INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI®
THE DETERMINATION OF
A CONSTITUTIVE EQUATION FOR
POLYSTYRENE BEAD FOAM

by

François C. Meunier

Thesis submitted to the School of Graduate Studies
of the University of Ottawa in partial fulfillment
of the requirements for the degree of Master of
Applied Science in Mechanical Engineering

UNIVERSITY OF OTTAWA
OTTAWA, CANADA, 1977

© F.C. Meunier, Ottawa, Canada, 1977
INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI®

UMI Microform EC52286
Copyright 2007 by ProQuest LLC
All rights reserved. This microform edition is protected against unauthorized copying under Title 17, United States Code.

ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346
Polystyrene Bead (PSB) foam is often used as an energy dissipator to reduce the likelihood of serious injury to motor vehicle passengers during accidents.

The purpose of this study was to investigate and model the behaviour of PSB foam subjected to quasi-static and dynamic deformation. A wide range of tests was conducted. By factoring into separate functions the effects of thickness, density, strain, and initial and instantaneous strain rate, the response of the foam can be simulated. The resulting constitutive equation can be used to accurately describe both the quasi-static and the dynamic deformation responses. The validity of the model is demonstrated with experimental data.

The behaviour of PSB foam is briefly discussed in relation to the biomechanics of head injury.
ACKNOWLEDGEMENTS

The Author wishes to thank his advisor Dr. J.A. Newman who identified the problem to be researched, whose grant from the National Research Council of Canada funded the expenses incurred and who provided important advice for the completion of the study.

The Author also thanks Mr. Conrad Meunier for the important design advice and some of the special machining.

Joe Zika and George Spak played a major role in the fabrication of the apparatus; to them, the Author is grateful.

Also, the Author acknowledges the special qualities of his wife Lise and of his immediate family who lived through it all!

Finally, the Author recognizes that the contributions of many other individuals, too numerous to mention, made the present study not only possible but interesting also. All participants are hereby graciously thanked and the author hopes that no one will feel hurt at not being named specifically.
### TABLE OF CONTENTS

| Acknowledgements                             | i |
| Table of Contents                             | ii |
| List of Appendices                            | iii |
| List of Tables                                | iv |
| List of Figures                               | tv |
| Introduction                                  | 1 |
| Foam Samples                                  | 7 |
| Quasi-Static Tests                            | 9 |
| Analysis of the Quasi-Static Tests            |  |
| Density                                       | 17 |
| Strain Rate                                   | 20 |
| Strain                                        | 20 |
| Dynamic Tests                                 | 28 |
| Analysis of Dynamic Test Results              | 34 |
| Discussion of Results - The Constitutive Equation |  |
| Density Dependence                            | 69 |
| Strain Rate Dependence                        | 74 |
| Strain Dependence                             | 78 |
| Dynamic Effect                                | 81 |
| Thickness Dependence                          | 83 |
| Discussion of Results - General               | 84 |
| Conclusions                                   | 101 |
| References                                    | 102 |
LIST OF APPENDICES

Appendix A - Sample Production, Quality Control, and Selection

Appendix B - Design and Reliability of the Quasi-Static Testing Apparatus

Appendix C - Quasi-Static Testing Procedure

Appendix D - Design and Installation of the Dynamic Testing Apparatus

Appendix E - Energy Levels for the Dynamic Tests

Appendix F - Linear Transducer Installation, Static and Dynamic Calibration

Appendix G - Load Cell Technical Data

Appendix H - Accelerometer Technical Data

Appendix I - Oscilloscope Triggering Device

Appendix J - Dynamic Testing Procedure

Appendix K - Extraction of Data - Dynamic Tests

Appendix L - Determination of the Initial Velocity

Appendix M - Computer Program for PSB Foam Simulation and Plotting Routine
LIST OF TABLES

Table 1 - Table of Various Quasi-Static Deformation Rates 11

LIST OF FIGURES

Figure No.

1. Polystyrene Bead Foam Samples 8
2. Quasi-Static Testing Apparatus 10
3. Post Deformation Samples 13
4a. Typical Quasi-Static Force-Deformation Curve for Polystyrene Bead Foam in Compression 14
4b. Typical Quasi-Static Force-Deformation Curve for Polystyrene Foam in Compression 14
5. Typical Stress-Strain Curve for the Quasi-Static Compressive Loading of Polystyrene Bead Foam 16
6. Graphical Representation of the Density-Dependence of Polystyrene Bead Foam. 18
7. Determination of Constants - Density Dependence 19
8. Strain Rate Coefficient - Variation in Slope with Thickness 21
9. Comparison of Experimental Data with Stress-Strain Simulation for Quasi-Static Testing - 1.27 cm Samples 24
10. Comparison of Experimental Data with Stress-Strain Simulation for Quasi-Static Testing - 2.54 cm Samples 25
11. Comparison of Experimental Data with Stress-Strain Simulation for Quasi-Static Testing - 3.81 cm Samples 26
<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.</td>
<td>Comparison of Experimental Data with Stress-Strain Simulation for Quasi-</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Static Testing - 5.08 cm Samples</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>Dynamic Testing Apparatus</td>
<td>29</td>
</tr>
<tr>
<td>14.</td>
<td>Drop Frame</td>
<td>30</td>
</tr>
<tr>
<td>15.</td>
<td>Load Cell</td>
<td>31</td>
</tr>
<tr>
<td>16.</td>
<td>Permanent Record - Dynamic Test Data</td>
<td>33</td>
</tr>
<tr>
<td>17.</td>
<td>Flow Chart of the CSMP Simulation Program</td>
<td>35</td>
</tr>
<tr>
<td>18.</td>
<td>Variation between Simulated and Experimental Data - Dynamic Tests</td>
<td>36</td>
</tr>
<tr>
<td>19.</td>
<td>Variation between Simulated and Experimentally Determined Strain Rate</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>Dependence</td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>Comparison between Simulated and Experimental Data - Strain Rate Factor</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Corrected</td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td>Comparison between Instantaneous Velocity-Dependence Simulation and</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Experimental Data</td>
<td></td>
</tr>
<tr>
<td>22.</td>
<td>Dynamic Factor</td>
<td>42</td>
</tr>
<tr>
<td>23a.</td>
<td>Variation between Simulated and Experimental Acceleration Data for Excessively</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>High Strain Tests</td>
<td></td>
</tr>
<tr>
<td>23b.</td>
<td>Variation between Simulated and Experimental Deformation Data for</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Excessively High Strain Tests</td>
<td></td>
</tr>
<tr>
<td>24.</td>
<td>Flow Chart - Complete CSMP Simulation Program</td>
<td>46</td>
</tr>
<tr>
<td>25.</td>
<td>Variation in Acceleration Response with Variation in Input Velocity</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>High Input Energy</td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td>Variation in Acceleration Response with Variation in Input Velocity</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Low Input Energy</td>
<td></td>
</tr>
<tr>
<td>Figure No.</td>
<td>Description</td>
<td>Page No.</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>27. to 45.</td>
<td>Simulated versus Experimental Acceleration Response</td>
<td>50 - 68</td>
</tr>
<tr>
<td>46.</td>
<td>PSB Foam Structural Model</td>
<td>71</td>
</tr>
<tr>
<td>47.</td>
<td>Cross-Section of a Single Bead</td>
<td>71</td>
</tr>
<tr>
<td>48.</td>
<td>Bearing Area - $l_0^{1.5}$ Relationship</td>
<td>73</td>
</tr>
<tr>
<td>49.</td>
<td>Kosten and Zwicker Fluid Flow Model for Foam</td>
<td>75</td>
</tr>
<tr>
<td>50.</td>
<td>Comparison between Experimental and Theoretical Stress-Strain Rate Relationship</td>
<td>77</td>
</tr>
<tr>
<td>51.</td>
<td>Strain Dependence Simulation Curve</td>
<td>79</td>
</tr>
<tr>
<td>52.</td>
<td>Comparison of the 3 Strain Dependent Exponentians</td>
<td>80</td>
</tr>
<tr>
<td>53.</td>
<td>Comparison between Quasi-Static and Dynamic Stress-Strain Relationships</td>
<td>82</td>
</tr>
<tr>
<td>54.</td>
<td>Comparison of Response for 9 Energy Modes, for 2.54 cm thick, 0.048 gm/cm$^3$ density PSB Foam Samples</td>
<td>85</td>
</tr>
<tr>
<td>55.</td>
<td>Thickness Variation Effect</td>
<td>89</td>
</tr>
<tr>
<td>56.</td>
<td>Density Variation Effect - Low Energy Input</td>
<td>90</td>
</tr>
<tr>
<td>57.</td>
<td>Density Variation Effect - High Energy Input</td>
<td>91</td>
</tr>
<tr>
<td>58.</td>
<td>Large Contact Area Acceleration Response</td>
<td>93</td>
</tr>
<tr>
<td>59.</td>
<td>Wayne State University Cerebral Concussion Tolerance Curve</td>
<td>95</td>
</tr>
<tr>
<td>60.</td>
<td>High Spike Acceleration Response</td>
<td>96</td>
</tr>
<tr>
<td>61.</td>
<td>High Rate of Onset Response</td>
<td>98</td>
</tr>
<tr>
<td>62.</td>
<td>High Rate of Onset - High Rate of Deceleration Response</td>
<td>98</td>
</tr>
<tr>
<td>63.</td>
<td>High Rate of Onset - High Spike Response</td>
<td>98</td>
</tr>
<tr>
<td>64a &amp; b</td>
<td>Comparison of Two Acceleration Traces with Identical Average Acceleration</td>
<td>99</td>
</tr>
</tbody>
</table>
INTRODUCTION

The amount of protection afforded to motor vehicle passengers during accidents by protective devices such as padded dashboards and motorcycle helmets is dependent upon the degree to which energy is dissipated.

Impact protection analysis has been hampered by the lack of relevant information on the load-deformation properties of polymers and plastics which might be used for energy absorption. Foamed polymeric materials exhibit properties which make them applicable as impact absorbers. They can undergo large compressive deformations and absorb relatively large energies during a deformation cycle. Rebound is minimal. In many instances, energy absorbing materials are selected by empirical, trial and error procedures, rather than by analytical techniques, because information relating the energy absorbing characteristics to the important foam variables such as density, thickness, cell size, etc. is not available.

New federal standards [1 to 7] emphasize high levels of protection for passengers of automotive vehicles (cars, motorcycles...). Modelling of protective materials has become essential.

The purpose of the present work, is to investigate and model the response of polystyrene bead (PSB) foam under quasi-static and dynamic loading, to investigate the effects of the individual foam parameters and finally to determine the effect of various mass-velocity combinations
on the response.

A good number of papers have been published describing specific mechanical properties of foams. Rinde [8] has shown that for 0.05 and 0.10 g/cc polystyrene bead (PSB) foam in compression, Poisson's ratio below yield strain is .25; at higher strains, the value is in the range of .03 to .07. His work has also shown that PSB foam in compression is density dependent i.e. higher stresses result from higher densities. Hoge and Wasley [9] have conducted investigations on PSB foam and Melvin and Roberts [10] on polystyrene; results show that strength at yield or higher strain levels is sensitive to strain rate; increasing strain rates result in greater strength. Their results also substantiate Rinde's conclusions on the density dependence of PSB foam.

Several attempts have been made to derive structure-property relations for foams. Chan, Lee and Nakamura [11] have taken the geometry of each bead into account and investigated the compression behavior of two foam beads in contact.

Patel and Finnie [12] have developed a model to explain many aspects of the mechanical behavior of rigid cellular plastics, using the relative sizes and mutual arrangements of the struts and cell walls of a unit cell. Quantitative relations between the properties of the plastic and the properties of the foam have been established on the basis of their model by assuming a unit cell in the form
of a pentagonal dodecahedron. The values obtained by Rinde and Hoge [13] for the yield stress density dependence of PSB foam in shear are within the same range as the values obtained by Patel and Finnie [12] for the density-dependence of the same material in compression. Traeger and Hermansen [14] have shown that the density-dependence of polyurethane foams varies over a wide range. Dement'ev, Tarakanov, and Seliverstov [15] have established a model of the mechanical properties of various foamed plastics. By assuming universal mechanisms for foaming and the formation of a macrostructure; they have suggested the superposition of structural and mechanical properties to account for variations in strength and elastic properties of high density foamed polymers.

Important attempts to predict polymeric foam properties from a constitutive equation of the matrix material and the foam structure include the cubic lattice model by Gent and Thomas [16] and the concentrically loaded plates model of closed cell walls by Matonis [17].

Some models use constant rate deformation data to predict impact behavior. Hinkley and Yang [18] have tested polyurethane foams and proposed a perfectly plastic, strain rate dependent model based on constant strain rate tests with additional empirical input from density and strain rate considerations. Integration techniques are proposed to account for the time-dependent response to impact. Although this model simulates the shape of the response
curve quite accurately, numerical values can deviate up to 25% from experimental results, even in the plateau region where the bulk of the energy is usually dissipated.

In the Rusch model [19] the compressive deformation stress is considered to be the product of a function of strain alone and another function dependent on the properties of the foam structure and the matrix material. The experimentally evaluated strain function, however, cannot be expressed exactly by any simple analytical expression; the approximate expression Rusch proposes is dependent on curve fitting constants determined by slopes and intercepts on the logarithmic plot of the strain function versus strain.

This method, as pointed out by Meinecke and Schwaber [20] is limited in its accuracy. In fact, they claim that the agreement of this representation with experimental results is not accurate enough to predict the time-dependent variation of velocity during impact deformation. They have shown that the impact behavior of foams with a rate independent modulus can accurately be predicted by quasi-static data. They reason that since the stress-strain relationship is constant regardless of rate, the energy input need only be equated to the proper area under the stress-strain curve. In a subsequent paper [21] they claim that impact behavior can be predicted for rate-dependent foams from constant rate of strain response which must be factorized into a strain dependent function and a rate-dependent modulus function. In that study, the rate dependence was artificially induced.
by means of various exterior coatings of the samples. As they have pointed out, the proposed rate-dependent modulus function is rather inaccurate. The scatter of data points is small enough that only a single straight line description can be justified; it appears that the three straight line function was used to minimize the error of the predicted values.

In a later paper, Meinecke, Schwaber and Chiang [22] have verified their analysis on two rubbery foams, one rate dependent the other rate independent. They point out that the analysis applies only to materials which lose energy by viscoelastic effects of the polymer matrix only. The normalization of the rate dependent curve by dividing the stress at any strain by the initial modulus is not a reliable method since it can be difficult to measure the initial slope precisely; the little scatter shown by the data points obtained in this fashion could be coincidental. Finally, the scope of their approach is limited. In their own words: "It is not possible to predict high speed impact processes (>13.9 fps) from constant rate experiments, if the rates of deformation are not comparable."

In the present work it will be shown that the impact behavior can be predicted from slow rate deformation data for closed cell brittle foams that exhibit pneumatic damping characteristics. The effects of thickness, density, strain, strain rate and the additional effects of variable strain rate impacts can be factored into separate functions. Their product yields the stress function. The resulting
constitutive equation can be used to accurately describe both quasi-static and dynamic deformation response.
FOAM SAMPLES

The polystyrene bead (PSB) foam used for the present analysis was manufactured by Morval Durofoam of Kitchener, Ontario. In a typical molding cycle, a mold is loaded with pre-expanded beads and steam is injected into the mold to slightly further expand the beads and to fill the voids. The steaming process sets the beads in place in two minutes. The foam is very soft and supple immediately after production and an estimated time lapse of three weeks is necessary before full strength is reached. The foam for the present study was tested one full year after production. It was manufactured in 30.48 cm (12 in.) square slabs using Dow Chemical Co. "Pelaspan" polystyrene beads. Five (5) nominal densities were specified: 0.04 gm/cm$^3$ (2-1/2 lb/ft$^3$), 0.048 gm/cm$^3$ (3 lb/ft$^3$), 0.056 gm/cm$^3$ (3-1/2 lb/ft$^3$), 0.064 gm/cm$^3$ (4 lb/ft$^3$) and 0.072 gm/cm$^3$ (4-1/2 lb/ft$^3$).

For each specified density, slabs of five (5) nominal thicknesses were produced: 1.27 cm (1/2 in.), 2.54 cm (1 in.), 3.81 cm (1-1/2 in.), 5.08 cm (2 in.) and 7.62 cm (3 in.).

Square samples of consistent aspect ratio thickness to side of 1:1.15 as shown in figure 1 were cut from the foam plates with a band saw. The cut surfaces were scuffed to ensure a uniform surface finish. All samples were measured and weighed and their density was calculated. Samples for which the density varied from nominal by more than ±3% were rejected. Detailed descriptions of sample production, quality control and sample selection can be found in Appendix A.
QUASI-STATIC TESTS

The term 'Quasi-Static' describes the relatively slow (compared to impact) constant rate deformation tests performed on the foam, using the Instron testing machine. Figure 2 shows the quasi-static testing apparatus mounted on the Instron. The parallel plates could be moved vertically to deform in compression the foam samples placed between them. The design and reliability of the apparatus is described in Appendix B.

Because 5 thicknesses of samples were used and because a restricted variety of cross-head velocities is possible on the Instron, all samples could not be tested at the same strain rates. Table 1 lists the various cross-head speeds and strain rates used for each thickness of sample. These tests were performed for each of the 5 densities available.

All the quasi-static tests were conducted at ambient temperature, 21° C (70° F).

All samples were tested in compression until the deformation force reached 2000 Kg. They were immediately unloaded at the same cross-head velocity. The force-deformation data was recorded on a strip chart pen recorder calibrated according to the manufacturers recommendations. The quasi-static testing procedure is outlined in Appendix C. To verify the repeatability of the foam behavior and the consistency of the apparatus and test procedure, three (3) tests were performed for each set of conditions.
FIGURE 2 — QUASI-STATIC TESTING APPARATUS.
<table>
<thead>
<tr>
<th>Thickness (cm)</th>
<th>1.27</th>
<th>2.54</th>
<th>3.81</th>
<th>5.08</th>
<th>7.62</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00083(0.00066)</td>
<td>0.00166(0.00066)</td>
<td></td>
<td>0.0033(0.00066)</td>
<td>0.005(0.00066)</td>
<td></td>
</tr>
<tr>
<td>0.00166(0.0013)</td>
<td>0.0033(0.0013)</td>
<td>0.005(0.0013)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0083(0.0066)</td>
<td>0.0166(0.0066)</td>
<td></td>
<td>0.033(0.0066)</td>
<td>0.05(0.0066)</td>
<td></td>
</tr>
<tr>
<td>0.0166(0.013)</td>
<td>0.033(0.013)</td>
<td>0.05(0.013)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.083(0.066)</td>
<td>0.166(0.066)</td>
<td></td>
<td>0.33(0.066)</td>
<td>0.5(0.066)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5(0.13)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 - Table of the various cross-head speeds (cm/sec.) used for each thickness of sample and the resulting strain rates (in brackets, cm/cm/sec.) at which they were tested.
(thickness, density, strain rate). A total of 285 samples of PSB foam were tested in the quasi-static mode. Typical post-deformation samples are shown in figure 3.

