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Partial DS-CDMA for Mobile Satellite Data Networks

by

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A thesis submitted to the
School of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering
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June 1994

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Abstract

In recent years, random access protocols have received considerable attention as useful multiple access techniques for systems with high peak to average traffic ratios (bursty traffic). In most conventional random access protocols, when a single packet is transmitted in a given time period, it is received correctly, while if two or more transmissions occur, then all the packets are destroyed. If direct sequence code division multiple access (DS-CDMA) is used, this need not to be the case, because the DS-CDMA system allows many packets to coexist in a given slot. Thus packets are correctly received with certain probability which depends on the simultaneous number of packets in the slot. This suggests the possibility of using the DS-CDMA in a random access mode of operation. In addition, it is well known that CDMA transmission can provide other benefits, such as greater immunity to interference and interception; and its inherent resistance to multipath fading.

In this thesis we propose to use the DS-CDMA in a random access mode of operation for the mobile satellite data packet networks. First, we analyze the DS-CDMA over a multipath fading channel and develop new and accurate approximations of the probability of bit error and probability of packet success. In these approximations an improved Gaussian process is used to model the multiple access interference (MAI) and the bit-to-bit dependence caused by the MAI and fading is taken into account. These approximations are used to investigate the performance of the DS-CDMA packet networks.

The second stage of our work is to propose a new DS-CDMA system referred to as partial DS-CDMA in which a limited number of spread spectrum codes is required. It is well known that a major problem in the design of a DS-CDMA packet networks
is the required large number of spreading codes when the network is aimed to serve a large population of users. The proposed partial DS-CDMA copes with this problem and suggests the use of only a limited number of spreading codes at the price of relatively small degradation on the performance.

The last stage of this work is to propose a collision resolution algorithm (CRA) as random access protocol to be used in conjunction with the proposed partial DS-CDMA system. The combined protocol is analyzed under Poisson traffic and the delay-throughput performance is analyzed for different system parameters and under different fading conditions. By combining the partial DS-CDMA with the proposed CRA we obtain a new protocol which is stable, implementable and has many advantages as we prove throughout the thesis.
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To my family
Acknowledgments

I wish to express my gratitude to Dr. A. Yongaçoğlu, my thesis supervisor, for his continuous encouragement and invaluable suggestions throughout the research. I thank him not only for teaching me the knowledge and problem solving ability, but also for helping me to develop scientific attitude. Many thanks to him for being as a good friend over the whole period of my graduate study.

I would like to express my sincere appreciation to Dr. J-Y Chouinard for reading this thesis and for his helpful suggestions over the whole period of my graduate study. Special thanks to Drs. S. Mahmoud, C. Loo and O. Yang for reading this thesis and for their suggestions and encouragement. Special thanks to Drs. M. Kavehrad, B. Felstead and M. El-Tanany for their suggestions and encouragement to continue the research.

I would like to express my sincere thanks to the Tunisian government for its scholarship program. I would also like to acknowledge the partial funding from Natural Sciences and Engineering Research Council (NSERC) and Telecommunications Research Institute of Ontario (TRIO) grants.

My epitome thanks go to my family for their patience, sacrifice, and support during all those years of my studies.
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Acronyms and Definitions

AWGN: Additive White Gaussian Noise
BCH: Bose, Chaudhuri and Hocquenghem
BER: Bit Error Rate
BPSK: Binary Phase Shift Keying
CDMA: Code Division Multiple Access
Cov: Covariance
CRA: Collision Resolution Algorithm
CRI: Collision Resolution Interval
CRP: Collision Resolution Point
CSMA: Carrier Sense Multiple Access
d.b.e: dependent bit error
DS: Direct Sequence
DS-CDMA: Direct Sequence Code Division Multiple Access
DS-SS: Direct Sequence Spread Spectrum
FCC: Federal Communication Commission
FDMA: Frequency Division Multiple Access
FEC: Forward Error Correction
FH: Frequency Hopping
FH-CDMA: Frequency Hopping Code Division Multiple Access
FH-SS: Frequency Hopping Spread Spectrum
FM: Frequency Modulation
FSK: Frequency Shift Keying
GMSK: Gaussian Minimum Shift Keying
GSM: Groupe Special Mobile
i.b.e: independent bit error
JPL: Jet Propulsion Laboratory
LOS: Line Of Sight
MAI: Multiple Access Interference
MSAT: Mobile Satellite
NASA: National Aeronautics and Space Administration
PASS: Personal Access Satellite System
pdf: probability density function
PCN: Personal Communication Network
PER: Packet Error Rate
PN: Pseudo Noise
QPSK: Quadrature Phase Shift Keying
RS: Reed-Solomon
SDMA: Space Division Multiple Access
SNR: Signal to Noise Ratio
SS: Spread Spectrum
SSMA: Spread Spectrum Multiple Access
TDM: Time Division Multiplexing
TDMA: Time Division Multiple Access
Var: Variance
UHF: Ultra High Frequency
VHF: Very High Frequency
VSAT: Very Small Aperture Terminal
Nomenclature

\( a_k(t) \): spreading sequence
\( b_k^j \): \( j \)’th data bit of the \( k \)’th signal
\( b_k^{\tau_k} \): the data bit of the \( k \)’th signal during \([0, \tau_k]\)
\( b_k^T_k \): the data bit of the \( k \)’th signal during \([\tau_k, T_k]\)
\( \beta_{i,k} \): the \( l \)’th path gain of the \( k \)’th user
\( B \): quantity equals to \((N - 1 - C)/2\).
\( B_c \): coherence bandwidth
\( B_f \): the fading bandwidth
\( b_k(t) \): data sequence
\( C \): discrete aperiodic autocorrelation of the signature sequence
\( c \): the nonfaded component of a Rician process
\( c_i \): spreading code of the \( i \)’th receiver
\( c_i \): the speed of light
\( c_k \): the \( k \)’th user nonfaded component of a Rician process
\( \tilde{C}_K \): number of packets successfully received by the central station, given \( K \) simultaneous transmissions
\( \bar{C}_K \): \( C_K \) for conventional DS-CDMA
\( \bar{C}_K^p \): \( C_K \) for partial DS-CDMA
\( CL \): constraint length of a convolutional code
\( \varepsilon \): relative error
\( E(\cdot) \): expectation function
\( \delta(\cdot) \): the Kronecker delta
\( \Delta \): length of the window utilized by the CRA
\( \Delta_{opt} \): optimum window size of the CRA
\( D_n \): delay experienced by the \( n \)’th packet
\( d_p \): round trip propagation delay
\( E_b \): energy per information bit
\( E_c \): energy per coded bit
\( E(L|\Delta) \): expected length of a CRI, given that it starts with an examined interval of length \( \Delta \)
\( E(X|\tau) \): expected value of the random variable \( X \) given that the examined interval has length \( \tau \)
\( F_1[\cdot, \cdot, \cdot] \): the confluent hypergeometric function
\( f_c \): carrier frequency
\( f^1 \): Gaussian random variable (I-channel) of a Rician process
\( f^Q \): Gaussian random variable (Q-channel) of a Rician process
\( \gamma_0 = \gamma \): the SNR for MAI is Gaussian given by eq (3.37)
\( \gamma_1 \): SNR1 for improved Gaussian MAI given by eq (3.76)
\( \gamma_2 \): SNR2 for improved Gaussian MAI given by eq (3.77)
\( \Gamma[\cdot] \): the gamma function
\( h \): step size of the Taylor series approximation, \( h = \sqrt{3} \sigma \psi \)
\( h_d \): number of slot needed to return to lag equals to one
when starting from a collision resolution instant with lag \( d \)
\( h_k(t) \): low-pass equivalent impulse response of the passband fading channel
\( I_0(\cdot) \): modified Bessel function of zeroth order
\( J \): number of packets transmitted in the CRI that starts at time \( t \)
\( K \): number of simultaneous users
\( k_c \): number of information bits in a codeword
\( K_f \): the direct-to-scattered signal power ratio
\( K_{max} \): maximum number of simultaneous transmissions beyond which
almost all transmitted packets collide
\( K_{opt1} \): maximum number of simultaneous transmissions a CDMA
system can accommodate without any collisions
\( K_{opt2} \): maximum number of simultaneous transmissions
that provides the maximum value of \( \bar{C}_K \)
\( L \): number of paths for a multipath fading channel
\( l \): length of a conflict resolution interval
\( l_j \): is the \( j \)'th user of the \( i \)'th group
\( \lambda \): Poisson traffic input rate
\( \lambda_{max} \): maximum throughput
\( L_c \): coded packet length
\( L_K \): expected length of a collision resolution interval
\( L_{n,K-n} \): expected number of slots needed by the CRA for the successful
transmission of \( K \) packets, given that \( n \) of those packets
have counter value equal to one and \( (K-n) \) of the packets
have counter values equal to two
\( L_p \): information packet length
\( L_i^t \): \( i \)'th users group
\( M \): number of receivers (codes or groups) at the central station
\( m \): number of users per group
\( \mu \psi \): mean of \( \Psi \)
\( N \): sequence length (spreading factor)
\( N_o \): one-sided noise spectral density
\( n_c \): blocklength of a codeword
\( n_o \): half the free distance of a convolutional code
\( n(t) \): Gaussian noise
\( \Omega \): cumulative delay of the \( J \) packets before the beginning of the CRP \( t \)
\( p_{\text{r}}(t) \): rectangular pulse of duration \( T_{b} \)

\( p_{\text{r}}(t) \): rectangular pulse of duration \( T_{c} \)

\( P \): transmitted signal power

\( P_{b} \): probability of information bit error

\( P_{\text{ch}} \): probability of channel bit error

\( P_{c}(K) \): Probability of packet success from a designated user given that there are \( K \) simultaneous transmissions

\( P_{d}(\cdot) \): probability of delay capture

\( P_{d}(\tau) \): power-delay profile

\( \phi_{l,k} \): the \( l \)'th path phase of the \( k \)'th user

\( \varphi_{l,k} \): parameter equals to \( \phi_{l,k} + \theta_{k} - u_{c}\tau_{l,k} \)

\( P_{j}(M) \): probability that \( K \) packets come from \( j \) different groups

\( P_{M,K}(j) \): probability that \( j \) packets are successfully transmitted in a transmission involving \( K \) packets in a system consisting of \( M \) different groups

\( P_{\text{sig}} \): power of the desired signal

\( P_{\text{noise}} \): power of the thermal noise

\( P^{n}(x) \): \( n \)'th derivative of \( P(x) \) with respect to \( x \)

\( \Psi \): conditional variance of the MAI

\( P_{u} \): union bound on the first event error probability

\( P(x|\tau) \): probability that the CRI has length \( x \) given that the examined interval has length \( \tau \)

\( Q \): the capture ratio which is \( T_{\text{cap}}/T_{u} \)

\( Q(\cdot) \): \( Q \)-function

\( r \): FEC code rate

\( R_{b} \): information bit rate

\( r(t) \): received signal

\( R_{1,k}(\tau_{l,k}) \): the continuous time partial cross-correlation function of the \( k \)'th and 1'st spectral spreading waveforms defined in \( [0, \tau_{l,k}] \)

\( \bar{R}_{1,k}(\tau_{l,k}) \): the continuous time partial cross-correlation function of the \( k \)'th and 1'st spectral spreading waveforms defined in \( [\tau_{l,k}, T_{b}] \)

\( R_{i} \): number of successfully transmitted packets in \( (0, T_{i}] \)

\( \bar{R}_{i} \): number of successfully transmitted packets in \( (T_{i}, T_{i+1}] \)

\( \bar{R} \): mean cycle length which is the average value of \( \bar{R}_{1}(E(\bar{R}_{1})) \)

\( \tau_{l} \): counter cycle length at time \( t \)

\( \sigma^{2} \): variance of \( f^{l} \) and \( f^{Q} \).

\( s_{k}(t) \): transmitted signal of the \( k \)'th user for a DS-CDMA system

\( S_{l,k} \): parameter equals to \( \tau_{l,k}/T_{c} - \tau_{l,k} \)

\( \sigma_{\Psi}^{2} \): variance of \( \Psi \)

\( \sigma_{\text{int2}}^{2} \): variance of the second term of the MAI

\( \sigma_{\text{int3}}^{2} \): variance of the third term of the MAI

\( t \): error correcting capability
\( \tau_{l,k} \): the \( l \)th path time delay of the \( k \)th user

\( T_b \): information bit period

\( T_c \): chip period

\( T_{\text{cap}} \): the capture time

\( T(D) \): the transfer function of a convolutional code

\( T(D, N_c) \): the augmented transfer function of a convolutional code

\( t_l \): sequence of successive collision resolution points

\( T_i \): ending point of a length one lag

\( T_m \): maximum multipath delay spread

\( T_p \): packet duration

\( T_s \): slot duration

\([0, T_u] \): randomization time interval

\( U \): total number of users

\( V \): the vehicle speed

\( W \): mean cumulative delay during \((T_i, T_{i+1})\)

\( W_{l,k} \): parameter equals to \( b_k^{-1} R_{l,k}(\tau_{l,k}) + b_k^0 R_{l,k}(\tau_{l,k}) \)

\( w_d \): cumulative delay experienced by all the packets that were successfully transmitted during the \( h_d \) slots

\( X_i \): the lag at \( t_i \)

\( X_{l,k} \): parameter equals to \( \beta_k^2 \cos^2(\varphi_{l,k}) \left[ 2(2B + 1)(S_{l,k}^2 - S_{l,k}) + N \right] \)

\( x_t \): feedback that corresponds to a slot \( t \)

\( x_t = C \): corresponds to collision slot \( t \)

\( x_t = E \): corresponds to an empty slot \( t \)

\( x_t = S \): corresponds to successful slot \( t \)

\( \xi \): parameter equals to \( (1 - e^{-1})/(e^{\frac{1}{T_1}} - 1) \)

\( y_k(t) \): the output signal produced by the multipath fading channel

\( z \): cumulative delay of the \( J \) packets after the CRP \( t \)

\( Z_{l,1} \): the decision statistic of the receiver 1 when locked on to the first path
Chapter 1

Introduction

1.1 Motivation

In this age of information technology, it is becoming essential to communicate information to people who are on the move and may be difficult to locate precisely. In the vicinity of cities, cellular systems can be used for this purpose. However, there is an urgent need to have global communication networks serving not only urban areas but remote and rural areas as well. Because of their wide coverage and their multiple access capability, satellites are very well suited to fulfill this demand.

In recent years, extensive research has been made in the field of mobile satellite communications and several systems are expected to become operational in the near future around the world, especially in the United States, Canada and Australia [1]-[14]. A mobile satellite network consists of a large fleet of mobiles, a communication satellite and one or many fixed earth stations. The transmission of information from the mobiles towards the fixed earth station takes place on a channel called the return channel. Since this channel is shared by all the system users, a multiple access protocol regulating the access to the channel is necessary. The forward channel (from earth station to mobiles) operates in a broadcast mode carrying the reverse traffic and
control information. Generally, because of the low gain of the antennas mounted on
the mobiles, a direct mobile to mobile communication is not possible and it requires
the information to be relayed by the earth station.

In digital communication systems, there are different multiple access techniques,
such as frequency division multiple access (FDMA), time division multiple access
(TDMA), code division multiple access (CDMA) and space division multiple access
(SDMA). The capacities of FDMA, TDMA, CDMA and SDMA systems have been
extensively studied in the past decades for different applications [15]-[22].

There have been analyses published [21]-[22] that show that CDMA is not as effi-
cient as FDMA or TDMA for channels that do not involve signal fading. However, it
was suggested in [21] that CDMA may be useful in fading channels. Other publica-
tions [19]-[20] show that by properly controlling the mutual interference, the capacity
of a CDMA system can be increased.

CDMA is an application of spread spectrum techniques, and is also called spread
spectrum multiple access (SSMA). It is well known that spread spectrum signals
can mitigate the multipath effects, and thus make CDMA more robust in mobile
environment [49]-[64]. Spread spectrum modulation is universally accepted in military
applications. In consumer applications, spread spectrum modulation is not widely
accepted yet. In recent years, the FCC is actively encouraging spread spectrum
development in commercial applications [23], [27], [32].

There are several different forms of spread spectrum. The two primary tech-
niques commonly used are direct sequence (DS) and frequency hopping (FH). In
DS-CDMA, several independent users simultaneously access the channel by modulating
preassigned signature waveforms, known as PN sequences or spreading codes. The
incoming signal at the receiver is therefore a superposition of such signals. Upon ob-
ervation of this composite signal and equipped with the knowledge of the spreading code, the receiver demodulates and decodes the information. In FH-CDMA, different frequency-hopped patterns are used by different users to access the channel. Although a number of different spread spectrum protocols have been discussed in the literature [65]-[78], the design and analysis of these protocols is still an active area for research.

In light of all of this, we will investigate the use of DS-CDMA for the case of mobile satellite communication systems.

1.2 System Model

In our study, a simple model for the mobile satellite communication system is considered. This model consists of a large population of users, each having a portable set, communicating over a satellite channel with a common destination referred to as the central station. This central station may represent either a satellite earth station or an onboard processing satellite. We first present the user model; then the channel model and finally the multiple access protocol considered in our system.

1.2.1 User Model

We consider a large number of independent identical users that collectively generate data traffic satisfying the Poisson statistic with intensity $\lambda$ packets per time unit. The data traffic generated by all the system users is considered to be bursty, i.e., the user requires the communications resources infrequently, but when he does, he may require a rapid response. That is, there is an inherently large peak to average ratio in the required data transmission rate.
1.2.2 Channel Model

A single channel is shared by all users. The channel is assumed to be slotted; that is, the channel time is divided into equal segments called slots and the users synchronize their transmissions so that the packets fall into slots. The channel is corrupted by thermal noise, fading and MAI induced by the CDMA system.

1.2.3 Multiple Access Protocols

Since the satellite channel is shared by all the users, a multiple access protocol regulating the access to the channel is necessary. The basic question here is how to choose this multiple access protocol to achieve a high throughput efficiency while the message delays are within an acceptable range.

The various known alternatives fall into the following categories.

(i) Fixed assignment techniques: This refers to preassigning to each user, a portion of the channel bandwidth if FDMA is used, or a time slot if TDMA is used. This assignment is fixed and does not depend on how active the users are at different instants.

(ii) Demand assignment techniques: In this case the resource is allocated as needed, so that a user transmits only when he has traffic. The control of the channel can either be done by a central station or all users executing a distributed algorithm.

In centrally controlled demand assignment techniques FDMA or TDMA could be used in a way that the bandwidth is divided into FDMA subchannels or TDMA slots which are assigned on demand. Two types of channels can be distinguished in these techniques. Signalling (reservation or request) channels and transmission channels. Signalling channels are used for the transmission of accessing request and the transmission channels are used for the transmissions of data.
In demand assignment with distributed control, the users share some information regarding the demand on the channel and its usage so that an algorithm executed by each one of them independently will result in some sort of coordination in their action. There exists many examples of such category, one of them is the token ring [132] which is used for computer communications applications.

(iii) Polling is another technique where a central controller interrogates all the users sequentially, one-by-one asking them to transmit any pending messages they may have.

(iv) Random access techniques: It has long been recognized that fixed allocation of a scarce communication resource is extremely wasteful when large population of users with bursty traffic is considered. In this case it may be appropriate to let all the users transmit randomly. If collisions between messages occur, then these messages should be retransmitted. Some schemes of this categories are ALOHA, carrier sense multiple access (CSMA) and tree type algorithms.

Although fixed assignment techniques have been widely used in satellite communications to transmit TV signals, voice traffic or large amount of data, these techniques are not adequate for mobile satellite communications where the traffic is coming from a large number of bursty users.

In demand assigned techniques, we can exclude the use of schemes with distributed control because of the difficulty of any possible coordination among the mobile users. This problem does not arise for centrally controlled demand assignment techniques where the earth station, with its large antenna gain can play the role of the central controller. These techniques may be suitable for the transmission of voice and long data file. Since these techniques use two types of channels, the random access is suitable for the signalling channel [101] and deterministic access for the transmission.
channel [101]. One of the major drawbacks of these schemes is the complicated message controlling mechanisms, especially, for large population of bursty users networks.

Polling techniques have their advantages and disadvantages. In some applications where the number of users is relatively small and the data transfer need not to be so urgent as compared to real time voice or television transmission and the objective is to have the data transferred reliably, polling techniques could be a good choice.

In applications involving messages that are short and transmitted infrequently (which is the case for MSAT systems [1]-[14]), the deployment of random access techniques are desirable for many reasons. The random access techniques are implemented independently by each user, they are insensitive to changing users population; and they induce low delays for low input rates.

In the conventional narrowband random access protocols, when two or more packets are transmitted at the same time and same frequency, then all the packets are destroyed or only one packet is correctly received (if some capture technique is employed [89]-[90]). If CDMA is employed, coding permits the simultaneous transmissions with some error rate of two or more users' data packet. This suggests the suitability of CDMA in a random access mode of operation.

In our research we propose to use the random DS-CDMA as a multiple access technique in the context of mobile satellite communications. In this study, our interest will be focused on the transmission of data using the proposed protocol from the users to the central station. The following assumptions and parameters are considered throughout our study.

- Messages are transmitted as single packets of fixed size on the order of one hundred to one thousand information bits [1]-[14].
• The transmission and processing time of the acknowledgments at the central station is negligible.

• DS/BPSK modulation scheme, block and convolutional codes are considered.

• The mobile users information bit rate is considered to be on the order of 2400 bps to 9600 bps [1]-[14].

1.3 Objectives and Thesis Contributions

One of the major difficulties in analyzing the bit error rate (BER) and packet error rate performance of a DS-CDMA system is the modeling of the MAI. Various analytical approaches have been taken to model the MAI. One of the most straightforward approach is to assume that the MAI is Gaussian. It became well known that the BER computations based on the Gaussian assumption is not accurate enough, especially for small number of simultaneous users. Many other approaches such as the moment space and the characteristic function approaches have been adopted by many researchers to model the MAI. However, most of these approaches yield complex approximations for the probability of bit error. Recently [41], it was shown that a good accuracy improvement on the probability of bit error and probability of packet success for DS-CDMA over an AWGN channel was obtained by modeling the MAI as an improved Gaussian process. However, the approximation developed in [41] is computationally complex, it requires a lot of numerical integrations and convolutions. Also only block codes were considered in [41]. In our previous work [45] the approximation developed in [41] was significantly simplified with a good accuracy for both block and convolutional codes. In this thesis we extend the work in [45] and we have been able to develop new approximations for the probability of bit error and
probability of packet success for DS-CDMA over multipath fading channel.

Most of the techniques that have been developed over the last decade to calculate the probability of bit error and probability of packet success did not take into account the effect of bit-to-bit dependence caused by the MAI and the multipath fading. In this thesis the approximations of the probability of bit error and probability of packet success that were developed take into account the effect of bit-to-bit dependence caused by the MAI and the multipath fading while modeling the MAI as an improved Gaussian process.

Most of the analysis of DS-CDMA protocol is based on the assumption that each system user has a unique spreading code, and there is a receiver for each user at the central station [69]-[78]. Therefore a system with very large number of users implies a very complex central station and requires a large number of spreading codes. Our objective is to develop a more efficient DS-CDMA protocol for mobile satellite communication networks referred to as partial DS-CDMA protocol, which reduce the central station complexity in terms of required number of receivers while maintaining the channel throughput at an acceptable level. One way of reducing the central station complexity is to divide the system users, $U$, into $M$ groups, with $m$ users in each group. If all users within a particular group use the same code, then the number of receivers in the central station can be reduced from $U$ to only $M$.

In the previous analysis of random DS-CDMA protocol, direct sequence spread spectrum is combined with certain characteristic of ALOHA system [70]-[85]. This is referred to as ALOHA/DS-CDMA protocol. In this protocol, when a source has a new packet, it transmits the packet and waits for an acknowledgment. If after a certain timeout period the source does not receive an acknowledgment, it assumes that the packet has collided and retransmits it after a random delay. The retransmission takes
place at a randomly selected time so that the conflicting packets may not collide again. It is well known that an ALOHA type system is inherently unstable [93]-[95], and statistical fluctuations may easily cause its saturation. This means that it reaches a situation where the number of retransmitting sources tends to infinity and the throughput tends to zero. Therefore, although the delay and throughput properties might be satisfactory in the short term, they are quite poor when observed over a long interval of time. Since the appearance of Capetanakis' pioneer paper [106] the subject of collision resolution algorithm (CRA) has attracted the attention of many researchers, who believe that CRAs are more efficient than ALOHA type system. Furthermore, the CRA is stable even for an infinite population of users. For this reason, in our study the partial DS-CDMA protocol is combined with a CRA to avoid the problem of instability which occurs with ALOHA. One of the main contribution of our study is the investigation of the CRA for CDMA systems which is rarely addressed in the literature.

1.4 Thesis Organization

In Chapter 2 we present a literature survey of the major subjects related to our study. Different mobile satellite systems under design are introduced. The CDMA technology is reviewed and comparison between CDMA and many other access techniques are presented. Finally, different collision resolution algorithms that have been proposed so far are reviewed.

In Chapter 3 we analyze the BER performance of DS-CDMA system over the mobile satellite fading channel. A new approximation for the probability of bit error for asynchronous DS-CDMA using convolutional and block codes over the multipath fading channel is developed.
In Chapter 4 we analyze the packet error rate performance of a DS-CDMA system over the mobile satellite fading channel. We develop new approximations for the probability of packet error which use different models for the MAI and take into account the effect of bit-to-bit error dependence due to fading and MAI. These approximations are used to investigate the throughput-delay performance of DS-CDMA satellite packet network.

In Chapter 5 we propose a new partial DS-CDMA protocol for the mobile satellite data packet network which reduces the central station complexity.

In Chapter 6 we propose a new CRA which is used in conjunction with the proposed DS-CDMA to resolve collisions when it occurs.

Finally in Chapter 7 we summarize our results and suggest some future work.

In this thesis, some detailed derivations for equations are put in the appendices. In Appendix A, mathematical derivations of the variance of the conditional variance of the MAI are given. In Appendix B, the delay analysis for the CRA for partial DS-CDMA is given. Finally, in Appendix C, the accuracy of the improved Gaussian model for the MAI on the computation of the probability of bit error for asynchronous DS-CDMA system is discussed.
Chapter 2

Literature Survey

2.1 Introduction

In this chapter we present an overview of the major topics related to our research. First an overview of the existing and planned mobile satellite communication systems is discussed. Then we present the different models used to characterize the mobile satellite channel. Since our interest is in the study of multiple access techniques, we review three basic multiple access schemes, FDMA, TDMA and CDMA in the context of mobile satellite communication systems. In this overview, emphasis is on the advantages of using spread spectrum CDMA over FDMA or TDMA. In our study CDMA is used in random access mode of operation, therefore we discuss some possible random access techniques that could be used in conjunction with CDMA for transmitting data in the mobile satellite communication systems.

2.2 Overview of MSAT Systems

The mobile satellite communication system became very popular this decade. The initial strength of mobile satellite communication system is its ability to provide services to vast regions where such services could not possibly be reliably provided
by purely terrestrial systems due to geographical reason. Mobile satellite system is capable of delivering a range of services to a wide variety of terminal types. Example of mobile satellite terminal platforms include land vehicles, aircrafts, marine vessels, remote data collection and control sites. Additionally, services can be provided to portable terminals, which are currently about the size of a briefcase, but may be reduced to "handheld" size for future systems. This section includes an overview of present and planned mobile satellite systems.

The first commercial availability of mobile satellite communications services began in early 1976 with the introduction of Inmarsat-A developed by the International Maritime Satellite Organization (Inmarsat). The Inmarsat-A standard evolved into the Inmarsat Aeronautical in 1990 and into Inmarsat-M and Inmarsat-B in 1993 [10].

The MOBILESAT system developed by AUSSAT [6] probably is the first domestic mobile satellite system offering both voice and data services. Communications capacity is provided with transponders on two AUSSAT B1 satellites. By focusing the transmitted signal only on Australia, the AUSSAT satellite antennas can have higher gain than those of the Inmarsat satellites which must view the entire world. Therefore, the antennas for the MOBILESAT terminals can have a lower gain. Consequently, they are smaller and less expensive than those for Inmarsat-M.

Telesat Mobile Inc. of Canada and the American Mobile Satellite Corporation of the United States are jointly developing the North American MSAT system [8], [9]. Canada's MSAT satellite will be launched first, probably at the end of 1994, with that of the United States being launched shortly after. All the systems discussed above are based on FDMA or a combination of TDMA and FDMA.

The OmniTRACS system is the first two-way mobile Ku-band satellite communications system developed by Qualcomm Inc., and has been operational in the United
States since 1989 [1]. This system provides two-way data messaging, position reporting, fleet broadcasting, call accounting and message confirmation services to mobile users throughout the United States. Typical users of the system include those involved in public safety, transportation, construction, agriculture, private fleets and others who have a need to send and receive information to vehicles, marine vessels or aircrafts enroute. In this system, a moderate rate of 5-15 kbps data stream from the hub to all the mobile terminals (forward link) in the system is used. On the return link, a low power and low data rate transmissions ranging from 55 to 165 bps is implemented. Direct sequence spread spectrum at a rate of 1 Megachips per second was used to mitigate the interference on the return link. Demand assignment multiple access was considered, where a mobile terminal cannot transmit unless commanded to do so by the hub, either as a direct request or as a response to a carefully defined and limited group poll.

The National Aeronautics and Space Administration Jet Propulsion Laboratory (NASA/JPL) has been studying the technical feasibility of a personal access satellite system (PASS) since 1988 [2]-[5]. The Ka-band (20/30 GHz) is considered for the PASS because this band permits small user terminals suited for personal communications, particularly for handheld operation. Also, ample bandwidth is currently available in that band. The system under design would be capable of handling data rates ranging from less than 100 bps for emergency and other low-rate services, to several kbps for computer file transfers on the return link. For the forward link a data rate higher than 100 kbps is considered. One of the early architectures proposed for PASS called for TDMA in the forward direction, and FDMA in the return direction [2]. An alternative proposed architecture employs DS-CDMA for the return channel. Recently, it was shown that the use of spread spectrum is beneficial in the context
of PASS, and the use of DS-CDMA for the return channel in the PASS was highly recommended [5].

