2 (1-x)(1-s)\} \]  \hfill (4.4)

From equation (4.3), we know that the equity share of revenue demanded by the
LSF is lower than that demanded by the private venture capitalist:

\[ s^{LF} < s^{VC} \]  \hfill (4.5)

Applying the relations in (4.3) and (4.5) to equation (4.4), we find the private
venture capitalist has a higher average expected gross return than the LSF:\(^{61}\)

\[ E[R]^{LSF} < E[R]^{VC} \]  \hfill (4.6)

As project selection skill increases, better projects are funded at the cost of fewer low return
projects and less entrepreneur effort provided to those low return projects. The increase in
high return projects offsets the reduction in low return projects. The LSF invests in more
projects; however the overall quality, determined by the average expected gross return of
projects, is lower. High quality projects receive less advice, but more effort. Low quality
projects receive more effort. The project selection is lower for the LSF. The quality of
advice provided by the LSF is lower than the private venture capitalist, but this is not what
causes the relatively lower quality of LSF projects. The difference in quality of the
portfolios of the LSF and private venture capitalist stem from the project selection skills.

\(^{61}\) The result holds as long as the return on successful high quality projects is greater than twice the expected
return on low quality projects - a realistic assumption given evidence of the distribution of project returns in
venture capital markets.
The project selection effect means more low quality projects are funded in equilibrium. The overall quality, the average expected gross return, of projects is lower for LSF.

**Proposition 4:**

(i) The LSF invests less in project selection skills and demands fewer equity shares from entrepreneurs relative to a private venture capitalist. (ii) The LSF finances more projects than a private venture capitalist. (iii) For high quality projects, the probability of success is identical to the private venture capitalist case. The entrepreneur invests more effort and offsets the lower advice provided by the LSF. (iv) Low quality projects have a higher probability of success compared to the private venture capitalist. The LSF seeds a larger share of returns to the entrepreneur and increases the effort it provides. (v) The average expected gross return on projects, or the quality of projects, for LSF is lower than the private venture capitalist. The lower quality of the pool of projects is created by the lower selection capabilities of the LSF.

The results of the comparison between private venture capitalist and LSF (LSVCC) are consistent with observations of the venture capital market in Canada. LSF’s are seen to be of lower quality that private venture capital firms (for instance Cumming and MacIntosh (2006) and Brander et. al. (2008)). In our model, the lower quality of LSF venture capital comes from two sources: project selection and managerial advice. The LSF invests less in both sets of skills. The poor performance of LSVCC (Cumming (2007)) can be explained, in part, by the lower quality of projects that result from the low investment of LSVCCs in project selection skills and managerial advice. The results are also consistent with the findings of Ayayi (2002), that questions the experience and decisions of LSVCC managers, and links these to the poor performance of LSVCC funds. In our model, the LSF is
motivated by profit maximizing and a labour (union) component, that lead to poor investments in project selection and management skills.

### 3.5 The Social Optimum

The social optimum is defined as the sum of venture capitalist profits, entrepreneur’s profits, and the ages paid to labour. The total social surplus, \( SS \), is given by equation (5.1).

The social optimum takes into account the cost of effort provided by the entrepreneur and the wage bill of labour. Labour required for projects are the third player in the model and their benefits count towards the social optimum. The social optimum investment in project evaluation skills is determined and compared to the private venture capitalist and LSF investments. We also compare the number of projects financed, the advice of the venture capitalist, the effort of the entrepreneur and the total expected return of projects.

\[
SS = \beta(a_H^* + e_H^*)R_Hnx + \beta(e_L^*)R_in(1-x) - \frac{a_H^{*2}}{2}nx - \frac{e_L^{*2}}{2}nx - \frac{e_H^{*2}}{2}n(1-x) + wn
\]

The social optimum advice and effort are:

\[
a_H^{**} = \beta R_H \\
a_L^{**} = 0 \\
e_i^{**} = \beta R_i \quad \forall \quad i = H, L
\]

Advice and effort are independent of the equity share of revenue. The social optimum advice and effort are greater than provided by the private venture capitalist and labour fund, because of the independence of advice and effort from equity share of revenue.

\[
a_H^{**} > a_H^i, e_i^{**} > e_i^j \quad \forall i = H, L \quad j = VC, LF
\]
Proposition 5:

(i) The private venture capitalist and LSF under-provide advice relative to the social optimum. Both demand an equity share of revenue that reduces the advice and effort provided to a project.