The curve shown in figure 4a is representative of the force-deformation behavior of PSB foam samples subjected to quasi-static loading. It differs from the force-deformation behavior of polystyrene foam shown in figure 4b. Different density distributions probably account for the difference in behavior. The density is essentially uniform in polystyrene foam. For PSB foam, a distribution in density was observed, as discussed in Appendix A; yielding of the cell walls occurs over a wide range of loads and the plateau region is modified.

All PSB foam samples exhibit an initial elastic region of very limited extent (in the vicinity of 2% deformation), and a region of almost constant force that gradually develops into a quickly-rising exponential at high strains (70-80%). The area under the unloading curve is always small compared to the area under the loading curve.

A comparison of similar tests revealed that in the plateau region, maximum variations from test to test were of the order of ± 5% while most variations were within ± 2%. In the exponential region ± 12% variations were observed.

The analysis in this study relates to the loading portion only. Unloading characteristics form a field of study on their own and could be the subject of future work.

A series of curves representative of the average of each set of 3 tests was established; the force values were
Figure 4b – Typical Quasi-Static Force-Deformation Curve for Polystyrene Foam in Compression
corrected for the elasticity of the testing apparatus.
(The deformation recorded on the strip chart included the
deformation of the apparatus. Therefore, true sample
deformation was equal to the recorded deformation minus
the deformation of the apparatus. A graphical representa-
tion of the elasticity of the apparatus can be found in
Appendix B, figure B2)

Stress-strain relationships were determined for all
corrected curves. Figure 5 shows a typical stress-strain
curve.
FIGURE 5 — TYPICAL STRESS-STRAIN CURVE FOR THE QUASI-STATIC COMPRESSIVE LOADING POLYSTYRENE BEAD FOAM.
ANALYSIS OF THE QUASI-STATIC TESTS

In this section, it will be shown that the effects of density, strain rate and thickness can be factored into separate functions whose product is the stress function.

Density

For specific values of strain, straight lines were fitted to the ln stress versus ln density data; their slope was almost identical* as shown in figure 6. The average slope was determined to be 1.5. The equation

\[ \sigma(\varepsilon) = C_1 A(\varepsilon) \rho^{1.5} \]  

(1)

hence describes the density dependence of stress at any given strain. \( \sigma \) is the stress in megapascals, \( C_1 \) is a constant of proportionality equal to .098, \( \ln A(\varepsilon) \) is the intercept of the \( \ln \) (stress) - \( \ln \) (density) relationship shown in figure 7, and \( \rho \) is the density of the foam in gm/cm\(^3\).

A straight line of slope 1.5 was fitted, by the method of least squares, to the \( \ln \) stress - \( \ln \) density data and from the intercept, values for \( A(\varepsilon) \) were determined for

*The slope for the 5.08 cm thick sample of 0.04, 0.064 0.072 gm/cm\(^3\) nominal density and all 7.62 cm thick samples however, was found to be significantly different (±15% variation). Their cell size distribution was observed to be much wider than the other samples and it is believed that this accounts for their different behaviour. They were eliminated from further analysis and future reference to 'all samples' does not include them.
FIGURE 6 – GRAPHICAL REPRESENTATION OF THE DENSITY-DEPENDENCE OF POLYSTYRENE BEAD FOAM.
FIGURE 7 — DETERMINATION OF CONSTANTS — DENSITY DEPENDENCE.
10% incremental values of strain up to 80% strain for all test conditions. Using the calculated values for $A(\varepsilon)$ and the proper values of density in equation (1) above, approximate values of stress were calculated for all samples and their standard deviation from experimental values was determined to be 3.71%.

**Strain Rate**

Plots of $\ln A(\varepsilon)$ versus $\ln$ strain rate were drawn for all sample thicknesses and 10% incremental values of strain up to 80% strain. The best straight lines, the slopes and the intercepts were determined for all sets of conditions by the method of least squares.

The slope of the straight lines was generally constant for any one thickness, the greater the thickness, the steeper the slope. Average slopes were determined for every thickness of sample and the expression

$$ 0.026 + .00276 T_0 $$

was developed to approximate the variation in slope. ($T_0$ is equal to the initial thickness of the foam sample.)

Thus, the equation

$$ A(\varepsilon) = C_2 B(\varepsilon) \dot{\varepsilon}_0^{(a + b T_0)} \quad (2) $$

was obtained. $C_2$ is a constant of proportionality equal to unity, $\ln B(\varepsilon)$ is the intercept of the $\ln A(\varepsilon)$ - $\ln \dot{\varepsilon}_0$ plot for any value of strain, $\dot{\varepsilon}_0$ is the strain rate in cm/cm/minute, $a$ is a constant equal to .026, $b$ is a constant equal to .00276
COEFFICIENT = $\varepsilon_o^{(a + bT_o)}$

**FIGURE 8** – STRAIN RATE COEFFICIENT – VARIATION IN SLOPE WITH THICKNESS
and $T_0$ is the initial thickness of the foam sample. Equation (2) is shown in graphical form in figure 8.

**Strain**

Values for $B$ were determined for all conditions of strain rate, thickness and 10% incremental values of strain up to 80% strain.

It was found that for any given value of strain, the values of $B$ were almost identical for all thicknesses. The average values were determined.

Eight values of $B$ were obtained, one for each of the 10% incremental values of strain up to 80%.

Two exponentials were determined to approximate the variation of $B$ with strain:

$$B(\varepsilon) = 415.72 \exp(0.88\varepsilon) + 0.01343 \exp(13.9\varepsilon)$$  \hspace{1cm} (3)

thus the total expression for the prediction of stress for samples of PSB foam subjected to quasi-static compression:

$$\sigma = f(\varepsilon) \cdot f(\rho) \cdot f(\dot{\varepsilon}_0, T_0)$$  \hspace{1cm} (4)

where

$\sigma$ = stress in megapascals

$f(\varepsilon) = 415.72 \exp(0.88\varepsilon) + 0.01343 \exp(13.9\varepsilon)$ (megapascals)

$f(\rho) = C_1 \rho^{1.5}$

$f(\dot{\varepsilon}_0, T_0) = C_2 \dot{\varepsilon}_0^{0.026 + 0.00276T_0}$

$\varepsilon$ = strain

$\rho$ = density (gm/cm$^3$)

$\dot{\varepsilon}_0$ = initial strain rate (cm/cm/min)

$T_0$ = initial thickness (cm)

$C_1 = 0.098$ and $C_2 = 1.$ when $\rho, \dot{\varepsilon}_0$ and $T_0$ are given in the units shown above.
The equation does not account for the initial elastic behaviour of PSB foam. Because the model being developed would be used primarily to predict response to relatively high levels of energy input (i.e. high strains would result from the input), the reduced contribution of the elastic region to absorption of energy was ignored; plasticity (permanent deformation) was assumed to be concurrent with the onset of deformation.

Figures 9, 10, 11 and 12 show experimental values as well as values predicted from equation (3) above for various combinations of thickness and strain rate.
**EXPERIMENTAL**

**SIMULATED**

STRAIN RATE = 0.00066 cm/cm/sec (0.0396 cm/cm/min)
THICKNESS = 1.27 cm (NOMINAL)

FIGURE 9 - COMPARISON OF EXPERIMENTAL DATA WITH STRESS-STRAIN SIMULATION FOR QUASI-STATIC TESTING - 1.27 CM SAMPLES.
**Figure 10** — Comparison of experimental data with stress-strain simulation for quasi-static testing — 2.54 cm samples

- Experimental
- Simulated

Strain rate: 0.00066 cm/cm/sec (0.0396 cm/cm/min)

Thickness: 2.54 cm (Nominal)
FIGURE 11 – COMPARISON OF EXPERIMENTAL DATA WITH STRESS-STRAIN SIMULATION FOR QUASI-STATIC TESTING – 3.81 CM SAMPLES.
FIGURE 12 — COMPARISON OF EXPERIMENTAL DATA WITH STRESS-STRAIN SIMULATION FOR QUASI-STATIC TESTING — 5.08 CM SAMPLES.
DYNAMIC TESTS

The term 'dynamic' is used to describe impact testing during which the velocity was time dependent.

Figures 13 and 14 show the dynamic testing apparatus. It consists of a rigid steel frame onto which is affixed a 13.54 cm (5.25 in.) diameter aluminum impactor. Various smooth surface weights can be solidly bolted onto the impactor to vary the impacting mass from a minimum of 2.1 Kg to a maximum of 126 Kg. The vertical path of the impactor assembly is prescribed by two guide rods. These guide rods as well as the load cell assembly had been installed for a previous study [23]. The design of the dynamic apparatus is explained in detail in Appendix D. A review of the different modes and levels of energy used for the tests can be found in Appendix E. A linear transducer assembly was made part of the apparatus to monitor the vertical movement of the impactor at any time during a test.

Appendix F contains all the information pertinent to the linear transducer: installation, static calibration, and dynamic calibration.

A load cell, shown in figure 15, was placed under the foam support plate for force-acceleration calibration purposes only, for possible future reference. The load cell technical data has been included in Appendix G.

During all tests, the deformation of the sample and the acceleration of the impactor were monitored and temporarily recorded on a Tektronix oscilloscope #5103S.
Technical data as well as calibration procedures for the accelerometer can be found in Appendix H. The oscilloscope triggering device is explained in Appendix I.

The data was permanently recorded on 35 mm high contrast panchromatic motion picture-type film Eastman #5369. The film was developed using a microfilm developing process. Figure 16 gives an example of the permanent record which includes all pertinent information.

To ensure the repeatability of the testing technique, three (3) tests of each set of conditions were performed. Altogether, 685 samples of PSB foam were subjected to impact loading. The testing procedure for the dynamic impact tests is outlined in Appendix J.
<table>
<thead>
<tr>
<th>TEST</th>
<th>SAMPLE</th>
<th>IMPACTOR</th>
<th>SWEEP RATE</th>
<th>TRACE 1</th>
<th>TRACE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>T</td>
<td>g Mass</td>
<td>Drop H</td>
<td>Trace 1</td>
<td>Trace 2</td>
</tr>
<tr>
<td>394</td>
<td>852</td>
<td>5.18 cm</td>
<td>60.1 Kg/m³</td>
<td>5.32 Kg</td>
<td>1.5 m</td>
</tr>
</tbody>
</table>

FIGURE 16 — PERMANENT RECORD — DYNAMIC TEST DATA.
ANALYSIS OF DYNAMIC TEST RESULTS

Appendix K outlines the procedure used to extract data from the oscilloscope traces.

It was hypothesized that the descriptive equation, obtained on the basis of the quasi-static test data, could be used to predict dynamic impact behavior provided higher strain rates and velocity variation could be accounted for. To acquire a proper description of the dynamic response to impact, the time independent stress expression developed for the quasi-static response was transformed into a time-dependent function as follows:

\[
\text{Force} = \text{stress} \times \text{area} \quad \quad (5)
\]

\[
\text{Force} = \text{mass} \times \text{acceleration} \quad \quad (6)
\]

\[
\text{acceleration} = \ddot{x} = \frac{\text{Force}}{\text{mass}} = \frac{\text{stress} \times \text{area}}{\text{mass}} \quad \quad (7)
\]

\[
\text{velocity} = \dot{x} = \int \text{acceleration} \, (dt) \quad \quad (8)
\]

\[
\text{deformation} = x = \int \text{velocity} \, (dt) \quad \quad (9)
\]

A CSMP (Continuous system modelling program)[24] was used to solve for the velocity and deformation expressions. A flow chart for this program is shown in figure 17. Figure 18 shows a typical curve, obtained by solving equations 7, 8 and 9, compared to experimental data. In the early stages of deformation, the simulated acceleration values are higher than the experimental results and because the model assumed such a large dissipation of energy at the onset, low values are predicted for the latter stages.
\[ f(\rho) = C_1 \rho^{1.5} \]
\[ f(v_0, T_0) = C_2 \left( \frac{v_0}{T_0} \right) \]
\[ f(x) = 415.72 \exp(0.88\varepsilon) + 0.01343 \exp(13.9\varepsilon) \]

\( v_0 = \text{INITIAL VELOCITY} \)
\( T_0 = \text{INITIAL THICKNESS} \)
\( x_0 = \text{INITIAL DEFORMATION} \)
\( \text{IC} = \text{INITIAL CONDITION} \)

\( f(x) \) is dependent on units for \( \rho, v_0 \), and \( T_0 \)
\( \varepsilon = \text{STRAIN} = \frac{x}{T_0} \)

FIGURE 17 — FLOW CHART OF THE CSMP SIMULATION PROGRAM.
FIGURE 18 — VARIATION BETWEEN SIMULATED AND EXPERIMENTAL DATA — DYNAMIC TESTS.
This behaviour was determined to be caused by an inappropriate strain rate function. The range over which the function had been determined was small compared to the magnitude of the dynamic strain rates; for the dynamic tests, the value of the coefficient could not be determined accurately with the present function.

Four of the dynamic experimental results were used to calculate the strain rate coefficient for four high strain rates. The values obtained with equation (2) were always higher (≤ 15%) than the experimentally-determined coefficients, as shown in figure 19.

By trial and error, the expression

$$f(\dot{\varepsilon}_0) = C_3 \sinh^{-1} \left( C_4 \dot{\varepsilon}_0 \right)$$

(10)

was devised to approximate the correct values for all initial strain rates. $C_3$ is equal to 0.03057 and $C_4$ is equal to 3.75 x $10^{13}$ when $\dot{\varepsilon}_0$, the initial strain rate unit is strain/minute. With this function, the strain rate coefficients obtained, for the range of the quasi-static tests, varied no more than 5% from the values obtained with the original expression.

For the range of velocities of the dynamic tests, the new function determined strain rate coefficients within 3% of the experimental numbers.

Figure 20 shows a simulated curve using the corrected strain rate function and actual data. Based on quasi-static behaviour, an ever-increasing curve was obtained because the acceleration expression as it now appeared did not account
\[ f(\dot{\varepsilon}, T_0) = \frac{v_0}{T_0} \left[ 0.028 + 0.000278 \times T_0 \right] \]

\[ \begin{align*} &X \ T = 1 \text{ cm} \\
&+ \ T = 3 \text{ cm} \\
&* \ T = 5 \text{ cm} \end{align*} \]

- EXPERIMENTAL, ANY T
- \( 0.03057 \sinh^{-1}(\dot{\varepsilon} \times 3.75 \times 10^{13}) \)

**FIGURE 19** — VARIATION BETWEEN SIMULATED AND EXPERIMENTALLY-DETERMINED STRAIN RATE DEPENDENCE.
FIGURE 20 — COMPARISON BETWEEN SIMULATED AND EXPERIMENTAL DATA — STRAIN-RATE FACTOR CORRECTED.
for the dynamic character of impact during which the velocity of the striker gradually decays, from an initial high, to zero at maximum deformation.

When the instantaneous strain rate was substituted for the initial strain rate, (figure 21) the predicted values were only slightly closer to the experimental results for the major portion of the loading process; the expression became equal to zero when the strain rate approached zero. However, the experimental results show that when the strain rate becomes zero the acceleration is near or is at a maximum.

An additional coefficient was needed to account for the velocity variation of the impactor \(f(v)\). The difference between one experimental result and the simulation based on quasi-static data (figure 20) was plotted against \(v/v_0\) where \(v\) is the instantaneous velocity and \(v_0\) is the initial velocity of the impactor. A graph similar to figure 22 was obtained and by the method of least squares a straight line was fitted, yielding the approximation

\[
f(v) = 0.796 + 0.204\frac{v}{v_0} \tag{11}
\]

During the quasi-static tests, the instantaneous velocity is equal to the initial velocity and the dynamic expression is equal to one (1).

The resulting equation was adequate to simulate most of the dynamic test results. For some of the tests, however, where the input energy was high enough to produce
FIGURE 21 — COMPARISON BETWEEN INSTANTANEOUS-VELOCITY-DEPENDENT SIMULATION AND EXPERIMENTAL DATA.
FIGURE 22 — THE DYNAMIC FACTOR.
deformations larger than 85%, the peak acceleration and the maximum deformation could not be predicted accurately. The simulation always resulted in a much lower peak acceleration than the experimental results and the predicted deformation was always appreciably larger than for the actual tests as shown in figures 23a and 23b. This behaviour had not been predicted because such high deformations were not possible during the quasi-static tests due to the 2000Kg limit of the load cell. An average strain dependent response from five high deformation tests was determined. The difference at any strain between actual and simulated results was used to determine a third exponential: $1.25 \times 10^{-13} \exp(42 \varepsilon)$.

For strains lower than 80%, the effect of this term is negligible. For higher strains, however, the predicted acceleration is increased considerably and the simulated maximum deformation is reduced, so that the model and the actual results are in better agreement.

The following formula can thus be used to determine the stress response to both quasi-static and dynamic tests:

$$\sigma = f(\varepsilon) \cdot f(\rho) \cdot f(\dot{\varepsilon}_0) \cdot f(v)$$ \hspace{1cm} (12)

where

- $\sigma$ = stress in megapascals
- $f(\varepsilon) = 415.72 \exp(0.88\varepsilon) + 0.01343 \exp(13.9\varepsilon) + 1.25 \times 10^{-13} \exp(42.0\varepsilon)$ (megapascals)
- $f(\rho) = C_1 \rho^{1.5}$
- $f(\dot{\varepsilon}_0) = C_3 \sinh^{-1}(C_4 \dot{\varepsilon}_0)$
**FIGURE 23a** — VARIATION BETWEEN SIMULATED AND EXPERIMENTAL ACCELERATION DATA FOR EXCESSIVELY HIGH STRAIN TESTS.