As of today, three other mobile satellite communications systems are being developed by different communications industries. Odyssey is a mobile satellite system being developed by TRW to provide worldwide personal communications systems [14]. Services to be provided include voice, data and paging. Odyssey will use 12 satellites in medium altitude to provide communications to mobile users in all major population area worldwide. Odyssey will use CDMA as multiple access technique.

Globalstar is another mobile satellite system being developed by Loral Qualcomm satellite services which is intended to provide worldwide extension to terrestrial wireless cellular services [13]. To provide worldwide services Globalstar will use either a 48 satellites at low earth orbit or 4 satellites at geostationary orbit. The fundamental requirement for Globalstar is voice and data at rates of up to 9.6 kbps. CDMA is also chosen to be the multiple access technique to be used for Globalstar.

Finally, Motorola is designing a mobile satellite system, called Iridium, to provide commercial, low-density, portable service employing low-profile antennas to millions of users throughout the world by using 66 satellites at a low earth orbit [12]. The multiple access technique to be used in the Iridium system has not yet been reported in the literature.

### 2.3 Fading Models for MSAT Channel

One of the main characteristic of mobile environment is multipath fading. Depending on the amplitude distribution of the received signal, the fading can be classified as, for example, Rayleigh, Rician or log-normal. Depending on the channel coherence bandwidth relative to the signal bandwidth, the fading can be classified as frequency
selective or frequency nonselective. Depending on the doppler spread or the fading rate relative to the signalling interval, the fading can be classified as slow or fast fading. Generally, the mobile satellite channel is either modeled as Rician fading model [125]-[127] or lognormally shadowed Rician fading model [122]-[124].

In the Rician fading model the received multipath signal consists of the sum of a direct or line-of-sight (LOS) component and a fading component due to reflections from the nearby scatters. The direct component is received without distortion other than an attenuation due to free space loss and a doppler shift due to movement of the vehicle. The multipath component is Rayleigh distributed in amplitude and uniformly distributed in phase. The direct component and multipath components combined give a received envelope which has the Rician distribution.

In the shadowed Rician fading model, the LOS component is subjected to a log-normal transformation due to shadowing, instead of being constant as in Rician model and the scatter path or multipath is Rayleigh distributed.

It was found in [64] and [131] that the BPSK bit error rate performance over lognormally shadowed Rician model is quite similar to the performance obtained over the Rician model with $K_f$ factor taken in the range of 3-10 dB. In this thesis we consider only Rician fading model with $K_f$ factor in the range of 3 to 10 dB.

If spread spectrum is used with a chip duration less than the delay spread of the channel, then the fading channel is considered to be frequency selective [49]-[64]. Therefore in this thesis both frequency selective and frequency nonselective fading channels are considered. As we will show in Chapter 4, the rapidity of fading depends on the vehicle speed. As the vehicle speed decreases the fading becomes slower, for that reason in this thesis we investigate also the effect of fading rate variation by considering fast and slow fading channel.
2.4 Overview of CDMA Technology

2.4.1 Introduction

CDMA is an application of spread spectrum techniques (SS). There are two basic SS techniques. The first one is known as direct sequence (DS) and the second one is called frequency hopping (FH).

In a DS-SS system, the baseband signal is spread by a pseudo-noise (PN) sequence (known also as the spreading code) in the transmitter. The resultant signal is at much higher rate than the baseband signal rate. At the receiver end, the same PN sequence is used to despread the received signal as a reference. The ratio of the PN sequence rate to the symbol rate is known as the spreading factor. If each data symbol contains a PN sequence of length \( N \), then the spreading factor equals \( N \).

In a FH-SS system, the signal carrier frequency hops periodically to different frequency cells according to a PN sequence pattern. In contrast to the DS system, where the spreading code is used one bit at a time, the spreading code here is used \( k \) bits at a time to choose one of the \( 2^k \) frequency tones. In the receiver, the hopped signal is dehopped by the same PN sequence.

Each of the two techniques, DS and FH has its own advantages and disadvantages and selection of one over the other is dependent only on the requirements and the particularities of a given application. For instance, a DS-SS system is generally simpler than a FH-SS system, while FH-SS system is more robust than a DS-SS system against near-far effects. However, the near-far problem is not as important in a mobile satellite network as it is in a fully connected network [51] or in a mobile cellular network [20]. In a mobile satellite system, the mobile users are almost equally distant from the satellite. Therefore, the near-far problem in mobile satellite environment is
mainly caused by fading and shadowing. Although, all users are equally distant from
the satellite some of them are “far” in a radio propagation sense because they are
faded and shadowed, and some are “near” because they are less faded or shadowed.
It has been shown that the near-far problem in mobile satellite environment can be
easily solved in the following manner [51], [20]. The forward link includes a pilot
carrier modulated by a separate, short PN code common to all users. The mobile
users measure the received pilot carrier power level to determine an estimate of the
mobile’s path loss for setting the return link transmitter power. Thus, the receivers
at the central station are subjected to the same average power from each transmitter
and the near-far problem is solved. In this thesis only DS-SS scheme is considered
in the analysis of mobile satellite packet networks and the adaptive power control
method proposed in [51] and [20] is expected to work.

Until recently, spread spectrum technology was used primarily by the military for
the purpose of overcoming intentional interference by hostile jamming radios. Because
of its noiselike characteristics, it has also been used to hide the transmitted signal
from detection. These antijamming and low probability of intercept properties of
spread spectrum radios can be very useful in commercial applications for the purpose
of allowing many spread spectrum radio users with different spreading codes to share
the same channel. Multiple users sharing the same channel are referred to as CDMA
as opposed to the more commonly used FDMA and TDMA techniques where signals
are assigned noninterfering frequency or time slots. Recently the FCC is actively
encouraging spread spectrum development in commercial applications [23], [32], [27].
Many field tests in North America have been conducted to investigate the technical
feasibility of using CDMA for cellular radio and mobile satellite communications
systems, in addition to the intensive research which is going on in this field worldwide.
Previous analysis of the performance of DS-CDMA systems falls into one of two groups. The first group focuses on the DS-CDMA channel model, deriving bit error rate and probability of packet success for a given spreading code and number of interfering transmissions. The other group focuses on the random access mode of operation of DS-CDMA by determining network performance, using measures such as throughput and delay. In what follows a description of these two issues are presented.

2.4.2 DS-CDMA Channel Models

In the computation of the throughput-delay performance of a DS-CDMA protocol two basic DS-CDMA channel models are usually considered. The first is a simple model in which no transmission errors occur if the number of simultaneous transmissions does not exceed a certain threshold known as system capacity, and transmission errors occur with a probability equals to one if the number of simultaneous transmissions exceeds the system capacity. This is referred to as the step function channel model, as the conditional probability of packet success versus the number of interfering transmissions is a step function [70]-[75]. There have been many publications on the estimation of the DS-CDMA system capacity [17]-[22]. The system capacity is found to be a function of the processing gain, required BER, modulation and FEC techniques. With this model, we can see that the use of the modulation and FEC techniques are implicitly incorporated in the performance evaluation of a DS-CDMA protocol.

The second DS-CDMA channel model is a more accurate one in which modulation and FEC techniques are explicitly incorporated in the performance of a DS-CDMA protocol. This means that the probability of packet success is not a step function of the number of simultaneous transmissions but it is a more general function that
depends on the channel environment, the type of modulation, error control coding and processing gain used. DS/BPSK is the modulation technique considered most of the time. The type of coding usually considered is convolutional coding with Viterbi decoding as well as block codes such as RS and BCH codes. Although the first model is very simple, in this thesis we consider only the second model because it is more accurate.

2.4.3 DS-CDMA Systems

There are two types of DS-CDMA systems, i.e., sequence synchronous and sequence asynchronous. In a sequence synchronous system, all the users transmit with their sequence period time aligned. This condition is difficult to maintain in most applications, particularly in a mobile environment. In a sequence asynchronous system no attempt is made to align the sequence period. This type of DS-CDMA is assumed in this thesis.

In addition to AWGN and fading which also exist in FDMA and TDMA systems, in CDMA system there is also MAI. For the BER performance analysis of DS-CDMA systems, much effort has been made in the evaluation of the MAI. The exact evaluation of the MAI is computationally difficult, so emphasis has been on approximations and bounds.

Different methods were used to determine the BER of a CDMA system. The method of moments [49], [34] and [37] and the characteristic function method [36], [57], [40] were widely used. The disadvantage of these approaches is the computational complexity. Techniques employing upper and lower bounds on the probability of bit error have also been discussed in [35] and [38]. The Gaussian approximation for the MAI is very attractive and was widely used by many researchers. Unfortu-
nately, that approximation is not generally accurate enough. An improved Gaussian approximation with good accuracy was presented in [41]-[42] for AWGN channel. In this thesis we extend the work in [41]-[42] and we analyze the BER performance of a DS-CDMA system over the multipath fading channel by modeling the MAI as an improved Gaussian process.

Since in this thesis packet communications is considered, we also investigate the packet error rate performance. The computation of the probability of packet success in a DS-CDMA system is even more difficult than the computation of the BER. This is due to the addition problem of the bit-to-bit dependence within a packet. It was shown in [41]-[42] that when asynchronous sequence DS-CDMA is considered we can no longer assume that bit errors within a packet are independent and one should take this effect into account in the computation of the probability of packet success. Furthermore, when the channel is corrupted by fading we expect that the bit-to-bit dependence increases and its effect on the probability of packet success is significant. This issue has not been investigated in the literature and in this thesis we examine this point in details.

2.4.4 DS-CDMA in a Random Access Mode of Operation

In the analysis of DS-CDMA network performance, direct sequence spread spectrum is combined with certain characteristics of ALOHA system. When DS-CDMA is combined with slotted ALOHA, this is referred to as slotted ALOHA/DS-CDMA, and if unslotted ALOHA is considered, this is referred to as unslotted ALOHA/DS-CDMA. In slotted ALOHA/DS-CDMA system, time on the channel is organized as a sequence of slots, in which case the users are required to wait until the beginning of some time slot to transmit or retransmit their packets, whereas in the unslotted
ALOHA/DS-CDMA protocol, the restriction of transmitting within a slot is removed. In both protocols, if after a given timeout an acknowledgment is not received, the user retransmits the same packet after a random delay.

Recently, researchers have concentrated on the unslotted ALOHA/DS-CDMA systems [71]-[76]. Unslotted systems are relatively easy to implement because no synchronization is required but more difficult to analyze because the number of interfering transmissions is not constant throughout the packet transmission time. Musser and Daigle [73] analyzed the throughput of an unslotted ALOHA/DS-CDMA network accounting for the multi-user interference with a threshold approximation. Daigle and Simon [72] provided a method of directly computing the throughput of an unslotted DS-CDMA system. Their methodology forms a basis for assessing the performance improvement which may be gained by providing a mechanism for sloting time. Abdelmonem and Saadawi [74] studied the threshold effect of channel load on the performance of an unslotted ALOHA/DS-CDMA network. They analyzed channel load sensing protocol under which a node refrains from transmitting if the channel load exceeds the channel threshold. Story and Tobagi [71] combined a Markovian network model and a model of a direct sequence BPSK radio network with convolutional coding in the throughput analysis of an unslotted ALOHA/DS-CDMA network. Lim and Lee [75] analyzed the throughput and packet delay of an unslotted ALOHA/DS-CDMA packet radio network, and considered the stability conditions of the network.

The early throughput analysis of random DS-CDMA protocol, such as the works by Raychaudhuri [77], Polydoros and Silvester [78], considered time slotted systems, in which the number of interfering transmissions is constant throughout the entire packet. The analysis of slotted ALOHA/DS-CDMA is relatively simple, however from practical implementation viewpoint, the synchronization requirement adds complexity
to the hardware and to the protocol.

In our study, we shall deal exclusively with slotted systems mainly because of their ease of analysis. It should be pointed out, however, that many military CDMA systems employ slotted systems since security considerations (such as periodic code changes) imply common clocks and timing [78]. Furthermore, the difference in efficiency between slotted and unslotted ALOHA/DS-CDMA systems is much less significant than their narrow-band ALOHA counterparts [72].

Most of the works mentioned above did not deal with the question of the number of codes to be used in the DS-CDMA system when it is designed to serve a large population of users. In this thesis we propose a new partial DS-CDMA protocol in which a small number of codes is required.

Furthermore most of the works mentioned above on the random DS-CDMA were based on the conjunction of DS-CDMA with the ALOHA protocol. It is well known that an ALOHA type system is inherently unstable [93]-[95], and statistical fluctuations may easily cause its saturation. This means that it reaches a situation where the number of retransmitting sources tends to infinity and the throughput tends to zero. It becomes well known that the collision resolution algorithms are stable and also have higher throughput than ALOHA. Collision resolution algorithms for CDMA systems are rarely addressed in the literature and worth investigation. Hu and Chang are the only ones who addressed this issue in [116]. However, a simple CDMA system in which perfect orthogonal codes are available and a perfect synchronization could be achieved in such a way the MAI is completely neglected. Neither fading, AWGN, modulation, nor error control coding were considered in their study. In this thesis we will propose a new collision resolution algorithm which can be used in conjunction with the proposed partial DS-CDMA protocol.
2.4.5 Spreading Codes

There is a large number of codes which can be used for direct sequence spread spectrum. The choice of which code to use in a given direct sequence spread spectrum system depends on the system application. A spreading code must meet several requirements: A code must have a full period autocorrelation function which has a sharp narrow peak and a high peak to sidelobe ratio. This is necessary for proper synchronization. The code must be balanced, i.e., must possess an equal number of zeros and ones. This is to suppress the chip rate from appearing on the power spectrum of the signal. If the direct sequence spread spectrum system is to operate in a multiple access environment, then the codes must exhibit low cross-correlation properties in order to minimize mutual interference. For highly secure systems a code must have an appropriate length. For example, be long enough to be non-repetitive within the duration of transmission.

Spreading codes are generally generated by shift register and can be classified into two categories: (i) linear codes which use linear feedback operators (modulo two adders) and (ii) Non-linear codes which use non-linear feed forward or feed back operators such as AND or OR gates. These codes are very difficult to reconstruct, hence, they are recommended for highly secure systems. Linear codes are the most commonly used ones for direct sequence spread spectrum systems. Common linear codes are m-sequences [28], Gold codes [116], [93] and composite codes [28], [32].

For a description of the large variety of codes available, one could refer to [28]-[32]. Since our study is not directly involved with the properties of the spreading codes, the performance of DS-CDMA network is determined for users using spreading codes that are completely random from bit-to-bit. Although the chip sequence is usually deterministic and periodic (although the sequence appears random within one of its
periods), analysis is sometimes simplified by assuming that the spreading codes are completely random [41].

2.5 Why CDMA for MSAT Systems?

The claim is that spread spectrum radios have the capability to allow many more active users than conventional narrowband analog and digital radios using FDMA or TDMA. The primary advantage of spread spectrum radios is their ability to tolerate a fair amount of interfering signals compared to conventional radios. From a system design point of view, this can be a considerable advantage. It was shown that CDMA could be used in a frequency band that has existing users [23]. As a result of the interference tolerance of CDMA, the system issues of frequency band assignment, adjacent channel interference and adjacent beam interference are greatly simplified when using spread spectrum technology. Moreover, flexibility in system design and development are greatly improved since some interference to others is not a problem. On the other hand, conventional radios must be carefully assigned a frequency or time slot to make sure that there is no interference with other similar radios and sophisticated filtering and guard band protection is required with FDMA and TDMA technologies. A guard time between time slots is required in TDMA. The guard time does occupy the time period for certain bits. These bits could be used to improve quality performance in TDMA. A guard band is required in FDMA to reduce the adjacent channel interference. Unlike FDMA and TDMA, CDMA does not require either guard band or guard time.

In many publications, it has been reported that the use of spread spectrum is beneficial in fading and shadowing environment [52]-[62]. Many field tests conducted recently have shown that the effect of fading due to multipath is negligible for CDMA
Many applications need secure high quality communications. At present, cellular and related mobile communications systems using FDMA and TDMA can be easily monitored by eavesdroppers. Many researchers suggest the use of some digital encryption techniques to offer higher degree of security. However, it was shown in [128] that some encryption techniques perform badly in fading channels. In that sense CDMA systems are very promising as they have the potential of providing some degree of privacy.

For mobile satellite applications, it has been shown in [20] that further capacity improvements for CDMA result from the use of spatial discrimination provided by satellite multibeam antennas, discrimination between multiple satellites providing co-coverage and cross-polarization frequency reuse. Spatial discrimination is equivalent to sectorization technique in cellular systems. The CDMA systems can reuse the entire frequency band again by utilizing the two opposite sense of circular polarization. Frequency reuse is possible because the mutual interference affecting a given channel is the sum of the mutual interference generated by the users with the same polarization plus the mutual interference generated by the opposite polarization attenuated by the cross-polarization isolation. This will increase capacity by a factor dependent on the cross-polarization attenuation level.

When all these systems issues are taken into account, the overall capacity advantage of using CDMA with spread spectrum is evident. Overall, CDMA spread spectrum will result in a greater net capacity. This capacity advantage will, however, depend to a large degree on the quality of the specific spread spectrum radio and the design of the entire system. Practical system issues such as the CDMA protocol over the whole network, type of spreading codes to be used, number of spreading codes
available, time synchronization among all users and code management, etc, will play a critical role in achieving the potential advantages of CDMA.

2.6 Overview of Collision Resolution Algorithms

As mentioned before the use of CDMA in a random access mode of operation is beneficial when a large population of bursty data traffic is considered. Most of the time ALOHA protocol as a random access is used in conjunction with CDMA. It is well known that ALOHA protocol is inherently unstable. If we look into the philosophy behind the ALOHA protocol, we notice that there is no serious attempt to resolve collisions among packets as soon as they occur. Instead, the attempts to resolve collisions are always deferred to the future, with the hope that things will then work out, somehow, but they never do.

In this section we review another type of random access protocol which is stable even for an infinite number of users; as far as the packet arrival rate does not exceed the rate at which collisions are resolved. These protocols are most of the time called collision resolution algorithm (CRA). In contrast to ALOHA type protocol, the efforts in the CRA are concentrated on resolving collisions as soon as they occur. Moreover, in most versions of these protocols, new packets that arrive to the system are inhibited from being transmitted while the resolution of collisions is in progress. This ensures that if the rate of arrival of new packets to the system is smaller than the rate at which collisions can be resolved (the maximal rate of departing packets), then the system is stable. The basic idea behind these algorithms is to exploit in a more sophisticated manner the feedback information that is available to the users in order to control the retransmission process so that collisions are resolved more efficiently and without chaotic events.
Most CRAs are based on assumptions identical to those assumed for the slotted ALOHA protocol. The channel is slotted and the users can transmit packets only at the beginning of slots. New packets arrive to the system according to a Poisson process with rate $\lambda$ packets per slot. In a conventional system (where neither CDMA nor capture techniques are used) if two or more packets are transmitted in a slot, a collision occurs and only the packets involved in the collision are allowed to retransmit in the next slots. At the end of each slot, the system users know what happened in that slot, namely, whether the slot was idle, or contained a successful transmission or there was a collision. This is known as the ternary feedback model. For some CRA, it suffices for the users to know whether the slot contained a collision or not. The latter is referred to as the binary feedback model.

The most basic CRA is called the binary tree algorithm and was proposed almost concurrently by Capetanakis [106], Tsybakov [107], and Hayes [108]. But in North America this algorithm is mostly known as Capetanakis' algorithm. According to this algorithm, when a collision occurs in slot $k$, all users that are not involved in the collision wait until the collision is resolved. The users involved in the collision split randomly into two subsets, by (for instance) each flipping a coin. The users in the first subset, those that flipped 0, retransmit in slot $k + 1$ while those that flipped 1 wait until all those that flipped 0 transmit successfully their packets. If slot $k + 1$ is either idle or contains a successful transmission, the users of the second subset those that flipped 1 retransmit in slot $k + 2$. If slot $k + 1$ contains another collision, then the procedure is repeated, i.e., the users whose packets collided in slot $k + 1$ flip a coin again and operate according to the outcome of the coin flipping, and so on.

Capetanakis [106] showed that this algorithm has a maximum throughput of 0.43 and is stable for all input rates less than 0.43. The maximum throughput attainable
with tree algorithms was later increased to 0.46 due to simple improvement first suggested by Massey [109]. The next improvement in throughput was due to Gallager [110] and also Tsybakov [111]. This algorithm eliminates the tree structure entirely and it is known in the literature as the Gallager's algorithm or part and try algorithm.

All the protocols mentioned above are first designed in an environment where every user must monitor the channel feedback at all times, even if that user has no packet to transmit. This is necessary because a newly generated packet can be transmitted for the first time only at the end of the collision resolution interval (CRI), requiring every user to positively determine the end of the CRI. This kind of feedback monitoring is known as full sensing. This mode of operation is quite impractical because a user that crashed can never again join the system and, furthermore, a user that due to some fault did not receive properly all signals may actually disturb the others and decrease the efficiency of the algorithm. Thus, it is desirable that users monitor the feedback signals only during limited periods, preferably after having generated a packet for transmission and until the packet is transmitted successfully. This kind of monitoring is knowing as limited sensing, and algorithms with such monitoring are referred to as algorithms operating in a limited sensing environment.

Not all the designed collision resolution algorithms can be operated in a limited sensing environment. For example, the Capetanakis' algorithm operates only in a full sensing environment and this is one of the drawbacks of this algorithm. Several collision resolution algorithms have been devised for the limited sensing environment. The simplest but not most efficient one, is the free access algorithm analyzed by Mathys [113] and by Fayolle [114]. To date the most efficient algorithm for a limited sensing environment is the one introduced by Humblet [115] and Georgiadis [103] which is essentially an adaptation of the full sensing Gallager's algorithm. However,
this algorithm is very sensitive to the channel feedback errors. Recently, Paterakis [99] developed a simple algorithm which is robust against feedback errors and operates in a limited sensing environment. In this thesis this algorithm will be modified to be used in conjunction with the partial DS-CDMA.
Chapter 3

DS-CDMA System over Multipath Fading Channels

3.1 Introduction

Different methods have been proposed in the literature to evaluate the probability of bit error for DS-CDMA systems. The characteristic function method was used in [36]. This method was further employed to analyze the DS-CDMA over fading channels [57] and binary/quaternary DS-CDMA systems [40]. The amount of calculations required for this method is proportional to $KN^2$ ($K$ is the number of simultaneous users and $N$ is the sequence length) which is very difficult for large $K$ and $N$ since a lot of integrations and convolutions are involved. The method of moments was used in [34], [37] and [49]. The disadvantage of these approaches is that the computational complexity is exponentially dependent on the number of system users. Techniques employing upper and lower bounds on the probability of bit error have also been discussed in [35] and [38]. The Gaussian approximation for the MAI is very attractive for its simplicity. Unfortunately, that approximation is not generally accurate enough. In [41], it was shown for AWGN channel, that the Gaussian approximation agrees with the exact result for a large number of simultaneous users $K$, conforming with
the central limit theorem, but gives optimistic results when $K$ is small (say $K < 15$). An improved Gaussian approximation with good accuracy was presented in [41]-[42] for AWGN channel.

In this chapter we analyze the performance of DS-CDMA over the multipath fading channel. First we evaluate the probability of bit error for asynchronous DS-CDMA without using error control coding. We investigate the accuracy of the approximation of the probability of bit error based on Gaussian model by comparing it to the one based on the improved Gaussian model. Then we consider the performance of DS-CDMA system using error control coding over the multipath fading channel. Both block and convolutional codes are considered. It is well known that the use of error correction coding improves the performance of a digital communication system. However, for mobile communications and moreover, when DS-CDMA is considered, fading and MAI induce channel memory that can degrade the coded performance. Most of the calculation on the probability of bit error using error correction coding for DS-CDMA over fading channel ignore the effect of bit-to-bit dependence caused by the fading and the MAI, by assuming fast fading and independent bit errors. In this chapter we also investigate the effect of bit-to-bit dependence on the probability of bit error when error control coding is used.

In Section 3.2 we describe the DS-CDMA system model considered in our study. First we describe the transmitter model, then we present the multipath fading model and the considered receiver model. In Section 3.3 we derive the probability of bit error for asynchronous DS-CDMA without error control coding over the multipath fading channels based on Gaussian and the improved Gaussian processes. In Section 3.4 and 3.5 we derive the probability of bit error for asynchronous DS-CDMA using block and convolutional codes respectively over the slow and fast fading channel. Finally
in Section 3.6 we summarize the conclusions of this chapter.

3.2 DS-CDMA System Model

The DS-CDMA system model used in our analysis is shown in Figure 3.1. This system consists of \( K \) simultaneous users communicating to a central station. The central station employs \( K \) parallel receivers, one for each user. In Figure 3.1 only the receiver of user 1 is shown because in this chapter we are interested in the performance of a single receiver given \( K \) simultaneous transmissions. The signal received at the desired receiver experiences fading, MAI and corrupted by additive white Gaussian noise (AWGN). In the following subsections we describe each element of the system shown in Figure 3.1.

3.2.1 Transmitter Model

The transmitted signal of the \( k \)'th user for a DS-CDMA system with BPSK modulation can be expressed as:

\[
s_k(t) = \sqrt{2P} \ b_k(t) \ a_k(t) \ \cos (\omega_c t + \theta_k) \tag{3.1}
\]

where \( b_k(t) \) is the data sequence of unit amplitude, positive and negative, rectangular pulses of duration \( T_b \), \( a_k(t) \) is the spreading signal which consists of a periodic sequence of unit amplitude, positive and negative, rectangular pulses of duration \( T_c \), \( P \) is the signal power, \( \omega_c \) represents the common angular center frequency and \( \theta_k \) is the phase of the \( k \)'th carrier. The data signal of the \( k \)'th user can be expressed as:

\[
b_k(t) = \sum_j b_k^j \ p_{T_b}(t - j T_b) \tag{3.2}
\]

and the spreading signal as

\[
a_k(t) = \sum_j a_k^j \ p_{T_c}(t - j T_c) \tag{3.3}
\]
Figure 3.1: DS-CDMA system model.
where $b_k^i$ and $a_k^j \in \{+1, -1\}$ and $p_r(t) = 1$ for $0 \leq t < \tau$ and $p_r(t) = 0$, otherwise. We assume that each bit is encoded with $N$ chips and that the signature sequence $(a_k^j)$ has a period $N$ where $N$ is known as the sequence length.

### 3.2.2 Fading Channel Model

In Figure 3.1, the link between each user transmitter and its receiver is corrupted by multipath fading. Although our study is mainly directed to mobile satellite applications, in this chapter we consider a general fading channel model which characterizes the mobile satellite channels as well as many other mobile communications channels. Therefore our results will be more general and useful for many applications. Upon the choice of the model parameters we can have AWGN channel, nonselective Rayleigh, nonselective Rician, selective Rayleigh etc.

This channel model is represented by a $L$ discrete multipath fading as shown in Figure 3.2. In this model we assume that the first path of the channel consists of a direct component (nonfaded component) and several nonselective fading components with delays approximately equal to the delay of the direct component. The differential delays for the components of the first path are much smaller than the chip duration $T_c$. Thus, the fading on the first path is modeled as nonselective Rician fading. Each of the remaining $L - 1$ paths consists of several nonselective fading channel components with small differential delays. The delays for the components of the $l'$th channel are all approximately $\tau$, so the fading on the $l'$th path may be modeled as a nonselective Rayleigh fading.

The low-pass equivalent impulse response of the passband fading channel for the link between the $k$'th user transmitter and its receiver is

$$h_k(t) = \sum_{l=1}^{L} \beta_{l,k} \delta(t - \tau_{l,k}) e^{j\phi_{l,k}} \quad (3.4)$$
Figure 3.2: Multipath fading channel.
where $\delta(\cdot)$ is the Kronecker delta, $\beta_{l,k}$, $\phi_{l,k}$ and $\tau_{l,k}$ are the $l$'th path gain, phase and time delay respectively of the $k$'th user. The difference between the maximum and minimum values of $\tau_{l,k}$ is called the maximum multipath delay spread and can be denoted by $T_m$.