The social optimal number of projects is determined, as are the private and LSF number of projects, by equation (3.5). In calculating the social optimum, the same constraints that face the private and LSF apply. The funds available must be spent either on project selection skills or investment in projects (equation 2.7).

Maximizing equation (5.1) with respect to project selection ability, subject to equations (2.8), (5.2), (5.3) and (5.4), gives the social optimum skills investment:

$$\frac{\partial SS}{\partial x} = R_n^2 n^* + \frac{\partial n^*}{\partial x} R_L^2 x - R_L^2 n^* + \frac{\partial n^*}{\partial x} R_L^2 (1 - x) \frac{R_L^2}{2} n^* + \frac{\partial n^*}{\partial x} \frac{R_L^2}{2} (1 - x) + \frac{w}{\beta^2} \frac{\partial n^*}{\partial x} = 0$$

(5.6)

It is ambiguous as to whether the private venture capitalist or LSF over-, under-, or optimally invests in project selection skills relative to the social optimum. It is therefore ambiguous whether there is over, under-, or optimal number of projects by the private venture capitalist and LSF. Examining equation (5.6) and comparing to equations (3.8) and (4.2), we can see the different components that create this ambiguity of project selection skills and number of projects. The private venture capitalist does not take into consideration the benefits to labour at all, while the labour fund only considers a portion of the wage bill. Thus, examining only the wage bill effect, both tend to under-invest in the number of projects. In fact, the private venture capitalists under-investment in projects (and corresponding over-investment in project selection skills) is further from the social optimum than the labour venture capitalist, if only the wage bill effect were considered. The
distortion created by the share of revenue captured (and required) by the private and LSF decreases the benefit of additional projects. They capture only a share of the benefits while bearing the entire cost associated with investment. Project selection skills influence this distortion (proposition 2). An increase in project selection skills reduces the under-investment in managerial advice by the private venture capitalist and LSF. The effect is relatively too much investment in project selection skills relative to the social optimum, holding number of projects fixed. However, more investment in project selection skill decreases the number of projects funded. The venture capitalist faces a cost trade-off in its funds available, between investment in project selection skills and number of projects. The relative costs of investing in project selection skills and number of projects also determine whether private venture capitalists and LSFs over, under-, or optimally invest relative to the social optimum. In summary, we identify several factors that cause the distortion in the investment by the venture capitalist and LSF in project selection skills and number of projects. The first is they do not fully consider the benefit to workers of additional projects. The second is the distortion cause by equity share of revenue creates.

**Proposition 6:**

(i) If the private venture capitalist or LSF under-invest (over-invest) in number of projects relative to the social optimum, they over-invest (under-invest) in project selection skills relative to the social optimum.

While it is clear there is under provision of advice and effort by the private venture capitalist and LSF relative to the social optimum, it is unclear whether project selection and number of projects are above, below, or at the social optimum levels. The “portfolio” effect favors the private venture capitalist, as its quality is higher than the labour fund.
3.6 Conclusion

Venture capitalists finance high-risk, high-return projects. In addition to financing projects, venture capitalists possess two skills that enhance the expected return of projects: (1) project selection skill, and (2) managerial advice. The first increases the probability of selecting high-quality projects. The second increases the probability the project is successful. Entrepreneurs seek venture capitalist funding for two main reasons. They cannot secure financing from other sources (e.g. banks) due to their riskiness and they desire the project enhancing skills of the venture capitalist.

Modeling a private venture capitalist and LSVCC, separately, we find that differences in the objectives of the firms drive different selection of portfolio enhancing skills. LSVCC have a labour objective that causes its investment in project selection skills and managerial advice to be smaller than the private venture capitalist investments in skill and advice. The quality of venture capital, defined as the average gross expected return on a project, is lower for the LSF. Smaller investment in project selection skills leads to less advice and the total quality of LSFs is lower than the private venture capitalists. Comparing to the social optimum, we find that both the private venture capitalist and labour fund under-invest in managerial advice. The result is consistent with the existing literature on equity contracts. The results for investment in project selection skills and number of projects is ambiguous. There may be over-investment, under-investment, or optimal investment by the private venture capitalist and labour fund relative to the social optimum. The ambiguous result stems from the allocation of funds between project selection skills and investment in projects and the relative costs and benefits of project selection skills versus number of projects.
For future research, there are several possible extensions to the paper. Examining competition between a labour fund and a private venture capitalist would lead to a better understanding of how the two types of funds compete and the implications for market structure. An extension introducing different probability of successful project functions for high and low projects would shed light on the issue of where marginal advice and effort are most effective. A more structured model of fund raising (private venture capital from large financiers, and LSVCC from many small financiers) would reveal insights into how the structure of fund raising impacts investments decisions, both in terms of number of projects and skills, of venture capitalists.
Conclusion