**FIGURE 23b** — VARIATION BETWEEN SIMULATED AND EXPERIMENTAL DEFORMATION DATA FOR EXCESSIVELY HIGH STRAIN TESTS.
\[ f(v) = 0.796 + 0.204v/v_0 \]

\[ \varepsilon = \text{strain} \]

\[ \dot{\varepsilon}_0 = \text{initial strain rate} = v_0 / T_0 \]

\[ \rho = \text{density (g/cm}^3 \text{)} \]

\[ v = \text{instantaneous velocity (cm/min)} \]

\[ v_0 = \text{initial velocity (cm/min)} \]

\[ T_0 = \text{initial thickness of the foam (cm)} \]

\[ C_1 = 0.098, \ C_2 = 0.03057 \text{ and } C_3 = 3.75 \times 10^{13} \text{ when } \rho, v, v_0 \text{ and } T_0 \text{ are given in the units shown above.} \]

Values obtained for the quasi-static mode using this expression were found to have a standard deviation of 5.3% with the experimental values.

The equation was used with CSMP to model the behaviour of PSB foam subjected to impact. A flow chart of this program is shown in figure 24. The program along with the plotting commands can be found in Appendix M.

It should be pointed out that the acceleration response is very sensitive to the initial velocity of the impactor as shown in figures 25 and 26. For a high input energy, a variation in the velocity considerably varies the value of the peak acceleration and the time duration. The early part of the response shows no appreciable change. For a low input energy, the effect is felt over a greater portion of the response. Still, small changes in velocity alter the acceleration response significantly. The time duration is slightly affected.
\( f(x) = 415.72 \exp(0.88\varepsilon) + 0.01343 \exp(13.9\varepsilon) + 1.25 \times 10^{-8} \exp(42\varepsilon) \)

\( f(\rho) = c_1 \rho^{1.5} \)

\( f(v_0, t_0) = c_3 \sinh^{-1}(c_4 \frac{v_0}{t_0}) \)

\( f(\dot{x}) = 0.796 + 0.204 \frac{\dot{x}}{v_0} \)

\( \rho = \text{DENSITY} \)

\( \varepsilon = \text{STRAIN} = \frac{x}{t_0} \)

\( c_1, c_3 \& c_4 = \text{CONSTANTS DEPENDENT ON UNITS FOR } \rho, v_0, \dot{x} \text{ AND } t_0 \)

**Figure 24 — Flow Chart — Complete CSMP Simulation Program.**
\( V_{th} \) = THEORETICAL VELOCITY = 626 cm/sec
MASS = 2.374 Kg

FIGURE 25 — VARIATION IN ACCELERATION RESPONSE WITH VARIATION IN INPUT VELOCITY — HIGH INPUT ENERGY.
\[ V_{th} = \text{THEORETICAL VELOCITY} = 313.2 \text{ cm/sec} \]
\[ \text{MASS} = 2.374 \text{ Kg} \]

**FIGURE 26** — VARIATION IN ACCELERATION RESPONSE WITH VARIATION IN INPUT VELOCITY — LOW ENERGY INPUT.
To compare the response predicted by the constitutive equation with the experimental results, the initial velocity of the impactor was determined by integration of the experimental acceleration trace. The method used for integration is discussed in Appendix L. The next few pages, figures 27 to 45, are representative of the results plotted by the computer. A variety of densities, thicknesses, impact modes etc... have been included to show the comprehensiveness and the accuracy of the constitutive equation. The response of the foam will be discussed later. The simulation values are usually within 5% of the experimental data; at high strains, the error can be greater because of the asymptotic nature of the response in this region.
TEST NO. 0127
SAMPLE NO. 571
INITIAL DENSITY OF SAMPLE = 0.0488 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.59 CM
INITIAL VELOCITY OF IMPACTOR = 291.7 CM/SEC
MASS OF IMPACTOR = 2.374 KG

+ EXPERIMENTAL
— SIMULATED

**FIGURE 27**
TEST NO. D167
SAMPLE NO. 586
INITIAL DENSITY OF SAMPLE = 0.0485 G/CLU.CM
INITIAL THICKNESS OF SAMPLE = 2.60 CM
INITIAL VELOCITY OF IMPACTOR = 2946 CM/SEC
MASS OF IMPACTOR = 3.167 KG

+ EXPERIMENTAL
— SIMULATED

FIGURE 28
TEST NO. 0246
SAMPLE NO. 917
INITIAL DENSITY OF SAMPLE = 0.0487 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.58 CM
INITIAL VELOCITY OF IMPACTOR = 300.0 CM/SEC
MASS OF IMPACTOR = .4748 KG

+ EXPERIMENTAL
--- SIMULATED

DEFORMATION VS TIME

VELOCITY VS TIME

PERCENT

100

80

60

40

20

0

0.0 12.0 16.0 20.0

MILLISECONDS

PERCENT

100

80

60

40

20

0

0.0 12.0 16.0 20.0

MILLISECONDS

ACCELERATION VS TIME

STRESS VS STRAIN

PERCENT

10

8

6

4

2

0

0 20 40 60 80 100

PERCENT

FIGURE 29
TEST NO. D87
SAMPLE NO. 584
INITIAL DENSITY OF SAMPLE = 0.0478 G/CM³
INITIAL THICKNESS OF SAMPLE = 2.58 CM
INITIAL VELOCITY OF IMPACTOR = 4121 CM/SEC
MASS OF IMPACTOR = 2.374 KG

+ EXPERIMENTAL
--- SIMULATED

**DEFORMATION VS TIME**

**VELOCITY VS TIME**

**ACCELERATION VS TIME**

**STRESS VS STRAIN**

FIGURE 30
TEST NO. D46
SAMPLE NO. 912
INITIAL DENSITY OF SAMPLE = 0.0477 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.58 CM
INITIAL VELOCITY OF IMPACTOR = 516.7 CM/SEC
MASS OF IMPACTOR = 2374 KG

+ EXPERIMENTAL
--- SIMULATED

DEFORMATION VS TIME

VELOCITY VS TIME

ACCELERATION VS TIME

STRESS VS STRAIN

FIGURE 31
TEST NO. D308
SAMPLE NO. 581
INITIAL DENSITY OF SAMPLE = 0.0489 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.59 CM
INITIAL VELOCITY OF IMPACTOR = 306.7 CM/SEC
MASS OF IMPACTOR = 3.436 KG

+ EXPERIMENTAL
— SIMULATED

DEFORMATION VS TIME

VELOCITY VS TIME

ACCELERATION VS TIME

STRESS VS STRAIN

FIGURE 32
TEST NO. D289
SAMPLE NO. 899
INITIAL DENSITY OF SAMPLE = 0.0475 G/CM
INITIAL THICKNESS OF SAMPLE = 2.59 CM
INITIAL VELOCITY OF IMPACTOR = 433.8 CM/SEC
MASS OF IMPACTOR = 4.748 KG

+ EXPERIMENTAL
--- SIMULATED

DEFORMATION VS TIME

VELOCITY VS TIME

ACCELERATION VS TIME

STRESS VS STRAIN

FIGURE 33
TEST NO. D207
SAMPLE NO. 895
INITIAL DENSITY OF SAMPLE = 0.0482 g/cu.cm
INITIAL THICKNESS OF SAMPLE = 2.59 cm
INITIAL VELOCITY OF IMPACTOR = 520.8 cm/sec
MASS OF IMPACTOR = 3.167 kg

+ EXPERIMENTAL
— SIMULATED

DEFORMATION VS TIME

ACCELERATION VS TIME

VELOCITY VS TIME

STRESS VS STRAIN

FIGURE 34
TEST NO. D10
SAMPLE NO. 911
INITIAL DENSITY OF SAMPLE = 0.0485 G/CM³
INITIAL THICKNESS OF SAMPLE = 2.58 CM
INITIAL VELOCITY OF IMPACTOR = 575.8 CM/SEC
MASS OF IMPACTOR = 2.374 KG

+ EXPERIMENTAL
--- SIMULATED

**FIGURE 35**
TEST NO. D603
SAMPLE NO. 1274
INITIAL DENSITY OF SAMPLE = 0.0521 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 5.15 CM
INITIAL VELOCITY OF IMPACTOR = 305.3 CM/SEC
MASS OF IMPACTOR = 18.984 KG

EXPERIMENTAL
SIMULATED

FIGURE 36
TEST NO. D498
SAMPLE NO. 1093
INITIAL DENSITY OF SAMPLE = 0.0527 G/CM^3
INITIAL THICKNESS OF SAMPLE = 3.78 CM
INITIAL VELOCITY OF IMPACTOR = 307.1 CM/SEC
MASS OF IMPACTOR = 10.677 KG

+ EXPERIMENTAL
— SIMULATED

DEFORMATION VS TIME

VELOCITY VS TIME

ACCELERATION VS TIME

STRESS VS STRAIN

FIGURE 37
TEST NO. D251
SAMPLE NO. 615
INITIAL DENSITY OF SAMPLE = 0.0538 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.60 CM
INITIAL VELOCITY OF IMPACTOR = 305.3 CM/SEC
MASS OF IMPACTOR = 4.478 KG

+ EXPERIMENTAL
--- SIMULATED

**DEFORATION VS TIME**

**VELOCITY VS TIME**

**ACCELERATION VS TIME**

**STRESS VS STRAIN**

FIGURE 38
TEST NO. D271
SAMPLE NO. 1514
INITIAL DENSITY OF SAMPLE = 0.0522 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 1.26 CM
INITIAL VELOCITY OF IMPACTOR = 295.8 CM/SEC
MASS OF IMPACTOR = 4.748 KG

+ EXPERIMENTAL
--- SIMULATED

**DEFORMATION VS TIME**

**VELOCITY VS TIME**

**ACCELERATION VS TIME**

**STRESS VS STRAIN**

FIGURE 39
TEST NO. 0131
SAMPLE NO. 938
INITIAL DENSITY OF SAMPLE = 0.0538 G/CM³
INITIAL THICKNESS OF SAMPLE = 2.59 CM
INITIAL VELOCITY OF IMPACTOR = 301.7 CM/SEC
MASS OF IMPACTOR = 2.374 KG

+ EXPERIMENTAL
- SIMULATED

FIGURE 40
TEST NO. D135
SAMPLE NO. 979
INITIAL DENSITY OF SAMPLE = 0.0689 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.60 CM
INITIAL VELOCITY OF IMPACTOR = 3046 CM/SEC
MASS OF IMPACTOR = 2.374 KG

+ EXPERIMENTAL
--- SIMULATED

DEFORMATION VS TIME
VELOCITY VS TIME

ACCELERATION VS TIME
STRESS VS STRAIN

FIGURE 41
TEST NO. D138
SAMPLE NO. 1018
INITIAL DENSITY OF SAMPLE = 0.0846 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 2.58 CM
INITIAL VELOCITY OF IMPACTOR = 306.7 CM/SEC
MASS OF IMPACTOR = 2.374 KG

+ EXPERIMENTAL
—— SIMULATED

DEFORMATION VS TIME

ACCELERATION VS TIME

VELOCITY VS TIME

STRESS VS STRAIN

FIGURE 42
TEST NO. D396
SAMPLE NO. 1179
INITIAL DENSITY OF SAMPLE = 0.0830 G/CU.CM
INITIAL THICKNESS OF SAMPLE = 3.79 CM
INITIAL VELOCITY OF IMPACTOR = 540.8 CM/SEC
MASS OF IMPACTOR = 5.339 KG

--- EXPERIMENTAL
--- SIMULATED

FIGURE 43
TEST NØ. D393
SAMPLE NØ. 1106
INITIAL DENSITY OF SAMPLE = 0.0605 G/CL. CM
INITIAL THICKNESS OF SAMPLE = 3.78 CM
INITIAL VELOCITY OF IMPACTOR = 528.3 CM/SEC
MASS OF IMPACTOR = 5.339 KG

FIGURE 44.
TEST NO. D388
SAMPLE NO. 1057
INITIAL DENSITY OF SAMPLE = 0.0485 G/CM³
INITIAL THICKNESS OF SAMPLE = 3.78 CM
INITIAL VELOCITY OF IMPACTOR = 5242 CM/SEC
MASS OF IMPACTOR = 5.339 KG

+ EXPERIMENTAL
--- SIMULATED

FIGURE 45
DISCUSSION OF RESULTS - THE CONSTITUTIVE EQUATION

The previous pages have described the procedure used to determine the variables of the constitutive equation of PSB foam. In the next pages, the results obtained will be discussed and plausible explanations for the observed behaviour will be offered. Each variable will be discussed separately.

Density Dependence

The present work has determined that compressive stress for PSB foam is a function of density $^{1.5}$. This is in good agreement with the published results of Patel and Finnie (12) who found a value of 1.46 for the exponent. The slight variation between the two results may well be the outcome of a different number of tests, different quality control and different sample production techniques.

A model similar to that of Matonis (17) can relate the bearing area and the volume of the individual foam beads to the density of the foam. The following assumptions were used: the non-expanded beads consist solely of bulk styrene material and no voids are present; all beads have the same unit weight and volume; cubic cells are formed by expanding the beads and all the original material is used to form the cell walls; the voids in the center of the cells are cubic (a cube within a cube); the wall density is constant; and voids, other than the voids within the cubic cells, are small.
Let

\[ V = \text{the volume of a non-expanded bead.} \]
\[ V_o = \text{the total volume of an expanded bead.} \]
\[ V_i = \text{the volume of voids within an expanded bead.} \]
\[ l = \text{dimension of a non-expanded bead.} \]
\[ l_o = \text{outside dimension of an expanded bead.} \]
\[ l_i = \text{inside dimension of an expanded bead.} \]
\[ W_o = \text{weight of each bead.} \]
\[ N = \text{number of beads per unit volume.} \]
\[ \rho_o = \text{density of an expanded bead.} \]
\[ \rho_s = \text{density of a sample.} \]
\[ \text{BA} = \text{bearing area of an expanded bead.} \]
\[ t = \text{thickness of a cell wall.} \]

Figure 46 is a representation of the proposed model and Figure 47 shows the cross-section of a unit cell.

From the definitions:

\[ V_o = V + V_i = l^3 + l_i^3 = l_o^3 \]  \hspace{1cm} (13)
\[ \rho_o = \frac{W_o}{V_o} \]  \hspace{1cm} (14)
\[ \rho_s = \frac{W_o \times N}{V_o \times N} = \frac{W_o}{V_o} = \rho_o = \frac{W_o}{l_o^3} \]  \hspace{1cm} (15)

Therefore:

\[ \rho_s^{1.5} = \frac{W_o^{1.5}}{l_o^{4.5}} \]  \hspace{1cm} (16)

Now,

\[ \text{BA} = l_o^2 - l_i^2 \]  \hspace{1cm} (17)
\[ l_i = \sqrt[3]{V_i} \]  \hspace{1cm} (18)
FIGURE 46 — PSB FOAM STRUCTURAL MODEL.

FIGURE 47 — CROSS-SECTION OF A SINGLE BEAD.
\[ V_i = V_0 - V \]  
(19)

\[ l_i = \sqrt[3]{V_0} - V \]  
(20)

\[ l_0 = \sqrt[3]{V_0} \]  
(21)

\[ V_0 = l_0^3 \]  
(22)

\[ l_i = \sqrt[3]{l_0} - V \]  
(23)

Therefore:

\[ BA = l_0^2 - (\sqrt[3]{l_0^3} - V)^2 \]  
(24)

Figure 48 shows the relationship between BA and \( l_0^{-1.5} \).

For a good portion of the curve, the relationship can be approximated with a straight line (dotted line). The linearity breaks down for high values of bearing area. Matonis has suggested that the cell walls act as compression columns. For relatively thick cell walls, the column analogy breaks down and massive compression of the bulk styrene is initiated immediately. (For the range of densities used of the present work, extensive buckling of the cell walls must take place before compression of the bulk styrene becomes appreciable.)

The linearity of the \( BA-l_0^{-1.5} \) relationship also breaks down at low values. As the volume of the bead is increased, the cell walls become thinner and consequently, the bearing area is decreased. At the limit, the cell wall becomes a very thin membrane that will yield immediately to loading. The intercept of the dotted line defines this limiting bearing area. The scope of densities for the present work is well into the linear section. For the range of interest then, there is a linear relationship between the bearing
FIGURE 48 -- BEARING AREA - $I_0^{-1.5}$ RELATIONSHIP.
area and $l_0^{-1.5}$. The stress is proportional to density$^{1.5}$. As shown earlier:

density$^{1.5}$ is proportional to $l_0^{-4.5}$

$l_0^{-3}$ is proportional to volume$^{-1}$

and $l_0^{-1.5}$ is proportional to the Bearing area.

Therefore, density$^{1.5}$ is proportional to $\frac{\text{Bearing Area}}{\text{Volume}}$ and Yield Stress is also proportional to $\frac{\text{Bearing Area}}{\text{Volume}}$.

The strength of a bead is directly proportional to its bearing area; however, when the volume of the bead is increased, the number of beads per unit volume is decreased and allowable stress per unit volume of sample is proportionally decreased.

This analysis is in agreement with Blair's investigations [25] that indicate that the strength of a foam increases with decreasing cell size.

**Strain Rate Dependence**

The equation:

$$f(\dot{\varepsilon}_0) = 0.03057 \sinh^{-1}(\text{Initial strain rate} \times 3.75 \times 10^{13})$$

(10)

was devised to approximate the strain rate dependence of PSB foam. The variation in yield stress versus the strain rate was shown in figure 21; the yield stress increases rapidly at low strain rates and slowly at higher strain rates.

Kosten and Zwikker [26] have proposed the model shown in figure 49 to explain this phenomenon. As the piston is pushed in, fluid exits through the small opening.
FIGURE 49 – KOSTEN AND ZWIKKER FLUID FLOW MODEL FOR FOAM.
Flow losses increase with rate in the low rate region up to a maximum. At higher compression rates, the flow resistance becomes large enough so that the fluid of the system gets compressed before it has a chance to flow.

A theoretical analysis was performed using gas tables [27] to investigate the Kosten-Zwikker model. It was assumed that a minimum stress must be applied to the piston to initiate movement. It was also assumed that yield always occurs at the same value of strain; thus for increasing strain rates, the time to achieve yield is decreased, and the volume of gas removed before yield occurs is also decreased. The remaining volume of air within the cell is compressed to yield strain. For higher strain rates the resultant stress is consequently higher when yield is achieved. A graphical representation of this analysis is shown in figure 50. The form of the curve is in general agreement with the experimental results, suggesting that the model is valid for PSB foam subjected to various rates of deformation.