For any input signal $s_k(t)$ that can be written as in eq. (3.1), the output signal $y_k(t)$ produced by the composite multipath fading channel model consists of a sum of delayed, phase shifted, attenuated replicas of the input signal, and therefore can be expressed as

$$y_k(t) = \sum_{l=1}^{L} \Re \left\{ \beta_{l,k} s_k(t - \tau_{l,k}) e^{j\phi_{l,k}} \right\} \tag{3.5}$$

where $\Re$ denotes the real part. Substituting (3.1) into (3.5) one obtains

$$y_k(t) = \sum_{l=1}^{L} \sqrt{2P} \beta_{l,k} b_k(t - \tau_{l,k}) a_k(t - \tau_{l,k}) \cos (\omega_c t + \varphi_{l,k}) \tag{3.6}$$

where $\varphi_{l,k} = \phi_{l,k} + \theta_k - \omega_c \tau_{l,k}$. In eq. (3.6), $\tau_{l,k}$ is taken as uniform in $[0, T_b]$ and $\varphi_{l,k}$ is Gaussian for $l = 1$ and uniform in $[0, 2\pi]$ for $l \neq 1$. As mentioned before, $\beta_{l,k}$ is a Rician random variable for $l = 1$ and all $k$, and a Rayleigh random variable for $l \neq 1$ and all $k$. Figure 3.3 shows the block diagram for generating the Rician and Rayleigh random processes. As shown in Figure 3.3, $\beta$ can be expressed as

$$\beta = c + f^t + jf^Q \tag{3.7}$$

where $c$ is a constant representing the nonfaded component, $f^t$ and $f^Q$ are zero mean statistically independent Gaussian random variables each having a variance $\sigma^2$. From [32] the probability density function (pdf) of $\beta$ could be expressed as

$$p(\beta) = \frac{\beta}{\sigma^2} e^{-\frac{(\beta^2 + c^2)}{2\sigma^2}} I_0 \left( \frac{\beta c}{\sigma^2} \right), \quad \beta > 0. \tag{3.8}$$

where $I_0(\cdot)$ is the modified Bessel function of zeroth order. The severity of fading is determined by the direct-to-scattered components power ratio known as $K_f$ factor.
Figure 3.3: Block diagram for generating a Rician and Rayleigh fading processes.
and defined as

\[ K_f = \frac{c^2}{2\sigma^2} \]  

(3.9)

If \( c = 0 \) which corresponds to \( K_f = 0 \) then \( \beta \) becomes a Rayleigh distributed and its pdf is given by:

\[ p(\beta) = \frac{\beta}{\sigma^2} e^{-\frac{\beta^2}{2\sigma^2}}, \quad \beta > 0. \]  

(3.10)

If \( \sigma^2 = 0 \) which corresponds to \( K_f \to \infty \), then \( \beta \) is a constant and thus we have an AWGN channel. As mentioned before, the multipath fading model considered in our study is a general one and upon the choice of the parameters \( c, \sigma^2 \) and \( L \) one can have different channel models.

Before ending this subsection and describing the receiver model, we shall give some information relevant to the choice of the required number of paths \( L \) in the fading channel model considered. It is well known that the number of paths depends on the channel coherence bandwidth and the multipath spread delay \( T_m \) which are in turn depends on the environment in which the mobile operates. For mobile radio communications, the multipath delay spread can be as large as 5-6\( \mu \)s in urban and metropolitan area [60], [62] and 0.5-1\( \mu \)s in a rural environment [120]. The existence of different time delays in two fading signals that are closely spaced in frequency can cause the two signals to become correlated. The frequency spacing that allows this condition to prevail is dependent upon the delay spread. This frequency interval is called the coherence bandwidth and denoted by \( B_c \). The precise definition of the coherence bandwidth often differs from one reference to another and tends to depend on the extend of the correlation, over which the two signals are correlated. A typical definition is \( B_c = 1/T_m \) [32]. The coherence bandwidth is typically in the range of 100 kHz to 2 MHz for mobile radio environment. For mobile satellite communications, there is no serious measurement for \( T_m \) and \( B_c \). Most of the researchers approximate
\( T_m \) and \( B_c \) for mobile satellite application by the one considered for mobile radio in a rural environment which is in the order of 0.5-1 \( \mu s \) [64]. Now if we consider the values of the delay spread, \( T_m \) in the range of 0.5-2 \( \mu s \) and we assume a data rate in the range of 2.4-9.6 kbps and a sequence length in the order of 31-255, one can find that \( L = \lceil T_m / T_c \rceil + 1 \) [32] varies from 1 to 6. Such values are considered in our study.

### 3.2.3 Receiver Model

The channel output \( y_k(t) \) is further corrupted by multiple access interference as well as system thermal noise which is modeled as additive white Gaussian noise \( n(t) \) of two-sided spectral density \( N_0/2 \), therefore the received signal, \( r(t) \) is given by:

\[
r(t) = \sum_{k=1}^{K} y_k(t) + n(t)
\]

\[
= \sum_{k=1}^{K} \sum_{l=1}^{L} \sqrt{2P} \beta_{l,k} b_k(t - \tau_{l,k}) a_k(t - \tau_{l,k}) \cos (w_c t + \varphi_{l,k}) + n(t) \quad (3.11)
\]

where \( K \) is the number of simultaneous users. Let us assume that the reference receiver 1 can ideally lock onto the first path. For mathematical simplification and with no loss in generality let us also assume that \( \varphi_{1,1} = 0 \) and \( \tau_{1,1} = 0 \). Such an assumption is widely taken in the literature [57]. Therefore, if the received signal \( r(t) \) is the input to the correlation receiver 1 matched to \( s_1(t) \), then the output is:

\[
Z_{1,1} = \int_{0}^{T_h} r(t) a_1(t) \cos(w_c t) \, dt \quad (3.12)
\]

\( Z_{1,1} \) represents the decision statistic of the receiver 1 when locked onto the first path. Substituting (3.11) into (3.12) one obtains

\[
Z_{1,1} = \sum_{k=1}^{K} \sum_{l=1}^{L} \sqrt{2P} \beta_{l,k} \int_{0}^{T_h} b_k(t - \tau_{l,k}) a_k(t - \tau_{l,k}) a_1(t) \cos (w_c t + \varphi_{l,k}) \cos(w_c t) \, dt
\]

\[
+ \int_{0}^{T_h} n(t) a_1(t) \cos(w_c t) \, dt \quad (3.13)
\]
Developing the previous equation we get

\[
Z_{1,1} = \sqrt{2P} \beta_{1,1} \int_0^{T_b} b_1(t) a_1^2(t) \cos^2(w_c t) \, dt \\
+ \sum_{k=2}^K \sqrt{2P} \beta_{1,k} \int_0^{T_b} b_k(t - \tau_{1,k}) a_k(t - \tau_{1,k}) a_1(t) \cos(w_c t) \cos(w_c t + \varphi_{1,k}) \, dt \\
+ \sum_{k=1}^K \sum_{l=2}^L \sqrt{2P} \beta_{l,k} \int_0^{T_b} b_k(t - \tau_{l,k}) a_k(t - \tau_{l,k}) a_l(t) \cos(w_c t + \varphi_{l,k}) \cos(w_c t) \, dt \\
+ \int_0^{T_b} n(t) a_1(t) \cos(w_c t) \, dt
\] (3.14)

In eq. (3.14) the first term represents the desired signal to be detected, the second and third term represent the self-interference and multiuser interference and finally the last term is the Gaussian random variable with zero mean and variance \(N_0 T_b/4\).

In a more compact form \(Z_{1,1}\) can be expressed as

\[
Z_{1,1} = \sqrt{P/2} \beta_{1,1} b_1^0 T_b + \sum_{k=2}^K \sqrt{P/2} \beta_{1,k} [b_k^{-1} R_{1,k}(\tau_{1,k}) + b_k^0 \hat{R}_{1,k}(\tau_{1,k})] \cos(\varphi_{1,k}) \\
+ \sum_{k=1}^K \sum_{l=2}^L \sqrt{P/2} \beta_{l,k} [b_k^{-1} R_{1,k}(\tau_{l,k}) + b_k^0 \hat{R}_{1,k}(\tau_{l,k})] \cos(\varphi_{l,k}) + \eta
\] (3.15)

where \(b_k^{-1}\) is the data bit of the \(k\)'th signal during \([0, \tau_k]\), \(b_k^0\) is the data bit of the \(k\)'th signal during \([\tau_k, T_b]\), \(R_{1,k}(\tau_{l,k})\), \(\hat{R}_{1,k}(\tau_{l,k})\) are the continuous time partial cross-correlation functions of the \(k\)'th and 1'st spectral spreading waveforms defined as:

\[
R_{1,k}(\tau_{l,k}) = \int_0^{\tau_{l,k}} a_k(t - \tau_{l,k}) a_1(t) \, dt,
\] (3.16)

\[
\hat{R}_{1,k}(\tau_{l,k}) = \int_{\tau_{l,k}}^{T_b} a_k(t - \tau_{l,k}) a_1(t) \, dt
\] (3.17)

and

\[
\eta = \int_0^{T_b} n(t) a_1(t) \cos(w_c t) \, dt
\] (3.18)

If we denote \(W_{l,k} = [b_k^{-1} R_{1,k}(\tau_{l,k}) + b_k^0 \hat{R}_{1,k}(\tau_{l,k})]\), then \(Z_{1,1}\) could be rewritten as

\[
Z_{1,1} = \sqrt{P/2} \beta_{1,1} b_1^0 T_b + \sqrt{P/2} \sum_{k=2}^K \beta_{1,k} W_{1,k} \cos(\varphi_{1,k})
\]

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\[ + \sqrt{P/2} \sum_{k=1}^{K} \sum_{l=2}^{L} \beta_{i,k} W_{i,k} \cos(\varphi_{i,k}) + \eta \]  

(3.19)

3.3 Probability of Bit Error for Uncoded System

Assuming that the data bits are equiprobable, and assuming that a 1 is transmitted, the bit error probability can be expressed as

\[ P_b = Pr \left( Z_{1,1} < 0 \mid b_i^0 = 1 \right) \]  

(3.20)

where \( Z_{1,1} \) is given by eq. (3.19). Below we determine the probability of bit error for two cases. First we model the MAI as a Gaussian process, then we consider the case where the MAI is an improved Gaussian process.

3.3.1 Gaussian Approximation

The Gaussian assumption is to take all the interference as Gaussian noise. We will base our calculation of the probability of bit error on eq. (3.19). To compute the probability of bit error we first find the probability of bit error conditioned on \( \beta_{1,1} \). The first term in eq. (3.19) is the signal term and for a fixed \( \beta_{1,1} \) it has an average power of

\[ P_{\text{sig}} = \frac{PT_b^2}{2} \beta_{1,1}^2 \]  

(3.21)

The last term in eq. (3.19) is a Gaussian random variable having zero mean and variance (power)

\[ P_{\text{noise}} = \frac{N_0T_b}{4} \]  

(3.22)

The second and third terms of eq. (3.19) represent the MAI. Since all terms of the summation are independent then the mean of the MAI reduces to zero and the average power which is also equal to the variance is given by the sum of the variances of the
second and third term. The variance of the second term is given by

\[ \sigma^2_{\text{int2}} = \sum_{k=2}^{K} E \left[ \left( \sqrt{\frac{P}{2}} \beta_{1,k} W_{1,k} \cos(\varphi_{1,k}) \right)^2 \right] = \frac{P}{2} \sum_{k=2}^{K} E \left[ (W_{1,k})^2 \right] E \left[ (\beta_{1,k} \cos(\varphi_{1,k}))^2 \right] \] (3.23)

and the variance of the third terms is given by

\[ \sigma^2_{\text{int3}} = \sum_{k=1}^{K} \sum_{l=2}^{L} E \left[ \left( \sqrt{\frac{P}{2}} \beta_{l,k} W_{l,k} \cos(\varphi_{l,k}) \right)^2 \right] = \frac{P}{2} \sum_{k=1}^{K} \sum_{l=2}^{L} E \left[ (W_{l,k})^2 \right] E \left[ (\beta_{l,k} \cos(\varphi_{l,k}))^2 \right] \] (3.24)

As mentioned before, \( \beta_{1,k} \) is a Rician random variable, thus

\[ E \left[ (\beta_{1,k} \cos(\varphi_{1,k}))^2 \right] = \frac{E(\beta_{1,k}^2)}{2} = \frac{c_k^2}{2} + \sigma_{1,k}^2 \] (3.25)

and \( \beta_{l,k} \) for \( l \neq 1 \) is Rayleigh random variable, thus

\[ E \left[ (\beta_{l,k} \cos(\varphi_{l,k}))^2 \right] = \frac{E(\beta_{l,k}^2)}{2} = \sigma_{l,k}^2 \quad \text{for} \quad l \neq 1 \] (3.26)

Pursley [33] has shown for Gold codes with sequence length \( N \) that

\[ E \left[ (W_{i,k})^2 \right] = \frac{2T_b^2}{3N} \] (3.27)

Substituting (3.25), (3.26), (3.27) into (3.23) and (3.24) one obtains

\[ \sigma^2_{\text{int2}} = \frac{PT_b^2}{6N} \sum_{k=2}^{K} (c_k^2 + 2\sigma_{1,k}^2) \] (3.28)

and

\[ \sigma^2_{\text{int3}} = \frac{PT_b^2}{3N} \sum_{k=1}^{K} \sum_{l=2}^{L} \sigma_{l,k}^2 \] (3.29)

If we assume that all \( c_k \)'s are equal and \( \sigma_{l,k}^2 \)'s are equal, i.e., \( c_k = c \) and \( \sigma_{l,k}^2 = \sigma_1^2 \) for all \( k \) and we also assume that all paths have equal variances, i.e., \( \sigma_{l,k}^2 = \sigma_1^2 \), then we have

\[ \sigma^2_{\text{int2}} = \frac{(K-1)PT_b^2}{6N} (c^2 + 2\sigma_1^2) \] (3.30)
\[ \sigma_{\text{int3}}^2 = \frac{K(L-1)PT_b^2}{3N} \sigma_1^2 \]  

Finally, the signal to noise ratio based on equal path variances is given by

\[
SNR = \frac{P_{\text{sig}}}{P_{\text{noise}} + \sigma_{\text{int2}}^2 + \sigma_{\text{int3}}^2} = \frac{\beta_{1,1}^2PT_b^2}{N_0T_b + \frac{(K-1)PT_b^2}{3N}(c^2 + 2\sigma_1^2) + \frac{K(L-1)PT_b^2}{3N}2\sigma_1^2} 
= \frac{\beta_{1,1}^2}{N_0E_b + \frac{(K-1)}{3N}(c^2 + 2\sigma_1^2) + \frac{K(L-1)}{3N}2\sigma_1^2} \]  

(3.32)

and the conditional probability of bit error is given by:

\[
P_b(\beta_{1,1}) = Q\left(\frac{\beta_{1,1}}{\sqrt{\frac{N_0}{2E_b} + \frac{(K-1)}{3N}(c^2 + 2\sigma_1^2) + \frac{K(L-1)}{3N}2\sigma_1^2}}\right) \]  

(3.33)

where \( E_b = PT_b \) is the energy per bit and the Q-function is defined as

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \]  

(3.34)

To obtain the probability of bit error when \( \beta_{1,1} \) is random, we must average \( P_b(\beta_{1,1}) \) over the pdf of \( \beta_{1,1} \) which is denoted as \( p(\beta_{1,1}) \). That is we must evaluate the integral

\[
P_b = \int_0^\infty P_b(\beta_{1,1}) p(\beta_{1,1}) \, d\beta_{1,1} 
= \int_0^\infty Q\left(\frac{\beta_{1,1}}{\sqrt{\frac{N_0}{2E_b} + \frac{(K-1)}{3N}(c^2 + 2\sigma_1^2) + \frac{K(L-1)}{3N}2\sigma_1^2}}\right) \, p(\beta_{1,1}) \, d\beta_{1,1} \]  

(3.35)

where \( p(\beta_{1,1}) \) is given by eq. (3.8) with \( \beta = \beta_{1,1} \).

If the first path of all users \( \beta_{1,k} \) is also modeled as a Rayleigh random variable, then the pdf of \( \beta_{1,1} \) is simply given by eq. (3.10) and the probability of bit error given
by eq. (3.35) reduces simply to

\[ P_b = \frac{1}{2} \left[ 1 - \sqrt{\frac{\gamma}{\gamma + 1}} \right] \]  

(3.36)

where

\[ \gamma = \frac{N_0}{(2\sigma_1^2)^2} + \frac{2(KL-1)}{3N} \]  

(3.37)

Eq. (3.36) is similar to the one developed in [50].

In eq. (3.31) we assumed that the variance for the path gain is the same for all paths and users; and is denoted by \( \sigma_1^2 \) which corresponds normally to the variance of the first path (i.e., for \( 1 \leq l \leq L \) and for all \( k \), \( \sigma_{1,k}^2 = \sigma_1^2 \)). In most physical situations, one should expect a reduction in the average path power (path variance) [51] and [64]. It has been reported in [64] that if spread spectrum modulation is used with a chip duration less than the delay spread of the channel, the multipath power is partially reduced by the correlation operation in the receiver. The parameters of the various path distributions can be found if the power-delay profile is known. In [64], it was considered that the power-delay profile is given by:

\[ P_d(\tau) = \frac{\sigma_1^2}{T_m} e^{-\frac{\tau}{T_m}} \]  

(3.38)

and due to the correlation operation, the multipath power is reduced. For path \( l \), the variance can be approximated as [64]:

\[ \sigma_{1,k}^2 = \sigma_1^2 \left( e^{-\frac{T_C}{T_m}}(l-1) \right) = \sigma_1^2 \left( e^{-\frac{T_C}{T_m}}(l-1) \right) \]  

for \( 2 \leq l \leq L \)  

(3.39)

To find the probability of bit error when the paths have different variances, we should recalculate the variance of the third term in \( Z_{1,1} \). By substituting (3.39) into (3.29), the variance of the third term becomes

\[ \sigma_{\text{int3}}^2 = \frac{PT_b^2}{3N} \sum_{k=1}^{K} \sum_{l=2}^{L} \sigma_{1,k}^2 = \frac{KPT_b^2}{3N} \sigma_1^2 \sum_{l=2}^{L} e^{-\frac{T_C}{T_m}}(l-1) = \frac{K\xi PT_b^2}{3N} \sigma_1^2 \]  

(3.40)
where

\[ \xi := \frac{1 - e^{-\frac{\Delta I_L}{l_i^L}}}{e^{-\frac{\Delta I_L}{l_i^L}} - 1} \quad (3.41) \]

If we approximate \( T_m/T_c \) by \( L - 1 \), then \( \xi \) becomes:

\[ \xi = \frac{1 - e^{-1}}{e^{-1} - 1} \quad (3.42) \]

and the SNR based on different path variances is:

\[ SNR = \frac{\beta_{1,1}^2}{\frac{N_0}{2E_b} + \frac{(K - 1)}{3N} (c^2 + 2\sigma_1^2) + \frac{K \xi}{3N} 2\sigma_1^2} \quad (3.43) \]

and the probability of bit error based on different path variances is given by:

\[ P_b = \int_0^\infty Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{N_0}{2E_b} + \frac{(K - 1)}{3N} (c^2 + 2\sigma_1^2) + \frac{K \xi}{3N} 2\sigma_1^2}} \right) p(\beta_{1,1}) \, d\beta_{1,1} \quad (3.44) \]

where \( \xi \) is given by eq. (3.42) and \( p(\beta_{1,1}) \) is given by eq. (3.8).

### 3.3.2 Improved Gaussian Approximation

An improved Gaussian approximation was presented in [41]-[42] and it was shown that this approximation provides accurate values for the probability of bit error for asynchronous DS-CDMA using random sequences in AWGN channel. In this subsection we extend the work in [41] and we evaluate the probability of bit error for asynchronous DS-CDMA in multipath fading channel using the improved Gaussian approximation. To follow the analysis in [41], let us normalize \( Z_{1,1} \) by \( T_c \) and set \( P = 2 \), thus \( Z_{1,1} \) becomes

\[ Z_{1,1} = \beta_{1,1} \, N + \sum_{k=2}^{K} \beta_{1,k} \, W_{1,k} \cos(\varphi_{1,k}) + \sum_{k=1}^{K} \sum_{l=2}^{L} \beta_{l,k} \, W_{l,k} \cos(\varphi_{l,k}) + \frac{\eta}{T_c} \quad (3.45) \]

Where \( W_{l,k} \) is also normalized by \( T_c \). As before, the first term in eq. (3.45) represents the desired signal, the second and third terms represent the MAI and finally the fourth
term represents the Gaussian noise. We see that if $\beta_{l,k}$ is fixed and $L = 1$, $Z_{1,1}$ given
by eq. (3.45) is equivalent to eq. (2) in [41]. So our model is a generalization of the
one adopted in [41]. As in [41] let $\Psi$ denote the variance of the MAI conditioned on
the path gains, delays and phases of all interfering signals and of the desired sequence
structure through a quantity $B = (N - 1 - C)/2$, where $C$ is the discrete aperiodic
autocorrelation of the signature sequence of receiver 1 and is given by:

$$C = \sum_{j=0}^{N-2} a_j^1 a_{j+1}^1$$

(3.46)

Since the MAI has zero mean and the terms of summation are independent, then the
conditional variance $\Psi$ is given by:

$$\Psi = \text{Var}[MAI \mid \tau_{1,k}, \varphi_{1,k}, \beta_{1,k}, B]$$

$$= E \left[ \left( \sum_{k=2}^{K} \beta_{1,k} W_{1,k} \cos(\varphi_{1,k}) + \sum_{k=1}^{K} \sum_{l=2}^{L} \beta_{l,k} W_{l,k} \cos(\varphi_{l,k}) \right)^2 \mid \tau_{1,k}, \varphi_{1,k}, \beta_{1,k}, B \right]$$

$$= \sum_{k=2}^{K} E \left[ (W_{1,k} \beta_{1,k} \cos(\varphi_{1,k}))^2 \mid \tau_{1,k}, \varphi_{1,k}, \beta_{1,k}, B \right]$$

$$+ \sum_{k=1}^{K} \sum_{l=2}^{L} E \left[ (W_{l,k} \beta_{l,k} \cos(\varphi_{l,k}))^2 \mid \tau_{1,k}, \varphi_{1,k}, \beta_{1,k}, B \right]$$

(3.47)

If we define $X_{1,k} = E \left[ (W_{l,k} \beta_{l,k} \cos(\varphi_{l,k}))^2 \mid \tau_{1,k}, \varphi_{1,k}, \beta_{1,k}, B \right]$, then $\Psi$ becomes simply

$$\Psi = \sum_{k=2}^{K} X_{1,k} + \sum_{k=1}^{K} \sum_{l=2}^{L} X_{l,k}$$

(3.48)

$X_{l,k}$ could be rewritten as:

$$X_{l,k} = E \left[ (\beta_{l,k} \cos(\varphi_{l,k}))^2 \mid \varphi_{l,k}, \beta_{l,k} \right] E \left[ W_{l,k}^2 \mid \tau_{1,k}, B \right]$$

(3.49)

The first term in eq. (3.49) is simply equal to $\beta_{l,k}^2 \cos^2(\varphi_{l,k})$ and the second term was
shown in [41] to be equal to $2(2B + 1)(S_{l,k}^2 - S_{l,k}) + N$. Thus, $X_{l,k}$ becomes

$$X_{l,k} = \beta_{l,k}^2 \cos^2(\varphi_{l,k}) \left[ 2(2B + 1)(S_{l,k}^2 - S_{l,k}) + N \right]$$

(3.50)
where \( S_{i,k} = \tau_{i,k} / T_c - \tau_{i,k} \) and \( \gamma_{i,k} = [\tau_{i,k} / T_c] \) [41]. Since \( \tau_{i,k} \) is a random variable that is uniformly distributed on \([0, T_c]\), \( S_{i,k} \) is a random variable uniformly distributed in \([0, 1]\). If the MAI is modeled as Gaussian, then the variance of the MAI can be found by averaging eq. (3.48) with respect to \( \tau_{i,k}, \phi_{i,k}, \beta_{i,k} \) and \( B \). We found that \( E(X_{i,k}) = N/3 \cdot E(\beta_{i,k}^2) \), and thus the variance of the MAI based on Gaussian approximation is

\[
\text{Var}[\text{MAI}] = \frac{N}{3} \sum_{k=2}^{K} E(\beta_{i,k}^2) + \frac{N}{3} \sum_{k=1}^{K} \sum_{l=2}^{L} E(\beta_{i,k}^2) \tag{3.51}
\]

If all the paths have equal variances, then

\[
\text{Var}[\text{MAI}] = \frac{(K - 1)N}{3} (c^2 + 2\sigma_1^2) + \frac{K(L - 1)N}{3} 2\sigma_1^2 \tag{3.52}
\]

and if all the paths have different variances as shown previously, then

\[
\text{Var}[\text{MAI}] = \frac{(K - 1)N}{3} (c^2 + 2\sigma_1^2) + \frac{K\xi N}{3} 2\sigma_1^2 \tag{3.53}
\]

where \( \xi \) is given by eq. (3.42). For equal path variances the \( \text{SNR} \) is given by:

\[
\text{SNR} = \frac{\beta_{i,1}^2 N^2}{\frac{N_0 T_b}{4T_c^2} + \frac{(K - 1)N}{3} (c^2 + 2\sigma_1^2) + \frac{K(L - 1)N}{3} 2\sigma_1^2} \tag{3.54}
\]

and for different path variances the \( \text{SNR} \) is given by:

\[
\text{SNR} = \frac{\beta_{i,1}^2 N^2}{\frac{N_0 T_b}{4T_c^2} + \frac{(K - 1)N}{3} (c^2 + 2\sigma_1^2) + \frac{K\xi N}{3} 2\sigma_1^2} \tag{3.55}
\]

If we substitute \( E_b = P T_b = 2 T_b \) into (3.54) and (3.55), we get the same \( \text{SNR} \)'s developed in the previous subsection. Let us return to the main objective of the evaluation of the probability of bit error using the improved Gaussian approximation.

As mentioned before, the conditional MAI variance \( \Psi \) is a function of the delays and phases of all interfering signals and the desired sequence structure, consequently,
each outcome $\Psi = \psi$ is produced by specific outcomes of $\tau_{l,k}$, $\varphi_{l,k}$, $\beta_{l,k}$ and $B$. In what follows, first we assume that MAI is the major source of errors, later the effect of the thermal noise is also taken into account. In [41] it has been shown that an accurate probability of bit error can be found by evaluating

$$
P_b(\beta_{1,1}) = \int_0^\infty P_b(\beta_{1,1} \mid \psi) p_\Psi(\psi) \, d\psi$$  \hspace{1cm} (3.56)

where $p_\Psi(\psi)$ is the probability density function of $\Psi$ and $P_b(\beta_{1,1} \mid \psi)$ is the probability of bit error conditioned on $\psi$ for a fixed $\beta_{1,1}$ and is given by:

$$
P_b(\beta_{1,1} \mid \psi) = Q\left(\frac{\beta_{1,1} N}{\sqrt{\psi}}\right)$$  \hspace{1cm} (3.57)

To simplify the notation, in the sequel we denote $P_b(\beta_{1,1} \mid \psi)$ as $P_b(\psi)$. Morrow and Lehner [41]-[42] have evaluated eq. (3.56) for AWGN channel by finding the probability density function of $\Psi$. The computations required to evaluate the probability density function of $\Psi$ are very cumbersome, especially in our case multipath fading is considered. Holtzman [39] used a simple approximation for the computation of probability of bit error for asynchronous DS-CDMA in AWGN channel which does not require the probability density function of $\Psi$ directly. By using a similar approach, we have computed the probability of bit error over the multipath fading. The objective here is to compute eq. (3.56) without carrying out the integration. This can be done by expanding $P_b(\Psi)$ using the Taylor series, i.e.,

$$
P_b(\Psi) = P_b(\mu_\Psi) + (\Psi - \mu_\Psi) P_b'(\mu_\Psi) + \frac{(\Psi - \mu_\Psi)^2}{2!} P_b''(\mu_\Psi) + \ldots + \frac{(\Psi - \mu_\Psi)^n}{n!} P_b^n(\mu_\Psi)$$  \hspace{1cm} (3.58)

where $\mu_\Psi$ is the mean of $\Psi$ and $P_b^n(\mu_\Psi)$ is the $n$'th derivative of $P_b(\Psi)$ evaluated at $\Psi = \mu_\Psi$. Taking the expectation of (3.58) with respect to $\Psi$ one obtains

$$
E\{P_b(\Psi)\} = P_b(\mu_\Psi) + \frac{\sigma_\Psi^2}{2!} P_b''(\mu_\Psi) + \ldots + \frac{\mu_\Psi^n}{n!} P_b^n(\mu_\Psi)$$  \hspace{1cm} (3.59)

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where \( \sigma_0^2 \) is the variance of \( \Psi \) and \( \mu_{\Psi,n} \) is the \( n \)’th moment given by \( E((\Psi - \mu_{\Psi})^n) \). We know that the first derivative of any function, \( P_b(\Psi) \) in which \( \Psi \) is a random variable could be approximated by:

\[
P'_b(\Psi) = \frac{P_b(\Psi + h) - P_b(\Psi - h)}{2h}
\]  

(3.60)

and the second derivative by:

\[
P''_b(\Psi) = \frac{P_b(\Psi + h) - 2P_b(\Psi) + P_b(\Psi - h)}{h^2}
\]

(3.61)

where \( h \) is the step size, \( h = \sqrt{3} \sigma_\Psi \) [39]. In what follows, we assume that only the first two terms in eq. (3.59) are important. Substituting (3.61) into (3.59) one obtains:

\[
E(P_b(\Psi)) = \frac{2}{3} P_b(\mu_{\Psi}) + \frac{1}{6} P_b(\mu_{\Psi} + \sqrt{3} \sigma_\Psi) + \frac{1}{6} P_b(\mu_{\Psi} - \sqrt{3} \sigma_\Psi)
\]

(3.62)

The mean of \( \Psi \) is given by eq. (3.52) for equal path variances and by eq. (3.53) for different path variances. To compute (3.62) we need to find \( \sigma_0^2 \). In (3.48) if we denote

\[
\omega_1 = \sum_{k=2}^{K} X_{1,k}
\]

(3.63)

and

\[
\omega_2 = \sum_{k=1}^{K} \sum_{l=2}^{L} X_{l,k}
\]

(3.64)

then the variance of \( \Psi \) is given by the sum of the variance of \( \omega_1 \), \( \omega_2 \) and twice the covariance of \( \omega_1 \) and \( \omega_2 \), so

\[
\sigma_0^2 = \text{Var}(\omega_1) + \text{Var}(\omega_2) + 2 \text{Cov}(\omega_1, \omega_2)
\]

(3.65)

For equal path variances we found that

\[
\text{Var}(\omega_1) = (K-1) \left[ \frac{7N^2 + 2N - 2}{40} E(\beta_{1,k}^2) - \frac{N^2}{9} E^2(\beta_{1,k}^2) + (K - 2) \frac{N - 1}{36} E^2(\beta_{1,k}^2) \right]
\]