The thesis explores several different aspects of venture capital markets. The impact of public policy initiatives, taxes, subsidies, and government venture capitalists, on venture capitalist quality, the quality of venture capital, market power, and number of investments in projects are the main themes of the thesis. The prominence of public policy initiatives, targeting venture capital, and the structure of venture capital markets motivates this research. Each chapter identifies, explores, and finds interesting insights in the impact of public policies on venture capital markets.

In the first chapter, *Public Policy and Venture Capital*, the contracting relationship between venture capitalist and entrepreneur is modeled. The ownership (property rights) of the venture matters when evaluating the impact of capital gains tax and entrepreneur revenue tax. The main results concern the impact of public policy initiatives on the quality and size of the venture capital market. Capital gains tax reduces the number of projects funded, regardless of whether the venture capitalist or the entrepreneur has market power. If the venture capitalist has market power, quality of venture capitalist advice decreases in capital gains tax. If entrepreneurs have the market power, the managerial advice of the venture capitalist decreases on high return projects and remains the same on low return projects. Public support programs for venture capital increase the quality of venture capitalists and the number of projects funded if the programs are complimentary to venture capitalist advice. Government venture capital crowds-out private venture capital if they compete.
The second chapter, * Imperfect Competition and Entry Deterrence in Venture Capital Markets*, models a three stage game of fund raising, project investment, and project advising. Using an entry deterrence framework, the implications of taxes and subsidies on market structure are examined. The main results concern the impact of tax and subsidy policy on market power and quality of projects. A tax on venture capitalist revenue increases the likelihood of entry deterrence and decreases the number of projects funded, but has an ambiguous effect on quality (venture capitalist advice and entrepreneur effort). A targeted fund raising subsidy on the incumbent has no impact on the likelihood of entry deterrence, the number of projects funded, or the quality of projects.

The third chapter, *Project Selection and Venture Capital*, presents a model where venture capitalist can invest in project selection skill and managerial skill. A private venture capitalist and a labour-sponsored venture capitalist are modeled separately. Labour-sponsored venture capital has a different objective than the private venture capitalist. While the private venture capitalist has a purely profit-maximizing objective, the labour-sponsored fund has a union component, derived from the union sponsorship required to operate the fund. The main results concern the investment of a venture capitalist in project selection skill, projects, and managerial advice. The labour-sponsored fund invests less in project selection skill and managerial skill than the private venture capitalist. The quality of a labour-sponsored fund, defined as the net expected return on a project, is less than the private venture capitalist. Relative to the social optimum, both the private venture capitalist and labour-sponsored fund under-invest in advice. The results of investment in project selection skill and number of projects is ambiguous relative to the social optimum.
Appendix for Chapter 1

Appendix A(1)

Differentiating the share of revenue function, equation (5.4):

\[(1 - 2s_i)(1 - t)(1 - \tau)R_i^{-2} < 0 \quad \forall s_i > \frac{1}{2}\]
\[(1 - 2s_i)(1 - t)(1 - \tau)R_i^{-2} > 0 \quad \forall s_i < \frac{1}{2}\]

The second order condition is:

\[-2(1 - t)(1 - \tau)R_i^{-2} < 0\]

Thus, the shares function is strictly concave.

Appendix A(2)

From equation (5.4), defining an implicit function (F):

\[F \equiv s_i(1 - s_i) = \frac{2(w + v)}{(1 - t)(1 - \tau)R_i^{-2}}\]
\[s_i(1 - s_i)(1 - t)(1 - \tau)R_i^{-2} - 2(w + v)\]
\[\frac{dF}{ds} = (1 - s_i) - s_i\]

When s=0 and s=1, F=0. Thus if a solution exists for shares it will be in the interval zero to one. The function is concave in s. As s moves from 0 to \(\frac{1}{2}\) (the diminishing returns on probability parameter), the slope of F is positive. As s moves from \(\frac{1}{2}\) to 1, the slope of F is negative. At s=1/2, the slope of F is zero and the function F reaches its maximum. Assuming an interior solution, there are two solutions for s. The higher of these values is the one selected by the VC (select largest shares that maximize profits). Thus the slope of the implicit function F with respect to s is negative.