Krausz and Eyring [28] have demonstrated that for polymeric materials, there exists a relationship of the form:

\[
\text{Strain rate} = A \sinh \phi \sigma
\]

(25)

where \( \phi \) and \( A \) are constants and \( \sigma \) is the applied stress. This equation describes the deformation process "when the plastic flow rate is controlled by activation over a single symmetrical energy barrier and when the unit density does not change".
FIGURE 50 — COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL STRESS-STRAIN RATE RELATIONSHIP.
From this equation we have

\[ \sigma = c \sinh^{-1}(B \times \text{strain rate}) \]  \hspace{1cm} (26)

where

\[ c = 1/\phi \]

and \( B = 1/A \)

This relationship is of the same form as the strain rate dependency equation derived from the present work, a probable indication that activation occurs over a single symmetrical energy barrier.

Strain Dependence

The three exponentials used to describe the strain functions can be explained as the mathematical representation of three different mechanisms that take place during deformation; namely, buckling of the cell walls, crushing of the cell wall and compression of the bulk styrene. The strain function is shown in figure 51 and the three exponentials have been plotted separately in figure 52.

The exponential described by 415.72 \( \exp(0.80\varepsilon) \) is of a form that would be expected to describe the buckling of the walls. A certain value of stress must be reached before buckling is initiated. Further buckling requires additional load because wall movement is restrained more and more by adjacent walls that come into contact with one another. During this stage of deformation, the gas entrapped within each bead is forced out.

The expression 0.01343 \( \exp(13.9\varepsilon) \) is the second exponential of the strain function; its form is consistent with the type
FIGURE 51 – STRAIN DEPENDENCE SIMULATION CURVE.

\[ 415.72 \exp(0.88 \varepsilon) + 0.01343 \exp(13.9 \varepsilon) + 1.25 \times 10^{13} \exp(42 \varepsilon) \]
of loading that would be expected to crush the cell walls. At low values of strain, very little compression of the walls takes place as all the deformation results in buckling of the cell walls. However, at higher values of strain, buckled walls gradually come into contact with one another, and further deformation results in compression of the cell walls. During this phase, most of the additional loading forces out any gas within the cell walls.

The third exponential, $1.25 \times 10^{-13} \exp(42\epsilon)$ simulates compression of the styrene once all voids within the walls are removed, a condition often referred to as 'bottoming out'. During this phase, solid styrene is deformed and results in very high loads compared to the previous phases. This bulk styrene deformation is essentially non-existent during the previous two phases; its effect becomes noticeable at strains higher than 80% only.

**Dynamic Effect**

Figure 53 shows stress-strain relationships for quasi-static and dynamic deformation. For constant strain rate, the stress is an ever-increasing function of strain. Under dynamic conditions however, the change in stress becomes zero at zero velocity. This behavior is accounted for by the dynamic factor $0.796 + 0.204 \frac{v}{v_0} \ (11)$ where $v$ equals the instantaneous velocity and $v_0$ is the initial velocity of the impactor. It is plausible to assume that because the strain rate becomes smaller during dynamic loading, more gas is allowed to escape during the deformation.
FIGURE 53 – COMPARISON BETWEEN QUASI-STATIC AND DYNAMIC STRESS-STRAIN RESPONSES.
process than would have escaped if the initial strain rate had been maintained. Consequently, the entrapped gas is compressed to a lower extent and for identical strain, a lower level of stress results.

**Thickness Dependence**

The original thickness of the sample appears only in the strain rate function, i.e. the only effect of the thickness is to modify the strain rate. This is in agreement with the analysis by Rusch [19].
DISCUSSION OF RESULTS - GENERAL

In this section, the response to impact of PSB foam is discussed. The effect of the different variables: density, thickness, strain rate, energy input and momentum are reviewed. Finally, the observed behavior is examined in relation to the biomechanics of head injury, only one example of the possible applications of PSB foam as a protective medium.

The results of figures 27 to 45 have shown time-dependent responses of various test conditions. Figures 27 to 35 show the response of 2.5 cm thick, 0.048 gm/cm³ density samples to 9 different impact modes. The curve shown in figure 27 is close to the ideal constant deceleration behavior proposed by Rusch [19]. The energy level and momentum are sufficient to buckle the cell walls but the deformation response does not leave the plateau region and very little crushing of the cell walls takes place. Figures 28 to 31 show responses to higher energy levels; deformation beyond the plateau region takes place and higher peak accelerations are observed. Figures 32 to 35 exhibit a behavior very similar to a quasi-static deformation response; the energy level and the momentum are sufficiently high to cause excessive deformation of the foam sample, well into the styrene compression region as evidenced by the high spikes.

For comparison, the acceleration traces of figures 27 to 35 were corrected to reflect response to theoretical energy inputs and have been plotted on the same page, figure 54.
FIGURE 54 — COMPARISON OF RESPONSE FOR 9 ENERGY MODES FOR 2.54 CM THICK, .048 G/CC DENSITY, PSB FOAM SAMPLES.
The mass, velocity, energy input, momentum and peak transmitted force for each test condition is shown.

Analysis of these curves reveals that the most important contributor to the acceleration is the energy input. Higher energies lead to greater peak accelerations.

The two components of the energy input, mass and velocity, affect the resultant acceleration response in a different way. Scrutiny of figures 54A, D and I reveals that the input velocity has a greater effect than the mass on the response. In figure 54D, the mass is 2.374 Kg and the impact velocity is 305.3 cm/sec.; a low acceleration response was obtained. The high spike response shown in figure 54A, was achieved by doubling the velocity to 610.8 cm/sec. The response shown in figure I was obtained by increasing the mass to 9.496 Kg, four times that of figure 54D. The energy of figures A and I is the same, 88.6 joules (65.3 ft.-lb). Yet, the peak acceleration in figure 54A is five times greater than that of figure 54I; the velocity has a greater influence than the mass on the peak acceleration.

Similarly, figures 54A, E, G and I exhibit different acceleration responses even though their energy input was identical. Closer examination shows that for greater momentum, the peak acceleration is smaller. Intuitively, the opposite might be expected. Indeed, if the momentum is greater, the deformation required to arrest the impactor might be expected to be greater; and, since the response becomes assymtotic at high strain, greater acceleration could result. Analysis of the experimental data shows that there is no appreciable difference in the force-deformation
response for equal energy levels regardless of the mass-velocity combination. (Of course combinations of very small masses and very high velocity that would cause local penetration, such as a bullet fired at high velocity, are excluded.) Only the time duration is affected. These results are in agreement with the following theoretical considerations. It has been observed that the strain rate does not markedly affect the force-deformation response above a certain strain rate. Also, the area under the force-deformation curve is equal to the energy input. Consequently, equal force-deformation responses should result from equal input energies regardless of the mass-velocity combination. Only the time duration should be affected.

The effect of identical masses impacting similar samples of PSB foam at various velocities is shown in figures 54A, B, C and D. For higher velocities, the energy and the momentum are greater; deformation of the foam is increased and higher spikes result, and the time duration of the impact is reduced.

The effect of various masses impacting similar samples of PSB foam at the same velocity is shown in figures 54D, F, H and I. Smaller masses can be decelerated more quickly during the initial stages of deformation, thus leaving less energy to be absorbed during the latter stages; consequently, lower peak accelerations are observed.
The thickness of the foam samples affects the acceleration response of the impactor. The acceleration trace of figures 36 to 39 have been plotted on the same page, figure 55. All four tests were conducted on samples of similar density with identical mass per unit area and identical impact velocity.

Because of the variation in thickness, the effective strain rate is modified. Lower strain rates (larger thicknesses) result in lower peak accelerations. At lower strain rates, more of the available energy is absorbed in the plateau region; the strain is reduced and the assymptotic region is not reached; consequently, a lower acceleration results. As shown in figure 55, much can be gained in respect to head protection by increasing the thickness of the protective foam layer. Present motorcycle helmets are lined with a foam shell approximately 2.5 cm (1") thick. Peak accelerations could be reduced by as much as 60% if the thickness was increased to 3.8 cm (1½ in.). Relatively little more would be gained by increasing the thickness to 5 cm. Certainly reducing the thickness from the present standards should not be contemplated.

Density plays a major role in the acceleration response. Figures 40 to 42 and figures 43 to 45 have been grouped in figures 56 and 57. Figure 56 shows typical responses for 3 densities of foam and low regimes of energy input. The curve is relatively constant with a characteristic slow rise and a small drop near the end. The greater the density, the higher the acceleration and the shorter the time duration.
FIGURE 55 — THICKNESS VARIATION EFFECT.
FIGURE 56 — DENSITY VARIATION EFFECT — LOW ENERGY INPUT.
FIGURE 57 – DENSITY VARIATION EFFECT – HIGH ENERGY INPUT.
With higher input energies however, the opposite is true; lower densities produce higher peaks as shown in figure 57. In deciding which density of foam to use in protective devices, it becomes important to determine the range of input energies over which protection will be afforded. Certainly, it is difficult to effectively protect motor vehicle passengers against very high input energy impacts.

Further, if the impacting area is increased, the retarding force is also increased, and there is evidence that the initial deceleration could be such that crushing of the styrene does not occur.

One series of tests was conducted with a low mass impactor, relatively high approach velocity and a large impacting area. The results show unequivocally that by increasing the retarding force (by increasing the area), appreciable crushing of the styrene can be prevented. In fact, if the contact area is large enough, no permanent deformation of the styrene is produced, but the peak acceleration is very high (figure 58). In such cases, the initial acceleration can be beyond the human concussion tolerance level [29, 30]. This is in agreement with the outcome of accidents where the impact energy was distributed over a wide area [31]. Although no permanent deformation of the energy absorbing layer was apparent, the acceleration was high enough to cause serious brain injury.

Furthermore, the stresses associated with the high acceleration spikes in the latter stages of some loading responses, suggest that the fracture tolerance levels of the
FIGURE 58 – LARGE CONTACT AREA ACCELERATION RESPONSE.
cranium could be considerably exceeded [32]. Under the proper loading conditions, this behavior would be expected if PSB foam was used in padded dashboards; their generally flat shape is consistent with the testing done in the present study. In a recent study of motorcycle helmets [33], this type of high spikes was observed when the impact was localized over a small effective area.

The Wayne State University Cerebral Concussion Tolerance (WST) curve [29] shown in figure 59 relates the variation of the tolerable effective acceleration of the head to the impact load duration. The use of PSB foam is an attempt to reduce the loading on the head so that the effective acceleration lies below the WST curve.

The assertion by Gurdjian et al. [34] that "...high amplitude spikes of short duration (less than 1 millisecond) should be disregarded" is questionable. Figure 60 shows the acceleration response of an impactor of relatively low mass travelling at high velocity. Although the acceleration spike resulting from the high velocity input is of short duration, surely, its magnitude is of such proportions that it cannot be ignored.

Newman [35] suggests that the likelihood of head injury increases with an increase in the kinetic energy of the blow. Indeed, as pointed out earlier, the energy input is the most important contributor to the peak acceleration.

The rate of change of acceleration has also been considered as a possible yardstick of head injury. It has been established that living organisms are sensitive to the
FIGURE 59 — WAYNE STATE UNIVERSITY CEREBRAL CONCUSSION TOLERANCE CURVE
FIGURE 60 – HIGH SPIKE ACCELERATION RESPONSE.
rate of change of acceleration, [36] although Viano and Gadd [37] have offered some evidence that it cannot always be related to head injury. One must remember that it is not the rate of change of acceleration which does or does not cause injury. It is only used as a tool to measure some mechanical response of the head which may produce injury. Lunenfeld et al. [38] and Newman [35] have proposed $2 \times 10^5 \text{ g/sec.}$ as a reasonable upper limit for the rate of change of acceleration. Naturally, the value of the acceleration should be considered too. The trace shown in figure 61 exhibits a rate of change of $2.2 \times 10^5$ at the onset of impact. The level of acceleration is low, however, and the shape of the response is close to an ideal acceleration pulse. The probability of injury is low. In figure 58 however, the high initial rate of change coupled with the high acceleration value is much more severe.

It is possible that an equally serious effect results from a high rate of change of acceleration during the unloading portion, or at any time during the response. Perhaps successive sudden changes in acceleration as shown in figures 62 and 63 have a cumulative effect on brain injury. Certainly, the sudden changes in the acceleration of the unloading portion should not be discounted without proper testing.

The entire acceleration-time history could have important effects on head injury. Consider the acceleration time responses shown in figures 64a and 64b. The time duration and the average acceleration for the two traces is
FIGURE 61 — HIGH RATE OF ONSET RESPONSE.

FIGURE 62 — HIGH RATE OF ONSET — HIGH RATE OF DECELERATION RESPONSE.

FIGURE 63 — HIGH RATE OF ONSET — HIGH SPIKE RESPONSE.
FIGURE 64a

FIGURE 64b

COMPARISON OF TWO ACCELERATION TRACES WITH IDENTICAL AVERAGE ACCELERATION.
is essential the same yet few people would agree that their severity is the same.

Rayne (39) has suggested that it is the total change in velocity (i.e. impact and rebound) that should be considered. Indeed, if rebound velocity is high, it is because the relative energy absorption is low and the energy given back to the head can be excessive. Also, the pulse duration is increased, thus increasing the chances of injury as shown by the WST curve.
CONCLUSIONS

It has been shown that impact energy absorbing characteristics for closed cell foams exhibiting pneumatic damping behaviour can be predicted using slow constant rate compression data.

The density exponent is a constant depending on the polymer characteristics. The strain rate factor, dependent on both fluid flow and activation energy characteristics can be approximated by an inverse hyperbolic sine function. The strain function can be expressed as the sum of three exponentials which relate to different but concurrent deformation mechanisms. A straight line function accounts for the dynamic behaviour of impacts.

This analysis has been restricted to polystyrene bead foam. It is possible that other closed cell foams could be modelled using the present analysis.
REFERENCES


APPENDIX A

Sample Production, Quality Control and Selection

Foam slabs 30.5 cm x 30.5 cm (12 in. x 12 in.) were manufactured by Morval Durofoam of Kitchener Ontario in 5 nominal densities: 0.04 gm/cm$^3$ (2.5 lb/ft$^3$), 0.048 gm/cm$^3$ (3.0 lb/ft$^3$), 0.056 gm/cm$^3$ (3.5 lb/ft$^3$), 0.064 gm/cm$^3$ (4.0 lb/ft$^3$) and 0.072 gm/cm$^3$ (4.5 lb/ft$^3$) and 5 nominal thicknesses: 1.27 cm (1/2 in.), 2.5 cm (1 in.), 3.81 cm (1-1/2 in.), 5.08 cm (2 in.) and 7.62 cm (3 in.).

Sample Production

The samples were cut from the foam slabs with a well-sharpened high speed bandsaw. Each sample (figure A1) was cut to the same thickness to side ratio of 1:1.15.* It was

*On the basis of the energy input considerations (see Appendix E) very small masses were required for dynamic testing of the 1.27 cm (1/2 in.) thick samples. The basic weight of the drop frame plus the impactor, was too large to satisfy these requirements. For dynamic testing of the 1.27 cm (1/2 in.) thick samples only, the thickness to side ratio was therefore modified to 1:2.3. Two 1.27 cm thick samples of 1:1.15 and 1:2.3 aspect ratios are shown in figure A3. This modification was considered valid for two reasons. First, Poisson's ratio is about 0.03 for polymeric foams in the plastic region as shown by Rinde (1). This indicates that a variation in area
FIGURE A1 — TYPICAL PSB FOAM SAMPLE.
would not greatly affect the stress-strain performance. Second, the results of one preliminary test (figure A4) confirmed that appreciable variations in the thickness to side ratio produced only a minimal effect on the stress-strain performance.

desireable to obtain samples that were as close to homogeneous as possible at least in the two directions perpendicular to the loading axis. A minimum of 2.9 cm. (1.15 in.) was removed from the periphery of each slab because of the variation in density near the surfaces of the slabs. A typical transverse cut from a slab of foam showing the density (bead size) distribution is given in figure A2.

**Sample Quality Control**

During the cutting process, heat was generated and most of it was absorbed by the saw blade because the foam (being an insulator) absorbed very little heat. That portion of the foam that was shredded by the blade generally melted and adhered to the newly cut surfaces where it resolidified. Very rough irregular surfaces resulted. The particles were removed from the cut surfaces because their presence would cause erroneous measurements of dimensions and because their high density would affect the measured weight. Each newly cut surface of every sample was briskly scraped with a sharp blade a minimum of 24 strokes to a side, 6 strokes in every direction.

To ensure consistency, all samples were prepared
FIGURE A2 — DENSITY DISTRIBUTION — TRANSVERSE CUT OF A SLAB OF PSB FOAM.
FIGURE A3 — 1/2" THICK PSB FOAM SAMPLES.
0.072 gm/cm$^3$ (4.72 lb/ft$^3$) DENSITY

+ 7.62 x 7.62 x 5.08 cm (3" x 3" x 2")

- 5.08 x 7.62 x 5.08 cm (2" x 2" x 2")

FIGURE A4 — PRELIMINARY TEST EFFECT OF THICKNESS TO SIDE RATIO.
the same way by the same person. The dimensions of each sample were determined to the nearest 0.001 inch, and the average of 3 readings was used for each measurement. Each sample was weighed on a Mettler analytical scale previously calibrated according to the manufacturer's specifications. The density of each specimen was calculated. Large variations in density were found to exist from sample to sample. A maximum of 6% variation (± 3%) from the 5 densities was allowed. All samples which did not conform were not used for testing. Small imprints left by the mold on the foam slabs made it possible to identify a consistent orientation for each specimen.

Sample Selection

When selecting test samples, different guidelines were followed for the quasi-static and the dynamic conditions:

a) Quasi-static: Of those samples whose density fell within the allowable 6% variation, 3 samples with a density as close to one another as possible were selected for any one set of 3 tests. By using 3 samples of very similar densities, the effect of density variation would be minimized and the repeatability of the tests could be verified.

b) Dynamic: The response to these tests would be simulated by the equation developed from the quasi-static data, and it was important that the results obtained be totally unbiased and untampered with. Samples were randomly selected within the ± 3% allowable variation from a new series of samples. The following procedure was used: all
samples of any one nominal density were placed in a box and mixed thoroughly - these samples included the ones outside the ± 3% variation limit. Samples were removed from the box at random and their identification numbers registered in order of selection. Once all the samples had been removed, those outside the allowable range were eliminated. What was left were the samples in the random order they would be used for dynamic testing.
APPENDIX B

Design and Reliability of the Quasi-Static Testing Apparatus

Design

The apparatus is shown in figure Bl. It was designed for use with the Instron testing machine. The top and bottom fittings were built to fit the standard Instron grips. With this system, the samples could be compressed while using the Instron in tension.