(3.66)
\[
\text{Var}(\omega_2) = K(L-1) \left[ \frac{7N^2 + 2N - 2}{40} E(\beta_{1,k}) - \frac{N^2}{9} E^2(\beta_{1,k}) \right] + (K(L-1) - 1) \frac{N-1}{36} E^2(\beta_{1,k}) \] for \( l \neq 1 \) \quad (3.67)
\
and
\
\text{Cov}(\omega_1, \omega_2) = K(K-1)(L-1) \left( \frac{N-1}{36} E(\beta_{1,k}) E(\beta_{1,k}) \right) \] for \( l \neq 1 \) \quad (3.68)

The derivation of these equations are given in Appendix A. As mentioned before, \( \beta_{1,k} \) is Rician for all \( k \) and \( \beta_{l,k} \) is Rayleigh for \( l \neq 1 \) and all \( k \), thus the \( j \)'th moment for \( \beta_{1,k} \) and \( \beta_{l,k} \) \( (l \neq 1) \) is given by:
\[
E(\beta_{1,k}^j) = (2\sigma_1^2)^{j/2} e^{-c^2/2\sigma_1^2} \Gamma \left[ j + \frac{2}{2} \right] F_1 \left[ \frac{j+2}{2}, 1, c^2/2\sigma_1^2 \right] \] \quad (3.69)

and
\[
E(\beta_{l,k}^j) = (2\sigma_l^2)^{j/2} \Gamma[1 + j/2] \] \quad (3.70)

where \( \Gamma[p] \) is the gamma function defined as \((p-1)!\) when \( p \) is a positive integer and \( F_1[a, b, x] \) is the confluent hypergeometric function defined as [32]:
\[
F_1[a, b, x] = \sum_{k=0}^{\infty} \frac{\Gamma(k + a) \Gamma(b)}{\Gamma(a) \Gamma(k + b) \k!} \] \quad (3.71)

For equal path variances if we substitute (3.57), (3.52) into (3.62) one can find that the probability of bit error conditioned on \( \beta_{1,1} \) is given by:
\[
P_b(\beta_{1,1}) = \frac{2}{3} Q \left( \frac{\beta_{1,1}}{\sqrt{(K-1)\left( c^2 + 2\sigma_1^2 \right) + K(L-1)2\sigma_1^2}} \right) \\
+ \frac{1}{6} Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{3\sigma_\psi}{N^2} + \frac{(K-1)\left( c^2 + 2\sigma_1^2 \right) + K(L-1)2\sigma_1^2}} \right) \\
+ \frac{1}{6} Q \left( \frac{\beta_{1,1}}{\sqrt{-\frac{3\sigma_\psi}{N^2} + \frac{(K-1)\left( c^2 + 2\sigma_1^2 \right) + K(L-1)2\sigma_1^2}} \right) 
\] \quad (3.72)
where $\sigma_\psi$ is given by (3.65). If we take into consideration the effect of thermal noise, the probability of bit error conditioned on $\beta_{1,1}$ becomes:

\[
P_b(\beta_{1,1}) = \frac{2}{3} Q\left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2E_b} + \frac{(K-1)}{3N}(c^2 + 2\sigma^2) + \frac{K(L-1)}{3N}2\sigma^2}} \right)
+ \frac{1}{6} Q\left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2E_b} + \frac{\sqrt{3}\sigma_\psi}{N^2} + \frac{(K-1)}{3N}(c^2 + 2\sigma^2) + \frac{K(L-1)}{3N}2\sigma^2}} \right)
+ \frac{1}{6} Q\left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2E_b} - \frac{\sqrt{3}\sigma_\psi}{N^2} + \frac{(K-1)}{3N}(c^2 + 2\sigma^2) + \frac{K(L-1)}{3N}2\sigma^2}} \right)
\]

Finally, the probability of bit error using improved Gaussian approximation with equal path variances is

\[
P_b = \frac{2}{3} \int_0^\infty Q\left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2E_b} + \frac{(K-1)}{3N}(c^2 + 2\sigma^2) + \frac{K(L-1)}{3N}2\sigma^2}} \right) p(\beta_{1,1}) \, d\beta_{1,1}
+ \frac{1}{6} \int_0^\infty Q\left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2E_b} + \frac{\sqrt{3}\sigma_\psi}{N^2} + \frac{(K-1)}{3N}(c^2 + 2\sigma^2) + \frac{K(L-1)}{3N}2\sigma^2}} \right) p(\beta_{1,1}) \, d\beta_{1,1}
+ \frac{1}{6} \int_0^\infty Q\left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2E_b} - \frac{\sqrt{3}\sigma_\psi}{N^2} + \frac{(K-1)}{3N}(c^2 + 2\sigma^2) + \frac{K(L-1)}{3N}2\sigma^2}} \right) p(\beta_{1,1}) \, d\beta_{1,1}
\]

(3.74)

If the first path of all users $\beta_{1,k}$ is also a Rayleigh random variable, then the pdf of $\beta_{1,1}$ is simply given by eq. (3.10) and the probability of bit error given by eq. (3.74)
reduces simply to

\[ P_b = \frac{1}{3} \left[ 1 - \sqrt{\frac{\gamma_0}{\gamma_0 + 1}} \right] + \frac{1}{12} \left[ 1 - \sqrt{\frac{\gamma_1}{\gamma_1 + 1}} \right] + \frac{1}{12} \left[ 1 - \sqrt{\frac{\gamma_2}{\gamma_2 + 1}} \right] \]  \hspace{1cm} (3.75)

where \( \gamma_0 = \gamma \) given by eq. (3.37),

\[ \gamma_1 = \frac{1}{N_o \left( \frac{2\sigma_r^2}{E_0} + \frac{2\sqrt{3} \sigma \phi}{N^2} + \frac{2(KL - 1)}{3N} \right)} \]  \hspace{1cm} (3.76)

and

\[ \gamma_2 = \frac{1}{N_o \left( \frac{2\sigma_r^2}{E_0} + \frac{2\sqrt{3} \sigma \phi}{N^2} + \frac{2(KL - 1)}{3N} \right)} \]  \hspace{1cm} (3.77)

with \( \sigma \phi \) in this case is given by:

\[ \sigma_r^2 = (KL - 1) \left[ \frac{7N^2 + 2N - 2}{20} - \frac{N^2}{9} + (KL - 2) \frac{N - 1}{36} \right] \]  \hspace{1cm} (3.78)

Now if the first path of some of the users is Rician random variable and the first path of the rest of the users is Rayleigh random variable, therefore the probability of bit error will be lower bounded by eq. (3.74) and upper bounded by eq. (3.75). This means that the BER performance in this case is in between the performance for all users having the first path Rician distributed and the performance for all users having the first path Rayleigh distributed.

3.3.3 Numerical Results

Before starting the analysis of the numerical results, it is important to mention that the accuracy of the probability of bit error based on the improved Gaussian model for the MAI has been studied in [41] for AWGN channel; and it was shown that a significant improvement on the accuracy of the probability of bit error was obtained by modeling the MAI as an improved Gaussian process. In what follows we will not repeat this analysis and we shall concentrate more on the comparison
of our equations with the one developed in [41], then investigate the accuracy of the probability of bit error based on Gaussian process versus the one based on the improved Gaussian process under different fading conditions and for different system parameters. However, for completeness reason we shall give a brief discussion on the accuracy of the probability of bit error based on the improved Gaussian model for the MAI in Appendix C.

Figure 3.4 represents a comparison of the probability of bit error by modeling the MAI as an improved Gaussian process for asynchronous DS-CDMA over nonfaded channel developed by Morrow and Lehnert in [41] and the one developed in this section. In this figure we take $N = 63$ and $E_b/N_0 = \infty$. As we can see that our equation for the probability of bit error is in good agreement with the one developed in [41]. Recall that our equation is much simpler to compute, it does not require the computation of the pdf of $\Psi$. In the sequel we discuss the probability of bit error for different system parameters using only the equations developed in this section.

Figure 3.5 compares the results of the probability of bit error for asynchronous DS-CDMA over nonfaded channel with different sequence length $N$ based on the standard Gaussian and improved Gaussian models for the MAI. The standard Gaussian model seems to be accurate only for large number of simultaneous users $K$ which corresponds to high probability of bit error. We note also from this figure that the probability of bit error based on Gaussian approximation becomes very optimistic as $N$ grows. Consequently, using the standard Gaussian approximation to model the MAI could result in a bit error rate that is lower than the actual value. These results have been also found in [41] and [39]. Now let us discuss the accuracy of the standard Gaussian approximation when the channel is corrupted by multipath fading. The following results are considered to be the contribution of the analysis presented in this chapter.
Figure 3.4: Probability of bit error for asynchronous DS-CDMA over nonfaded channel with \( N = 63 \) and \( E_b/N_o = \infty \) (Pb1 from [41] and Pb2 this chapter).
Figure 3.5: Probability of bit error for asynchronous DS-CDMA over nonfaded channel with different $N$ and $E_b/N_0 = \infty$. 
Figure 3.6 shows the probability of bit error for asynchronous DS-CDMA over Rician fading channel \((L = 1)\) with different \(K_f\) factors, \(N = 31\) and \(E_b/N_o = \infty\). The main interpretation from this figure is that the accuracy of the standard Gaussian approximation depends on the \(K_f\) factor. As \(K_f\) becomes smaller (less than 5 dB), the accuracy of the standard Gaussian approximation becomes acceptable. This could be explained by the fact that as \(K_f\) decreases, the probability of bit error becomes higher and as we said earlier the Gaussian approximation gives accurate results for high probability of bit error.

Figure 3.7 shows the probability of bit error for asynchronous DS-CDMA over Rician fading channel \((L = 1)\) with different \(N\), for \(K_f = 10\) dB and \(E_b/N_o = \infty\). From this figure we can say that the accuracy of the standard Gaussian approximation does not depend on \(N\) as found for the nonfaded channel. The discrepancy between the standard Gaussian approximation and the improved Gaussian approximation remains almost the same when \(N\) is changed.

Figure 3.8 shows the probability of bit error for asynchronous DS-CDMA over Rician fading channel with different number of paths \(L\) for \(K_f = 10\) dB, \(N = 31\) and \(E_b/N_o = \infty\). Only equal path variances is considered in this figure. As we can see from the figure as \(L\) increases the probability of bit error increases. This is an expected result. Also we can say that as \(L\) increases the discrepancy between the standard Gaussian approximation and the improved Gaussian approximation reduces and for large \(L\) the standard Gaussian approximation gives accurate results. This result is reasonable and conform with the central limit theorem.
Figure 3.6: Probability of bit error for asynchronous DS-CDMA over Rician fading channel ($L = 1$) with different $K_f$ factors, $N = 31$ and $E_b/N_0 = \infty$. 

N=31, L=1, No=0

- standard Gaussian
- improved Gaussian

Simultaneous users

Probabilty of bit error

$10^{-2}$

$10^{-3}$

$10^{-4}$

2 3 4 5 6 7 8 9 10 11 12

5 dB

10 dB

$K_f=15$ dB
Figure 3.7: Probability of bit error for asynchronous DS-CDMA over Rician fading channel \((L = 1)\) with different \(N\) for \(K_f = 10\) dB and \(E_b/N_o = \infty\).
Figure 3.8: Probability of bit error for asynchronous DS-CDMA over Rician fading channel with different number of paths, for $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. 

$N=31, K_f=10$ dB, $N_o=0$

--- standard Gaussian

--- improved Gaussian
Figure 3.9 shows the probability of bit error for asynchronous DS-CDMA over Rician fading channel \((L = 1)\) with different \(E_b/N_0\), for \(K_f = 10\) dB and \(N = 31\). From this figure we can say that the thermal noise increase significantly the probability of bit error and consequently the discrepancy between the two models for the MAI reduces. Such result was also found by the author in [45] and [79], where it was recommended that the effect of thermal noise should be taken into account in the design of DS-CDMA network.

Figure 3.10 shows the probability of bit error for asynchronous DS-CDMA over Rayleigh fading channel \((\beta_{1,k} \text{ is Rayleigh})\) with different \(N\), \(L = 1\) and \(E_b/N_0 = \infty\). This figure shows that the BER performance over Rayleigh fading channel is much worse than the performance over Rician fading channel, and this is an expected result. Therefore some techniques to combat the fading effect must be used. This figure also shows that the standard Gaussian approximation gives accurate results for Rayleigh fading channel which confirms the results of Figure 3.6 and Figure 3.9 which prove that both models for the MAI give similar results for high probability of bit error.
Figure 3.9: Probability of bit error for asynchronous DS-CDMA over Rician fading channel ($L = 1$) with different $E_b/N_0$, for $K_f = 10$ dB and $N = 31$. 
Figure 3.10: Probability of bit error for asynchronous DS-CDMA over Rayleigh fading channel \((L = 1)\) with different \(N\) and \(E_b/N_0 = \infty\).
3.4 Probability of Bit Error using Block Codes

It is well known that the performance of a digital communication system can usually be improved by using error control coding. In an asynchronous DS-CDMA system, it is well known that the MAI induces channel memory [41]. Furthermore, for mobile communications, the fading also induces channel memory. This channel memory, or the bit-to-bit dependence can degrade the coded performance significantly. Traditionally, this is remedied by using enough interleaving to destroy the channel memory. Unfortunately, a large interleaving causes a large delay which is not acceptable in some applications [63].

In a mobile environment, the fading rate depends on vehicle speed. At high speeds, the fading rate is fast enough so that a small amount of interleaving will effectively destroy the channel memory [129]-[131]. However, at low vehicle speeds, the channel memory induced by the fading is important and a large amount of interleaving is required. As mentioned before, in some applications the delay requirements preclude the use of such large amount of interleaving, so the channel will have memory. The effect of bit-to-bit dependence caused by the MAI and the fading on the performance of error correction codes is illustrated here by considering $t$ error correcting BCH $(n_c,k_c)$ codes. Two extreme cases are considered, slow fading and fast fading. The relevant criteria of these two fading conditions are discussed in more details in Chapter 4. In what follows we assume that for the slow fading case, the bit energy to total noise ratio is random but constant over the duration of an entire codeword. In the case of fast fading, the channel variations are fast relative to the signalling interval. Thus each signalling bit undergoes fading independently from other bits. For both slow and fast fading we model the MAI as either Gaussian or an improved Gaussian
process.

3.4.1 Fast Fading and Gaussian Approximation

Assume that the channel bit errors are statistically independent, i.e., the bit errors at the block decoder input are independent. Which means that we ignore the effect of bit-to-bit dependence caused by the fading and the MAI. Modeling the MAI as Gaussian process, the probability of information bit error (at the decoder output) for asynchronous DS-CDMA over the multipath fading channel using \((n_c, k_c)\) block codes that can correct \(t\) or fewer errors is given by [32]:

\[
P_b = \frac{1}{n_c} \sum_{i=t+1}^{n_c} \binom{n_c}{i} [P_{cb}]^i [1 - P_{cb}]^{n_c-i}
\]  

(3.79)

where \(P_{cb}\) is the channel bit error probability which can be obtained from eq. (3.35) by replacing \(E_b\) by \(rE_b\) where \(r\) is the code rate defined as \(r = k_c/n_c\).

3.4.2 Fast Fading and Improved Gaussian Approximation

By assuming independent bit errors and modeling the MAI as an improved Gaussian, the probability bit error in this case is still given by eq. (3.79) where the channel bit error probability \(P_{cb}\) is given by eq. (3.74) with \(E_b\) is replaced by \(rE_b\).

3.4.3 Slow Fading and Gaussian Approximation

With slow fading, the probability of bit error given by eq. (3.79) is conditioned on \(\beta_{1,1}\); then this conditional probability is averaged with respect to \(\beta_{1,1}\) to find the unconditional probability of bit error, which takes into account the bit-to-bit dependence due to fading. Thus, we have to evaluate the following integral

\[
P_b = \int_0^\infty \frac{1}{n_c} \sum_{i=t+1}^{n_c} \binom{n_c}{i} [P_{cb}(\beta_{1,1})]^i [1 - P_{cb}(\beta_{1,1})]^{n_c-i} p(\beta_{1,1}) d\beta_{1,1}
\]  

(3.80)
where $P_{cb}(\beta_{1,1})$ is the conditional channel bit error probability and is given by eq. (3.33) with replacing $E_b$ by $rE_b$.

### 3.4.4 Slow Fading and Improved Gaussian Approximation

To take into account the effect of bit-to-bit error dependence caused by the MAI and fading, and to use an improved Gaussian approximation for the MAI, we condition the probability of bit error given by eq. (3.79) on $\Psi$ and $\beta_{1,1}$, then we average the conditional probability of bit error with respect to $\Psi$ and $\beta_{1,1}$. Following the same steps as before we found that the probability of bit error for asynchronous DS-CDMA using block codes is given by:

$$
P_b = \frac{2}{3n_c} \sum_{i=l+1}^{n_c} i \binom{n_c}{i} \int_0^\infty [P_{cb1}(\beta_{1,1})]^i [1 - P_{cb1}(\beta_{1,1})]^{n_c-i} p(\beta_{1,1}) \, d\beta_{1,1}
+ \frac{1}{6n_c} \sum_{i=l+1}^{n_c} i \binom{n_c}{i} \int_0^\infty [P_{cb2}(\beta_{1,1})]^i [1 - P_{cb2}(\beta_{1,1})]^{n_c-i} p(\beta_{1,1}) \, d\beta_{1,1}
+ \frac{1}{6n_c} \sum_{i=0}^{l} i \binom{n_c}{i} \int_0^\infty [P_{cb3}(\beta_{1,1})]^i [1 - P_{cb3}(\beta_{1,1})]^{n_c-i} p(\beta_{1,1}) \, d\beta_{1,1}
$$

(3.81)

where $p(\beta_{1,1})$ is given by eq. (3.8), $P_{cb1}(\beta_{1,1})$, $P_{cb2}(\beta_{1,1})$ and $P_{cb3}(\beta_{1,1})$ are given by:

$$
P_{cb1}(\beta_{1,1}) = Q\left(\frac{\beta_{1,1}}{\sqrt{\frac{N_c}{2rE_b} + \frac{(K-1)(c^2 + 2\sigma_t^2)}{3N} + \frac{K(L-1)}{3N}2\sigma_t^2}}\right)
$$

(3.82)

$$
P_{cb2}(\beta_{1,1}) = Q\left(\frac{\beta_{1,1}}{\sqrt{\frac{N_c}{2rE_b} + \frac{\sqrt{3}\sigma_{\Psi}}{N} + \frac{(K-1)(c^2 + 2\sigma_t^2)}{3N} + \frac{K(L-1)}{3N}2\sigma_t^2}}\right)
$$

(3.83)

$$
P_{cb3}(\beta_{1,1}) = Q\left(\frac{\beta_{1,1}}{\sqrt{\frac{N_c}{2rE_b} - \frac{\sqrt{3}\sigma_{\Psi}}{N} + \frac{(K-1)(c^2 + 2\sigma_t^2)}{3N} + \frac{K(L-1)}{3N}2\sigma_t^2}}\right)
$$

(3.84)

65
3.4.5 Numerical Results

The block codes used in this section are (7,4), (15,7) and (31,16) BCH codes with error correcting capability $t$ equals to 1, 2, and 3 respectively. The code rate for these codes is approximately equal to 0.5. Figure 3.11 shows the performance of these different codes for asynchronous DS-CDMA over the nonfaded channel with $N = 31$ and $E_b/N_o = \infty$. The results are shown for the MAI modeled as Gaussian process as well as improved Gaussian process. As we can see from this figure these codes improve significantly the bit error rate performance. As $t$ increases the improvement also increases (i.e., (31,16) has better performance than (15,7) and (15,7) has better performance than (7,4)).

Figure 3.12 shows the performance of different BCH codes for asynchronous DS-CDMA over the fast Rician fading channel ($L = 1$) with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. As with the nonfaded case, the BCH codes significantly improve the BER performance for fast fading channel. Also this improvement increases by increasing $t$. This figure also shows that Gaussian model yields a bit optimistic results on the BER, especially for small number of simultaneous user ($K < 15$).

Figure 3.13 shows the performance of different BCH codes for asynchronous DS-CDMA over the slow Rician fading channel ($L = 1$) with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. The BCH codes do not improve the BER performance significantly for slow fading channel. Also this improvement does not increase when $t$ increases. This is an expected results because the slow fading induces channel memory which degrades the code performance. For comparison purposes we plot in Figure 3.14 the BER for asynchronous DS-CDMA over the slow and fast Rician fading channels ($L = 1$) for (15,7) BCH code with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. This figure shows that the channel memory degrades significantly the performance of this code.
Figure 3.11: Probability of bit error for asynchronous DS-CDMA over nonfaded channel using different BCH codes with \( N = 31 \) and \( E_b/N_0 = \infty \).
Figure 3.12: Probability of bit error for asynchronous DS-CDMA using different BCH codes over the fast Rician fading channel ($L = 1$) with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. 
Figure 3.13: Probability of bit error for asynchronous DS-CDMA using different BCH codes over the slow Rician fading channel ($L = 1$) with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. 
Figure 3.14: Probability of bit error for asynchronous DS-CDMA using (15,7) BCH code over the fast and slow Rician fading channels ($L = 1$) with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$. 
3.5 Probability of Bit Error using Convolutional Codes

In this section, following a similar analysis to the one given for block codes, we develop the probability of bit error expression for asynchronous DS-CDMA using convolutional codes over the multipath fading for different cases.

3.5.1 Fast Fading and Gaussian Approximation

If we assume that the channel bit errors are statistically independent, then a tight upper bound for the probability of bit error at the Viterbi decoder output is given by [43]:

\[
P_b = \left( \frac{2^{n_o} - 1}{n_o} \right) 2^{-2n_o} \frac{d}{dN_c} \left\{ T(D, N_c) + T(-D, N_c) \right\} + \right.
\]

\[
D \left[ T(D, N_c) - T(-D, N_c) \right] \bigg|_{D = 2\sqrt{rE_b}N_c = 1}
\]  

(3.85)

where \( P_{cb} \) is the channel bit error probability and \( T(D, N_c) \) is the augmented transfer function of the convolutional code and \( n_o \) is half the free distance of the code (or half the “free distance plus one”, if the distance is odd). If the MAI is a standard Gaussian process then \( P_{cb} \) is given by eq. (3.35) with replacing \( E_b \) by \( rE_b \).

3.5.2 Fast Fading and Improved Gaussian Approximation

If we assume independent bit errors and model the MAI as an improved Gaussian process, then \( P_b \) is given by eq. (3.85) where \( P_{cb} \) is given by eq. (3.74) with replacing \( E_b \) by \( rE_b \).

3.5.3 Slow Fading and Gaussian Approximation

It has been shown in [43]-[44] for AWGN channel that the bit errors at the output of Viterbi decoder cluster within a short segment of decoded data. These clusters
are known as error events. These error events are statistically independent. The computation of the probability of bit error at the Viterbi decoder output eq. (3.85) was based in the assumption that the bit errors are also independent within these error events. For slow fading condition this assumption is no longer valid. To take into account the bit-to-bit dependence due to fading, we assume that fading is slow during the length of these error events, thus we have to evaluate

\[ P_e = \int_0^\infty P_b(P_{cb}(\beta_{1,1})) \ p(\beta_{1,1}) \ d\beta_{1,1} \] (3.86)

with the MAI is Gaussian \( P_b(\cdot) \) and \( P_{cb}(\beta_{1,1}) \) are given by eq. (3.85) and eq. (3.82) respectively.

3.5.4 Slow Fading and Improved Gaussian Approximation

The probability of bit error for asynchronous DS-CDMA using convolutional codes which takes into account the effect of bit-to-bit error dependence caused by fading and the MAI and uses an improved Gaussian approximation for the MAI is given by:

\[ P_e = \int_0^\infty \left[ \frac{2}{3} P_b(P_{cb1}(\beta_{1,1})) + \frac{1}{6} P_b(P_{cb2}(\beta_{1,1})) + \frac{1}{6} P_b(P_{cb3}(\beta_{1,1})) \right] p(\beta_{1,1}) \ d\beta_{1,1} \] (3.87)

where \( P_b(\cdot) \) is given by eq. (3.85), \( P_{cb1}(\beta_{1,1}) \), \( P_{cb2}(\beta_{1,1}) \) and \( P_{cb3}(\beta_{1,1}) \) are given by eqs. (3.82), (3.83) and (3.84) respectively.

3.5.5 Numerical Results

Figure 3.15 shows the probability of bit error for asynchronous DS-CDMA using a convolutional code with rate \( r = 1/2 \) and constraint length \( CL = 3 \) over the fast and slow Rician fading channel (\( L = 1 \)) with \( K_f = 10 \) dB, \( N = 31 \) and \( E_b/N_0 = \infty \). This code shows similar performance to the (15,7) BCH code for both slow and fast fading conditions using Gaussian and improved Gaussian models for the MAI.
Figure 3.15: Probability of bit error for asynchronous DS-CDMA using a convolutional code with rate $r = 1/2$ and constraint length $CL = 3$ over the fast and slow Rician fading channels ($L = 1$) with $K_f = 10$ dB, $N = 31$ and $E_b/N_0 = \infty$. 
3.6 Summary

In this chapter we analyzed the performance of uncoded and coded asynchronous DS-CDMA systems over multipath fading channels. The fading channel model adopted is a general one, where upon the choice of the fading parameters we can have different fading models, such as Rician fading, Rayleigh fading, etc. Using these fading models, we developed the probability of bit error for the asynchronous DS-CDMA under different assumptions.

For the uncoded system, the probability of bit error was computed by two methods. In the first method we assumed that the MAI is Gaussian and in the second one we modeled the MAI as an improved Gaussian. The main conclusion is that the computation of the probability of bit error based on the Gaussian approximation for the MAI is not accurate for low probability of bit error. However, it was shown that when the channel exhibits a deep fading the standard Gaussian approximation and the improved Gaussian approximation give more or less the same results. This is due to the fact that for deep fading the probability of bit error is quite high. Table 3.1 summarizes some of the results obtained for the uncoded system.

For coded system, we investigated further the effect of the bit-to-bit dependence on the computation of the probability of bit error by considering a slow fading. The main conclusion we reached is that the bit-to-bit dependence caused by the MAI and the slow fading degrade significantly the bit error rate performance. One should expect that the bit-to-bit effect to be more important on the performance of packet transmission. Table 3.2 summarizes some of the results obtained for the coded system.

Since our objective is the study of DS-CDMA packet networks, we should carefully investigate the effect of the bit-to-bit dependence within a packet as well as the
accuracy of the Gaussian assumption on the computation of the probability of packet success which is going to be the topic of the next chapter.
Table 3.1: Comparison of the probability of the error for DS-CDMA without FEC based on standard Gaussian approximation $P_e$ and improved Gaussian approximation $P_e^*$ (e is the relative error).

| $N$ | $P_e$ | $P_e^*$ | $P_e$ | $P_e^*$ | $P_e$ | $P_e^*$ | $P_e$ | $P_e^*$ | $P_e$ | $P_e^*$ |
|-----|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|
| 1,617 | 2.516 | 1.512 | 2.516 | 1.512 | 2.516 | 1.512 | 2.516 | 1.512 | 2.516 | 1.512 |
| 1,281 | 1.962 | 1.223 | 1.962 | 1.223 | 1.962 | 1.223 | 1.962 | 1.223 | 1.962 | 1.223 |
| 1,080 | 1.536 | 1.023 | 1.536 | 1.023 | 1.536 | 1.023 | 1.536 | 1.023 | 1.536 | 1.023 |
| 908.0 | 1.211 | 0.816 | 1.211 | 0.816 | 1.211 | 0.816 | 1.211 | 0.816 | 1.211 | 0.816 |
| 762.0 | 0.987 | 0.687 | 0.987 | 0.687 | 0.987 | 0.687 | 0.987 | 0.687 | 0.987 | 0.687 |
| 639.0 | 0.875 | 0.589 | 0.875 | 0.589 | 0.875 | 0.589 | 0.875 | 0.589 | 0.875 | 0.589 |

Note: The values in the table are for $K = 15$ and $K = 10$.
Table 3.2: Comparison of the probability of bit error for DS-CDMA using KOU codes and convolutional code with relative error.

<table>
<thead>
<tr>
<th>System</th>
<th>$K$ = 20</th>
<th>$K$ = 15</th>
<th>$K$ = 10</th>
<th>$K$ = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conv</td>
<td>0.11</td>
<td>0.06</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>4.56-5</td>
<td>0.25</td>
<td>0.15</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>9.45e-5</td>
<td>0.42</td>
<td>0.25</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>12.70-4</td>
<td>0.60</td>
<td>0.37</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Slow Fading Channel $K$ = 10 DB, $N$ = 31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conv</td>
<td>0.07</td>
<td>0.04</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>4.45e-5</td>
<td>0.18</td>
<td>0.11</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>9.34e-5</td>
<td>0.36</td>
<td>0.22</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>12.54-4</td>
<td>0.55</td>
<td>0.33</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>Fast Fading Channel $K$ = 10 DB, $N$ = 31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conv</td>
<td>0.03</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4.35e-5</td>
<td>0.08</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>9.24e-5</td>
<td>0.24</td>
<td>0.15</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>12.45-4</td>
<td>0.43</td>
<td>0.27</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>Non-Delayed Channel $N$ = 31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conv</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4.25e-5</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>9.14e-5</td>
<td>0.10</td>
<td>0.07</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>12.34-4</td>
<td>0.29</td>
<td>0.19</td>
<td>0.07</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Chapter 4

Probability of Packet Success for DS-CDMA over Multipath Fading Channels

4.1 Introduction

There have been many publications on the computation of the probability of packet success for DS-CDMA systems [41]-[45]. The exact calculation is computationally difficult, so emphasis has been on approximations and bounds. One particularly attractive approximation is based on the assumptions that MAI is Gaussian and bit errors within a packet are independent. It was shown in the previous chapter that the computation of the probability of bit error based on the assumption that MAI is Gaussian is not generally accurate. Furthermore, when asynchronous DS-CDMA is considered and the channel experiences fading, we can no longer assume that bit errors within a packet are independent. Morrow and Lehnert [41]-[42] developed an accurate approximation for the probability of packet success for asynchronous DS-CDMA over AWGN channel where an improved Gaussian model for the MAI was considered and the effect of bit-to-bit error dependence caused by the interfering signal relative delays and phases was taken into account. It has been shown in [41] that the assumptions
that the MAI is Gaussian and the bit errors are independent produces an optimistic estimate of packet success for a lightly loaded channel and then becomes pessimistic as the MAI increases. The major drawback of the approximation developed in [41]-[42] is the computational complexity. In our previous work we simplified this approximation and maintained good accuracy [45].