Differentiating the implicit function (F) with respect to all exogenous and endogenous variables yields:
\[(1-t)(1-\tau)[R_{i}^{-2} - 2s,R_{i}^{-2}]ds + (1-t)(1-\tau)2s(1-s)RdR\]
\[-(1-t)s(1-s)R^2 d\tau - (1-\tau)s(1-s)R^2 dt - 2dw - 2dv = 0\]

We observe how the endogenous variable (s) varies with exogenous variables (t, \(\tau\), w, v). The following are the comparative static’s for shares.

\[
\frac{ds}{dR} = \frac{2s - 2s^2}{R(1-2s)} > 0, \quad \frac{ds}{dv} = \frac{ds}{dw} = \frac{2}{(1-t)(1-\tau)R^2(1-2s)} < 0, \quad \frac{ds}{d\tau} = \frac{s - 2s}{(1-t)(1-2s)} < 0, \quad \frac{ds}{dt} = \frac{s - 2s}{(1-\tau)(1-2s)} < 0
\]

**Appendix A(3)**

Using equation (5.6) we compute the comparative static relationship between the number of projects funded and exogenous cost (v, w, I) and policy variables (\(\tau\), \(\sigma\)).

\[G \equiv (1-\tau)s_j\alpha_j^{1/2}R_j - (1-\sigma)I - \alpha_j = 0 \quad \forall j = \tilde{R}\]

Totally differentiating:

\[d\tau \left[ \frac{\partial s}{\partial \tau} (1-\tau)a^{1/2}\tilde{R} + \frac{s(1-\tau)a^{-1/2}\tilde{R}}{2} \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) - sa^{1/2}\tilde{R} - \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) \right]
\]
\[+ dt \left[ \frac{\partial s}{\partial t} (1-\tau)a^{1/2}\tilde{R} + \frac{s(1-\tau)a^{-1/2}\tilde{R}}{2} \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) - \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) \right]
\]
\[+ d\tilde{R} \left[ \frac{\partial s}{\partial \tilde{R}} (1-\tau)a^{1/2}\tilde{R} + \frac{s(1-\tau)a^{-1/2}\tilde{R}}{2} \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) + (1-\tau)a^{1/2}\tilde{R} - \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) \right]
\]
\[+ dv \left[ \frac{\partial s}{\partial v} (1-\tau)a^{1/2}\tilde{R} + \frac{s(1-\tau)a^{-1/2}\tilde{R}}{2} \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) - \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) \right]
\]
\[+ dw \left[ \frac{\partial s}{\partial w} (1-\tau)a^{1/2}\tilde{R} + \frac{s(1-\tau)a^{-1/2}\tilde{R}}{2} \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) - \left( \frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial s} \frac{\partial s}{\partial \tau} \right) \right]
\]
\[+ d\sigma[I] + dI[-(1-\sigma)] = 0\]

The FOC of the private VC yields the result (equation 5.2):

\[\frac{(1-\tau)s_i\alpha_i^{-1/2}}{2} - 1 = 0\]
This eliminates several of the terms from the linearized implicit function resulting in
the simplified result:

\[
\begin{align*}
&d\tau \left[ \frac{\partial s}{\partial \tau} (1 - \tau) a^{1/2} \tilde{R} - sa^{1/2} \tilde{R} \right] + dt \left[ \frac{\partial s}{\partial t} (1 - \tau) a^{1/2} \tilde{R} \right] d\tilde{R} \left[ \frac{\partial s}{\partial \tilde{R}} (1 - \tau) a^{1/2} \tilde{R} + (1 - \tau) a^{1/2} \tilde{R} \right] + dv \left[ \frac{\partial s}{\partial v} (1 - \tau) a^{1/2} \tilde{R} \right] \\
&+ dw \left[ \frac{\partial s}{\partial w} (1 - \tau) a^{1/2} \tilde{R} \right] + d\sigma[I] + dI[-(1 - \sigma)] = 0
\end{align*}
\]