The main frame of the apparatus was a 15.2 cm. (6 in) steel box section .95 cm. (3/8 in.) thick. A system of 3 parallel plates constituted the movable crushing mechanism. The center steel plate formed one of the compressive surfaces and it was extended so that it acted as a sliding guide for the vertical movement of the corner posts. The exact attitude of this plate could be adjusted with the help of 4 adjustment screws so that the lower and middle plates were parallel. (The orientation of the two plates was adjusted and the variation in the distance between them, measured at the 4 corner posts was half a thousandth of an inch). To help maintain the two plates in their original attitude during testing, a center piston-cylinder was built-in.

Reliability

Because the quasi-static testing apparatus was new, it was deemed necessary to ensure that the readings obtained would be accurate and repeatable. For this purpose, the
FIGURE B1 – QUASI-STATIC TESTING APPARATUS.
-B3-

apparatus was subjected to successive loading-unloading cycles until 2 consecutive force-deformation traces were identical. The last of these curves was kept as the elasticity curve of the testing apparatus; it was used to correct data and obtain true sample deformation. The elasticity curve is given in Figure B2.
FIGURE B2 — ELASTICITY OF THE QUASI-STATIC TESTING APPARATUS.
APPENDIX C

Quasi-Static Testing Procedure

A set sequence was followed during testing of the quasi-static tests to ensure consistency throughout.

The Instron was set at the desired cross-head velocity. Calibration of the strip chart pen recorder was carried out according to the manufacturer's specifications. Each specimen was positioned in the center of the compressing plates by measuring equal distances from the sample to the edge of the plates. Each specimen was oriented the same way according to the data collected at the time of production as discussed in Appendix A. All pertinent data: sample number, density of sample, thickness of sample, velocity of cross-head, speed of strip chart etc... was recorded directly on the strip chart.

The cross head movement was initiated and the compressive deformation of the foam sample was allowed to proceed until the force registered on the strip chart recorder was equal to 2000 Kg, the maximum allowable for the load cell. The cross-head movement was immediately reversed and the specimen was unloaded at the same cross-head velocity as for the loading. The specimen was removed from the apparatus and identified with a quasi-static test number. This procedure was repeated for all quasi-static tests.
APPENDIX D

Design and Installation of the Dynamic Testing Apparatus

For the dynamic tests, various energy levels were to be imparted to the foam samples by altering the impacting mass and varying the drop height, as discussed in Appendix F. On this basis, a range of impacting masses of 2.374 Kg to 42.712 Kg was needed.

It was desirable that all samples be tested with the same testing apparatus to avoid the necessity for cross-calibration between testing units. The apparatus was designed in such a way that its mass could be adjusted to any value (± 0.002 Kg) within the required range by adding various weights to the basic testing unit.

As shown in figure D1, the unit consisted basically of a rigid box-type steel frame onto which was bolted an aluminum impactor. Special attention was given to the design of the frame to reduce stress concentrations in corners and bends and to ensure rigidity while maintaining light weight.

Figure D2 shows how the drop frame and the impactor were bolted together; also shown in this figure is the accelerometer mounted directly onto the impactor to ensure proper monitoring of its response to impact.

Figure D3 shows the various weights that were used. Their shape and sizes were arrived at primarily out of considerations for stability.

Each weight was designed so that its center of gravity would lie directly above the impactor to avoid tipping over
FIGURE D1 – DROP FRAME.
FIGURE D3 — WEIGHTS, DYNAMIC TESTING APPARATUS.
-D5-
during installation. The weights were also designed such that the center of gravity of the maximum mass was located below the mid-point between the two sliding points of contact with the guide bars; this was done because it was important for consistent results during all dynamic tests, that the impactor remained parallel to the sample support structure and that no excessive stresses be imposed upon the guide bars due to instability. The weights had therefore to be as thin as possible. Segments were used to satisfy both horizontal and vertical center of gravity location restrictions. It was determined that with the present design, the center of gravity of a steel mass weighing 57 Kg (125 lb) would be 3 cm (1.18 in.) below the mid-point. Figure D4 shows the impactor almost fully loaded.

All weights were produced from hot rolled plates of steel. Each weight's surface was ground finished to 64 rms roughness to achieve good contact between them and to ensure that the apparatus-weights assembly would act as a single solid mass.

The weights were tied down onto the impactor by means of 4 threaded rods and nuts. Depending on the total height of the weights, short bars (12.5 cm - 5 in.) or long bars (22.9cm - 9 in.) were used. The long bars were not used when the height of the weights was small, in order to reduce vibration due to free height. Whenever the total height of the weights exceeded 5.08 cm (2 in.), a stiffening plate was added to ensure high rigidity.
FIGURE D4 — FULLY-LOADED IMPACTOR.
All weights were weighed to ± 0.0005 Kg on a Ohaus balance scale previously adjusted with calibration weights. The weights were grouped in pairs of equal weight and placed in a specific order so that weight increments as close as possible to 4.545 Kg (10 lbs), 0.455 Kg (1 lb), 0.046 Kg (0.1 lb) and 0.005 Kg (0.01 lb) could be achieved.

Each weight was stamped with an identification number to specify the sequence and the location (front or back) in which they would be used to achieve the various masses of the impactor unit. (Figure D5)

Table D1 shows the identification numbers and the value of every weight.

These values were entered into a computer program to simplify the selection and ensure the consistency of the order in which the weights would be selected according to the energy input requirements.

Table D2 shows a typical computer-selected series of weights for 26.154 joules (19.29 ft.-lb) of energy input for a mass dropped from a height of 0.5 meters (1.64 feet).

Figures D6 and D7 show the dynamic testing installation. A schematic diagram of this installation is shown in figure D8.
### Weights Used

<table>
<thead>
<tr>
<th>#</th>
<th>Newtons (Pounds)</th>
<th>Newtons (Pounds)</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>10A</td>
</tr>
<tr>
<td>20</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>20A</td>
</tr>
<tr>
<td>30</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>30A</td>
</tr>
<tr>
<td>40</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>40A</td>
</tr>
<tr>
<td>50</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>50A</td>
</tr>
<tr>
<td>60</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>60A</td>
</tr>
<tr>
<td>70</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>70A</td>
</tr>
<tr>
<td>80</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>80A</td>
</tr>
<tr>
<td>90</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>90A</td>
</tr>
<tr>
<td>100</td>
<td>22.326 (5.019)</td>
<td>22.326 (5.019)</td>
<td>100A</td>
</tr>
</tbody>
</table>

**Total: 130,880 (75,315) + 229,933 (49,633)**

**Basic Weight of Impactor:** 22.146 NT (49.985 lb)

**Total Weight:** 561.966 NT (126,334 lb)

---

Table D1 - Weights and Identification Numbers
***** WEIGHTS USED *****

<table>
<thead>
<tr>
<th>#</th>
<th>NEWTONS (POUNDS)</th>
<th>NEWTONS (POUNDS)</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0</td>
<td>0.0</td>
<td>10A</td>
</tr>
<tr>
<td>20</td>
<td>0.0</td>
<td>0.0</td>
<td>20A</td>
</tr>
<tr>
<td>30</td>
<td>0.0</td>
<td>0.0</td>
<td>30A</td>
</tr>
<tr>
<td>40</td>
<td>0.0</td>
<td>0.0</td>
<td>40A</td>
</tr>
<tr>
<td>50</td>
<td>0.0</td>
<td>0.0</td>
<td>50A</td>
</tr>
<tr>
<td>60</td>
<td>0.0</td>
<td>0.0</td>
<td>60A</td>
</tr>
<tr>
<td>70</td>
<td>0.0</td>
<td>0.0</td>
<td>70A</td>
</tr>
<tr>
<td>80</td>
<td>0.0</td>
<td>0.0</td>
<td>80A</td>
</tr>
<tr>
<td>90</td>
<td>0.0</td>
<td>0.0</td>
<td>90A</td>
</tr>
<tr>
<td>100</td>
<td>0.0</td>
<td>0.0</td>
<td>100A</td>
</tr>
<tr>
<td>110</td>
<td>0.0</td>
<td>0.0</td>
<td>110A</td>
</tr>
<tr>
<td>1</td>
<td>2.231</td>
<td>0.502</td>
<td>1A</td>
</tr>
<tr>
<td>2</td>
<td>2.226</td>
<td>0.500</td>
<td>2A</td>
</tr>
<tr>
<td>3</td>
<td>2.231</td>
<td>0.502</td>
<td>3A</td>
</tr>
<tr>
<td>4</td>
<td>2.226</td>
<td>0.500</td>
<td>4A</td>
</tr>
<tr>
<td>5</td>
<td>2.236</td>
<td>0.503</td>
<td>5A</td>
</tr>
<tr>
<td>6</td>
<td>2.221</td>
<td>0.499</td>
<td>6A</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>0.0</td>
<td>7A</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>0.0</td>
<td>8A</td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>0.0</td>
<td>9A</td>
</tr>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0A</td>
</tr>
<tr>
<td>S1</td>
<td>0.883</td>
<td>0.198</td>
<td>51A</td>
</tr>
<tr>
<td>S2</td>
<td>0.657</td>
<td>0.148</td>
<td>52A</td>
</tr>
<tr>
<td>S3</td>
<td>0.432</td>
<td>0.097</td>
<td>53A</td>
</tr>
<tr>
<td>S4</td>
<td>0.0</td>
<td>0.0</td>
<td>54A</td>
</tr>
<tr>
<td>A1</td>
<td>0.091</td>
<td>0.021</td>
<td>A1A</td>
</tr>
<tr>
<td>A2</td>
<td>0.0</td>
<td>0.0</td>
<td>A2A</td>
</tr>
<tr>
<td>A3</td>
<td>0.0</td>
<td>0.0</td>
<td>A3A</td>
</tr>
<tr>
<td>A4</td>
<td>0.023</td>
<td>0.005</td>
<td>A4A</td>
</tr>
</tbody>
</table>

TOTALS: 15.457 (3.475) 15.448 (3.473)
BASIC WEIGHT OF IMPACTOR: 21.458NT (4.824LB)
TOTAL WEIGHT: 32.362NT (11.771LB)

***** TEST VARIABLES *****

- MASS OF IMPACTOR: 5.339KG (0.366SLUGS)
- DROP HEIGHT: 0.500METERS (1.640FEET)
- THEORETICAL VELOCITY: 3.130M/SEC (10.266FT/SEC)
- THEORETICAL ENERGY INPUT: 26.154JOULES (19.291FT-LB)

Table D2 - Typical Computer Selected Weights
FIGURE D7 – DYNAMIC TESTING INSTALLATION.
FIGURE D8 – DYNAMIC TESTING APPARATUS INSTALLATION SCHEMATIC DIAGRAM.
APPENDIX E

Energy Levels for the Dynamic Tests

Various levels of input energy were required to determine the PSB foam response to impact. Because the dynamic behavior of this foam was still unknown, a systematic approach was used whereby the separate effect of specific variables could be investigated. It would be possible to correct discrepancies in the constitutive equation. A triangular matrix of the form shown in table E1 was used. With this matrix the effect on the response, of the velocity, the mass, and the energy input of the striker could be investigated.

The average curve from 3 quasi-static tests ('s 28, 29, 30, 0.0033cm/sec., 2.5 lb/ft\(^3\) nominal density, 2" thickness) was used to determine the maximum level of energy input for every thickness of specimen. This curve is shown in figure E1. From the area under the curve obtained by graphical integration, it was determined that 186 joules (137.2 ft.-lb) were required to achieve 90% deformation. The equivalent maximum energy input per unit area (5.45 joules/cm\(^2\)) was selected. With this value and a constant thickness to side ratio of 1:1.15 the maximum energy level for every thickness was developed as listed in table E2.

Four specific drop heights were selected and the four theoretical impact velocities were thereby specified as listed in table E3.

By using the velocities, the maximum levels of energy inputs and the relation \(E = \frac{1}{2}mv^2\), four masses were determined
Table E1 - Energy Input Triangular Matrix
FIGURE E1 — AVERAGE FORCE-DEFORMATION CURVE, QUASI-STATIC TESTS NOS. 28, 29, 30.
<table>
<thead>
<tr>
<th>Thickness</th>
<th>Energy Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27 cm (½ in.)</td>
<td>11.63 joules (8.58 ft-lb)</td>
</tr>
<tr>
<td>2.54 cm (1 in.)</td>
<td>46.51 joules (34.31 ft-lb)</td>
</tr>
<tr>
<td>3.81 cm (1½ in.)</td>
<td>104.70 joules (77.19 ft-lb)</td>
</tr>
<tr>
<td>5.08 cm (2 in.)</td>
<td>186.00 joules (137.20 ft-lb)</td>
</tr>
<tr>
<td>7.62 cm (3 in.)</td>
<td>418.60 joules (308.80 ft-lb)</td>
</tr>
</tbody>
</table>

Table E2 - Maximum Levels of Energy
<table>
<thead>
<tr>
<th>DROP HEIGHT</th>
<th>THEORETICAL VELOCITY AT IMPACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 m (6.56 ft)</td>
<td>$v_1$ - 6.26 m/sec. (20.55 ft/sec.)</td>
</tr>
<tr>
<td>1.5 m (4.92 ft)</td>
<td>$v_2$ - 5.42 m/sec. (17.80 ft/sec.)</td>
</tr>
<tr>
<td>1.0 m (3.28 ft)</td>
<td>$v_3$ - 4.43 m/sec. (14.50 ft/sec.)</td>
</tr>
<tr>
<td>0.5 m (1.64 ft)</td>
<td>$v_4$ - 3.13 m/sec. (10.30 ft/sec.)</td>
</tr>
</tbody>
</table>

Table E3 - Theoretical Impact Velocities
for each thickness of foam, as listed in table E4.

The basic weight of the drop frame plus the impactor was too large, however, to satisfy the small weights required to test the 1.27 cm (½ in.) specimens. The thickness to side ratio for these samples was therefore modified to 1:2.3 as mentioned in Appendix A. The masses required to test the new 1.27 cm (½ in.) thick samples were the same as for the 2.54 cm (1 in.) thick samples.

The maximum levels of energy per unit area were prescribed to the diagonal of the input energy matrices as shown in tables E5, E6, E7 and E8. Because the considerable mass of 85.4 Kg (188 lb) was required to satisfy the maximum input of energy from a height of 0.5 m (1.64 ft.) for the 7.62 cm (3 in.) thick samples, this test was eliminated.
\[
\begin{array}{cccccc}
1.27 \text{ cm} & 2.54 \text{ cm} & 3.81 \text{ cm} & 5.08 \text{ cm} & 7.62 \text{ cm} \\
(\frac{1}{2} \text{ in.}) & (1 \text{ in.}) & (1\frac{1}{2} \text{ in.}) & (2 \text{ in.}) & (3 \text{ in.}) \\
\hline
m_1^* & 0.593 \text{ Kg} & 2.374 \text{ Kg} & 5.339 \text{ Kg} & 9.492 \text{ Kg} & 21.36 \text{ Kg} \\
& (0.041 \text{ slugs}) & (0.162 \text{ slugs}) & (0.365 \text{ slugs}) & (0.650 \text{ slugs}) & (1.46 \text{ slugs}) \\
w_1 & 5.82 \text{ N} & 23.28 \text{ N} & 52.32 \text{ N} & 93.02 \text{ N} & 209.3 \text{ N} \\
& (1.31 \text{ lb}) & (5.23 \text{ lb}) & (11.77 \text{ lb}) & (20.92 \text{ lb}) & (47.07 \text{ lb}) \\
m_2 & 0.791 \text{ Kg} & 3.164 \text{ Kg} & 7.119 \text{ Kg} & 12.66 \text{ Kg} & 28.48 \text{ Kg} \\
& (0.054 \text{ slugs}) & (0.217 \text{ slugs}) & (0.487 \text{ slugs}) & (0.866 \text{ slugs}) & (1.95 \text{ slugs}) \\
w_2 & 7.76 \text{ N} & 31.03 \text{ N} & 69.76 \text{ N} & 124.0 \text{ N} & 279.1 \text{ N} \\
& (1.74 \text{ lb}) & (6.97 \text{ lb}) & (15.69 \text{ lb}) & (27.89 \text{ lb}) & (62.75 \text{ lb}) \\
m_3 & 1.187 \text{ Kg} & 4.746 \text{ Kg} & 10.68 \text{ Kg} & 18.98 \text{ Kg} & 42.71 \text{ Kg} \\
& (0.081 \text{ slugs}) & (0.325 \text{ slugs}) & (0.731 \text{ slugs}) & (1.30 \text{ slugs}) & (2.92 \text{ slugs}) \\
w_3 & 11.6 \text{ N} & 46.54 \text{ N} & 104.6 \text{ N} & 186.0 \text{ N} & 418.6 \text{ N} \\
& (2.62 \text{ lb}) & (10.46 \text{ lb}) & (23.53 \text{ lb}) & (41.84 \text{ lb}) & (94.13 \text{ lb}) \\
m_4 & 2.374 \text{ Kg} & 9.492 \text{ Kg} & 21.36 \text{ Kg} & 37.97 \text{ Kg} & 85.43 \text{ Kg} \\
& (0.162 \text{ slugs}) & (0.650 \text{ slugs}) & (1.46 \text{ slugs}) & (2.60 \text{ slugs}) & (5.85 \text{ slugs}) \\
w_4 & 23.28 \text{ N} & 93.09 \text{ N} & 209.3 \text{ N} & 372.1 \text{ N} & 837.1 \text{ N} \\
& (5.23 \text{ lb}) & (20.92 \text{ lb}) & (47.07 \text{ lb}) & (83.67 \text{ lb}) & (188.3 \text{ lb}) \\
\end{array}
\]

* \( m_1 = \text{mass}_1 \)  \\
\( w_1 = \text{weight}_1 \)