In this chapter, we extend the work in [45] and we derive an approximation for the probability of packet success in a DS-CDMA system over the fading channel. Both block and convolutional coding are considered. In the probability of packet success approximation a Gaussian as well as an improved Gaussian models for the MAI is considered and the effect of bit-to-bit error dependence caused by the fading and the MAI is taken into account.

In Section 4.2 we present the conditions in which the fading is considered to be fast or slow within the packet length. In Section 4.3 and 4.4 we derive the probability of packet success for asynchronous DS-CDMA over the multipath fading channel using block and convolutional codes respectively, considering both independent and dependent bit errors assumptions and using the standard Gaussian and the improved Gaussian approximations. In Section 4.5 we present some numerical results and finally in Section 4.6 we summarize this chapter.

4.2 Slow and Fast Fading Conditions

In the previous chapter, the effect of bit-to-bit dependence caused by the fading channel was studied by considering two extreme conditions: the fast fading condition and the slow fading condition. In the fast fading condition we considered that the fading is fast enough so that the signal to noise ratio of two adjacent bits are independent and therefore the bit errors of individual bits are mutually independent.
[138]. For a slow fading condition, the fading is assumed to be such that the signal to noise ratio does not change appreciably throughout the length of the codeword for block codes and the length of the error events for convolutional codes [138].

When packet error performance is considered, we have a different situation and the question is, how long should the fading be considered slow? In general, a packet could have either one or several codewords depending on the packet length used and the error correction codes applied. If we assume a packet length in the order of 100 to 1000 information bits, the packet could have either several small BCH codewords using codes such as (15,kc) and (31,kc) or one long BCH codeword using codes such as (511, kc) and (1023, kc) [41]. The question is, shall we consider the fading to be slow during the entire packet length? To answer such questions, we examine some fading parameters that determine the rapidity of fading.

It is well known that the rapidity of fading depends on the vehicle speed \( V \) and the carrier frequency \( f_c \). The fading bandwidth denoted as \( B_f \) is an important parameter that measures the fading rate and it is directly related to the vehicle speed \( V \) as:

\[
B_f = \frac{V}{C_l} f_c
\]

(4.1)

where \( C_l \) is the speed of light. Table 4.1 shows the normalized fading bandwidth (normalized to the bit duration \( T_b = 1/R_b \)) with different vehicle speeds for L-Band (1540-1660 MHz). \( B_{f1} \) and \( B_{f2} \) in Table 4.1 correspond to 1540 MHz and 1660 MHz respectively. This table indicates that as the vehicle speed decreases, the rapidity of fading reduces significantly and for data transmission rates in the order of 4800 to 9600 bps, the normalized fading bandwidth could be as small as 0.1%, which means that fading could be very slow and once it occurs it could last for several hundred bits. Another way to investigate this problem is by computing the average fade duration which measures the average time the signal remains below a certain signal.
<table>
<thead>
<tr>
<th>$V$ (km/h)</th>
<th>$B_{f1}$ (Hz)</th>
<th>$B_{f2}$ (Hz)</th>
<th>$B_{f1} T_b$</th>
<th>$B_{f2} T_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_b = 4800$ (bps)</td>
<td>$R_b = 9600$ (bps)</td>
<td>$R_b = 4800$ (bps)</td>
<td>$R_b = 9600$ (bps)</td>
</tr>
<tr>
<td>20</td>
<td>28.3</td>
<td>30.7</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>30</td>
<td>42.75</td>
<td>46.1</td>
<td>0.009</td>
<td>0.0045</td>
</tr>
<tr>
<td>40</td>
<td>57</td>
<td>61.4</td>
<td>0.012</td>
<td>0.006</td>
</tr>
<tr>
<td>50</td>
<td>71.25</td>
<td>76.8</td>
<td>0.0145</td>
<td>0.0072</td>
</tr>
<tr>
<td>60</td>
<td>85.5</td>
<td>92.17</td>
<td>0.0175</td>
<td>0.0087</td>
</tr>
<tr>
<td>70</td>
<td>99.75</td>
<td>107.5</td>
<td>0.021</td>
<td>0.0105</td>
</tr>
<tr>
<td>80</td>
<td>114</td>
<td>122.9</td>
<td>0.024</td>
<td>0.012</td>
</tr>
<tr>
<td>90</td>
<td>128.25</td>
<td>138.25</td>
<td>0.0265</td>
<td>0.0132</td>
</tr>
<tr>
<td>100</td>
<td>142.5</td>
<td>153.6</td>
<td>0.03</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 4.1: Normalized fading bandwidths as a function of vehicle speed and transmission rate for L-Band (1540-1660 MHz).

level. It was shown in [129]-[131] that a fade duration at a signal level of -10 dB could be as much as several hundred bits for normalized fading bandwidth smaller than 2%. As mentioned before, in our study a packet length $L_p$ in the order of 100 to 1000 information bits is considered, therefore we shall determine the probability of packet success for different conditions, in which the effect of bit-to-bit dependence within a packet is investigated for different cases and the Gaussian assumption is also examined. These cases will be numbered to make the discussion of the numerical results easier to follow.

### 4.3 Probability of Packet Success with Block Codes

In this section we present different methods of computing the probability of packet success for asynchronous DS-CDMA over the multipath fading channel with block codes. First we assume that the fading is fast, therefore the bit-to-bit dependence caused by fading is neglected. The MAI is modeled as Gaussian process and the bit-to-bit dependence due to MAI is neglected. This case is referred to as fast fading-
case 1 which is an ideal case where an ideal interleaving is employed and kills all the channel memory. For the same fading condition, we investigate the accuracy of the Gaussian assumption by modeling the MAI as an improved Gaussian process. Using the improved Gaussian model allows us to investigate the effect of bit-to-bit dependence caused by the MAI. When the improved Gaussian model is used and the effect of bit-to-bit dependence due to MAI is neglected, this is referred to as fast fading-case 2; and when the effect of bit-to-bit dependence due to MAI is taken into account, this is referred to as fast fading-case 3. Secondly, we consider the case where the fading is slow, which means that the bit-to-bit dependence caused by fading is taken into account. We determine the probability of packet success in this case with and without taking into account the effect bit-to-bit dependence caused by the MAI by modeling the MAI as either a Gaussian or improved Gaussian process. In other words the three cases considered for fast fading are considered.

4.3.1 Fast Fading and Gaussian Approximation

Fast fading-case 1:

In this case we assume a fast fading channel in which the bit-to-bit dependence caused by fading can be neglected. We also neglect the bit-to-bit dependence caused by the MAI. Furthermore, we assume that the bit errors at the decoder output are independent. Then, the probability of packet success for asynchronous DS-CDMA given \( K \) simultaneous transmissions over the fast fading channel using \( (n_c, k_c) \) block codes that can correct \( t \) or fewer errors is given by:

\[
P_c(K) = [1 - P_b(K)]^{L_p}
\]  

(4.2)

where \( L_p \) is the number of information bits per packet, and \( P_b(K) \) is the probability of bit error for \( K \) simultaneous users at the decoder output which is given by eq.
(3.79) and repeated here for convenience.

\[
P_b(K) = \frac{1}{n_c} \sum_{i=1}^{n_c} i \binom{n_c}{i} [P_{cb}(K)]^i [1 - P_{cb}(K)]^{n_c-i}
\]

where \( P_{cb}(K) \) is the channel bit error probability given by

\[
P_{cb}(K) = \int_0^\infty Q\left(\frac{\beta_{1,1}}{\sqrt{\frac{N_0}{2rL_b} + \frac{(K-1)}{3N} (c^2 + 2\sigma_f^2) + \frac{K(L-1)}{3N} 2\sigma_f^2}}\right) p(\beta_{1,1}) \, d\beta_{1,1}
\]

where \( p(\beta_{1,1}) \) is the pdf of \( \beta_{1,1} \) and is given by eq. (3.8). To compute the probability of packet success for uncoded system we replace \( P_b(K) \) in eq. (4.2) by \( P_{cb}(K) \) with \( r = 1 \).

4.3.2 Fast Fading and Improved Gaussian Approximation

As mentioned before, in this subsection we consider two cases. Since fast fading is considered, there is no bit-to-bit dependence due to fading in both cases.

Fast fading-case 2:

In this case we neglect also the effect of bit-to-bit dependence caused by the MAI and model it as an improved Gaussian process. The probability of packet success is still given by eq. (4.2) with \( P_b(K) \) given by eq. (4.3) but \( P_{cb}(K) \) in this case is given by eq. (3.74).

Fast fading-case 3:

In this case we model the MAI as an improved Gaussian process while taking into account the effect of bit-to-bit dependence. We condition the probability of packet success given by eq. (4.2) on \( \Psi \), and then average the conditional probability with respect to \( \Psi \). Following the same procedure as in the previous chapter, we found that

\[
P_c(K) = \frac{2}{3} [1 - P_{b1}(K)]^{L_p} + \frac{1}{6} [1 - P_{b2}(K)]^{L_p} + \frac{1}{6} [1 - P_{b3}(K)]^{L_p}
\]
where $P_b(K)$ for $i = 1, 2, 3$ are given by:

$$P_b(K) = \frac{1}{n_c} \sum_{i=1}^{n_c} \binom{n_c}{i} [P_{cb}(K)]^i [1 - P_{cb}(K)]^{n_c-i}$$

(4.6)

where $P_{cb1}(K)$ is given by eq. (4.4), $P_{cb2}(K)$ and $P_{cb3}(K)$ are given by:

$$P_{cb2}(K) = \int_0^\infty Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2r} + \frac{\sqrt{3} \sigma_p}{N^2} + \frac{(K-1)}{3N}(c^2 + 2\sigma_i^2) + \frac{K(L-1)}{3N}2\sigma_i^2}} \right) p(\beta_{1,1}) \, d\beta_{1,1}$$

(4.7)

and

$$P_{cb3}(K) = \int_0^\infty Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{N_o}{2r} + \frac{\sqrt{3} \sigma_p}{N^2} + \frac{(K-1)}{3N}(c^2 + 2\sigma_i^2) + \frac{K(L-1)}{3N}2\sigma_i^2}} \right) p(\beta_{1,1}) \, d\beta_{1,1}$$

(4.8)

where $\sigma_p$ is given by eq. (3.65).

### 4.3.3 Slow Fading and Gaussian Approximation

**Slow fading-case 1:**

In this case the fading is assumed to be slow during the entire packet. We neglect the bit-to-bit dependence caused by the MAI and a Gaussian process is used to model the MAI. To take into account the bit-to-bit dependence caused by fading within a packet, we first condition the probability of packet success given by eq. (4.2) on $\beta_{1,1}$, thus we have

$$P_c(K \mid \beta_{1,1}) = [1 - P_b(K \mid \beta_{1,1})]^{L_p}$$

(4.9)

where $P_b(K \mid \beta_{1,1})$ is the conditional probability of bit error and is given by:

$$P_b(K \mid \beta_{1,1}) = \frac{1}{n_c} \sum_{i=1}^{n_c} i \binom{n_c}{i} [P_{cb}(K \mid \beta_{1,1})]^i [1 - P_{cb}(K \mid \beta_{1,1})]^{n_c-i}$$

(4.10)
and $P_{cb}(K | \beta_{1,i})$ is given by

$$P_{cb}(K | \beta_{1,i}) = Q \left( \frac{\beta_{1,i}}{\sqrt{\frac{N_c}{2\pi E_b} + \frac{(K - 1)}{3N} (c^2 + 2\sigma^2) + \frac{K(L-1)}{3N} 2\sigma^2} } \right) \quad (4.11)$$

To find $P_c(K)$ when $\beta_{1,i}$ is random we have to evaluate

$$P_c(K) = \int_{0}^{\infty} [1 - P_b(K | \beta_{1,i})]^{L_p} p(\beta_{1,i}) \, d\beta_{1,i} \quad (4.12)$$

4.3.4 Slow Fading and Improved Gaussian Approximation

We repeat the same scenario as we did with the fast fading and improved Gaussian approximation subsection.

**Slow fading-case 2:**

As in the previous case we consider a slow fading, we neglect the bit-to-bit dependence caused by the MAI but we use the improved Gaussian process to model the MAI. In this case the probability of packet success is still given by eq. (4.12) with $P_b(K | \beta_{1,i})$ is given by eq. (4.10) but $P_{cb}(K | \beta_{1,i})$ is given by eq. (3.73).

**Slow fading-case 3:**

Finally we examine the situation where the fading is slow and we take into account the effect of bit-to-bit dependence caused by fading and MAI. We condition the probability of packet success given by eq. (4.2) on $\beta_{1,i}$ and $\Psi$, thus we have

$$P_c(K | \Psi, \beta_{1,i}) = [1 - P_b(K | \Psi, \beta_{1,i})]^{L_p} \quad (4.13)$$

where $P_b(K | \Psi, \beta_{1,i})$ is given by:

$$P_b(K | \Psi, \beta_{1,i}) = \frac{1}{n_c} \sum_{i=1}^{n_c} \binom{n_c}{i} [P_{cb}(K | \Psi, \beta_{1,i})]^i [1 - P_{cb}(K | \Psi, \beta_{1,i})]^{n_c-i} \quad (4.14)$$

and $P_{cb}(K | \Psi, \beta_{1,i})$ is given by

$$P_{cb}(K | \Psi, \beta_{1,i}) = Q \left( \frac{\beta_{1,i} N}{\sqrt{\Psi}} \right) \quad (4.15)$$
For keeping it simple, in eq. (4.15) we neglected the effect of thermal noise which we will consider later. To obtain the probability of packet success when \( \Psi \) is random, we must average \( P_e(K \mid \Psi, \beta_{1,1}) \) over the probability density function of \( \Psi \). That is, we have to evaluate

\[
P_e(K \mid \beta_{1,1}) = \int_0^\infty P_e(K \mid \Psi, \beta_{1,1}) p_\Psi(\psi) \, d\psi
\]  

(4.16)

By following the same procedure as in the previous chapter, we found that

\[
P_e(K \mid \beta_{1,1}) = \frac{2}{3} [1 - P_{cb1}(K \mid \beta_{1,1})]^L + \frac{1}{6} [1 - P_{cb2}(K \mid \beta_{1,1})]^L + \frac{1}{6} [1 - P_{cb3}(K \mid \beta_{1,1})]^L
\]  

(4.17)

where \( P_{cb1}(K \mid \beta_{1,1}), P_{cb2}(K \mid \beta_{1,1}) \) and \( P_{cb3}(K \mid \beta_{1,1}) \) are given by:

\[
P_{cb1}(K \mid \beta_{1,1}) = Q \left( \frac{\beta_{1,1}}{\sqrt{(K - 1)(c^2 + 2\sigma_i^2) + K(L - 1)2\sigma_i^2}} \right)
\]  

(4.18)

\[
P_{cb2}(K \mid \beta_{1,1}) = Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{\sqrt{3} \sigma_\Psi}{N^2} + (K - 1)(c^2 + 2\sigma_i^2) + K(L - 1)2\sigma_i^2}} \right)
\]  

(4.19)

\[
P_{cb3}(K \mid \beta_{1,1}) = Q \left( \frac{\beta_{1,1}}{\sqrt{-\frac{\sqrt{3} \sigma_\Psi}{N^2} + (K - 1)(c^2 + 2\sigma_i^2) + K(L - 1)2\sigma_i^2}} \right)
\]  

(4.20)

where \( \sigma_\Psi \) is given by eq. (3.65). If we take into consideration the effect of thermal noise, \( P_{cb1}(K \mid \beta_{1,1}), P_{cb2}(K \mid \beta_{1,1}) \) and \( P_{cb3}(K \mid \beta_{1,1}) \) becomes:

\[
P_{cb1}(K \mid \beta_{1,1}) = Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{N_b}{2\tau E_b} + (K - 1)(c^2 + 2\sigma_i^2) + K(L - 1)2\sigma_i^2}} \right)
\]  

(4.21)

\[
P_{cb2}(K \mid \beta_{1,1}) = Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{N_b}{2\tau E_b} + \frac{\sqrt{3} \sigma_\Psi}{N^2} + (K - 1)(c^2 + 2\sigma_i^2) + K(L - 1)2\sigma_i^2}} \right)
\]  

(4.22)
\[ P_{cb3}(K \mid \beta_{1,1}) = Q \left( \frac{\beta_{1,1}}{\sqrt{\frac{N_p}{2\pi \sigma_b^2}} - \frac{\sqrt{3} \sigma_w}{N^2} + \frac{(K - 1)}{3N}(c^2 + 2\sigma_1^2) + \frac{K(L - 1)}{3N}(2\sigma_1^2)} \right) \]  (4.23)

To find the probability of packet success when \( \beta_{1,1} \) is random we have to evaluate

\[ P_c(K) = \frac{2}{3} \int_0^\infty [1 - P_{b1}(K \mid \beta_{1,1})]^L p(\beta_{1,1}) \, d\beta_{1,1} \]
\[ + \frac{1}{6} \int_0^\infty [1 - P_{b2}(K \mid \beta_{1,1})]^L p(\beta_{1,1}) \, d\beta_{1,1} \]
\[ + \frac{1}{6} \int_0^\infty [1 - P_{b3}(K \mid \beta_{1,1})]^L p(\beta_{1,1}) \, d\beta_{1,1} \]  (4.24)

where \( P_{b_i}(K \mid \beta_{1,1}) \) for \( i = 1, 2, 3 \) are given by:

\[ P_{b_i}(K \mid \beta_{1,1}) = \frac{1}{n_c} \sum_{i=1}^{n_c} i \binom{n_c}{i} [P_{cb}(K \mid \beta_{1,1})]^i [1 - P_{cb}(K \mid \beta_{1,1})]^{n_c-i} \]  (4.25)

and \( P_{cb}(K \mid \beta_{1,1}) \) for \( i = 1, 2, 3 \) are given by eqs. (4.21), (4.22) and (4.23).

### 4.4 Probability of Packet Success with Convolutional Codes

In this section we develop an approximation for the probability of packet success for asynchronous DS-CDMA using convolutional codes with Viterbi decoder over the multipath fading channel. We determine this probability of packet success under fast and slow fading conditions; and considering the three cases studied above.

#### 4.4.1 Fast Fading and Gaussian Approximation

**Fast fading-case 1:**

Under fast fading conditions and if we consider that the bit errors are independent and the MAI is Gaussian, then the probability of packet success for asynchronous DS-CDMA over the multipath fading using convolutional codes is:

\[ P_c(K) = [1 - P_e(P_{cb}(K))]^L_p \]  (4.26)
where $P_e(.)$ is the probability of bit error at the decoder output which is given by eq. (3.85) and $P_{cb}(K)$ is the channel bit error probability given by eq. (4.4). It was shown in [43], [44] that replacing $P_e(.)$ in eq. (4.26) by the union bound on the first event error probability $P_u(.)$ leads to more accurate results on the probability of packet success. Thus $P_e(K)$ becomes:

$$P_e(K) = [1 - P_u(P_{cb}(K))]^{L_p} \quad (4.27)$$

There exists many approximations for $P_u(p)$. In our study we consider the bound taken in [43] because it represents a tight bound with respect to many other bounds. It is given by:

$$P_u(p) = \left( \begin{array}{c} 2n_o - 1 \\ n_o \end{array} \right) 2^{-2n_o-1} \left\{ [T(D) + T(-D)] + D \left[ T(D) - T(-D) \right] \right\} D=2\sqrt{p} \quad (4.28)$$

where $T(D)$ is the transfer function of the convolutional code and $n_o$ is as defined in the previous chapter.

### 4.4.2 Fast Fading and Improved Gaussian Approximation

**Fast fading-case 2:**

If we consider fast fading conditions and model the MAI as an improved Gaussian while neglecting the effect of bit-to-bit dependence due to MAI, then the probability of packet success is given by eq. (4.27) with $P_u(.)$ given by eq. (4.28) and $P_{cb}(K)$ in this case is given by eq. (3.74).

**Fast fading-case 3:**

If we consider the same conditions as fast fading-case 2 while taking into account the effect of bit-to-bit dependence due to MAI, the probability of packet success becomes:

$$P_e(K) = \frac{2}{3} [1 - P_u(P_{cb1}(K))]^{L_p} + \frac{1}{6}[1 - P_u(P_{cb2}(K))]^{L_p} + \frac{1}{6}[1 - P_u(P_{cb3}(K))]^{L_p} \quad (4.29)$$
where $P_{cb}(K)$ for $i = 1, 2, 3$ are given by eq. (4.4), eq. (4.7) and eq. (4.8) respectively.

### 4.4.3 Slow Fading and Gaussian Approximation

**Slow fading-case 1:**
To find the probability of packet success for slow fading with the MAI is Gaussian, we first condition the probability of packet success on $\beta_{1,1}$ as we did with block codes, then we average with respect to $\beta_{1,1}$. Thus, we have to evaluate the following integral:

$$P_c(K) = \int_0^\infty [1 - P_u(P_{cb}(K \mid \beta_{1,1}))]^L_p p(\beta_{1,1}) \, d\beta_{1,1} \quad (4.30)$$

where $P_{cb}(K \mid \beta_{1,1})$ is the conditional channel error probability and is given by eq. (4.11).

### 4.4.4 Slow Fading and Improved Gaussian Approximation

**Slow fading-case 2:**
For slow fading and to use an improved Gaussian approximation for the MAI while neglecting the bit-to-bit dependence caused by the MAI, the probability of packet success is still given by eq. (4.30) with $P_{cb}(K \mid \beta_{1,1})$ is given by eq. (3.73).

**Slow fading-case 3:**
Finally for slow fading and to use an improved Gaussian approximation for the MAI which takes into account the bit-to-bit dependence caused by the MAI, we condition the probability of packet success on $\Psi$, thus for a fixed $\beta_{1,1}$ the conditional probability of packet success for asynchronous DS-CDMA with convolutional codes is given by:

$$P_c(K \mid \Psi, \beta_{1,1}) = [1 - P_u(P_{cb}(K \mid \Psi, \beta_{1,1}))]^L_p \quad (4.31)$$

where $P_{cb}(K \mid \Psi, \beta_{1,1})$ is the conditional channel bit error probability and is given by eq. (4.14). To obtain the probability of packet success when $\Psi$ and $\beta_{1,1}$ are random
we have to average \( P_c(K | \Psi, \beta_{1,1}) \) with respect to \( \Psi \) and \( \beta_{1,1} \). Following the same procedure as before, we found that \( P_c(K) \) is given by:

\[
P_c(K) = \frac{2}{3} \int_0^\infty [1 - P_u(P_{cb1}(K | \beta_{1,1}))]^{L_p} p(\beta_{1,1}) \, d\beta_{1,1}
+ \frac{1}{6} \int_0^\infty [1 - P_u(P_{cb2}(K | \beta_{1,1}))]^{L_p} p(\beta_{1,1}) \, d\beta_{1,1}
+ \frac{1}{6} \int_0^\infty [1 - P_u(P_{cb3}(K | \beta_{1,1}))]^{L_p} p(\beta_{1,1}) \, d\beta_{1,1}
\tag{4.32}
\]

where \( P_{cbi}(K | \beta_{1,1}) \) for \( i = 1, 2, 3 \) are given by eqs. (4.21), (4.22) and (4.23). In our study, convolutional codes with rate 1/2 and constraint length \( CL = 3 \) and 7 are considered. For the convolutional code with \( r = 1/2 \) and \( CL = 3 \), the transfer function \( T(D) \) was taken as:

\[
T(D) = \frac{D^5}{1 - 2D}
\tag{4.33}
\]

and for \( CL = 7 \), \( T(D) \) is developed in [70]. The first four terms are given by:

\[
T(D) = 11D^{10} + 38D^{12} + 193D^{14} + 1331D^{16} + \ldots
\tag{4.34}
\]

Before presenting some numerical results, we summarize all the studied cases in Table 4.2 which is helpful in the analysis of the numerical results.

<table>
<thead>
<tr>
<th>Case</th>
<th>Bit-to-bit dependence</th>
<th>MAI Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bit-to-bit dependence</td>
<td></td>
</tr>
<tr>
<td>Fast fading-case 1</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Fast fading-case 2</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Fast fading-case 3</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Slow fading-case 1</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Slow fading-case 2</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Slow fading-case 3</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 4.2: Different cases used to study the accuracy of the Gaussian approximation as well as the effect of bit-to-bit dependence on the computation of the probability of packet success.
4.5 Numerical Results

Figure 4.1 shows the probability of packet success for uncoded asynchronous DS-CDMA over nonfaded channel with $N = 31$, $L_p = 252$ and $E_b/N_0 = \infty$ for the following three cases.

Case 1: using the standard Gaussian model for the MAI and independent bit errors assumption.

Case 2: using the improved Gaussian model for the MAI and independent bit errors assumption, denoted as improved Gaussian (i.b.e) in the figure.

Case 3: using the improved Gaussian model for the MAI and take into account the effect of bit-to-bit error dependence within a packet caused by the MAI, denoted as improved Gaussian (d.b.e) in the figure.

Comparing case 1 and case 2 we can see that the Gaussian approximation gives optimistic values for the probability of packet success especially for number of users less than 20. This is conform with the results found in the previous chapter with the probability of bit error. Now if we compare case 1 and case 3 we see that using the Gaussian model for the MAI and neglecting the bit-to-bit dependence caused by the MAI results in very optimistic values of the probability of packet success for small number of users ($K < 13$) and pessimistic values for large number of users ($K > 13$). Comparing case 2 and case 3 we see that as the number of interfering signals increases, the bit errors becomes more dependent and the computation of the probability of packet success based on independent bit error becomes pessimistic.

Similar results for the nonfaded channel was also found in [41].

This effect of bit-to-bit dependence caused by the MAI on the probability of packet success for the nonfaded channel motivates us to investigate the case where the
Figure 4.1: Probability of packet success for uncoded asynchronous DS-CDMA over nonfaded channel with $N = 31, L_p = 252$ and $E_b/N_0 = \infty$; (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
channel is corrupted by fading. Because fading adds additional dependency between the bits. In the analysis of numerical results we first examine the effect of the packet length on the probability of packet success. Then we discuss the three cases studied above for fast and slow fading conditions.

Figure 4.2 shows the probability of packet success for uncoded asynchronous DS-CDMA system over nonselective fast Rician fading channel with \( K_f = 10 \) dB for different packet lengths \( L_p \) with \( N = 31 \) and \( E_b/N_0 = \infty \). In this figure we consider only the case where the MAI is Gaussian, which corresponds to the fast fading-case 1. This figure indicates that the probability of packet success depends on the number of information bits per packet. As the packet length increases, the probability of packet success decreases.

Figure 4.3 shows the probability of packet success for uncoded asynchronous DS-CDMA over nonselective fast Rician fading channel with \( K_f = 10 \) dB, \( N = 31 \), \( L_p = 252 \), 1008 and \( E_b/N_0 = \infty \). All the three cases are considered for each packet length. This figure shows the same behavior as the nonfaded channel (Figure 4.1) for the three cases considered. As we can see from the figure, a comparison of case 1 and case 2 which corresponds to independent bit errors with MAI is Gaussian and improved Gaussian yields the same behavior results as the nonfaded case, i.e., the Gaussian assumption gives optimistic results especially for number of users less than 15 and 9 for \( L_p = 252 \) and \( L_p = 1008 \) respectively. Recall that this number was 20 for the nonfaded case. Comparison of case 1 and case 3 indicates that the independent bit errors and Gaussian assumptions give optimistic values of the probability of packet success for number of users less than 9 and 6 for \( L_p = 252 \) and \( L_p = 1008 \) respectively, and pessimistic values for number of users larger than 9 and 6 for \( L_p = 252 \) and \( L_p = 1008 \) respectively.
Figure 4.2: Probability of packet success for uncoded asynchronous DS-CDMA system over nonselective fast Rician fading channel with $K_f = 10$ dB for different packet lengths $L_p$ with $N = 31$ and $E_b/N_o = \infty$. 
Figure 4.3: Probability of packet success for uncoded asynchronous DS-CDMA over nonselective fast Rician fading channel with $K_f = 10$ dB, $N = 31$, $L_p = 252$ and 1008 and $E_b/N_o = \infty$; (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.4 shows the probability of packet success for uncoded asynchronous DS-CDMA over nonselective fast Rician fading channel with $K_f = 10$ dB, $N = 63$, $L_p = 252$ and different $E_b/N_o$. As indicated in Chapter 3, the thermal noise has a significant effect on the probability of bit error as well as the probability of packet success as it can be seen from this figure. This figure also indicates that the accuracy of the Gaussian approximation becomes acceptable as the $E_b/N_o$ decreases, however the effect of bit-to-bit dependence on the probability of packet success values remain the same and it is not affected by changing the values of the $E_b/N_o$. In practice, for a reliable data transmission a BER of $10^{-5}$ is required. To achieve this BER over fading channel a value of $E_b/N_o$ of at least 15 dB is required. This is the reason why we consider values of $E_b/N_o$ over 10 dB in this thesis.