The resulting comparative statics:

\[
\begin{align*}
\frac{d\tilde{R}}{d\tau} &= -\left[ \frac{\partial s}{\partial \tau} (1 - \tau) a^{1/2} R - sa^{1/2} R \right] > 0, \\
\frac{d\tilde{R}}{dt} &= -\left[ \frac{\partial s}{\partial t} (1 - \tau) a^{1/2} R + (1 - \tau) a^{1/2} \tilde{R} \right] > 0, \\
\frac{d\tilde{R}}{dv} &= -\left[ \frac{\partial s}{\partial v} (1 - \tau) a^{1/2} \tilde{R} \right] > 0, \\
\frac{d\tilde{R}}{dw} &= -\left[ \frac{\partial s}{\partial w} (1 - \tau) a^{1/2} \tilde{R} \right] > 0, \\
\frac{d\tilde{R}}{d\sigma} &= -\left[ \frac{\partial s}{\partial \sigma} (1 - \tau) a^{1/2} \tilde{R} \right] < 0, \\
\frac{d\tilde{R}}{dI} &= -\left[ \frac{\partial s}{\partial I} (1 - \tau) a^{1/2} \tilde{R} \right] > 0, \\
\frac{d\tilde{R}}{\sigma} &= -\left[ \frac{\partial s}{\partial \sigma} (1 - \tau) a^{1/2} \tilde{R} \right] > 0, \\
\frac{d\tilde{R}}{I} &= -\left[ \frac{\partial s}{\partial I} (1 - \tau) a^{1/2} \tilde{R} \right] > 0
\end{align*}
\]

**Appendix A(4)**

Substitute the participation constraint for the entrepreneur

\[
PC^E : \quad (1 - t) sa^{1/2} \tilde{R} = (1 - t) a^{1/2} \tilde{R} - v - w
\]

into equation (5.6):

\[
E \Pi^{VC} = (1 - t) a^{1/2} \tilde{R} - v - w - (1 - \sigma) I - a = 0 \quad (A.4)
\]

Substituting equation (5.2) into equation (A.4):

\[
(\tilde{R}^*)^2 = \frac{4((1 - \sigma) I + v + w)}{s(1 - \tau)(1 - t)}
\]
Comparing this function with the social optimum, the following results can be obtained:

If \( \tau = 0, t = 0, \) and \( \sigma = 0: \)

\[
(\tilde{R}^*)^2 = \frac{4(I + v + w)}{s} > (\tilde{R}^{**})^2
\]

If \( \tau > 0 \) or \( t > 0, \) and \( \sigma = 0: \)

\[
(\tilde{R}^*)^2 = \frac{4(I + v + w)}{s(1 - \tau)^2} > (\tilde{R}^{**})^2
\]

If \( \tau = 0, t = 0, \) and \( \sigma > 0: \)

\[
(\tilde{R}^*)^2 = \frac{4((1 - \sigma)I + v + w)}{s} < (\tilde{R}^{**})^2
\]

If \( \tau > 0 \) and/or \( t > 0 \) and \( \sigma > 0: \)

\[
(\tilde{R}^*)^2 = \frac{4((1 - \sigma)I + v + w)}{s(1 - \tau)^2} < (\tilde{R}^{**})^2
\]

**Appendix A(5)**

The general proof of maximizing equation 5.12 with a slack venture capitalist participation constraint:

Let \( \text{Pr}(a) = a^\alpha \) where \( 0 < \alpha < 1 \)

Using this more general functional form of probability, the venture capitalist selects advice:

\[
a^* = (\alpha(1 - \tau)sR)^{1-\alpha}
\]

Substitute \( a^* \) into \( \pi \) and maximize shares:

\[
\frac{\partial \pi}{\partial s} = \frac{\alpha}{1 - \alpha}(1 - s) - s = 0
\]

\[
s^* = \alpha
\]

The result states that the entrepreneur selects shares of returns equal to the exponent of the probability of success function.