Table E4 - Masses and Weights
\[ E_{11} = \frac{1}{2}m_1 v_1^2 = 46.51 \text{ joules} = (34.31 \text{ ft-lb}) \]
\[ E_{12} = \frac{1}{2}m_1 v_2^2 = 34.88 \text{ joules} = (25.73 \text{ ft-lb}) \]
\[ E_{13} = \frac{1}{2}m_1 v_3^2 = 23.23 \text{ joules} = (17.15 \text{ ft-lb}) \]
\[ E_{14} = \frac{1}{2}m_1 v_4^2 = 11.63 \text{ joules} = (8.58 \text{ ft-lb}) \]

\[ E_{22} = \frac{1}{2}m_2 v_2^2 = 46.51 \text{ joules} = (34.31 \text{ ft-lb}) \]
\[ E_{23} = \frac{1}{2}m_2 v_3^2 = 15.50 \text{ joules} = (11.44 \text{ ft-lb}) \]

\[ E_{33} = \frac{1}{2}m_3 v_3^2 = 46.51 \text{ joules} = (34.31 \text{ ft-lb}) \]
\[ E_{34} = \frac{1}{2}m_3 v_4^2 = 23.26 \text{ joules} = (17.15 \text{ ft-lb}) \]

\[ E_{44} = \frac{1}{2}m_4 v_4^2 = 46.51 \text{ joules} = (34.31 \text{ ft-lb}) \]

**Table E5** - Energy Input Matrix 1.27 cm and 2.54 cm Samples
\[ E_{11} = \frac{1}{2}m_1 v_1^2 = 104.6 \text{ joules} = (77.19 \text{ ft-lb}) \]

\[ E_{12} = \frac{1}{2}m_1 v_2^2 = 78.49 \text{ joules} = (57.89 \text{ ft-lb}) \]

\[ E_{13} = \frac{1}{2}m_1 v_3^2 = 52.32 \text{ joules} = (38.59 \text{ ft-lb}) \]

\[ E_{14} = \frac{1}{2}m_1 v_4^2 = 26.16 \text{ joules} = (19.30 \text{ ft-lb}) \]

\[ E_{22} = \frac{1}{2}m_2 v_2^2 = 104.6 \text{ joules} = (77.19 \text{ ft-lb}) \]

\[ E_{23} = \frac{1}{2}m_2 v_3^2 = 34.88 \text{ joules} = (25.73 \text{ ft-lb}) \]

\[ E_{33} = \frac{1}{2}m_3 v_3^2 = 104.6 \text{ joules} = (77.19 \text{ ft-lb}) \]

\[ E_{34} = \frac{1}{2}m_3 v_4^2 = 52.32 \text{ joules} = (38.59 \text{ ft-lb}) \]

\[ E_{44} = \frac{1}{2}m_4 v_4^2 = 104.6 \text{ joules} = (77.19 \text{ ft-lb}) \]

Table E6 - Energy Input Matrix 3.81 cm Samples
\begin{align*}
E_{11} &= \frac{1}{2}m_1v_1^2 = 186.0 \text{ joules} = (137.2 \text{ ft-lb}) \\
E_{12} &= \frac{1}{2}m_1v_2^2 = 139.5 \text{ joules} = (102.9 \text{ ft-lb}) \\
E_{13} &= \frac{1}{2}m_1v_3^2 = 93.02 \text{ joules} = (68.6 \text{ ft-lb}) \\
E_{14} &= \frac{1}{2}m_1v_4^2 = 46.51 \text{ joules} = (34.3 \text{ ft-lb}) \\
E_{22} &= \frac{1}{2}m_2v_2^2 = 186.0 \text{ joules} = (137.2 \text{ ft-lb}) \\
E_{24} &= \frac{1}{2}m_2v_4^2 = 62.01 \text{ joules} = (45.7 \text{ ft-lb}) \\
E_{33} &= \frac{1}{2}m_3v_3^2 = 186.0 \text{ joules} = (137.2 \text{ ft-lb}) \\
E_{34} &= \frac{1}{2}m_3v_4^2 = 93.02 \text{ joules} = (68.6 \text{ ft-lb}) \\
E_{44} &= \frac{1}{2}m_4v_4^2 = 186.0 \text{ joules} = (137.2 \text{ ft-lb})
\end{align*}

Table E7 - Energy Input Matrix 5.08 cm Samples
\[ E_{11} = \frac{1}{2} m_1 v_1^2 \quad E_{12} = \frac{1}{2} m_1 v_2^2 \quad E_{13} = \frac{1}{2} m_1 v_3^2 \quad E_{14} = \frac{1}{2} m_1 v_4^2 \]
\[ = 418.6 \text{ joules} \quad = 313.9 \text{ joules} \quad = 209.3 \text{ joules} \quad = 104.6 \text{ joules} \]
\[ = (308.8 \text{ ft-lb}) \quad = (231.6 \text{ ft-lb}) \quad = (154.4 \text{ ft-lb}) \quad = (77.2 \text{ ft-lb}) \]

\[ E_{22} = \frac{1}{2} m_2 v_2^2 \quad E_{23} = \frac{1}{2} m_2 v_3^2 \quad E_{24} = \frac{1}{2} m_2 v_4^2 \]
\[ = 418.6 \text{ joules} \quad = 209.6 \text{ joules} \quad = 139.5 \text{ joules} \]
\[ = (308.8 \text{ ft-lb}) \quad = (154.4 \text{ ft-lb}) \quad = (102.9 \text{ ft-lb}) \]

\[ E_{33} = \frac{1}{2} m_3 v_3^2 \quad E_{34} = \frac{1}{2} m_3 v_4^2 \]
\[ = 418.6 \text{ joules} \quad = 209.6 \text{ joules} \]
\[ = (308.8 \text{ ft-lb}) \quad = (154.4 \text{ ft-lb}) \]

\[ E_{44} = \frac{1}{2} m_4 v_4^2 \]
\[ = 418.6 \text{ joules} \]
\[ = (308.8 \text{ ft-lb}) \]

**Table E8 - Energy Input Matrix 7.62 cm Samples**
APPENDIX F

Linear Transducer Installation, Static and Dynamic Calibration

The linear transducer and core assembly are shown in figure F1. A funnel was placed directly above the transducer to facilitate penetration of the core at high velocities. A rigid clamping device was built to hold the transducer core assembly in a permanent position relative to the impactor during a test. A new transducer core assembly was built to provide more rigidity than offered by the original manufacturer-supplied assembly. With the threaded core support shown in figure F2, the vertical position of the core with respect to the impactor surface could be adjusted exactly.

Static Calibration of the Linear Transducer

The transducer calibration unit was supplied with 6 volts, as recommended by the manufacturer. A displacement to voltage ratio of 2.8 cm/volt was established. The following procedure was used to obtain a calibration curve. With the help of the threaded core support and a vertical measurement device equipped with a vernier, the core was displaced through the transducer by increments of 0.25 cm (0.1 inch). The voltage was read from the oscilloscope trace and converted into displacement. These values have been plotted in figure F3. The curve obtained was linear in the range of -2.8 cm to +3.05 cm displacement.
FIGURE F1 — LINEAR TRANSDUCER AND CORE ASSEMBLY.
FIGURE F2 - LINEAR TRANSDUCER THREADED CORE SUPPORT.
Dynamic Calibration of the Linear Transducer

The dynamic calibration was performed to ascertain that the linear transducer was sensitive enough to respond accurately to the high velocity movement of the core during the dynamic tests. Calibration impacts were carried out on a Modular Elastomer Programmer (MEP) pad shown in figure F4. It exhibits highly elastic properties, quick recovery from deformation and good repeatability of the response to impact.

The impactor assembly was positioned so that the striker just touched the surface of the MEP as shown in figure F5. This was defined as 'zero position'. The transducer core was displaced vertically until its position inside the transducer was in the linear displacement-voltage region, and it was locked in place. The displacement-voltage corresponding to that position was recorded as the 'zero position voltage'. The impactor assembly was dropped from selected heights to impact the MEP. The displacement-time response trace was stored on the oscilloscope. A typical curve is shown in figure F6. The difference between the 'zero position voltage' and the minimum voltage on the trace yielded the maximum deformation of the MEP during impact. Upon impact, the aluminum impactor left a faint galling imprint on the MEP. The difference in elevation between the center of the MEP and the edge of the imprint yielded a measurable approximation of the maximum deformation due to the impact. The dynamic calibration test results are presented in table F1.
<table>
<thead>
<tr>
<th>TEST #</th>
<th>DROP HEIGHT (m)</th>
<th>DISPLACEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Transducer (cm)</td>
</tr>
<tr>
<td>LT1</td>
<td>0.25</td>
<td>0.165</td>
</tr>
<tr>
<td>LT2</td>
<td>0.25</td>
<td>0.165</td>
</tr>
<tr>
<td>LT3</td>
<td>0.25</td>
<td>0.165</td>
</tr>
<tr>
<td>LT4</td>
<td>0.50</td>
<td>0.198</td>
</tr>
<tr>
<td>LT5</td>
<td>0.50</td>
<td>0.187</td>
</tr>
<tr>
<td>LT6</td>
<td>0.50</td>
<td>0.187</td>
</tr>
<tr>
<td>LT7</td>
<td>0.75</td>
<td>0.204</td>
</tr>
<tr>
<td>LT8</td>
<td>0.75</td>
<td>0.209</td>
</tr>
<tr>
<td>LT9</td>
<td>0.75</td>
<td>0.209</td>
</tr>
<tr>
<td>LT10</td>
<td>1.00</td>
<td>0.220</td>
</tr>
<tr>
<td>LT11</td>
<td>1.00</td>
<td>0.226</td>
</tr>
<tr>
<td>LT12</td>
<td>1.00</td>
<td>0.220</td>
</tr>
<tr>
<td>LT13</td>
<td>1.25</td>
<td>0.237</td>
</tr>
<tr>
<td>LT14</td>
<td>1.25</td>
<td>0.237</td>
</tr>
<tr>
<td>LT15</td>
<td>1.25</td>
<td>0.231</td>
</tr>
<tr>
<td>LT16</td>
<td>1.50</td>
<td>0.242</td>
</tr>
<tr>
<td>LT17</td>
<td>1.50</td>
<td>0.242</td>
</tr>
<tr>
<td>LT18</td>
<td>1.50</td>
<td>0.242</td>
</tr>
<tr>
<td>LT19</td>
<td>1.75</td>
<td>0.248</td>
</tr>
<tr>
<td>LT20</td>
<td>1.75</td>
<td>0.237</td>
</tr>
<tr>
<td>LT21</td>
<td>1.75</td>
<td>0.248</td>
</tr>
<tr>
<td>LT22</td>
<td>2.00</td>
<td>0.248</td>
</tr>
<tr>
<td>LT23</td>
<td>2.00</td>
<td>0.231</td>
</tr>
<tr>
<td>LT24</td>
<td>2.00</td>
<td>0.242</td>
</tr>
</tbody>
</table>

Average = -0.3

Table F1 - Dynamic Calibration - Linear Transducer
-F10-
Although an approximate technique was used to measure the deformation, the good correlation between the galling measurement and the linear transducer results showed that the linear transducer was effective within the range of impact velocities required.
DATA AND CALIBRATION SHEET
DISPLACEMENT TRANSDUCER RLL-2 SERIAL NO.1111

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Displacement (R.O.)</td>
<td>25 MM</td>
</tr>
<tr>
<td>Dimensions Transformer DXL</td>
<td>26X102 MM</td>
</tr>
<tr>
<td>Core DXL</td>
<td>4.8X79 MM</td>
</tr>
<tr>
<td>Weight of Core</td>
<td>12 G</td>
</tr>
<tr>
<td>Input Voltage, Recommended</td>
<td>6 V</td>
</tr>
<tr>
<td>Input Voltage, Min - Max</td>
<td>4.5 - 9 V</td>
</tr>
<tr>
<td>Cut Off Frequency (3dB)</td>
<td>230 Hz</td>
</tr>
<tr>
<td>Current At Rec. Input Voltage</td>
<td>40 MA</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>201 OHMS</td>
</tr>
<tr>
<td>Load Resistance</td>
<td>1000 OHMS</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-40 TO +60 DEG.C</td>
</tr>
<tr>
<td>Temperature Effect ( -40 TO +60 DEG.C )</td>
<td>ON OUTPUT VOLTAGE (SENSITIVITY) 0.05</td>
</tr>
<tr>
<td>On Zero Balance</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The table beside shows the relationship between output voltage V and core position S. The output voltage is measured with no load at +23 DEG.C. Input voltage 6.0 V.

Calibration Value | -18.90 MM |

The stated calibration value is valid with an accuracy of 0.5 O/D with input voltage 5-7 V within the temperature range 0 TO +40 DEG.C.

AB BOFORS, ELECTRONICS DIVISION
S-69020 BOFORS, SWEDEN

[Signature]
BOFORS 72-10-07
Connection and calibration of displacement transducer

<table>
<thead>
<tr>
<th>Connection</th>
<th>Pin on the connector Hexagon 126 - 217</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Supply</td>
<td>A - E</td>
</tr>
<tr>
<td>negative terminal</td>
<td>A</td>
</tr>
<tr>
<td>positive terminal</td>
<td>E</td>
</tr>
<tr>
<td>Output signal</td>
<td>D - E</td>
</tr>
<tr>
<td>(polarity according to fig. below)</td>
<td></td>
</tr>
</tbody>
</table>

The outer steel casing of the transducer is electrically insulated from the circuit and can be connected arbitrarily to the voltage source, the output signal or be left floating.

As core support only austenitic steel or an insulating material may be used.

For electrical calibration, the transducer must be connected via a calibrating unit of type K-5 and calibration is performed according to the following:

1. Push the 0-button, which gives a 0-signal by shorting the transducer output.

2. Push the CAL-button. This gives an output signal, which refers to the 0-signal corresponds to a certain displacement of the core. This calibration displacement value is given on page 1.
Displacement transducer

With the displacement transducer type RLL-2, static as well as dynamic positions and movements up to a maximum of ± 25 mm (approximately ± 1 inch) can be measured and registered. When designing the transducer, an entirely new principle has been adopted and its special features are:

- D.C. feeding of the transducer
- Indicating or recording instrument can be directly connected
- Very high resolution
- Can be electrically calibrated
- High reliability
- Independent of cable length

The transducer is of the inductive type, a so-called differential transformer and has a core, moving axially through a coil system. In order to eliminate an external, complex carrier system, this has been incorporated in the transducer. The transducer can thus be fed with d.c. and the electronic part first converts the d.c. supply voltage for feeding the transformer and thereafter rectifies the output voltage.

As only low frequency signals are transmitted in the measuring cables, the length of the cables will be of little influence. The output signal is suitable for direct connection to indicating or recording instruments and measuring galvanometers in oscillographs. The transducer output is safe against short-circuiting. For electric calibration, the transducer can be connected to a separate calibrating unit, which enables a simple method for calibrating.

All components in the transducer are housed in an outer casing of stainless steel making the transducer a rugged and reliable instrument, particularly suited for industrial use. The components have been chosen so that the electrical life of the transducer is indefinite.

### Typical applications:
- Static and dynamic position measurements.
- Dimensional and positional control for servo systems.
- Level supervision.
- Continuous measurement of thickness.
- Vibration measurement.
- Transducer for determining pressure, force a.s.o.

<table>
<thead>
<tr>
<th>Nom. measuring range mm</th>
<th>Transformer</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D mm</td>
<td>L mm</td>
</tr>
<tr>
<td>± 1.5</td>
<td>26</td>
<td>66</td>
</tr>
<tr>
<td>± 3</td>
<td>26</td>
<td>72</td>
</tr>
<tr>
<td>± 6</td>
<td>26</td>
<td>88</td>
</tr>
<tr>
<td>± 12</td>
<td>26</td>
<td>114</td>
</tr>
<tr>
<td>± 25</td>
<td>26</td>
<td>182</td>
</tr>
</tbody>
</table>
TYPE RLL-2   DISPLACEMENT TRANSUDCER
-F14-

TECHNICAL DESCRIPTION

The differential transformer consists of a coil with one primary and two secondary windings. The electronic part is placed at one end of the transformer. The transformer as well as the electronic part is embedded in silicone rubber and housed in a tube of stainless steel. The connecting cable is brought out in one of the gables, parallel to the long axis. The core, made of μ-metal and moving through a hole in the coil, is connected to the measuring object by means of a rod of non-magnetic material. Due to a special heat treatment, the core is shock stabilized. An oscillator converts the supply voltage into a square wave which is fed into the primary winding of the transformer. The amplitude is determined by the supply voltage. The secondary voltages which are a function of the core position, are rectified and filtered. The transducer thus delivers a d.c. voltage, the magnitude and polarity of which varies with the position of the core. A series diode in the input circuit protects the transducer against changing the polarity of the input voltage.

CALIBRATION

For calibrating the measuring system, a simple calibrating unit, type K-5, can be used. By means of components incorporated in the transducer, it will be possible to obtain (1) a zero signal and (2) a signal, related to a certain, definite position of the core. This method of calibrating takes into consideration possible variations in the supply voltage and the calibration can be accomplished independently of the core position.

TECHNICAL SPECIFICATIONS

<table>
<thead>
<tr>
<th>Property</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring range</td>
<td>±1.5, 3, 6, 12 and 25 mm (approx. ±1/16, 1/8, 1/4, 1/2, 1 inch)</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>4.5—9 Volt d.c. (nom. 6 Volt 35 mA)</td>
</tr>
<tr>
<td>Temperature range</td>
<td>-40°C to +70°C</td>
</tr>
</tbody>
</table>

The following data are valid at 6 Volt supply and a load resistance greater than 1 kΩ. Deviations given refer to total measuring range.

<table>
<thead>
<tr>
<th>Property</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output voltage (at nom. measuring range)</td>
<td>approx. ±1.0 Volt</td>
</tr>
<tr>
<td>Output impedance</td>
<td>approx. ±1.5 mm approx. 500 Ω</td>
</tr>
<tr>
<td>Frequency limit (3 dB)</td>
<td>at range ±1.5 mm 300 cps</td>
</tr>
<tr>
<td></td>
<td>at range ±3, ±6, ±12 700 cps</td>
</tr>
<tr>
<td></td>
<td>at range ±25 mm 230 cps</td>
</tr>
<tr>
<td>Linearity deviation</td>
<td>less than ±0.5% from best straight line through zero</td>
</tr>
<tr>
<td>Temperature effect on sensitivity</td>
<td>less than ±0.05%/°C</td>
</tr>
<tr>
<td>on zero shift</td>
<td>less than ±0.01%/°C</td>
</tr>
<tr>
<td>Dimensions</td>
<td>see sketch overhead</td>
</tr>
<tr>
<td>Connections</td>
<td>6 ft cable with connector</td>
</tr>
<tr>
<td></td>
<td>Amphenol-Hexagon 126—217</td>
</tr>
</tbody>
</table>

Calibrating Unit Type K-5

<table>
<thead>
<tr>
<th>Property</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>60×110×80 mm</td>
</tr>
<tr>
<td>Connections</td>
<td>Input and output: Terminals</td>
</tr>
<tr>
<td></td>
<td>Transducer: Amphenol-Hexagon 126—218</td>
</tr>
<tr>
<td>Calibration value</td>
<td>Stated individual calibration value is valid with 0.5% accuracy at supply voltage 5—7 V within the temperature range 0°C to +40°C</td>
</tr>
</tbody>
</table>

We reserve the right to modify technical specifications and data.