Figure 4.5 shows the probability of packet success for asynchronous DS-CDMA using (15,7) BCH code over the nonselective fast Rician fading channel with $K_f = 10$ dB, $N = 31$, $L_p = 252$, 1008 and $E_b/N_o = \infty$ (same conditions as Figure 4.3 except that coding is used in this case). The first interpretation from this figure is that error control coding improve the packet error rate performance. Comparing this figure with Figure 4.3 we see that when coding is used the discrepancy between case 1 and case 3 is increased especially for small number of users. This could be explained by the fact that coding introduces additional dependency between the bits within a packet.
Figure 4.4: Probability of packet success for un-coded asynchronous DS-CDMA over non-selective fast Rician fading channel with $K_f = 10$ dB, $N = 63$, $L_p = 252$ and different $E_b/N_0$; (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.5: Probability of packet success for asynchronous DS-CDMA using (15,7) BCH code over the nonselective fast Rician fading channel with $K_f = 10$ dB, $N = 31$, $L_p = 252$ and 1008 and $E_b/N_0 = \infty$; (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).

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Figure 4.6 shows the probability of packet success for asynchronous DS-CDMA using (15,7) BCH code over the nonselective slow Rician fading channel with $K_f = 10$ dB, $N = 31$, $L_p = 252$, 1008 and $E_b/N_0 = \infty$. The effect of using the improved Gaussian process with and without taking into account the effect of bit-to-bit dependence caused by the MAI is quite similar to the fast fading case. However, a significant difference on the probability of packet success is obtained for fast and slow fading channels.

Figure 4.7 and Figure 4.8 show a comparison of the probability of packet success for slow and fast fading for $L_p = 252$ and $L_p = 1008$ respectively. For each fading condition the three cases studied above are considered in the figures, so in each figure we have the six cases developed in the previous sections (fast fading-case 1 to fast fading-case 3 and slow fading-case 1 to slow fading-case 3). As we can see from these two figures, the slow fading channel increases significantly the bit-to-bit dependence within a packet. This bit-to-bit dependence caused by the slow fading has the same effect as the bit-to-bit dependence caused by the MAI except that this effect of bit-to-bit dependence caused by the slow fading is more significant. This explains why the probability of packet success for fast fading channel has larger values for small number of users and smaller values for large number of users than for slow fading channel. From these two figures we notice also that the effect bit-to-bit dependence caused by the slow fading increases with the packet length as expected.
Figure 4.6: Probability of packet success for asynchronous DS-CDMA using (15,7) BCH code over the nonselective slow Rician fading channel with $K_f = 10$ dB, $N = 31$, $L_p = 252, 1008$ and $E_b/N_0 = \infty$ (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.7: The probability of packet success for asynchronous DS-CDMA using $(15,7)$ BCH code for nonselective slow and fast Rician fading channel for $L_p = 252$ with $K_f = 10$ dB, $N = 31$ and $E_b/N_0 = \infty$ (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.8: The probability of packet success for asynchronous DS-CDMA using (15,7) BCH code for nonselective slow and fast Rician fading channel for $L_p=1008$ with $K_f = 10$ dB, $N = 31$ and $E_b/N_0 = \infty$ (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.9 and Figure 4.10 show a comparison of the probability of packet success based on fast fading-case 3 and slow fading-case 3 for \( L_p = 252 \) and \( L_p = 1008 \) respectively. These figures show clearly the additional effect of bit-to-bit dependence caused by fading. These figures will be referred to as the two extremes performance and will be used to analyze the throughput-delay performance of the DS-CDMA packet networks.

Finally, Figure 4.11 shows the probability of packet success fast fading-case 3 and slow fading-case 3 for asynchronous DS-CDMA using convolutional code with constraint length \( CL = 3 \) and code rate \( r = 1/2 \). This figure shows that the convolutional code considered gives similar behavior and performance as the (15,7) BCH code.
Figure 4.9: Comparison of the probability of packet success for asynchronous DS-CDMA using (15,7) BCH code for nonselective slow and fast Rician fading channels for $L_p = 252$ with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$; (d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.10: **Comparison** of the probability of packet success for asynchronous DS-CDMA using (15,7) **BCH code** for nonselective **slow and fast** Rician fading channels for $L_p = 1008$ with $K_f = 10$ dB, $N = 31$ and $E_b/N_o = \infty$; (d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 4.11: Probability of packet success for asynchronous DS-CDMA using convolutional code with $CL = 3$ and $r = 1/2$ over nonselective slow and fast Rician fading channels with $K_f = 10$ dB, $N = 31$, $L_p = 252$ and $E_b/N_0 = \infty$; (d.b.e: consider the bit-to-bit dependence due to MAI).
4.6 Summary

The probability of packet success for asynchronous DS-CDMA over the multipath fading channel has been investigated under different fading conditions and for different cases. We studied the effect of using the Gaussian versus the improved Gaussian process on the computation of probability of packet success as well as the effect of the bit-to-bit dependence within a packet. We studied the bit-to-bit dependence caused by fading and by the MAI originated from the chip and phase offsets between the desired and each interfering signal. The improved Gaussian approximation not only provides accurate values for the probability of bit error for any number of simultaneous users, but it also allows us to incorporate the effects of bit-to-bit dependence caused by the MAI into the calculations for the probability of packet success. The main conclusions that can be drawn from our analysis are summarized as following:

When bit errors are independent, the Gaussian assumption for the MAI gives optimistic values for the probability of packet success especially for small number of users in comparison to the more accurate improved Gaussian model; for both faded and nonfaded channels (this is a comparison of case 1 and case 2).

When the MAI is modeled as an improved Gaussian process we have two choices: either take into account the bit-to-bit dependence caused by the MAI or ignore it. Ignoring the effect of bit-to-bit dependence caused by the MAI, produces a lower bound on the actual probability of packet success when no coding is used for both faded and nonfaded channels. When coding 's used, then the actual packet performance will be worse than that calculated for small number of users and better for large number of users, for both faded and nonfaded channels (this is a comparison of case 2 and case 3).
When the MAI is modeled as Gaussian and the effect of bit-to-bit dependence due to MAI is ignored, the probability of packet success is found to be optimistic for small number of users and pessimistic for large number of users in comparison to the case where the MAI is modeled as an improved Gaussian and the effect of bit-to-bit dependence due to MAI is taken into account. This is true for both faded and nonfaded channels (this is a comparison of case 1 and case 3).

When slow fading is considered, it is found that the bit-to-bit dependencies within a packet have significantly increased. This increase of bit-to-bit dependencies adds further degradation on the packet performance for small number of users and enhances the improvement for large number of users.

Finally, we conclude that the effect of bit-to-bit dependence within a packet is significant in an asynchronous DS-CDMA system, especially when the channel experiences fading. Therefore, one should take into account this effect in the analysis of DS-CDMA packet networks. Furthermore, the improved Gaussian process adds additional accuracy on the computation of the probability of packet success, therefore in our study of the delay-throughput performance of the DS-CDMA packet networks, the improved Gaussian process will be used to model the MAI and the effect of bit-to-bit dependence will be taken into account. Both slow and fast fading condition will be considered.
Chapter 5

Proposed Partial DS-CDMA System

5.1 Introduction

In the previous chapters we analyzed the bit and packet error rate performance of the DS-CDMA system for a given receiver equipped with one spread spectrum code in the presence of a number of interfering transmissions. As mentioned before, the central station of the communication system considered in our study employs several receivers. The question is, how many receivers the central station should have to accommodate a given number of system users $U$? How does the choice of this number of receivers at the central station affect the whole DS-CDMA system performance? This issue has not been investigated in the literature. Most of the analyses of DS-CDMA system are based either on a single receiver performance or by considering a network where each system user has a unique code and there is a separate receiver for each user at the central station [69]-[78]. Such a network is referred to as the conventional DS-CDMA network and is illustrated in Figure 5.1.

The network considered in Figure 5.1 is not realistic and could not be implemented for the mobile satellite applications because such a network implies a very complex
Figure 5.1: Conventional DS-CDMA network.
central station when it serves a very large population of users which is the case for mobile satellite communications. In this chapter we propose a more realistic DS-CDMA network in which the number of required receivers at the central station is significantly reduced. Such a network is implementable and is referred to as partial DS-CDMA network [83].

In this chapter the performance of the DS-CDMA network is measured in terms of the average number of packets successfully received by the central station per slot given a number of simultaneous transmissions, denoted by \( \bar{C}_K \). For comparison reason \( \bar{C}_K \) is determined for the unrealistic conventional DS-CDMA network as well as the proposed partial DS-CDMA network.

In Section 5.2 we present the performance of the so called conventional DS-CDMA network. In Section 5.3 we derive the average number of packets successfully received by the central station for the proposed partial DS-CDMA network. In Section 5.4 we present some numerical results and finally in Section 5.5 we summarize the findings of this chapter.

5.2 Conventional DS-CDMA Network

As shown in Figure 5.1, in the conventional DS-CDMA network each system user has a unique code \( c_i \) and there is a separate receiver \( i \) for each user at the central station. Each receiver \( i \) is equipped with only one code \( c_i \). We assume that each receiver constantly monitors its code and it successfully receives the desired packet with certain probability denoted as \( P_o(K) \). This probability \( P_o(K) \) which represents the probability that a given receiver successfully receives a packet in the presence of \( K \) simultaneous transmissions using \( K \) different codes and was called simply the probability of packet success in the previous chapter.
For the conventional DS-CDMA network with $U$ system users, the average number of packets successfully received by the central station per slot given $K$ simultaneous transmissions is given by [77]:

$$
\bar{C}_K^c = K P_c(K) \quad \text{for} \quad K = 1, \cdots, U
$$

(5.1)

where $P_c(K)$ is the probability that a given receiver successfully receives a packet in the presence of $K$ simultaneous transmissions using $K$ different codes and was developed in the previous chapter. In eq. (5.1) we used a superscript $c$ in $C_K$ to denote the conventional network case. Later we use a superscript $p$ for the partial network.

### 5.3 Partial DS-CDMA Network

To reduce the central station complexity in terms of the required number of receivers, we group all system users $U$ into $M$ groups. Let us assume that each group has $m$ users (i.e., $U = mM$). If all users within a particular group use the same code, then we can reduce the number of receivers at the central station from $U$ to only $M$. Users from different groups use different codes, therefore the number of codes is equal to the number of receivers (groups) $M$. Figure 5.2 shows the network configuration of the proposed partial DS-CDMA system. Since all users in a given group use the same code to communicate with a particular receiver corresponding to that group, intra-group collisions occur. This means that each code is shared among a group of system users in a contention mode of operation. We assume that when several packets are transmitted from a specific group, in a given time slot, the receiver is able to capture only the first arriving packet with a certain probability and all the remaining packets are considered to be collided and treated as interference. This is
Figure 5.2: Proposed partial DS-CDMA network.
known in the literature as the delay capture phenomena and has been analyzed for the situation in which several transmitters use the same code for communicating with a single receiver [85].

We assume that if $K$ packets are transmitted in a given slot from different groups with $n$ ($n < K$) packets from a specific group $i$, then among the $n$ contending packets, only the first arriving one captures the receiver $i$ with certain probability referred to as delay capture probability and denoted as $P_d(n)$; and all other packets coming from the same group as well as other groups are rejected as interference. Any user from a given group $i$ who captures first the receiver $i$, he retains capture (all data packet is completely successful) with certain probability $P_c(K)$ which depends on the total number of transmissions in the slot. Therefore a receiver successfully receives a complete data packet with probability equals to the delay capture probability times the probability of retaining capture in the presence of $K$ transmissions.

In the performance of the proposed partial DS-CDMA network, first we assume ideal delay capture where the probability of delay capture is unity, then we investigate the effect of delay capture probability on the network performance.

### 5.3.1 Calculation of $\tilde{C}_K^p$: Ideal Case

Let us assume that there are $K$ users with packets to transmit in a given slot. If $K_1$ packets are from users of group 1, then only the first arrived packet of the $K_1$ packets can be acquired (lock onto receiver 1) and retains capture (i.e., the complete data packet successfully received) with probability $P_c(K_1)$ while the others are rejected as interference. Generally, if $K$ packets come from $j$ different groups, then $j$ of $K$ packets can be acquired and each of the $j$ packets retains the capture with probability $P_c(K)$. To determine the average number of packets successfully received by the
central station per slot given $K$ simultaneous transmissions denoted as $C_K$, we need to find the probability that $K$ packets come from $j$ different groups given $M$ receivers and this is denoted as $P_{K_j}^{(M)}$.

For a given number of receivers (codes) $M$, let us find $P_{K_j}^{(M)}$ for $j = 1, \ldots, \min(K, M)$. We define

$$
\mathcal{L}_1 = \{l_1^n, l_2^n, \ldots, l_m^n\} \rightarrow \text{group 1}
$$

$$
\mathcal{L}_2 = \{l_1^n, l_2^n, \ldots, l_m^n\} \rightarrow \text{group 2}
$$

$$
\vdots
$$

$$
\mathcal{L}_M = \{l_1^n, l_2^n, \ldots, l_m^n\} \rightarrow \text{group M}
$$

(5.2)

where $l_j^n$ is the $j^{th}$ user of the $i^{th}$ group. There is one code per group, and at the central station there is a receiver associated with each $\mathcal{L}_i$. Suppose that $K$ packets come from $j$ different groups, then

$$
K = \sum_{i=1}^{M} K_i
$$

(5.3)

where $K_i = 0$ or $K_i \in \mathcal{L}_i$. If $j = M$, then all $K_i$'s are greater than zero. If $j < M$, then we have $j$ of $K_i$'s $\in \mathcal{L}_i$ and $(M - j)$ of $K_i$'s as zero.

The probability that $K$ packets come from $j$ different groups with $K_1$ packets from group 1, $K_2$ packets from group 2, etc... is given by:

$$
pr(K_1, K_2, \ldots, K_M) = \frac{K!}{K_1! K_2! \ldots K_M!} p_1^{K_1} p_2^{K_2} \ldots p_M^{K_M}
$$

(5.4)

where $p_i$ is the probability that a packet comes from group $i$. Assuming that all $p_i$ are equal:

$$
p_i = p = \frac{1}{M} \quad \text{for all} \quad i,
$$

(5.5)

then eq. (5.4) becomes:

$$
pr(K_1, K_2, \ldots, K_M) = \frac{K!}{K_1! K_2! \ldots K_M!} p^K
$$

(5.6)

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Once we find all the combinations, then the probability of \( K \) packets coming from \( j \) different groups given a number of receivers \( M \) could be computed as:

\[
P_{Kj}^{(M)} = \begin{cases} \sum_i \Pr \{ I_{i|K,j} \} & \text{for } 1 \leq j \leq \min(M, K) \\ 0 & \text{otherwise,} \end{cases} \tag{5.7}
\]

where \( I_{i|K,j} \) is the \( i \)'th possible combination for a given \( j \) and \( K \) and is represented as \( I_{i|K,j} = (I_{1}^{i}, I_{2}^{i}, \ldots, I_{j}^{i}) \) where \( \sum_{s=1}^{j} I_{s}^{i} = K \). The probability of a given combination is given by:

\[
\Pr \{ I_{i|K,j} \} = \sum_{i_1}^{M} \sum_{i_2}^{M} \ldots \sum_{i_j}^{M} \Pr(K_{i_1} = I_{1}^{i}, K_{i_2} = I_{2}^{i}, \ldots, K_{i_j} = I_{j}^{i}) \tag{5.8}
\]

If \( I_{1}^{i} = I_{2}^{i} = \ldots = I_{j}^{i} \) then \( i_1 \) starts from 1, \( i_2 \) starts from \( i_1 + 1 \), etc. If \( I_{1}^{i} \neq I_{2}^{i} \neq \ldots \neq I_{j}^{i} \) then \( i_1 \) starts from 1, \( i_2 \) starts from 1 for all \( i_2 \neq i_1 \), etc. All the possible combinations and the calculation of the \( P_{Kj}^{(M)} \) for \( j = 1, \ldots, K \) could be found by using a computer program. The flowchart of the program we used is given in Figure 5.3.1. In what follows we give a simple example, to better understand the computation of the \( P_{Kj}^{(M)} \)'s for different \( j \) given \( K \) and \( M \). Suppose we want to compute the \( P_{Kj}^{(M)} \)'s for \( K = 4 \), thus we need to compute \( P_{4,1}^{(M)} \), \( P_{4,2}^{(M)} \), \( P_{4,3}^{(M)} \) and \( P_{4,4}^{(M)} \). In the computation of the probability that four packets come from one group, \( P_{4,1}^{(M)} \), the only possibility is all four packets come from a given group. Then

\[
P_{4,1}^{(M)} = \sum_{i_1=1}^{M} \Pr(K_{i_1} = 4) = \sum_{i_1=1}^{M} \frac{1}{M^4} = \frac{1}{M^3} \tag{5.9}
\]

In the computation of the probability that four packets come from two different groups, \( P_{4,2}^{(M)} \), there are two possibilities. (i) One packet comes from one group and three packets come from another group. (ii) Two packets come from one group and two other packets come from another group. Then

\[
P_{4,2}^{(M)} = \sum_{i_1=1}^{M} \sum_{\substack{i_2=1 \\ i_2 \neq i_1}}^{M} \Pr(K_{i_1} = 1, K_{i_2} = 3) + \sum_{i_1=1}^{M} \sum_{i_2=i_1+1}^{M} \Pr(K_{i_1} = 2, K_{i_2} = 2) \tag{5.10}
\]

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For $K = 1 : K_{\text{max}}$

For $j = 1 : K$

Find all possible combinations $(I_1^1, I_1^2, ..., I_1^j), (I_2^1, I_2^2, ..., I_2^j), ...$

where $\sum_{x=1}^{j} I_x^x = K$

Find the probability of each combination

$$\Pr \{ I_i | K_j \} = \sum_{i_1}^{M} \sum_{i_2}^{M} ... \sum_{i_j}^{M} \text{pr}(K_{i_1} = I_1^1, K_{i_2} = I_1^2, ..., K_{i_j} = I_1^j)$$

where $\text{pr}(K_1, K_2, ..., K_j) = \frac{K!}{K_1! K_2! ... K_j!} \frac{1}{M^K}$

$$P_{K_j}^{(M)} = \sum_i \Pr \{ I_i | K_j \}$$

where the summation is over all the possible combinations

Yes

$j < \min(K, M)$?

No

$M < K$?

Yes

$P_{K_j}^{(M)} = 0$

for $M < j \leq K$

No

$K < K_{\text{max}}$?

Yes

No

STOP

Figure 5.3: Algorithm for finding the values of $P_{K_j}^{(M)}$. 

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In the computation of probability that four packets come from three different groups, $P_{4,3}^{(M)}$, the only possibility is one packet comes from a given group, one packet comes from a second group and two other packets come from a third group. Then

$$P_{4,3}^{(M)} = \sum_{i_1=1}^{M} \sum_{i_2=i_1+1}^{M} \sum_{i_3 = 1}^{M} \sum_{i_4 \neq i_1 \neq i_2}^{M} \text{pr}(K_{i_1} = 1, K_{i_2} = 1, K_{i_3} = 2) \quad (5.11)$$

Finally, in the computation of the probability that four packets come from four different groups, $P_{4,4}^{(M)}$, the only possibility is all four packets come from four different groups. Then

$$P_{4,4}^{(M)} = \sum_{i_1=1}^{M} \sum_{i_2=i_1+1}^{M} \sum_{i_3=i_2+1}^{M} \sum_{i_4=i_3+1}^{M} \text{pr}(K_{i_1} = 1, K_{i_2} = 1, K_{i_3} = 1, K_{i_4} = 1) \quad (5.12)$$

For a given number of simultaneous transmissions $K$ and a given number of receiver at the central station, the average number of packet successfully received by the central station for the partial DS-CDMA network is given by:

$$\bar{C}_K^s = \sum_{j=1}^{K} j \cdot P_{Kj}^{(M)} P_e(K) \quad (5.13)$$

### 5.3.2 Effect of Delay Capture

In the previous subsection, we have assumed that if $n$ packets come from a given group, then one packet is captured with certainty (perfect capture). In other words, the probability that one packet is captured given that $n$ packets using the same code were sent from a given group, $P_d(n) = 1$. A system cannot achieve perfect delay capture for a number of reasons. In what follows we analyze the proposed partial DS-CDMA network by taking into account the delay capture phenomena.

The basic mechanism for capture considered in this paper is the ability of the receiver to synchronize with and lock onto the first arriving packet and subsequently
treat other overlapping packets as noise. This is possible if the first packet arrives at least $T_{cap}$ seconds earlier than the subsequent packets which use the same PN code. Here $T_{cap}$ denotes the capture time and it is approximately equal to the PN synchronization time. Conceptually, the capture time may be as little as few chip durations in a direct sequence spread spectrum system [85]. It is assumed that each user within a given group introduces its own random transmission delay to randomize its packet arrival time at the central station, but for slotted operation at the packet level, these random delays are required to be small compared to the packet duration $T_p$. Hence, the packet arrival times are assumed to be statistically independent and uniformly distributed over a randomization time interval $[0, T_u]$, where $T_u << T_p$. The slots are slightly longer in duration than packets $(T_s = T_u + T_p)$, however, significant increases in system throughput per slot can more than compensate for this small amount of overhead [100]. Figure 5.4 shows an example slot for this scheme wherein $n$ users from a given group $i$ attempt transmission to receiver $i$. In this example if $X_{1-2}$ is greater than the capture time $T_{cap}$, then we say that packet 1 captures the receiver.

The probability that the first arriving packet is captured, given that $n$ packets using the same code arrive at an intended receiver in a given slot, $P_d(n)$ has been derived in [85]. For a randomized time of arrival process with a randomization interval $T_u$, this probability is given by:

$$P_d(n) = \begin{cases} 
1 & n = 1 \\
(1 - Q)^n & n \geq 2 \\
0 & \text{otherwise,}
\end{cases} \quad (5.14)$$

where $Q = T_{cap}/T_u$ is called the capture ratio. In some recent personal communication systems, synchronization times on the order of 10 to 200 micro seconds has been
Figure 5.4: Time randomization capture.
reported [134]. The randomization time can typically be about 10% of a slot duration. Assuming that a typical slot duration is in the order of 20 to 100 milli-seconds [133]-[134], then capture ratios on the order of 0.01 to 0.1 are practically attainable. Similar capture ratio values were also reported in [85] and [88].

Now let’s find the average number of packets successfully received by the central station per slot given $K$ simultaneous transmissions for the partial DS-CDMA network in which we take into account the delay capture probability, $P_d(n)$. Let us start with a simple example to explain how the effect of $P_d(n)$ is taken into account in calculation of $\bar{C}_K^p$. Suppose we have four packets transmitted in a given slot and these packets come from two different groups ($K = 4$ and $j = 2$), thus we have two possible combinations:

(i) Combination 1: two packets come from one group and the other two packets come from another group ($I_{1|4,2} = (I_1^1 = 2, I_1^2 = 2)$).

(ii) Combination 2: one packet comes from one group and the other three packets come from another group ($I_{2|4,2} = (I_1^1 = 1, I_2^2 = 3)$).

When we computed $\bar{C}_K^p$ with the ideal delay capture phenomena, we assumed that in such a case two packets are captured and each of them retains capture with $P_c(4)$. Now, in the modified analysis the average number of packets captured will be a certain value denoted as $\alpha_{K,j}$ which depends on $P_d(n)$. In this case $\alpha_{4,2}$ is given by:

$$\alpha_{4,2} = (1 \times P_d(2) + 1 \times P_d(2)) \times \Pr(\text{comb1}) + (1 \times F_{\cdot \cdot \cdot 1 \cdot r \cdot 1 \times P_d(3)) \times \Pr(\text{comb2}),$$

where $\Pr(\text{comb1})$ and $\Pr(\text{comb2})$ are the probabilities of occurrence of the combination 1 and 2 respectively. In general, for a given $j$ and $K$, we have the average number of packets captured is given by:

$$\alpha_{K,j} = \sum_i \left[ \Pr(\text{comb1}) \sum_{s=1}^{j} P_d(I_s^1) \right]$$

(5.15)

and finally the average number of packets successfully received (captured and retains
captured) $\tilde{C}_K^p$ becomes:

$$\tilde{C}_K^p = \sum_{j=1}^{K} \alpha_{K_j} P_{K_j}^{(M)} P_e(K)$$  \hspace{1cm} (5.16)

### 5.4 Numerical Results

In what follows we shall first discuss the numerical results of the average number of packets successfully received per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system under different conditions. Then we compare these results with the ones obtained for partial DS-CDMA system. Unless stated differently, most of the figures are obtained for the fast Rician fading channel, the MAI is modeled as an improved Gaussian process and the effect of bit-to-bit dependence due to MAI is taken into account (this corresponds to the fast fading-case 3 in Chapter 4).

Figure 5.5 shows the average number of packets successfully received $\tilde{C}_K^p$ per slot given $K$ simultaneous transmissions at the central station for the conventional DS-CDMA system without error control coding for nonfaded and fast Rician fading channel with $K_f = 10$ dB, $N = 31$ and $L_p = 252$. From this figure we can see that the value of $\tilde{C}_K^p$ increases as $K$ increases until it reaches a maximum then it starts to decreases as $K$ increases. From the figure we can distinguish three parameters of importance. The first parameter which is denoted by $K_{opt1}$ represents the maximum number of simultaneous transmissions the system can accommodate without any collisions, which means that if $K < K_{opt1}$ all transmitted packets will be successful. From the figure we can see that $K_{opt1} = 5$ and 2 for nonfaded and faded channel respectively. The second parameter which is denoted by $K_{opt2}$ represents the maximum number of simultaneous transmissions that provides the maximum value of $\tilde{C}_K^p$. $K_{opt2} = 10$ and 7 for nonfaded and faded channel respectively. For the nonfaded channel with $K_{opt2} = 10$ we have $\tilde{C}_{K_{opt2}}^p$ equals approximately to 8 successful packets,
therefore two packets are collided. The third parameter which is denoted by $K_{\text{max}}$ represents the maximum number of simultaneous transmissions beyond which almost all transmitted packets are collided. This figure indicates that for $K > K_{\text{max}} = 17$ and 23 for nonfaded and faded channel; all transmitted packets are collided. Figure 5.6 shows the average number of collided packets ($K - \bar{C}_K$) per slot given $K$ simultaneous transmissions for the same conditions as Figure 5.5. This figure illustrates well the different parameters indicated above $K_{\text{opt1}}$, $K_{\text{opt2}}$ and $K_{\text{max}}$. The main conclusion we can draw from these two figures is that implementing a central station with $U$ (number of system users) codes (which is not realistic) is not beneficent because the system can not tolerate more than a limited number of simultaneous transmitted packets which is denoted here by $K_{\text{max}}$. The values of $K_{\text{opt1}}$, $K_{\text{opt2}}$ and $K_{\text{max}}$ depends basically as we will see later on the channel condition and different system parameters such as $N$, $L_p$ etc.

The effect of the error control coding and the effect of using different packet length on the values of the average number of successful packets and the average number of collided packets is shown in Figure 5.7 and Figure 5.8 respectively. A (15,7) BCH code and a packet length of 252 and 1008 information bits are considered. The same system and fading condition as the other figures are considered. This figure indicates that coding increases significantly the system performance. For $L_p = 252$, the (15,7) BCH code increases $K_{\text{max}}$ from approximately 17 to 41 users. These figures also show that increasing $L_p$ implies a reduction on the system performance. With (15,7) BCH code the value of $K_{\text{max}}$ reduces from 41 to 31 users by increasing $L_p$ from 252 to 1008.

Previous figures are shown for case where the MAI is modeled as an improved Gaussian process and the effect of bit-to-bit dependence due to MAI is taken into
Figure 5.5: Average number of packets successfully received per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system without coding for nonfaded and fast Rician fading channel with $K_f = 10$ dB, $N = 31$ and $L_p = 252$. 
Figure 5.6: Average number of collided packets per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system without coding for nonfaded and fast Rician fading channel with $K_f = 10$ dB, $N = 31$ and $L_p = 252$. 
Figure 5.7: Average number of packets successfully received per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system with and without coding for fast Rician fading channel with $K_f = 10$ dB, $N = 31$ and different packet length $L_p$. 
Figure 5.8: Average number of collided packets per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system with and without coding for fast Rician fading channel with $K_f = 10$ dB, $N = 31$ and different packet length $L_p$. 
account. For comparison reason, in Figure 5.9 we add the performance based on the assumption that the MAI is Gaussian and the effect of bit-to-bit dependence due to MAI is ignored. The fast Rician fading channel with $K_f = 10$ and 5 dB is considered. As the figure shows the computation of the average number of successful packets based on the assumptions that the MAI is Gaussian and bit errors are independents yields optimistic results for small number of simultaneous users and pessimistic results for large number of users. Similar behavioral results on the throughput performance of slotted ALOHA conventional DS-CDMA system were found in [42]. Finally, before moving to discuss the performance of the partial DS-CDMA, we shall investigate the performance under the slow fading condition. Figure 5.10 shows the average number of successful packets for conventional DS-CDMA with fast and slow fading channel. In both cases the MAI is modeled as an improved Gaussian process and the effect of bit-to-bit dependence due to MAI is taken into account. As the figure shows, the performance of the system is better with slow fading especially for large number of simultaneous users. This result agrees with the results found in [121] for the throughput of slotted ALOHA conventional DS-CDMA system, where it was shown that the throughput for slow fading channel is much higher than for fast fading channel.
Figure 5.9: Average number of packets successfully received per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system using (15,7) BCH code for fast Rician fading channel with $K_f = 10$ and 5 dB; $N = 31$ and different MAI models (i.b.e: ignore the bit-to-bit dependence due to MAI; d.b.e: consider the bit-to-bit dependence due to MAI).
Figure 5.10: Average number of packets successfully received per slot given $K$ simultaneous transmissions for the conventional DS-CDMA system with $(15,7)$ BCH code for fast and slow Rician fading channel with $K_f = 10$ dB; $N = 31$ and $L_p = 1008$. 
Figure 5.11 and Figure 5.12 show the average number of successful packets per slot given $K$ simultaneous transmissions for the uncoded and coded conventional DS-CDMA system as well as the partial DS-CDMA with different number of receivers (codes) $M$ at the central station. In these figures an ideal delay capture is assumed. As we can see from these figures a reduction on the required number of receivers at the central station results in a degradation on the system performance due to intra-group collisions. However, this degradation is not really significant if one considers that the required number of receivers is significantly reduced and thus making the system implementable. From these figures we can see that the partial DS-CDMA system performance increases by increasing $M$, and as $M$ approaches the number of system users $U$ we get the same performance as the conventional DS-CDMA system. However, these figures indicate that the degradation on the performance of the partial DS-CDMA with $M = 30$ is not significant and one could implement a central station with only 30 receivers for a large population of users $U$ with each of $(U/30)$ users share the same code in a contention mode.