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### Appendix B

Comparative Static Results: Entrepreneur has market power

<table>
<thead>
<tr>
<th>Shares:</th>
<th>Low Quality (VC&lt;sub&gt;PC&lt;/sub&gt; Binding)</th>
<th>High Quality (VC&lt;sub&gt;PC&lt;/sub&gt; Slack)</th>
</tr>
</thead>
<tbody>
<tr>
<td>returns</td>
<td>Negative</td>
<td>-</td>
</tr>
<tr>
<td>capital gains tax</td>
<td>Positive</td>
<td>-</td>
</tr>
<tr>
<td>investment cost</td>
<td>Positive</td>
<td>-</td>
</tr>
<tr>
<td>investment subsidy</td>
<td>Positive</td>
<td>-</td>
</tr>
<tr>
<td>Advice:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>returns</td>
<td>Positive</td>
<td>-</td>
</tr>
<tr>
<td>capital gains tax</td>
<td>Negative</td>
<td>Negative</td>
</tr>
<tr>
<td>investment cost</td>
<td>Positive</td>
<td>-</td>
</tr>
<tr>
<td>investment subsidy</td>
<td>Negative</td>
<td>-</td>
</tr>
<tr>
<td>Marginal Project (Return):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>entrepreneur cost/wage</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>capital gains tax</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>investment cost</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>Tax on entrepreneur revenue</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>investment subsidy</td>
<td>Negative</td>
<td>Negative</td>
</tr>
</tbody>
</table>

“-“ denotes no impact.
Appendix for Chapter 3

Appendix A

The general equation that determines the private venture capitalists optimal investment in project selection skills is:

\[
\frac{\partial \Pi^{vc}}{\partial x} = \frac{\partial s^*}{\partial x} \beta (a^*_H + e^*_H) R_H n^* x + s^* \beta (a^*_H + e^*_H) R_H n^* + \frac{\partial n^*}{\partial x} s^* \beta (a^*_H + e^*_H) R_H x
\]

\[
+ \frac{\partial s^*}{\partial x} \beta (e^*_L) R_L n^* (1 - x) + \frac{\partial e^*_L}{\partial x} s^* \beta (e^*_L) R_L n^* (1 - x) + \frac{\partial n^*}{\partial x} s^* \beta (e^*_L) R_L (1 - x) - s^* \beta (e^*_L) R_L n^*
\]

\[
- \frac{\partial n^*}{\partial x} \frac{(a^*_H)^2}{2} x - \frac{\partial a^*_H}{\partial x} n^* x - \frac{(a^*_H)^2}{2} n^* = 0
\]

(A.1)

We know from equations (3.1), (3.3), (3.5), and (3.6) the marginal impact project selection skills has on managerial advice, entrepreneur’s effort, number of investments, and share of revenues. Share of revenues and managerial advice increase, and number of investments and entrepreneur’s effort decrease, in project selection skills.

The private venture capitalist considers the marginal costs and benefits of investing in project selection skills, taking into account the effects on these variables.

Appendix B

The comparative static relationship on how the labour fund responds to changes in the exogenous variables (α, w, \(c'[x]\), I) are derived by how the additional term in the LSF’s optimal investment in project selection skill equation (4.2), \(\Phi\), responds.

Let \(\Phi = \frac{(1 - \alpha)wc'[x]}{\alpha \beta^2 (w + I)}\)

(B.1)

And let \(\frac{\partial LF}{\partial x} = \frac{\partial \Pi^{vc}}{\partial x} - \Phi = 0\).
The derivatives of $\Phi$ with respect to the exogenous parameters of interest are taken. If the derivative is negative (positive), the term $\Phi$ decreases (increases) as the parameter increases. Since the term is subtracted in the equation that determines the optimal investment by the LSF in project selection, a decrease (increase) in $\Phi$ leads to an increase (decrease) in skill relative to the private venture capitalist.

\[
\frac{\partial \Phi}{\partial \alpha} = - \frac{wc'[x]}{\alpha^3 \beta^2 (w + I)} < 0 \quad (B.2)
\]

\[
\frac{\partial \Phi}{\partial w} = \frac{(1 - \alpha)c'[x]I}{\alpha \beta^2 (w + I)^2} > 0 \quad (B.3)
\]

\[
\frac{\partial \Phi}{\partial c'[x]} = - \frac{(1 - \alpha)w}{\alpha \beta^2 (w + I)} > 0 \quad (B.4)
\]

\[
\frac{\partial \Phi}{\partial l} = - \frac{(1 - \alpha)c'[x]w}{\alpha \beta^2 (w + I)} < 0 \quad (B.5)
\]

The results are discussed in section 4. The changes in the optimal investment by LSF in project selection ability are relative to the private venture capitalist.
References