AKTIEBOLAGET BOFORS
ELECTRONICS DIVISION
Address: S-650 20 Bofors, Sweden
APPENDIX G

Load Cell Technical Data
The Model 2106E Force Gage is a new approach in dynamic force measurement. It incorporates three force transducers in the same plane which accurately simulate point loading. This Endevco development gives high force sensitivity while permitting the convenience of the center mounting hole. It also permits a more stable design in which the height is small compared to the diameter. This “flat” configuration practically eliminates concern about shear loads.

The entire top and bottom surfaces are used as load bearing members, and this large area results in a gage with stiffness of 2.7 \times 10^3 \text{ lb/in.}

High gage stiffness results in high natural frequency which permits accurate force measurements at high frequencies. High frequency response, high sensitivity, center mounting hole and “flat” configuration are benefits of the three transducer concept.

The transducers used in the Model 2106E Force Gage utilize self-generating piezoelectric elements requiring no external source of electrical excitation.

**Specifications for Model 2106E Force Gage**
(According to ANSI and ISA Standards)

**DYNAMIC**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge Sensitivity</td>
<td>20 pC/lb., ±50%</td>
</tr>
<tr>
<td>Voltage Sensitivity</td>
<td>3.8 mV/lb., nom.</td>
</tr>
<tr>
<td>Amplitude Linearity</td>
<td>Sensitivity increases approximately 1% per 250 lb., 0 - 5000 lb.</td>
</tr>
<tr>
<td>Frequency Response (±5%)</td>
<td>2 to 5000 Hz'</td>
</tr>
<tr>
<td>Allowable Compressive Load (Static)</td>
<td>25,000 lb., maximum</td>
</tr>
<tr>
<td>Stiffness</td>
<td>2.7 \times 10^3 \text{ lb/in.}, nominal, with mounting bolt removed.</td>
</tr>
<tr>
<td>Effective End Mass</td>
<td>0.3 lb., nominal</td>
</tr>
<tr>
<td>Transducer Capacitance</td>
<td>5000 pF, nominal</td>
</tr>
<tr>
<td>Resistance</td>
<td>20,000 MΩ, minimum, output to signal ground. 10 MΩ, minimum, signal ground to case ground.</td>
</tr>
</tbody>
</table>

**NOTES**

1. With 300 pF external capacitance.
2. Use ENDEVCO\textsuperscript{®} Model 2700 Series Charge Amplifiers, or voltage amplifier with high input resistance. See curves below.
3. Compressive preload must exceed dynamic tensile load by 1000 lb, or more.

**TYPICAL FREQUENCY RESPONSE**

The solid line shows the response when using a charge amplifier. The broken line shows the frequency response with 300 pF external capacitance and when using a voltage amplifier. The low frequency response depends on the input resistance of the voltage amplifier, as indicated by the curves.
Specifications for Model 2106E

Dimensions in inches.

**PHYSICAL**

**WEIGHT**
200 grams (7.1 oz.), nominal.

**CRYSTAL MATERIAL**
Piezite® Element Type P-8.

**CASE MATERIAL**
Stainless Steel and Aluminum.

**CONNECTOR TYPE**
Coaxial, 10-32 thread, mates with accessory cable.

**MOUNTING**
Hole provided for 1/4” bolt. High strength steel bolt recommended for high force applications. Recommended mounting torque: 30 ft.-lb. to measure up to ±1000 pk lb.; 125 ft.-lb. to measure up to ±5000 pk lb.

**GROUNDING**
Signal ground insulated from case.

**ACCESSORIES INCLUDED**
Model 39LA-120 Low Noise Cable Assembly,
10 ft., 300 Ω, nominal.
Model 10063 Tapered Adapter Block (173 grams).
Ground Lug and Screw.

**ENVIRONMENTAL**

**ACCELERATION LIMITS**
±1000 g, sinusoidal
±5000 g, shock

**FORCE LIMIT**
Limited by strength of mounting bolt

**TEMPERATURE RANGE**
-30°F to +200°F (−35°C to +94°C)

**ALTITUDE**
Not affected

**HUMIDITY**
Will withstand normal laboratory environmental conditions.

**SALT SPRAY**
Meets MIL-E-5272C (with sealed connector)

**APPLICATIONS**
The excellent design configuration of the ENDEVCO® Model 2106E brings new possibilities and conveniences in dynamic force measurement. The Model 2106E may be used to measure forces transmitted by liquid or solid fuel rocket engines; turbines of all kinds; or any rotating device, such as a generator, where the forces caused by unbalance must be measured.

When electrodynamic shakers are used as drivers of a complex structure, such as a wing section or automobile, the Model 2106E Force Gage may be used to monitor the input force. It can be used in measuring thrust forces in hydraulic, pneumatic, or explosive actuators. Other applications are the determination of dynamic loading characteristics of plastics and fibers, bulk modulus studies of plastic materials, and measurement of forces generated by ball bearings.

For mechanical impedance studies, the ENDEVCO® Model 2110E Impedance Head is recommended.

Continuing product improvement necessitates that Endevco reserve the right to modify these specifications and/or prices without notice.

**RELIABILITY:** Endevco maintains a program of constant surveillance over all products to ensure a high level of reliability. This program includes attention to reliability factors during product design, the support of stringent Quality Control requirements, and compulsory corrective action procedures. These measures, together with conservative specifications, have made the name Endevco synonymous with reliability.

**CALIBRATION:** Each force gage is dynamically calibrated for sensitivity with sinusoidal forces and its capacitance is measured and recorded. Other calibrations are available; see Calibration Service Bulletin No. 301.

**PRICES:** F.O.B. Pasadena

- **1 only**
  - $470.00
- **2 to 5**
  - $450.00
- **6 or more**
  - $425.00

Prices of ENDEVCO® products for export or purchased with intent to export beyond the territorial limits of the United States and Canada are subject to special quotation.
APPENDIX H

Calibration of the *Endevco* Accelerometer

To ensure that the supplier-provided sensitivity of 4.60 millivolts/g for the *Endevco* Accelerometer was correct, a series of impact tests on the MEP was performed to compare its response with that of other accelerometers.

For each of 8 drop heights (0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75 and 2.0 meters), 3 impact tests were performed for each of 5 accelerometers: the *Endevco* accelerometer and 4 *Bruel and Kjaer* (B & K) accelerometers. The impactor was adjusted to a mass of 5 Kg for all tests. The even-shaped pulse shown in figure H1 is typical of the results. The peak acceleration was determined for every test. For each accelerometer, the average of the three tests was plotted against drop height as shown in figure H2.

The difference in results between the 4 B & K accelerometers and the *Endevco* accelerometer was not significant enough to warrant using a different sensitivity than the 4.60 mv/g recommended by the manufacturer.
FIGURE H2 — CALIBRATION OF THE ENDEVCO ACCELEROMETER.
The Models 2271A and 2275-Accelerometers featuring the new ISOBASE® construction represent a major breakthrough in the state of the art of transducer development. ISOBASE® construction provides mechanical isolation of the seismic element from the base, resulting in very low strain sensitivity heretofore available only in shear-type construction.

The Models 2271A and 2275 are precision accelerometers for use in the laboratory or for airborne applications. They have extremely flat charge-temperature response over a broad temperature range, from \(-185\) °C to \(+260\) °C, and excellent stability with time. Their high internal capacitance permits operation directly into oscilloscopes or voltmeters.

In the Model 2271A, the signal ground is insulated from the case; in the Model 2275, signal ground is connected to the case. The Models 2271AM20 and 2275M15 with dynamic characteristics identical to the above, respectively, feature top connectors.

These accelerometers are self-generating piezoelectric transducers, require no external power for operation, and may be used with either charge or voltage amplifiers.

**SPECIFICATIONS FOR MODEL 2271A AND 2275 ACCELEROMETERS**

(Approximately ANSI and IESA Standards)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge Sensitivity</td>
<td>11.5 pC/g, nominal</td>
</tr>
<tr>
<td>Voltage Sensitivity</td>
<td>5.0 mV/g, nominal</td>
</tr>
<tr>
<td>Mounted Resonance Frequency</td>
<td>27,000 Hz ± 3 kHz</td>
</tr>
<tr>
<td>Frequency Response (±5%)</td>
<td>2 to 5500 Hz, see curves below</td>
</tr>
<tr>
<td>Transverse Sensitivity</td>
<td>2% maximum; 1% on special selection</td>
</tr>
<tr>
<td>Amplitude Linearity, Range</td>
<td>Sensitivity increases approximately 1% per 1000 g, 0 to 10,000 g</td>
</tr>
<tr>
<td>Transducer Capacitance</td>
<td>2000 pF, nominal</td>
</tr>
<tr>
<td>Transducer Resistance</td>
<td>20,000 MΩ, minimum at +75°F (24°C); 100 MΩ, minimum at +500°F (260°C)</td>
</tr>
<tr>
<td>Insulation Resistance (2271A)</td>
<td>10 MΩ minimum</td>
</tr>
</tbody>
</table>

**NOTES**

1. With 300 pF external capacitance.
2. In shock measurement, minimum pulse duration for half-sine or triangular pulses should exceed 0.01 ms, to avoid high frequency ringing. See ENDEVCO Piezoelectric Instruction Manual.
3. Use ENDEVCO® Charge Amplifier Series 2700 or 2600.

**TYPICAL TEMPERATURE RESPONSE**

The solid line shows the nominal charge-temperature response. The broken lines show the voltage-temperature response with the cable supplied and also with an external capacitance of 2000 pF.

**TYPICAL FREQUENCY RESPONSE**

The solid curve shows the charge-frequency response. The broken lines show the voltage-frequency response with the loads shown and cable as supplied.

Estimated Calibration Errors:
- 5 to 900 Hz: ±1.25% (300 pF)
- 900 to 10,000 Hz: ±2.5%
Specifications for Models 2271A and 2275

**PHYSICAL**

**DESIGN**  
Single-ended compression, ISOBASE*  

**WEIGHT**  
27 grams (1 ounce), nominal  

**CRYSTAL MATERIAL**  
Piezite® Element Type P-10  

**CASE MATERIAL**  
Stainless Steel  

**CONNECTOR TYPE**  
Coaxial, 10-32 thread, mates with accessory cable  

**MOUNTING**  
Base tapped for 10-32×1¼" ENDEVCO® Mounting Stud. Recommended mounting torque: 18 in.-lb. (20 kg-cm)  

**GROUNDING**  
2271A: Signal ground is isolated from case by 10 MΩ, minimum  
2275: Signal ground connected to case  

**ACCESSORIES INCLUDED**  
Model 2981-3 Mounting Stud (10-32), or Model 2981-4 (mm metric).  
Model 5090A-120 Low Noise Cable Assembly. 10 ft., 300 pF nominal capacitance  

**OPTIONS AVAILABLE**  
Model 2271AM20 with top connector.  
Model 2275M15 with top connector.  

**ENVIRONMENTAL**

**ACCELERATION LIMITS**  
Vibration: 1000 pk g, sinusoidal, in any direction  
Shock: 10,000 pk g, in any direction  

**TEMPERATURE**  
-270°C to +260°C (−450°F to +500°F). Short connector during storage at extreme temperatures.  

**BASE STRAIN SENSITIVITY**  
0.5 equivalent g, nominal, at 250 µin./in. strain  

**MAGNETIC SENSITIVITY**  
0.03 equivalent g, nominal, at 100 gauss, 60 Hz  

**STRAY VOLTAGE SENSITIVITY**  
Typically, 0.1 g/V applied between signal ground and case  

**ALTITUDE**  
Not affected  

**HUMIDITY**  
All welded hermetic seal  

**CHARGE SENSITIVITY STABILITY**  
Within calibration accuracy over 12-month period.¹  

**NOTE:** Endevco warrants only the free recalibration of this accelerometer if the sensitivity deviates beyond the normal ±1.5% accuracy of the original calibration.  

Continued product improvement necessitates that Endevco reserve the right to modify these specifications and/or prices without notice.  

**RELIABILITY:** Endevco maintains a program of constant surveillance over all products to ensure a high level of reliability. This program includes attention to reliability factors during product design, the support of stringent Quality Control requirements, and compulsory corrective action procedures. These measures, together with conservative specifications, have made the name Endevco synonymous with reliability.  

**CALIBRATION:** Each unit is calibrated at room temperature for charge and voltage sensitivity, capacitance of both transducer and cable, transverse sensitivity, and frequency response from 20 to 4000 Hz. Temperature calibration at −300°F, +75°F, −350°F and +500°F, and other calibrations are supplied on special order. See Calibration Service Bulletin No. 301.  

**PRICES:** F.O.B. Pasadena  

<table>
<thead>
<tr>
<th>Model</th>
<th>1 only</th>
<th>2 to 5</th>
<th>6 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>2271A</td>
<td>$225.00</td>
<td>$215.00</td>
<td>$200.00</td>
</tr>
<tr>
<td>2275</td>
<td>$215.00</td>
<td>$205.00</td>
<td>$190.00</td>
</tr>
<tr>
<td>2271AM20</td>
<td>$235.00</td>
<td>$225.00</td>
<td>$210.00</td>
</tr>
<tr>
<td>2275M15</td>
<td>$225.00</td>
<td>$215.00</td>
<td>$200.00</td>
</tr>
</tbody>
</table>

Prices of ENDEVCO® products to be exported or purchased with intent to export beyond the territorial limits of the United States and Canada are subject to special quotation.
Calibration Certificate

ACCELEROMETER MODEL 2371/1120 SERIAL NO. HH12

Charge Sensitivity at 100 Hz: 11.6 pC/g
Voltage Sensitivity with 300 pF external capacitance: 4.60 mV/g
Maximum Transverse Sensitivity: 17% 
Capacitance: 2239 pF at 100 Hz, 19 pF at 1000 Hz

Temperature response calibrations are provided as options when requested through Endevo Calibration Service, see Bulletin 301.

All calibrations are traceable to the National Bureau of Standards and in accordance with MIL-C-45662A. This certifies that this accelerometer meets all the performance, environmental, and physical characteristics listed in ENDEVCO specifications.

Temperature Response Calibration:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge Sensitivity</td>
<td>% deviation</td>
</tr>
<tr>
<td>Capacitance</td>
<td>pF at 1000Hz</td>
</tr>
<tr>
<td>Resistance</td>
<td>GΩ</td>
</tr>
</tbody>
</table>

BY PD
DATE 6-17-74
APPENDIX I

Oscilloscope Triggering Device

Because of the rapidity of an impact test, it was rather difficult to obtain a properly positioned trace on the oscilloscope while using a relatively fast sweep rate. To achieve a proper trace, a triggering mechanism shown in figure I1 was built. A blade mounted on the impactor frame cut a beam of light an instant before impact. The interruption of light to the photo-transistor caused a step change in voltage to initiate the sweep of the oscilloscope. A schematic diagram of the trigger circuit is shown in figure I2.
FIGURE 11 – OSCILLOSCOPE TRIGGERING DEVICE.
FIGURE 12 — OSCILLOSCOPE TRIGGERING DEVICE, SCHEMATIC DIAGRAM.
Dynamic Testing Procedure

A specific test procedure was used for all dynamic tests to ensure the consistency of the results. This procedure is outlined here.

1. Select an energy mode, velocity, mass and adjust the impactor weight accordingly.
2. Use proper sample selected according to random techniques and record all pertinent data, sample number, dimensions, density.
3. Select proper vertical and sweep rate scales for oscilloscope.
4. Record on oscilloscope strip all data: test #, sample number, thickness and density, mass and drop height of impactor, sweep rate, sensitivity for accelerometer and displacement trace.
5. Place sample in proper consistent orientation on upper plate.
7. Adjust vertical position of transducer core for 'zero position'.
8. Record this position on oscilloscope.
9. Raise impactor assembly to proper position and accurately measure distance from the impactor bottom surface to the top surface of the foam sample.
11. Release impactor and allow to impact foam sample.
12. Take photo of impact response recorded on oscilloscope.
13. Record photo #.
APPENDIX K

Extraction of Data - Dynamic Tests

Figure K1 shows 4 sample acceleration responses to impact. Oscillations of various frequency and amplitude can be observed; different masses produce distinctive oscillations. It was hypothesized that this phenomenon was due to the instability of the extra weights and tie-down bolts. Their vibration during impact caused the oscillations.

When extracting data from the trace, the curve was arbitrarily smoothed by using the mid-point of every oscillation.

Similarly, a characteristic 1000 cycle noise was produced by the linear transducer. The same smoothing procedure was used to extract data.
FIGURE K1 — 4 SAMPLE ACCELERATION RESPONSES.
APPENDIX L

Determination of the Initial Velocity

In order to obtain a precise value for the velocity of the striker at impact, integration of the acceleration response was performed in the following manner. A photograph of the acceleration trace was enlarged approximately 6 times (figure L1). One square enlarged division (black lines) was cut away and weighed on a Mettler analytical scale previously calibrated according to the manufacturer's recommendations. The area under the loading portion of the acceleration response was carefully cut out of the photograph and weighed. This weight divided by the weight of the square division yielded the area under the acceleration trace in square divisions. This area multiplied by the vertical and horizontal sensitivity of the division was equal to the approximation of the initial velocity of the impactor used with the constitutive equation.
FIGURE L1 — ENLARGED ACCELERATION RESPONSE — INITIAL VELOCITY DETERMINATION.
APPENDIX M

Computer Program PSB Foam Simulation
and Plotting Routine
***CONTINUOUS SYSTEM MODELING PROGRAM***

***PROBLEM INPUT STATEMENTS***

INCON XIDIC = 25900, XIIC = 0.0
DYNAMIC
AA = A*AREA/AMASS*(DENSITY**1.5)*(7.96+.204*FF)
BB = .03057*ALOG(XX+SQRT((XX**2)+1))
CC = 415.72*EXP(-.88*DD)+.01343*EXP(13.9*DD)+1.25E-13*EXP(42.*DD)
DD = XI/THICKN
FF = (ABS(XID))/XIDIC
XX = (XIDIC/THICKN)*3.75E13
XIDD = -AA*BB*CC - A
XI = INTGRAL(XIDIC,XIDD)
XI = INTGRAL(XIIC,XI)
TIMER FINITIM = .0003, DELT = .000001, OUTDEL = .000001666
FINISH XID = 0.0
CONST A = 3530532, AREA = 8.422, AMASS = 2.374, THICKN = 2.58, ...
DENSITY = .0478, TESTNC = 37, SMPLNC = 584
PREPARE XI, XIIC, XIDD
PRTPLT XIDD(XI, XI)
END
STOP
**STEP2** lit les variables mises sur disque par **STEP1** (CSMP), les réduit à la bonne échelle et en fait le graphe. Les titres et les coordonnées sont également produits par **STEP2**.