The effect of delay capture on the system performance is shown in Figure 5.13 for partial DS-CDMA with $M = 10$. Capture ratios of $Q = 0.1$ and 0.01 are considered. From this figure we can notice a small degradation on the performance due to delay capture probability for $Q = 0.1$. However, this degradation is negligible for $Q = 0.01$. Since it has been reported in [85] and [88] that a practical capture ratio is in the order of 0.01 to 0.1, then we can conclude that for those practical value of capture ratio, the degradation due to delay capture probability is very small.
Figure 5.11: Average number of packets successfully received per slot given $K$ simultaneous transmissions for the conventional and partial DS-CDMA system without coding for fast Rician fading channel with $K_f = 10$ dB; $N = 31$ and $L_p = 1008$. 
Figure 5.12: Average number of packets successfully received per slot given \( K \) simultaneous transmissions for the conventional and partial DS-CDMA system with \((15,7)\) BCH code for fast Rician fading channel with \( K_f = 5 \) dB; \( N = 31 \) and \( L_p = 1008 \).
Figure 5.13: Average number of packets successfully received per slot given $K$ simultaneous transmissions for the partial DS-CDMA system with $M = 10$ without coding for fast Rician fading channel with $K_f = 10$ dB; $N = 31$ and $L_p = 1008$. 

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5.5 Summary

In this chapter we proposed a new partial DS-CDMA system for land mobile satellite data networks that requires a limited number of spread spectrum codes.

The proposed partial DS-CDMA was analyzed and its performance was compared with the conventional DS-CDMA in which each system user should have a separate code, thus a large number of codes is required when the system is designed to serve a large number of users.

The comparison was done under different channel conditions and by considering different system parameters. Different results have been reported and different design parameters such as $K_{opt1}$, $K_{opt2}$ and $K_{max}$ were defined and presented. The knowledge of such parameters is essential in the design of the DS-CDMA networks.

It was shown that the number of collided packets increases as the number of simultaneous transmissions increases and if the number of transmissions exceeds certain threshold denoted by $K_{max}$, then all packets collide. Before retransmitting these packets, a way to avoid repeated collisions is needed. In this chapter we did not consider any protocols to resolve the collisions. In the next chapter we present a collision resolution algorithm which will be used in conjunction with partial DS-CDMA.
Chapter 6

CRA for Partial DS-CDMA Packet Networks

6.1 Introduction

In the literature when DS-CDMA is used in a random access mode of operation, direct sequence spread spectrum is combined with certain characteristic of ALOHA system [70]-[85]. This is referred to as ALOHA/DS-CDMA protocol. In this protocol, when a source has a new packet, it transmits and waits for an acknowledgment. If after certain timeout the source does not receive an acknowledgment, it assumes that the packet has collided and retransmits it after a random delay. The retransmission takes place at a randomly selected time so that the conflicting packets are unlikely to collide again. It is well known that an ALOHA type system is inherently unstable [93]-[95], and statistical fluctuations may easily cause its saturation. This means that it may reach a situation where the number of retransmitting sources tends to infinity and the throughput tends to zero. Therefore, although the delay and throughput properties might be satisfactory in the short term, they are quite poor when observed over a long interval of time. Actually if the source satisfies the Poisson source model, the average delay of ALOHA/DS-CDMA system is infinite [93].

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Since the appearance of Capetanakis' pioneer paper [106] the subject of collision resolution algorithm (CRA) has attracted the attention of many researchers, who consider that CRAs are more efficient than ALOHA type system. Furthermore, the CRA is stable even for an infinite population of users. In this chapter the proposed partial DS-CDMA is combined with a CRA to avoid the problem of instability which occurs with ALOHA. One of the main contributions of our study is the investigation of the CRA for CDMA systems which is rarely done in the literature. The CRA considered in our study is a modification of an algorithm proposed in [99]. This algorithm is simple and exhibits lower delay in comparison to other CRAs, such as the one proposed by Capetanakis [106] or Gallager [110].

As described in Chapter 5 in the partial DS-CDMA network, we assume that all system users $U$ are divided into $M$ groups with $m$ users in each group. All users within a given group communicate with one receiver at the central station, i.e., there exists a receiver at the central station for each group. All users within a particular group transmit the data packet using a common code. Two users from different groups use different codes. We assume that each receiver constantly monitors its code and it captures the first arriving packet if there is more than one packet transmitted from a given group. In the previous chapter we have shown that a number of transmitted packets suffers a collision in a given slot and thus need to be retransmitted. The proposed CRA is used to resolve these collisions for the partial DS-CDMA network considered in the previous chapter.

We assume that the first arriving packet captures the receiver and retains capture with a probability that depends on the number of simultaneous transmissions.

We assume that for each channel slot there are three possible outcomes.

(i) No packet transmission, in which case the slot is called empty (E).
(ii) Capture in either the presence or the absence of multiple transmissions, in which case the slot is called successful (S).

(iii) Multiple or single transmission with no capture, in which case the event is called collision (C).

We assume that the central station broadcasts one of those ternary feedback per slot to all users in the system.

The CRA is first analyzed for full feedback sensing environment in which each system user knows the overall feedback history. Then we investigate the operation of the algorithm in limited sensing environment.

In Section 6.2 we present the collision resolution algorithm. The throughput-delay analysis of this algorithm for partial DS-CDMA is given in Section 6.3. The algorithm is first analyzed for full sensing environment while neglecting the propagation delay. In Section 6.4 we present the algorithm in a limited sensing environment. In Section 6.5 the effect of the propagation delay on the algorithm performance is given. Finally in Section 6.6 we summarize the findings of this chapter.

6.2 The Collision Resolution Algorithm

To resolve a collision, a CRA is required. In what follows we describe the CRA that is used in our study. We consider that time is measured in slot units; slot $t$ occupies the time interval $[t, t+1)$ and $x_t$ denotes the feedback that corresponds to slot $t$. $x_t$ takes three possible values, $x_t = E$ corresponds to an empty slot; $x_t = S$ corresponds to successful slot and $x_t = C$ corresponds to collision slot.

The algorithm utilizes a window of length $\Delta$. Let $t$ be a time instant such that, for some $t_1 < t$ all the packets arrivals in $[0,t_1]$ have been successfully transmitted and there is no information regarding the arrival interval $(t_1,t]$, and such that $t$
corresponds to the beginning of some slot. The instant \( t \) is then called collision resolution point (CRP). The arrival interval \((0, t_1]\) is called "resolved interval", and the interval \((t_1, t]\) is called "the lag at \( t \)". In slot \( t \), the packet arrivals in \((t_1, t_2]\) (where \( t_2 = \min (t_1 + \Delta, t) \)) attempt transmission, and the arrival interval \((t_1, t_2]\) is then called the "examined interval". The examined interval is called resolved, when all the arrivals in it have been successfully received. Until \((t_1, t_2]\) is resolved, no arrivals in \((t_2, \infty)\) are allowed transmission. The time period required for the resolution of an examined interval is called the collision resolution interval, (CRI). The algorithm rules are as follows:

1) If the examined interval \((t_1, t_2]\) contains zero packets, then the CRI lasts one slot, and new examined interval \((t_2, t_3 = \min (t_2 + \Delta, t + 1)\) is selected at \( t + 1 \).

2) If the examined interval \((t_1, t_2]\) contains at least one packet and all the packets are successfully transmitted, then slot \( t + 1 \) is wasted, (with \( x_{t+1} = E \)), so that it becomes known to all users in the system that the examined interval has been resolved. Thus, the CRI lasts then two slots, and a new examined interval is selected at \( t + 2 \).

3) If the examined interval \((t_1, t_2]\) contains at least one packet and \( x_t = C \), then the CRI lasts at least three slots. During the time period that the CRI lasts, each involved user implements the algorithm rules independently, via the use of a counter. Given some user, the value of his counter at time \( t \) is denoted \( r_t \) where \( r_t \) equals either 1 or 2. The utilization and updating of the counter values and the identification of the slot when the CRI ends, are as follows:

3.1) The user transmits in slot \( t \), if and only if \( r_t = 1 \).

3.2) The counter values are updated as follows:

- If \( x_t = E \) or \( S \) and \( r_t = 2 \), then \( r_{t+1} = 1 \).
\begin{itemize}
\item If \( x_t = C \) and \( r_t = 2 \), then \( r_{t+1} = 2 \).
\item If \( x_t = S \) and \( r_t = 1 \), and the user identifies success for himself, then this successful packet departs the system.
\item If \( x_t = S \) and \( r_t = 1 \), and the user identifies no success for himself, then \( r_{t+1} = 1 \).
\item If \( x_t = C \) and \( r_t = 1 \), then,
\[
    r_{t+1} = \begin{cases} 
        1 & \text{with probability } 1/2 \\
        2 & \text{with probability } 1/2. 
    \end{cases}
\]
\end{itemize}

The CRI ends at the beginning of slot \( t \), if and only if \( x_{t-2} = E \) or \( S \) and \( x_{t-1} = E \), and there has been no empty or successful slot followed by an empty slot pattern previously occurred during the CRI. That is, the CRI ends the first time after its beginning, that a noncollision slot is followed by an empty slot.

### 6.3 Algorithm Analysis for the Partial DS-CDMA Network

To analyze the algorithm, we first need to compute \( L_K \), the expected length of a collision resolution interval given that it starts with a collision of multiplicity \( K \). But before starting the derivation of \( L_K \), let us define the following:

\( P_{cap}(K) \): is the probability that a receiver successfully receives a packet in the presence of \( K \) simultaneous transmissions. We have shown in the previous chapter that the effect of the delay capture probability is not significant. Thus, in this chapter we neglect the effect of the delay capture probability and consider only the probability of retaining capture which depends on the type of modulation, error control coding, channel environment etc., and denoted in Chapter 3 by \( P_c(K) \). Therefore \( P_{cap}(K) \) is simply given by \( P_c(K) \).

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\( P_{K_j}^{(M)} \): is the probability that \( K \) packets come from \( j \) different groups and it was derived in Chapter 5.

\( P_{M,K}(j) \): is the probability that \( j \) packets are successfully transmitted in a transmission involving \( K \) packets in a system consisting of \( M \) different groups (\( M \) different codes) and is given by:

\[
P_{M,K}(j) = \begin{cases} 
P_{K_j}^{(M)} P_{\text{exp}}(K) & \text{for } j = 1, 2, \ldots, \min(K, M) \\
1 - P_{\text{exp}}(K) & \text{for } j = 0 \text{ and } K > 0 \\
1 & \text{for } j = K = 0 \\
0 & \text{otherwise.}
\end{cases}
\]  

(6.2)

Let's now define \( L_{n,K-n} \) as the expected number of slots needed by the algorithm for the successful transmission of \( K \) packets, given that \( n \) of those packets have counter value equal to one and \( (K - n) \) of the packets have counter values equal to two. Notice that \( L_K = L_{K,0} \). The algorithm described in the previous section induces the following recursions:

\[
L_{0,0} = 1,
\]

\[
L_{0,K} = 1 \cdot L_{K,0} \text{ for } K \geq 1
\]
for $1 \leq n \leq K$ and $K \geq 1$, $L_{n,K-n}$ is given by:

$$
L_{n,K-n} = \begin{cases} 
1 + L_{K-1,0} & \text{w.p. } P_{M,n}(1) \quad (1 \text{ success given } n \text{ trans}) \\
1 + L_{K-2,0} & \text{w.p. } P_{M,n}(2) \quad (2 \text{ successes given } n \text{ trans}) \\
\vdots \\
1 + L_{K-n,0} & \text{w.p. } P_{M,n}(n) \quad (n \text{ successes given } n \text{ trans}) \\
1 + \sum_{i=0}^{n} \binom{n}{i} 2^{-n} L_{i,K-i} & \text{w.p. } P_{M,n}(0) \quad (0 \text{ success given } n \text{ trans})
\end{cases}
$$

(6.3)

In eq. (6.3) the abbreviations "w.p." represents with probability and "trans" represents transmission. By combining all these possible combinations, $L_{n,K-n}$ could be rewritten as:

$$
L_{n,K-n} = \sum_{j=1}^{n} (1 + L_{K-j,0}) P_{M,n}(j) + \left[ 1 + \sum_{i=0}^{n} \binom{n}{i} 2^{-n} L_{i,K-i} \right] P_{M,n}(0) \quad (6.4)
$$

for $1 \leq n \leq K$ and $K \geq 1$.

After an extensive manipulation and by using induction, $L_{n,K-n}$ can be expressed as:

$$
L_{n,K-n} = B_n + \sum_{i=0}^{n} A_n^{(i)} L_{K-i,0} \quad (6.5)
$$

for $1 \leq n \leq K$ and $K \geq 1$. 

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where $A_{n}^{(i)}$ for $0 \leq i \leq n$ are independent of $K$ and can be computed recursively as follows:

\[
A_{n}^{(i)} = \begin{cases} 
1 & \text{for } n = i = 0, \\
\frac{2^{-n} P_{M,n}(0)}{1 - 2^{-n} P_{M,n}(0)} \sum_{j=0}^{n-1} \binom{n}{j} A_{j}^{(i)} & \text{for } i = 0, n \geq 1, \\
\frac{P_{M,n}(i)}{1 - 2^{-n} P_{M,n}(0)} & \text{for } i \geq 1, n = i, \\
\frac{P_{M,n}(i)}{1 - 2^{-n} P_{M,n}(0)} + \frac{2^{-n} P_{M,n}(0)}{1 - 2^{-n} P_{M,n}(0)} \sum_{j=i}^{n-1} \binom{n}{j} A_{j}^{(i)} & \text{for } 1 \leq i < n.
\end{cases}
\]  

(6.6)

and $B_{n}$ is given by:

\[
B_{n} = \begin{cases} 
1 & \text{for } n = 0, \\
\frac{1}{1 - 2^{-n} P_{M,n}(0)} + \frac{2^{-n} P_{M,n}(0)}{1 - 2^{-n} P_{M,n}(0)} \sum_{j=0}^{n-1} \binom{n}{j} B_{j} & \text{for } n \geq 1.
\end{cases}
\]  

(6.7)

For $n = K$ eq. (6.5) gives

\[
L_{K} = L_{K,0} = \frac{1}{1 - A_{K}^{(0)}} \left( B_{K} + \sum_{i=1}^{K} A_{K}^{(i)} L_{K-i,0} \right) \text{ for } K \geq 1
\]  

(6.8)

### 6.3.1 Throughput Analysis

As mentioned before, a Poisson model is considered. Let $\lambda$ denotes the traffic intensity of the Poisson traffic process of all network users. Given the window size, $\Delta$ of the algorithm, let $E(L|\Delta)$ denotes the expected length of a CRI, given that it starts with an examined interval of length $\Delta$. Thus, we can write

\[
E(L|\Delta) = \sum_{K=0}^{\infty} E(L|\Delta, K) \exp(-\lambda\Delta) \frac{(\lambda\Delta)^{K}}{K!}
\]

\[
= \sum_{K=0}^{\infty} L_{K} \exp(-\lambda\Delta) \frac{(\lambda\Delta)^{K}}{K!}
\]  

(6.9)
Since $E(L|\Delta, K) = L_K$ and $L_K$ is given by eq. (6.8).

Assume that the system starts operating at time zero. Let $t_i, i \geq 1$ be the sequence of successive CRP's, and let $X_t$ be the lag at $t_i$. It has been shown in [99] that the sequence $X_t, i \geq 1$ is a Markov chain with state space $\mathbb{F}$. If $\Delta$ is rational, then it was shown in [99] that $\mathbb{F}$ is at most countable. The ergodicity condition in [105] gives that the Markov chain is ergodic and the system is stable if and only if

$$E(\ell|\Delta) < \Delta$$  \hspace{1cm} (6.10)

If we denote $x = \lambda \Delta$ then eq. (6.9) and eq. (6.10) yield

$$\frac{x}{\lambda} > \sum_{K=0}^{\infty} L_K e^{-x} \frac{x^K}{K!}$$  \hspace{1cm} (6.11)

By optimizing eq. (6.11), one can find the maximum value of the intensity traffic, $\lambda_{\text{max}}$, which is the maximum throughput as

$$\lambda < \lambda_{\text{max}} = \sup_{x} \frac{x}{\sum_{K=0}^{\infty} L_K e^{-x} \frac{x^K}{K!}}$$  \hspace{1cm} (6.12)

Then the optimum window size $\Delta_{\text{opt}}$ could be deduced as $\Delta_{\text{opt}} = x_{\text{opt}}/\lambda_{\text{max}}$.

For various values of the system parameters and different channel conditions, we computed the optimal window size $\Delta_{\text{opt}}$ as well as the maximum throughput $\lambda_{\text{max}}$ of the collision resolution algorithm for the partial DS-CDMA. The number of receiver $M$ at the central station is chosen to be 10, 20 and 30.

Table 6.1 and Table 6.2 represent the maximum throughput and optimal window size for the CRA for partial DS-CDMA system for nonfaded channel without considering error control coding with $N = 31$, $E_b/N_o = \infty$, $L_p = 252$ and 1008 respectively. From these tables we can see that as $M$ increases the maximum throughput increases. But this increase is not significant for $M$ greater than 30 which agrees with the results.
obtained in Chapter 5. We can see that increasing the packet length results in the reduction of the maximum throughput.

Table 6.3 shows the performance of a system similar to the one presented in Table 6.1 but with changing the spreading factor from \( N = 31 \) to 63. This table indicates that doubling the spreading factor results in an increase of the maximum achievable throughput by a factor approximately equals to 2.

The effect of the thermal noise on the throughput is shown in Table 6.4 where all system parameters are kept similar as in Table 6.1 with the \( E_b/N_o \) equals to 10 dB. From this table we can see that the degradation on the throughput performance is significant. Therefore, one should consider the effect of the thermal noise in the design of the mobile satellite network.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{max} )</td>
<td>( M = 10 ) ( M = 20 ) ( M = 30 )</td>
</tr>
<tr>
<td></td>
<td>2.84 3.27 3.45</td>
</tr>
<tr>
<td>( \Delta_{opt} )</td>
<td>5.93 5.66 5.51</td>
</tr>
</tbody>
</table>

Table 6.1: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers \( M \), nonfaded channel without coding, \( N = 31 \), \( L_p = 252 \) and \( N_o = 0 \).

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{max} )</td>
<td>( M = 10 ) ( M = 20 ) ( M = 30 )</td>
</tr>
<tr>
<td></td>
<td>2.35 2.63 2.75</td>
</tr>
<tr>
<td>( \Delta_{opt} )</td>
<td>5.5 5.247 5.01</td>
</tr>
</tbody>
</table>

Table 6.2: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers \( M \), nonfaded channel without coding, \( N = 31 \), \( L_p = 1008 \) and \( N_o = 0 \).

So far we have been analyzing the throughput performance of the proposed protocol in a channel which is corrupted only by the MAI and the thermal noise. As
<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>4.28</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>6.92</td>
</tr>
</tbody>
</table>

Table 6.3: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, nonfaded channel without coding, $N = 63$, $L_p = 252$ and $N_o = 0$.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>2.02</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>5.91</td>
</tr>
</tbody>
</table>

Table 6.4: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, nonfaded channel without coding, $N = 31$, $L_p = 252$ and $E_b/N_o = 10$ dB.

mentioned before, the mobile satellite channel is further corrupted by the multipath fading. In the following tables the effect of fading on the throughput performance of the proposed protocol is discussed. Both fast and slow fading are considered and the MAI is modeled as an improved Gaussian process which take into account the effect of bit-to-bit dependencies due to the MAI.

Table 6.5 and Table 6.6 represent the maximum throughput and optimal window size for the CRA for partial DS-CDMA system for fast Rician fading channel with $K_f = 10$ dB without considering error control coding with $N = 31$, $E_b/N_o = \infty$, $L_p = 252$ and 1008 respectively. By comparing the results of these two tables with the ones in Table 6.1 and Table 6.2 we can see that the fading channel reduces the throughput of the proposed protocol considerably. To combat the effect of the multipath fading we propose to use error control coding with the proposed protocol.

Table 6.7 and Table 6.8 represent the throughput performance of the proposed
protocol with the same system parameters as in Table 6.6 but error control coding is used in this case. Table 6.7 represents the throughput performance using the (7,4) BCH code and Table 6.8 represents the throughput performance using the (15,7) BCH code. These tables show that even for the small BCH codes considered, the maximum throughput of the proposed protocol is significantly increased. For example for \( M = 30 \), this maximum throughput goes from 1.63 to 3.56 and to 4.5 by using the (7,4) and (15,7) BCH codes respectively. Table 6.9 shows the throughput performance of the proposed protocol with the same system parameters as in Table 6.5 \( (L_p = 252) \) and using the (15,7) BCH code. Also this table indicate that the (15,7) BCH code improve the throughput performance significantly. For \( M = 30 \), the maximum throughput goes from 2.36 to 5.57.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>( \lambda_{\text{max}} )</th>
<th>( \Delta_{\text{opt}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M = 10 )</td>
<td>2.04</td>
<td>5.98</td>
</tr>
<tr>
<td>( M = 20 )</td>
<td>2.26</td>
<td>5.79</td>
</tr>
<tr>
<td>( M = 30 )</td>
<td>2.36</td>
<td>5.60</td>
</tr>
</tbody>
</table>

Table 6.5: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers \( M \), fast Rician fading channel \( (K_f = 10 \text{ dB}) \) without coding, \( N = 31 \), \( L_p = 252 \) and \( N_o = 0 \).

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>( \lambda_{\text{max}} )</th>
<th>( \Delta_{\text{opt}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M = 10 )</td>
<td>1.47</td>
<td>5.5</td>
</tr>
<tr>
<td>( M = 20 )</td>
<td>1.58</td>
<td>5.31</td>
</tr>
<tr>
<td>( M = 30 )</td>
<td>1.63</td>
<td>5.16</td>
</tr>
</tbody>
</table>

Table 6.6: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers \( M \), fast Rician fading channel \( (K_f = 10 \text{ dB}) \) without coding, \( N = 31 \), \( L_p = 1008 \) and \( N_o = 0 \).

Table 6.10 and Table 6.11 show the throughput performance of the proposed protocol with the same system parameters as in Table 6.6 and using the convolutional
<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>2.91</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>6.06</td>
</tr>
</tbody>
</table>

Table 6.7: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 10$ dB) with (7,4) BCH code, $N = 31$, $L_p = 1008$ and $N_o = 0$.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>3.51</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>6.33</td>
</tr>
</tbody>
</table>

Table 6.8: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 10$ dB) with (15,7) BCH code, $N = 31$, $L_p = 1008$ and $N_o = 0$.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>4.08</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Table 6.9: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 10$ dB) with (15,7) BCH code, $N = 31$, $L_p = 252$ and $N_o = 0$.  

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codes with rate $r = 1/2$ and constraint length $CL = 3$ and 7 respectively. As deduced with BCH codes the convolutional codes increase significantly the maximum throughput. This increase is more significant as the constraint length increases. These tables indicate also that the considered convolutional codes provide better performance than the (7,4) and (15,7) BCH codes.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>3.66</td>
</tr>
<tr>
<td>$\Delta_{\text{opt}}$</td>
<td>6.63</td>
</tr>
</tbody>
</table>

Table 6.10: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 10$ dB) with convolutional code, $r = 1/2$, $CL = 3$, $N = 31$, $L_p = 1008$ and $N_o = 0$.

<table>
<thead>
<tr>
<th>Maximum throughput &amp; Optimum window size</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>4.26</td>
</tr>
<tr>
<td>$\Delta_{\text{opt}}$</td>
<td>6.64</td>
</tr>
</tbody>
</table>

Table 6.11: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 10$ dB) with convolutional code, $r = 1/2$, $CL = 7$, $N = 31$, $L_p = 1008$ and $N_o = 0$.

To study the effect of the fading depth we computed the throughput performance for $K_f$ factor equals to 5 dB. Table 6.12 and Table 6.13 show the throughput performance of the proposed protocol with the same system parameters as in Table 6.8 and Table 6.9 with $K_f = 5$ dB instead of 10 dB. The (15,7) BCH code is considered in these table, $L_p = 252$ for Table 6.12 and $L_p = 1008$ for Table 6.13. By comparing these two tables with Table 6.8 and Table 6.9 we can say that the maximum throughput decreases as $K_f$ decreases.
<table>
<thead>
<tr>
<th>Optimum window size &amp; Maximum throughput</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>3.2</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>6.66</td>
</tr>
</tbody>
</table>

Table 6.12: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 5$ dB) with (15,7) BCH code, $N = 31$, $L_p = 252$ and $N_o = 0$.

<table>
<thead>
<tr>
<th>Optimum window size &amp; Maximum throughput</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>2.52</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>6.19</td>
</tr>
</tbody>
</table>

Table 6.13: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, fast Rician fading channel ($K_f = 5$ dB) with (15,7) BCH code, $N = 31$, $L_p = 1008$ and $N_o = 0$.

Finally Table 6.14 shows the throughput performance under the slow fading condition. The system parameters in this case are similar to the one in Table 6.8 except slow fading is considered in this case. By comparing these two tables, we can say that slow fading channels yield better performance than the performance of fast fading channels.

<table>
<thead>
<tr>
<th>Optimum window size &amp; Maximum throughput</th>
<th>Number of receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>3.77</td>
</tr>
<tr>
<td>$\Delta_{opt}$</td>
<td>7.96</td>
</tr>
</tbody>
</table>

Table 6.14: Maximum throughput and optimal window size for the CRA for partial DS-CDMA system with different number of receivers $M$, slow Rician fading channel ($K_f = 10$ dB) with (15,7) BCH code, $N = 31$, $L_p = 1008$ and $N_o = 0$. 

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6.3.2 Delay Analysis

Let the packets be labeled $1, 2, 3, \ldots$ according to the order of their arrival instants. We define the delay $D_n$, experienced by the $n$th packet as the time difference between its arrival and the end of its successful transmission. We are interested in evaluating the first moment of the steady state delay process, when it exists. Let the first lag corresponds to the empty slot zero; thus, $X_1 = 1$. Let the sequence $T_i; i \geq 1$ be defined as follows: Each $T_i$ corresponds to the beginning of some slot, and $T_1 = 1$. Also, each $T_i$ corresponds to the ending point of a length-one lag ($X_i = 1$). $T_{i+1}$ is then the first CRP after $T_i$ at which the lag has length one. From the description of the algorithm it can be seen that the induced delay process probabilistically restarts itself at the beginning of each slot $T_i, i \geq 1$. The interval $[T_i, T_{i+1})$ will be referred to as the $i$th session. Note that the sessions have lengths that are i.i.d random variables.

Let $R_i, i \geq 1$ denote the number of successfully transmitted packets in the interval $(0, T_i]$. Let $\bar{R}_i = R_{i+1} - R_i; i \geq 1$, be the number of packets successfully transmitted in the interval $(T_i, T_{i+1}]$. The sequence $\bar{R}_i, i \geq 1$ is a sequence of i.i.d random variables; thus, $R_i, i \geq 1$, is a renewal process. In addition, the delay process $D_n, n \geq 1$ is regenerative with respect to the process $R_i, i \geq 1$, with regeneration cycle $\bar{R}_i$; and the distribution of $\bar{R}_i$ is nonperiodic since $P(\bar{R}_i) > 0$. From the regenerative theorem 1 in [104], we conclude that if

$$\bar{R} = E(\bar{R}_i) < \infty$$  \hspace{1cm} (6.13)

and

$$W = E \left( \sum_{n=1}^{\infty} D_n \right) < \infty$$  \hspace{1cm} (6.14)

then the expected steady state delay, $D$ per successfully transmitted packet is given
by the following expression:

$$D = \frac{W}{\bar{R}}$$  \hspace{1cm} (6.15)

The average packet delay can be found by computing the quantities $W$ and $\bar{R}$. In Appendix B we developed systems of linear equations whose solution may be used to compute the mean cycle length $\bar{R}$ and the mean cumulative delay $W$.

For various values of the system parameters and different channel conditions, we computed the upper bound on the average packet delay for various Poisson rate $\lambda$ within the corresponding stability regions of the protocol.

Figures 6.1 and 6.2 represent the expected packet delay versus the channel throughput for the proposed protocol for nonfaded channel without considering error control coding with $N = 31$, $E_b/N_o = \infty$, $L_p = 252$ and 1008 respectively. The expected packet delay is computed in a slot unit. From these figures we can say that the expected delay induced by the proposed protocol is small (less than two slots) for light traffic (low input rate). This delay remains almost constant for certain traffic and when the input rate approaches the maximum throughput, the expected delay increases rapidly. This conforms with the nature of the random access protocol which induces low delay for low input rate when the traffic is bursty. From the figure we can also say that for a given input rate the expected delay increases by reducing the number of codes used. This is an expected result since a reduction in the number of required codes implies more intra-group contention.