DIMENSION GRIX(5), ORY(5), SPEC1(2), SPEC2(3), SPEC3(11),
  1SPEC4(10), SPEC5(11), SPEC6(7), LINEX(13), LINEY(13), AXISX(68),
  2AXISY(68), NAME(2), TITLE1(5), TITLE2(4), TITLE3(5),
  3TITLE4(5), DUNITS(3), VUNITS(5), AUNITS(1), FUNITS(3),
  4UNITS(3), AA(6), KB(6), KFNT(6), SEC(6), TIME(200), XI(200), KY(6),
  5X(200), YXDD(200), XMN(4), XMAX(4), FF(200), TH(200), VV(200), T(200),
  6FNT(6), AX(20), XXX(20), TTT(20), DENS(2), VVVV(2), MASS(2), XUNITS(2)
DIMENSION EXPER(3), SIMU(3)

**LES 4 PROCHAINES LIGNES COMMENCENT LA PROCEDURE **MILGO** **

CALL REREAD
CALL BPLT(4,2)
CALL PEN(1)
CALL INCM(0)
KKKK = 5

**LES PROCHAINES LIGNES LISENT LES DONNÉES ESSENTIELLES A LA PRODUCTION DES AXES, DES COORDONNÉES ET DES TITRES**

READ(KKKK, 102) (LINEX(I), LINEY(I), I = 1, 13)
READ(KKKK, 101) (AXISX(I), AXISY(I), I = 1, 68)
READ(KKKK, 101) (ORIX(I), ORIY(I), I = 1, 5)
READ(KKKK, 102) SPEC1
READ(KKKK, 103) SPEC2
READ(KKKK, 104) SPEC3
READ(KKKK, 105) SPEC4
READ(KKKK, 106) SPEC5
READ(KKKK, 107) SPEC6
READ(KKKK, 108) TITLE1
READ(KKKK, 109) TITLE2
READ(KKKK, 110) TITLE3
READ(KKKK, 111) TITLE4
READ(KKKK, 112) DUNITS
READ(KKKK, 113) VUNITS
READ(KKKK, 114) AUNITS
READ(KKKK, 115) FUNITS
READ(KKKK, 116) UNITS
READ(KKKK, 117) NAME
READ(KKKK, 118) NN
READ(KKKK, 119) (AXX(I), XXX(I), TTT(I), I = 1, NN)
READ(KKKK, 120) XUNITS
READ(KKKK, 121) PLUS
READ(KKKK, 122) EXPER
READ(KKKK,123) SIMU
101 FORMAT (F6.2,2X,F6.2)
102 FORMAT (2(A4))
103 FORMAT (3(A4))
104 FORMAT (11(A4))
105 FORMAT (10(A4))
106 FORMAT (11(A4))
107 FORMAT (7(A4))
108 FORMAT (5(A4))
109 FORMAT (4(A4))
110 FORMAT (5(A4))
111 FORMAT (5(A4))
112 FORMAT (3(A4))
113 FORMAT (5(A4))
114 FORMAT (A4)
115 FORMAT (3(A4))
116 FORMAT (3(A4))
117 FORMAT (2(A4))
118 FORMAT(I2)
119 FORMAT (3(F6.2))
120 FORMAT (2(A4))
121 FORMAT(A4)
122 FORMAT (3(A4))
123 FORMAT(3(A4))

***************************************************************************

* * INIT* EST LA SOURoutine QUI LIT LES DONNEES ENREGISTREES SUR DISQ
* PAR *STEP1*.
*  
*  *TESTNO* = LE NUMERO DU TEST.
*  *SMPLNO* = LE NUMERO DE L'ECHANTILLON
*  *THICKN* = L'EAISSEUR ORIGINALE DE L'ECHANTILLON
*  *DENSTY* = LA DENSITE ORIGINALE DE L'ECHANTILLON
*  *XIDIC* = LA VELOCITE ORIGINALE DU PROJECTILE
*  *AMASS* = LA MASSE DU PROJECTIL

***************************************************************************

CALL *INIT(XMIN,XMAX,TESTNO,SMPLNO,DENSTY,THICKN,XIDIC,AMASS,A)

***************************************************************************

* * ICOUNT* EST LE COMPTEUR QUI DETERMINE LE NOMBRE D'ITERATIONS
* * NECESSAIRES POUR QUE LA VELOCITE DEVIENNE ZERO.
*  
***************************************************************************

ICOUNT = 0
DO 200 I = 1,200

***************************************************************************

* * VALUE* EST LA SOURoutine QUI LIT LES VALEURS DES SPECIFICATIONS
* TEL QUE SPECIFIE PAR CSMP.
* TIME* EST LA VALEUR DU TEMPS ECOULE A CHAQUE ITERATION
* XI* EST LA DEFORMATION
* XID* EST LA VELOCITE
* XIDD* EST L'ACCELERATION
* CODE* : LORSQUE CODE = 1.0, LES VALEURS POUR TOUTES LES ITERATIONS ON
* ETE LUES DU DISQUE
* 
CALL VALUE(TIME(I),XI(I),XID(I),XIDD(I),CODE)
IF (CODE.EQ.1, ) ICOUNT = I-1
IF (ICOUNT.NE.0) GO TO 205
200 CONTINUE
205 CONTINUE
* LES PROCHAINES 103 LIGNES DETERMINE:
* 1) L'ORDRE DE GRANDEUR DES VARIABLES
* 2) LES VALEURS A INSCRIRE SUR LES AXES
* 3) LES FACTEURS PAR LESQUELS LES DONNEES DES VARIABLES
* SERONT MULTIPLIEES AFIN DE LES REDUIRE A LA BONNE
* ECHELLE POUR LE GRAPHE.
* 
* LA SECTION QUI SUIT OPERE SUR L'EPAISSEUR.
* 
T = THICKN
IF (T.GT.7.0) KT = 5
IF (T.GT.4.0 .AND. T.LT.7.0) KT = 4
IF (T.GT.3.0 .AND. T.LT.4.0) KT = 3
IF (T.LT.3.0) KT = 2
AA(I) = 0.
GO TO (1,2,3,4,5),KT
1 AINC = +3
XMULT = 20./3.
GO TO 6
2 AINC = +6
XMULT = 10./3.
GO TO 6
3 AINC = 1.0
XMULT = 2.
GO TO 6
4 AINC = 1.5
XMULT = 10./7.5
GO TO 6
EVEL 21    MAIN    DATE = 76229    17723/13

5 AINC = 2.0
XMULT = 1.
6 DO 7 I = 1,5
7 AA(I+1) = AA(I) + AINC

******************************************************************************
* ALPHA EST LA SOUCUTINE QUI CONVERTIT LES DONNEES NUMERIQUES EN *
* DONNEES ALPHANUMERIQUES; CETTE OPERATION EST NECESSAIRE POUR FAIRE *
* TRACER LES LETITIES PAR 'MILGO'.
******************************************************************************
* LA SECTION QUI SUIV OPERE SUR LA VELOCITE *
******************************************************************************
KB(1) = 0
6 DO 8 I = 1,5
8 KB(I+1) = KB(I) + 20.
CALL ALPHA(AA,KB,6,2)
******************************************************************************
* LA SECTION QUI SUIV OPERE SUR L'ACCELERATION *
******************************************************************************
G = ABS(XMIN(4)/A)
IF(G.GT.1000) KA = 6
IF(G.LE.1000 .AND. G.GT.500) KA = 5
IF(G.LE.500 .AND. G.GT.250) KA = 4
IF(G.LE.250 .AND. G.GT.100) KA = 3
IF(G.LE.100 .AND. G.GT.50) KA = 2
IF(G.LT.50) KA = 1
KC(1) = 0
GO TO (13,12,11,10,9,36),KA
36 KAINC = 250
AMULT = .008
GO TO 14
9 KAINC = 200
AMULT = .01
GO TO 14
10 KAINC = 100
AMULT = .02
GO TO 14
11 KAINC = 50
AMULT = .04
GO TO 14
12 KAINC = 20
AMULT = .1
GO TO 14
13 KAINC = 10
AMULT = .2
GO TO 14
14 DO 15 I = 1,5
15 KC(I+1) = KC(I) + KAINC
CALL ALPHA(AA,KC,6,2)
******************************************************************************
* LA SECTION QUI SUIV CONVVERTIT L'ACCELERATION EN FORCE EQUIVALENTE *
* OPERE SUR CETTE FORCE.
F = ABS(XMIN(4)/3.6E7)*AMASS/(THICKN)**2
IF(F GT 100) KF = 5
IF(F LT 50. AND. F LE 100) KF = 4
IF(F GT 25. AND. F LE 50) KF = 3
IF(F GT 5. AND. F LE 25) KF = 2
IF(F GT 2.5. AND. F LE 5) KF = 1
IF(F GT 1. AND. F LE 2.5) KF = 6
IF(F LE 1.) KF = 7
KFNT(1) = 0
FNT(1) = 0.0
GO TO (26, 19, 18, 16, 17, 16, 27, 26, 37), KF
37 KFINC = 40.0
FMULT = 0.05
GO TO 20
16 KFINC = 20.0
FMULT = 0.1
GO TO 20
17 KFINC = 10.0
FMULT = 0.2
GO TO 20
18 KFINC = 5
FMULT = 0.4
GO TO 20
19 KFINC = 2
FMULT = 1.0
GO TO 20
26 KFINC = 1
FMULT = 2.0
20 DO 21 I = 1, 5
21 KFNT(I+1) = KFNT(I) + KFINC
CALL ALPHA(AA, KFNT, 6, 2)
GO TO 35
27 FINC = 5
FMULT = 4.0
GO TO 29
28 FINC = 2
FMULT = 10.0
29 CONTINUE
DO 30 I = 1, 5
30 FNT(I+1) = FNT(I) + FINC
CALL ALPHA(FNT, KFNT, 6, 1)
35 CONTINUE

************************************************************
* * LA SECTION QUI SUIT OPERE SUR LE TEMPS *
************************************************************
TT = XMAX(1)*600.00
IF(TT GT 10.0) KTIME = 2
IF(TT LE 1.0) KTIME = 1
SEC(1) = 0.0
GO TO (22, 23), KTIME
22 SECINC = 2.0
TMULT = 1.0
GO TO 24
EVEL 21          MAIN          DATE = 76229          17/23/13

23  SECINC = 4.0
    TMULT = .5
24  DO 25 I = 1,5
25  SEC(I+1) = SEC(I) + SECINC
    CALL ALPHA(SEC,KFNT,6,1)

* LA LOUPE QUI SUISTE REDUIT LES DONNEES A L'ECHELLE APPROPRIEE
* CALCULEE PLUS HAUT.

*

DO 210 I = 1,ICOUNT
  XI(I) = (XI(I)/THICKN) * 10.
  XID(I) = (XID(I)/XIDIC) * 10.
  FF(I) = ((ABS(XIID(I)/3.657)*AMASS)/(THICKN*1.15)**2)*FMULT
  XIDD(I) = ABS(XIDD(I) * AMULT/A)
  TIME(I) = TIME(I)*60000.*TMULT

210 CONTINUE

DO 310 I = 1,NN
  AXX(I) = AXX(I) * AMULT -.2
  XXX(I) = (XXX(I)/THICKN)*10. -.2
  TTT(I) = TTT(I) * TMULT -.2

310 CONTINUE

* BETA EST LA SOUPUTINE QUI CONVERTIT LES VALEURS A ETRE INSCRIT
* DANS LES SPECIFICATIONS EN HAUT DU GRAPHE.

CALL BETA (TESTNO,SMPLNO,THICKN,DENSTY,VV,AMASS,TEST,
          SMPL,THIC,DENS,VVV,MASS)

* GRAPEH EST LA SOURUTINE QUI RENIIT TOUS LES APPELS NECESSAIRES PO
* FAIRE TRACER LES TITRES, LES AXES, LES CORDONNEES ET LES GRAPHE.

CALL GRAPH(ORIX,ORIY,SPEC1,SPEC2,SPEC3,SPEC4,
           1SPEC5,SPEC6,LINEX,LINEX,AXISX,AXISY,NAME,TITLE1,
           2TITLE2,TITLE3,TITLE4,DUNITS,VUNIT5,AUNIT5,UNIT5,
           3UNIT5,AA,KB,KC,KFNT,SEC,XI,XID,XIID,FF,TIME,ICOUNT,TH,V,TT,T01,
           4T02,V01,TEST,SMPL,THIC,DENS,MASS,VVV,FNT,
           SAXX,XXX,TTT,NN,PLUS,XUNIT5,EXPER,SINU)

* LES DEUX PROCHAINES LIGNES TERMINENT LA PROCEDURE DE MILGO.

CALL EPLT
CALL QUIT
STOP
END
SUBROUTINE ALPHA(A,K,N,IK)
DIMENSION A(6),K(6),D(6)
CALL SETB99(D,24)
GO TO (50,100),IK
50 DO 40 I = 1,6
   WRITE(99,51) A(I)
40 CONTINUE
GO TO 200
100 DO 60 I = 1,6
   WRITE (99,101) K(I)
60 CONTINUE
200 CONTINUE
RETURN
END

SUBROUTINE BETA(TESTNO,SMPLNO,THICKN,DENSTY,XIDIC,AMASS,
               TEST,SMPL,THIC,DENS,VVV,MASS)
DIMENSION D(6),DENST(2),VVV(2),MASS(2)
CALL SETB99(D,24)
WRITE(99,5) TESTNO
READ(99,11) TEST
WRITE(99,6) SMPLNO
READ(99,11) SMPL
WRITE(99,7) THICKN
READ(99,11) THIC
WRITE(99,8) DENSTY
READ(99,12) DENS
WRITE(99,9) XIDIC
READ(99,12) VVV
WRITE(99,10) AMASS
READ(99,12) MASS
5 FORMAT (F4.0)
6 FORMAT(F5.0)
7 FORMAT(F4.2)
8 FORMAT(F4.4)
9 FORMAT(F5.1)
10 FORMAT(F6,3)
11 FORMAT(A4)
12 FORMAT (2(A4))
RETURN
END
SUBROUTINE INIT (XMIN, XMAX, TESTNO, SMPLNO, DENSITY, THICKN, XIDIC, IMASS, A)
DIMENSION XMIN(4), XMAX(4)
REWIND 12
DO 10 LOOP = 1, 4
READ (12)
10 CONTINUE
READ (12) XMIN
READ (12) XMAX
DO 20 LOOP = 1, 3
READ (12)
20 CONTINUE
READ (12) TIME, DELT, DELMIN, FINTIM, PDEL, OUTDEL, XID, XI, XIDD, ZZ0003,
1XIDIC, XIIC, A, AREA, AMASS, THICKN, DENSITY, TESTNO, SMPLNO, AA, BB, CC,
2DD, FF, XX
READ (12)
RETURN
END

SUBROUTINE VALUE (TIME, XI, XID, XIDD, CODE)
READ (12) TIME, XI, XID, XIDD
IF (TIME . LT. 0.0) GO TO 10
CODE = 0.0
RETURN
10 CODE = 1.0
RETURN
END
SUBROUTINE SPEC(SPEC1, SPEC2, SPEC3, SPEC4, SPEC5, SPEC6)
DIMENSION SPEC1(2), SPEC2(3), SPEC3(11), SPEC4(10), SPEC5(11), SPEC6(7)
CALL LDR(-2.5, 3.9, 125, 0, 0, SPEC1, 6)
CALL LDR(-2.5, 3.65, 125, 0, 0, SPEC2, 10)
CALL LDR(-2.5, 2.65, 125, 0, 0, SPEC3, 42)
CALL LDR(-2.5, 2.9, 125, 0, 0, SPEC4, 43)
CALL LDR(-2.5, 2.0, 125, 0, 0, SPEC6, 28)
RETURN
END

SUBROUTINE CRDNTS(X, Y, AXISX, AXISY)
DIMENSION AXISX(68), AXISY(68)
CALL ORG(X, Y)
CALL LIN(AXISX(49), AXISY(48), 21)
DO 10 I = 4, 46, 2
   IF(I.GT.24) GO TO 5
   XX = 10-((I-4)/2)
   CALL SLEW(XX, 0, 0)
   GO TO 8
5   YY = (I-26)/2
   CALL SLEW(0, 0, YY)
   CONTINUE
   CALL LIN(AXISX(I), AXISY(I), 2)
10  CONTINUE
RETURN
END
SUBROUTINE LABEL(ALABEL, BLABEL).
DIMENSION ALABEL(6), BLABEL(6)
DO 10 I = 1, 6
   Y = -2.25 + 2*(FLOAT(I))
   CALL LDR(-2.15, Y, 1, 0, 0, ALABEL(I), 4)
10 CONTINUE
CALL SLEW(-1, 0, -.9)
DO 20 I = 1, 6
   X = -3.0 + 2*(FLOAT(I))
   CALL LDR(X, -.9, 1, 0, 0, BLABEL(I), 4)
20 CONTINUE
RETURN
END

SUBROUTINE CUTLIN
DIMENSION X(23), Y(23)
CALL SLEW(-4.25, 5.775)
DO 10 I = 1, 18
   J = I
   Y(I) = 5.525
10 X(I) = -5.0 + (FLOAT(J))/2.
   CALL LIN(X(I), Y(I), 18)
DO 20 I = 1, 23
   J = I
   X(I) = 4.0
20 Y(I) = 6.025 - (FLOAT(J))/2.
   CALL LIN(X(I), Y(I), 23)
DO 30 I = 1, 18
   J = I
   Y(I) = -5.475
30 X(I) = 4.5 - (FLOAT(J))/2.
   CALL LIN(X(I), Y(I), 18)
DO 40 I = 1, 23
   J = I
   X(I) = -4.5
40 Y(I) = -5.975 + (FLOAT(J))/2.
   CALL LIN(X(I), Y(I), 23)
RETURN
END