Figure 6.3 shows the expected packet delays for a system similar to the one in Figure 6.1, but in this case fast Rician fading channel with $K_f = 10$ dB is considered. This figure shows that fading does not only reduce the throughput of the protocol but also increases the expected delay. By comparing those two figures we can see that for a given throughput the expected delay a packet experiences in fading channel

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Figure 6.1: Average packet delay per slot for the CRA partial DS-CDMA without error control coding for nonfaded channel with $N = 31$, $E_b/N_0 = \infty$ and $I_p = 252$. 
Figure 6.2: Average packet delay per slot for the CRA partial DS-CDMA without error control coding for nonfaded channel with $N = 31$, $E_b/N_0 = \infty$ and $L_p = 1008$. 
(Figure 6.3) is higher than the delay in nonfaded channel.

The influence of the use of error control coding is shown in Figure 6.4. The (15,7) BCH code is considered. As we can see the code improves significantly the throughput as well as the delay characteristic of the protocol. By using coding the delay becomes almost constant for certain values of the input rate. The delay increases rapidly only when the input rate reaches the maximum achievable throughput.

Figure 6.5 shows the expected packet delay for a system similar to the one in Figure 6.4 with $K_f = 5$ dB instead of 10 dB. Comparing this figure with Figure 6.4 we can say that the expected packet delay increases when the fading becomes deeper which means that reducing the Rician factor results in higher packet delay.

Finally, Figure 6.6 shows the expected packet when the fading is slow. The same system parameters as in Figure 6.4 are considered here. This figure indicates clearly that the throughput-delay performance under slow fading conditions is better than the throughput-delay performance under fast fading conditions.

6.4 Operation of the CRA in the Limited Sensing Environment

In the limited sensing algorithm, the user monitors the channel feedback only from the time he generates a packet to the time this packet is successfully transmitted. The objective here is to prevent new arrivals from interfering with some collision resolution in progress. This is possible if each user recognizes if a collision resolution is in progress or not within a finite number of slots from the time he generates a new packet. As explained previously, the occurrence of a consecutive success and empty slots corresponds to the end of a CRI which started with a collision slot; and the occurrence of two consecutive empty slots corresponds to either two consecutive
Figure 6.3: Average packet delay per slot for the CRA partial DS-CDMA without error control coding for fast Rician fading channel with $K_f = 10 \text{ dB}$, $N = 31$, $E_b/N_o = \infty$ and $L_p = 1008$. 
Figure 6.4: Average packet delay per slot for the CRA partial DS-CDMA using (15,7) BCH code for fast Rician fading channel with $K_f = 10$ dB, $N = 31$, $E_b/N_o = \infty$ and $L_p = 1008$. 
Figure 6.5: Average packet delay per slot for the CRA partial DS-CDMA using (15,7) BCH code for fast Rician fading channel with $K_f = 5$ dB, $N = 31$, $E_b/N_0 = \infty$ and $L_p = 1008$. 
Figure 6.6: Average packet delay per slot for the CRA partial DS-CDMA using (15,7) BCH code for slow Rician fading channel with $K_f = 10$ dB, $N = 31$, $E_b/N_0 = \infty$ and $L_p = 1008$. 
unit-length CRI's or to the end of a CRI followed by an empty slot. Thus upon the observation of such an event, a user can decide that there is no collision resolution in progress. The algorithm proposed in [99] was also analyzed in the limited sensing environment by adopting a small modification to the algorithm with full sensing. In our case the same modification could be applied to operate the algorithm in a limited sensing environment. The modification of the algorithm could be summarized as follows:

The window size is the same as in the full feedback sensing case. However, the window slides through the unexamined interval from present to past and its edge is maintained one slot before the current time as illustrated in Figure 6.7. Within each window, the operations of the algorithm are the same as in the full feedback sensing case. The throughput of the algorithm remains identical to that under full feedback sensing, but the delay in this case increases [99].

Figure 6.8 shows the expected delay for the proposed protocol with only $M = 20$ under both full and limited feedback sensing. In this figure fast Rician fading channel with $K_f = 10$ dB and without error control coding with $N = 31$, $E_b/N_0 = \infty$ and $L_p = 252$ was considered. From this figure we can say that in the limited sensing environment, and for light input traffic, the protocol induces expected packet delay approximately to 2.7 as opposed to 1.75 for full sensing environment and as the rate of the input traffic increases, the expected packet delays approach those induced under full feedback sensing.

6.5 Effect of the Propagation Delay on the CRA

The collision resolution algorithm discussed in the previous subsection did not consider the propagation delay. In some applications, such as communication via satellite
Figure 6.7: Window selection of the CRA in limited sensing environment.
Figure 6.8: Average packet delay per slot for the CRA partial DS-CDMA with $M = 20$ under both full and limited feedback sensing without error control coding for fast Rician fading channel with $K_f = 10$ dB, $N = 31$, $E_b/N_0 = \infty$ and $L_p = 252$. 
the propagation delay is not negligible. In what follows the effect of propagation delay and alternative approach for coping with such delay are briefly discussed.

Suppose the round-trip propagation delay time and the transmission time for acknowledgment of information is equal to or longer than a unit slot, or this delay is $d_p$ slots. With $d_p$ taken into consideration, the result of transmission in slot $i$ should not affect transmissions in slot less than $i + d$ where

$$d = [d_p] + 1$$

(6.16)

where $[d_p]$ denotes the smallest integer equal to or less than $d_p$.

It was shown in [137] that a channel with a delay $d$ may be modeled as $d$ interleaved channels without delay. That is slots $0, d, 2d, \ldots$ of a channel with delay form slots $0, 1, 2, \ldots$ of the first interleaved channel without delay. Slots $1, d + 1, 2d + 1, \ldots$ of the channel with delay form slots $0, 1, 2, \ldots$ of the second interleaved channel without delay. By using the interleaving technique, we can deal with the algorithm as we did in the previous subsection by considering zero delay. The interleaving scheme is simple for coping with the propagation delay. The throughput remains the same, however the delay will be increased due to the extra delay induced by the interleaving process.

Another possibility for coping with the propagation delay is possible. Assume $d = 2$ and each slot in the channel is identified as even or odd. A user who has a packet to transmit, before doing so, he flips a binary fair coin. If the outcome is a 0, then he will transmit in the even slot. If the outcome is a 1, then he will transmit in the odd slot. A user who flips 0 and enter a collision session transmits and keeps track only on the odd slots and vice-versa. Like this we form two subchannels "acting as channels without delay".
6.6 Summary

In this chapter we proposed a collision resolution algorithm that is used in conjunction with the partial DS-CDMA protocol. The proposed protocol was analyzed in the presence of the Poisson user model and under both full feedback sensing and limited feedback sensing. A fast and slow Rician fading channel with different $K_f$ was considered. We considered also in the analysis of the proposed protocol both convolutional and block codes.

The main advantages of this protocol is its simplicity and it is an implementable protocol because it requires a limited number of spreading codes and can be operated in limited feedback sensing.

We have shown that fading has a significant effect on the throughput-delay performance of the proposed protocol. As fading becomes deep the degradation on the throughput of the protocol increases. To overcome this problem we suggested to use error control coding techniques, which have been found to be effective in combating the effect of fading. The throughput performance of the proposed protocol can be significantly improved by using some diversity combining techniques and more powerful error control codes.

In order to compare the performance of the proposed CRA/DS-CDMA protocol with the conventional protocols (which do not use spread spectrum) such as slotted ALOHA, the throughput must be normalized to the processing gain. This allows a direct comparison to the conventional systems. That is, given a certain bandwidth, the slotted ALOHA channel for example could transmit a message in $1/N$ times the amount of time required by the slotted CRA/DS-CDMA system, so normalization by the processing gain provides a fair comparison on a bit per unit bandwidth basis. For
instance assume the data transmission rate is 10 kbps. If we use the CRA/DS-CDMA with processing gain of 31 and we achieve a throughput of let say 7 packets per slot (it could be much more than that if some diversity combining techniques and more powerful error control codes are used), then the total system throughput will be 70 kbps. Now if we use a conventional protocol with 310 kHz of bandwidth, one may not achieve a total throughput of 70 kbps if ALOHA protocol is used; known that ALOHA protocol achieves a normalized throughput of 0.2-0.25 over fading channel [135], [136]. For more discussion on the performance comparison of the proposed protocol with many other existing protocols, one can refer to the author’s publications in [79], [80], [82] and [117].
Chapter 7

Conclusions and Suggestions for Future Research

7.1 Conclusions

In this thesis we proposed a new DS-CDMA protocol to be used as a multiple access technique for the mobile satellite data packet networks. We focused on the transmission of data from a large population of bursty users to a central station. The performance of the proposed protocol has been analyzed over AWGN and fading channels. Forward error correction coding was used to compensate for the degradations caused by fading and multiple access interference. In this section, we summarize the main contributions and conclusions of our study.

To analyze the bit and packet error rate for the DS-CDMA system over the mobile satellite channel, the adopted fading model permits the fast or slow Rayleigh or Rician fading processes. This model characterizes many communications channels such as the mobile satellite channel, mobile radio channel and many others. Although our study is mainly focused on the mobile satellite application, we preferred to consider a general fading model in such a way our result be applicable to many other applications.

To model the MAI, most of the time the Gaussian distribution is used because
of its simplicity. Morrow and Lehnert [41] have shown that modeling the MAI as an improved Gaussian process provides a significant improvement in the accuracy of the probability of bit error and probability of packet success. However, the approximation developed in [41] is computationally cumbersome and is only limited to the AWGN channel. In our early work [45] we simplified the approximation developed in [41] with good accuracy for the AWGN channel. In this thesis we have also developed new and simple approximations for the probability of bit error and probability of packet success for DS-CDMA over the proposed fading channels by modeling the multiple access interference as an improved Gaussian process.

The main conclusion that we draw on the BER analysis of the DS-CDMA over the multipath fading is that the computation of the probability of bit error based on the Gaussian approximation for the MAI is not accurate especially for small number of simultaneous users which corresponds to low probability of bit error. However, it was found that the discrepancy on the BER results based on the standard Gaussian and the improved Gaussian reduces as the fading becomes more pronounced, and it was shown that when the channel exhibits a Rayleigh fading the standard Gaussian approximation and the improved gaussian approximation give more or less the same results. This is due to the fact that for deep fading the probability of bit error is quite high. But when coding is used the probability of bit error decreases even for deep fading and the discrepancy on the BER results based on the standard Gaussian and the improved Gaussian increases.

Another phenomena which is most of the time neglected in the literature and well examined in this thesis is the bit-to-bit dependence. The bit-to-bit dependence is important especially when the packet error rate is considered and the channel is corrupted by fading. Morrow and Lehnert [41]-[42] have shown that when asynchronous
DS-CDMA is considered we cannot assume that the bit errors are independent due to the interfering signal relative delays and phases. It has been shown in [41] that the assumptions that the MAI is Gaussian and the bit error are independent produces an optimistic estimate of packet success for a lightly loaded channel and then becomes pessimistic as the MAI increases. However, the results found in [41]-[42] were limited to the AWGN channel and the approximations developed for the probability of packet success are quite cumbersome. In this thesis we extended the work in [41]-[42] by investigating the effect of bit-to-bit dependence on the the probability of packet success over the multipath fading channels. The approximations on the the probability of packet success developed in this thesis are relatively simple and very general ones in such a way that they are applicable to many channel conditions. Even for the AWGN channel case our approximation on the the probability of packet success is much simpler than the one developed in [41].

The main conclusions that we can draw from the analysis of the probability of packet success for the DS-CDMA systems over the multipath fading channel are as follows: When the MAI is modeled as Gaussian and the effect of bit-to-bit dependence due to MAI is ignored, the probability of packet success is found to be optimistic for small number of users and pessimistic for large number of user with respect to the case where the MAI is modeled as an improved Gaussian and the effect of bit-to-bit dependence due to MAI is taken into account. This is true for both faded and nonfaded channel. When slow fading is considered, it is found that the bit-to-bit dependencies within a packet has significantly increased. This increase of bit-to-bit dependencies add additional degradation on the packet performance for small number of users and enhance the improvement for large number of users.

The second stage of the work is to propose a new DS-CDMA system referred to
as partial DS-CDMA system in which a limited number of spread spectrum codes is used and hence solve the problem of the large number of codes required when the system is designed to serve a large population of users. To reduce the central station complexity in terms of the required number of receivers (codes), in Chapter 5 we proposed to group all system users $U$ into $M$ groups with each group having $m$ users (i.e., $U = mM$). All users within a particular group use the same code in a contention mode, and users from different groups use different codes. Therefore, we reduce the required number of codes from $U$ to only $M$.

The proposed partial DS-CDMA system was analyzed in Chapter 5 and its performance was compared with the unrealistic conventional DS-CDMA in which each system users should have a separate code. The comparison was done under different channel conditions and by considering different system parameters. It was shown that a reduction on the required number of receivers at the central station results in a degradation of the system performance due to intra-group collisions. However, this degradation is not really significant if one considers that the required number of receivers is significantly reduced and thus making the system more practical. The results in Chapter 5 indicate that the partial DS-CDMA system performance increases by increasing $M$ and as $M$ approaches the number of system users $U$ we get the same performance as the conventional DS-CDMA system. However, these results indicate that the degradation on the performance of the partial DS-CDMA is not considerable if one consider the significant reduction on the central station complexity.

The final stage of this thesis work was to propose a new random access protocol that could be used in conjunction with the partial DS-CDMA to solve collisions when they occur. In Chapter 6 we proposed to use a collision resolution algorithm as a random access protocol instead of an ALOHA type protocol to be used with the
partial DS-CDMA to avoid the problem of instability which occurs with ALOHA. One of the main contributions of our study is the investigation of the CRA for CDMA systems which is rarely addressed in the literature.

The proposed algorithm in conjunction with the partial DS-CDMA has been analyzed in the presence of the Poisson user model and under both full feedback sensing and limited feedback sensing. Fast and slow Rician fading channels with different $K_f$ has been considered. Both convolutional and block codes were considered in the analysis of the proposed protocol. We have shown that fading has a significant effect on the throughput-delay performance of the proposed protocol. As fading becomes deep the degradation on the throughput of the protocol increases. To overcome this problem we investigated the effect of using error control coding.

### 7.2 Suggestions for Future Research

There are many interesting techniques which could be investigated for the MSAT communications system which could not be pursued in the context of this thesis.

- In this thesis only BPSK as a modulation technique is considered. Although BPSK is a widely used modulation technique, it is worth investigating the proposed protocol with other modulation techniques such as DPSK, QPSK and MFSK.

- It was shown that FEC is a very important means to enhance the system performance. In our study only some simple BCH and convolutional codes were considered. More powerful nonbinary coding techniques such as RS codes or a concatenation of RS codes and convolutional codes can be employed to further increase the throughput of the proposed protocol.
• The receiver considered in this thesis did not use any combining techniques. This is basically due to the fact that the first path represents the strongest path and has a nonfaded component onto which the receiver locks. Although the first path has more power than the other paths, we still can gain some improvement by combining the $L$ paths.

• In the analysis of partial DS-CDMA system, a single packet capture model is assumed, i.e., when several packets were transmitted from a given group using the same spread spectrum code only the first arriving one will be captured by the receiver. It has been reported in [86] that multiple packet capture is feasible. In [86] a multiple capture model is presented in a centralized packet radio system with common direct sequence spread spectrum modulation. It was shown that the multiple capture system has significantly higher system throughput than a single capture system. Recently, it was also claimed in [87] that multiple capture is possible in the centralized packet radio system with common direct sequence spread spectrum. Applying the multiple capture model to our proposed protocol would be interesting since it is expected to increase the throughput performance.

• In this thesis, only delay capture phenomenon was considered. Other capture phenomena are possible to be employed in the partial DS-CDMA system. It is worth investigating the performance of the partial DS-CDMA system using power capture.

• In this thesis the proposed protocol was intended to be used for data applications. This protocol can be easily modified to accommodate multimedia traffic.
Appendix A

Derivation of the Variance of $\Psi$

The variance of $\Psi$ is given by the sum of the variance of $\omega_1$, $\omega_2$ and twice the covariance of $\omega_1$ and $\omega_2$, i.e.,

$$\sigma_\Psi^2 = \text{Var}(\omega_1) + \text{Var}(\omega_2) + 2 \text{Cov}(\omega_1, \omega_2) \quad (A.1)$$

where

$$\omega_1 = \sum_{k=2}^{K} X_{i,k} \quad (A.2)$$

and

$$\omega_2 = \sum_{k=1}^{K} \sum_{l=2}^{L} X_{i,k} \quad (A.3)$$

with $X_{i,k}$ is given by:

$$X_{i,k} = \beta_{i,k}^2 \cos^2(\varphi_{i,k}) \left[2(2B + 1)(S_{i,k}^2 - S_{i,k}) + N\right] \quad (A.4)$$

To find the variance of $\omega_1$ and $\omega_2$ we should first find the variance of $X_{i,k}$ which is given by:

$$\text{Var}(X_{i,k}) = E(X_{i,k}^2) - E^2(X_{i,k}) \quad (A.5)$$

where $E(X_{i,k}^2)$ can be evaluated from

$$E(X_{i,k}^2) = E(\beta_{i,k}^4) \cdot \text{E}^{4}(\cos^2(\varphi_{i,k})) \cdot \text{E}\left\{\left[2(2B + 1)(S_{i,k}^2 - S_{i,k}) + N\right]^2\right\} \quad (A.6)$$
Since $S_{i,k}$ is a uniform random variable in $[0,1]$, then

$$E(S_{i,k}) = \frac{1}{2},$$  \hspace{1cm} (A.7)  \\
$$E(S^2_{i,k}) = \frac{1}{3},$$  \hspace{1cm} (A.8)  \\
$$E(S^3_{i,k}) = \frac{1}{4}$$  \hspace{1cm} (A.9)

and

$$E(S^4_{i,k}) = \frac{1}{5}$$  \hspace{1cm} (A.10)

It was shown in Chapter 3 that $B = (N - 1 - C)/2$, where $C$ is the discrete aperiodic autocorrelation of the signature sequence of receiver 1 and is given by:

$$C = \sum_{j=0}^{N-2} a^i_j a^{i+1}_j$$  \hspace{1cm} (A.11)

From the above equations we have

$$E(B) = \frac{N - 1}{2},$$ \hspace{1cm} (A.12)  \\
$$E(B^2) = \frac{N(N - 1)}{4}$$ \hspace{1cm} (A.13)

and

$$E(C^2) = N - 1$$ \hspace{1cm} (A.14)

Developing eq. (A.6) and using previous equations one obtains

$$E(X^2_{i,k}) = \frac{7N^2 + 2N - 2}{40} \cdot E(\beta^4_{i,k})$$ \hspace{1cm} (A.15)

and

$$\text{Var}(X_{i,k}) = \frac{7N^2 + 2N - 2}{40} \cdot E(\beta^4_{i,k}) - \frac{N^2}{9} \cdot E^2(\beta^2_{i,k})$$ \hspace{1cm} (A.16)

The covariance of $(X_{i,k}, X_{i,j})$ for any $j \neq k$ can be computed by following the same procedure as we did with the variance, thus we obtain

$$\text{Cov}(X_{i,k}, X_{i,j}) = \frac{N - 1}{36} \cdot E^2(\beta^2_{i,k})$$ \hspace{1cm} (A.17)
The variance of $\omega_1$ is given by:

$$Var(\omega_1) = (K - 1) [Var(X_{1,k}) + (K - 2)Cov(X_{1,k}, X_{1,j})]$$  \hspace{1cm} (A.18)

and

$$Var(\omega_2) = K(L - 1) [Var(X_{l,k}) + (K (L - 1) - 1)Cov(X_{l,k}, X_{l,j})]$$  \hspace{1cm} (A.19)

Finally, the $\text{Cov}(\omega_1, \omega_2)$ is given by

$$\text{Cov}(\omega_1, \omega_2) = K(K - 1)(L - 1) \left( \frac{N - 1}{36} E(\beta_{i,k}^2) E(\beta_{l,k}^2) \right), \hspace{1cm} l \neq 1.$$  \hspace{1cm} (A.20)
Appendix B

Delay Analysis for the CRA

B.1 Bounds on the Quantities $\bar{R}$ and $W$

Let $\bar{R}$ and $W$ be as in Chapter 6, and for the sequence $\{T_i\}$ being as in Section 6.4, let us define

$$H = E(T_2 - T_1)$$  \hspace{1cm} (B.1)

For the computation of the expected values of $\bar{R}$ and $W$, we also need the computation of the expected value $H$. Towards that, let us define the following quantities:

$l$: length of a conflict resolution interval.

$E(X|\tau)$: expected value of the random variable $X$, given that the examined interval has length $\tau$.

$P(\tau|\tau)$: the probability that the conflict resolution interval has length $\tau$, given that the examined interval has length $\tau$.

$J$: number of packets transmitted in the CRI, that starts at time $t$.

$x$: cumulative delay of the $J$ packets, after the CRP $t$.

$\Omega$: cumulative delay of the $J$ packets, before the beginning of the CRP $t$.

$h_d$: the number of slot needed to return to lag equals to one, when starting from a collision resolution instant with lag $d$. 

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$w_d$: cumulative delay experienced by all the packets that were successfully transmitted during the $h_d$ slots.

Let's also define

$$H_d = E(h_d) \quad \text{(B.2)}$$

and

$$W_d = E(w_d) \quad \text{(B.3)}$$

Note that by definition

$$W_1 = E \left( \sum_{n=1}^{R_1} D_n \right) = W \quad \text{(B.4)}$$

and $H = H_1$. If $H_1 < \infty$, then we have:

$$\tilde{R} = \lambda H_1 \quad \text{(B.5)}$$

Therefore the determination of $W_1$ and $H_1$ will permit the computation of the mean packet delay.

### B.2 Computation of $H_1$

The operation of the algorithm induces the following recursions:

$$1 \leq d \leq \Delta; \quad h_d = \begin{cases} 
  l & l = 1 \\
  l + h_l & l > 1 
\end{cases} \quad \text{(B.6)}$$

and for

$$d > \Delta; \quad h_d = l + h_{d-\Delta+l} \quad \text{(B.7)}$$

Taking the expectation of (B.6) and (B.7) yields:

$$H_d = \begin{cases} 
  E(l|d) + \sum_{l=2}^{\infty} H_l \, P(l|d) & 1 \leq d \leq \Delta \\
  E(l|\Delta) + \sum_{l=1}^{\infty} H_{d-\Delta+l} \, P(l|\Delta) & d > \Delta 
\end{cases} \quad \text{(B.8)}$$
This is a denumerable system of linear equations. Of interest to us is the element $H_1$ of a particular solution of this system. We now proceed in the development of this system, following the methodology in [104]. It was shown in [104] that in general $H_d$ could be bounded as following:

$$H_{d,l}^0 \leq H_{d,l}^1 \leq H_d \leq H_{d,u}^1 \leq H_{d,u}^0$$  \hfill (B.9)

where $u, l$ stands for upper and lower bounds respectively. It was shown [104] that

$$H_{d,u}^0 = \alpha_u d + \beta_u$$  \hfill (B.10)

$$H_{d,l}^0 = \alpha_l d + \beta_l$$  \hfill (B.11)

with $\alpha, \beta$ are constant to be determined and

$$H_{d,u}^1 = \begin{cases} 
E(l|d) + \sum_{i=2}^{\infty} H_{d,u}^0 P(l|d) & 1 \leq d \leq \Delta \\
E(l|\Delta) + \sum_{i=1}^{\infty} H_{d-\Delta+l,u}^0 P(l|\Delta) & d > \Delta
\end{cases}$$ \hfill (B.12)

$H_{d,l}^1$ is similar to $H_{d,u}^1$ except that we use $H_{d,l}^0$ instead of $H_{d,u}^0$ in (B.12). Let us first find the upper bound for $H_d$, then we can deduce the lower bound. By substituting (B.10) into (B.12) we have

$$H_{d,u}^1 = \begin{cases} 
E(l|d) + \sum_{i=2}^{\infty} (\alpha_u l + \beta_u) P(l|d) & 1 \leq d \leq \Delta \\
E(l|\Delta) + \sum_{i=1}^{\infty} (\alpha_u (d - \Delta + l) + \beta_u) P(l|\Delta) & d > \Delta
\end{cases}$$ \hfill (B.13)

Let's solve this system equation by equation

$$H_{d,u}^1 = E(l|d) + \beta_u \sum_{i=2}^{\infty} P(l|d) + \alpha_u \sum_{i=2}^{\infty} l P(l|d) ; \quad 1 \leq d \leq \Delta$$  \hfill (B.14)
We have

\[
\sum_{l=2}^{\infty} P(l|d) = 1 - P(l = 1|d) = 1 - e^{-\lambda d}
\]  

(B.15)

Note that \( P(l = 1|d) \) corresponds to the probability that we have zero packet in \( d \); and

\[
\sum_{l=2}^{\infty} l P(l|d) = E(l|d) - P(l = 1|d) = E(l|d) - e^{-\lambda d}
\]  

(B.16)

By substituting (B.15) and (B.16) into (B.14) we obtain

\[
H_{d,u}^1 = H_{d,u}^0 + E(l|d) + \alpha_u [E(l|d) - d - e^{-\lambda d}] - \beta_u e^{-\lambda d}; \quad 1 \leq d \leq \Delta
\]  

(B.17)

By proceeding in the same manner, the second equation of the system (B.12) yields:

\[
H_{d,u}^1 = H_{d,u}^0 + E(l|\Delta) + \alpha_u [E(l|\Delta) - \Delta]; \quad d > \Delta
\]  

(B.18)

Thus system (B.12) becomes

\[
H_{d,u}^1 = \begin{cases} 
H_{d,u}^0 + E(l|d) + \alpha_u [E(l|d) - d - e^{-\lambda d}] - \beta_u e^{-\lambda d} & 1 \leq d \leq \Delta \\
H_{d,u}^0 + E(l|\Delta) + \alpha_u [E(l|\Delta) - \Delta] & d > \Delta 
\end{cases}
\]  

(B.19)

Since the condition stability states that \( E(l|\Delta) < \Delta \) and using the fact that \( H_{d,u}^1 \leq H_{d,u}^0 \), the second equation of the system (B.19) yields to

\[
\alpha_u = \frac{E(l|\Delta)}{\Delta - E(l|\Delta)}
\]  

(B.20)

and the first equation of the system yields to

\[
\beta_u = \max \left\{ -\alpha_u, \sup_{1 \leq d \leq \Delta} (\rho(d)) \right\}
\]  

(B.21)
with
\[ \rho(d) = \frac{\alpha_u [E(l|d) - d - e^{-\lambda d}] + E(l|d)}{e^{-\lambda d}} \]  \hspace{1cm} (B.22)

If we use a similar method for the development of a lower bound, we find that
\[ \alpha_u = \alpha_l \]  \hspace{1cm} (B.23)

and
\[ \beta_l = \inf_{1 \leq d \leq \Delta} \rho(d) \]  \hspace{1cm} (B.24)

Finally for \( d = 1 \) we have
\[ \alpha_l + \beta_l \leq H_{1,l} \leq H_1 \leq H_{1,u} \leq \alpha_u + \beta_u \]  \hspace{1cm} (B.25)

where
\[ H_{1,u} = E(l|1) + \alpha_u [E(l|1) - e^{-\lambda}] + \beta_u (1 - e^{-\lambda}) \]  \hspace{1cm} (B.26)

and
\[ H_{1,l} = H_{1,u} - (\beta_u - \beta_l) (1 - e^{-\lambda}) \]  \hspace{1cm} (B.27)

Recall that
\[ E(l|x) = \sum_{k=0}^{\infty} L_k \frac{(\lambda x)^k}{k!} e^{-(\lambda x)} \]  \hspace{1cm} (B.28)

### B.3 Computation of \( W_1 \)

From the algorithm we have the following recursions:
\[ 1 \leq d \leq \Delta; \quad w_d = \begin{cases} \Omega + z & l = 1 \\ \Omega + z + w_l & l > 1 \end{cases} \]  \hspace{1cm} (B.29)

and for
\[ d > \Delta; \quad w_d = \Omega + z + (d - \Delta) J + w_{d-\Delta+l} \]  \hspace{1cm} (B.30)
Taking the expectation of (B.29) and (B.30) yields:

\[
W_d = \begin{cases} 
E(\Omega|d) + E(z|d) + \sum_{l=2}^{\infty} W_l P(l|d) & 1 \leq d \leq \Delta \\
E(\Omega|\Delta) + E(z|\Delta) + (d - \Delta) E(J|\Delta) + \sum_{l=1}^{\infty} W_{d-\Delta+l} P(l|\Delta) & d > \Delta 
\end{cases}
\]  
(B.31)

Following the methodology in [104] we can show that

\[
W_{d,u}^0 = \mu_t d^2 + \nu_t d + \epsilon_t \leq W_{d,l}^1 \leq W_d \leq W_{d,u}^1 \leq W_{d,u}^0 = \mu_u d^2 + \nu_u d + \epsilon_u \]  
(B.32)

where \( \mu, \nu, \epsilon \) are constant to be determined and

\[
W_{d,u}^1 = \begin{cases} 
W_{d,u}^0 + E(\Omega|d) + E(z|d) + \mu_u [E(l^2|d) - d^2 - e^{-\lambda d}] & 1 \leq d \leq \Delta \\
+ \nu_u [E(l|d) - d - e^{-\lambda d}] - \epsilon_u e^{-\lambda d} & 1 \leq d \leq \Delta \\
W_{d,u}^0 + E(\Omega|\Delta) + E(z|\Delta) + (d - \Delta) E(J|\Delta) & d > \Delta \\
+ \mu_u [2d E(l|\Delta) - 2d\Delta + E((l - \Delta)^2|\Delta)] + \nu_u [E(l|\Delta) - \Delta] & d > \Delta 
\end{cases}
\]  
(B.33)

\( W_{d,u}^1 \) is similar to \( W_{d,u}^1 \) with changing \( u \) by \( l \) everywhere in the system. By following the same procedure as in the previous section we found

\[
\mu_l = \mu_u = \frac{\lambda \Delta}{2(\Delta - E(l|\Delta))} 
\]  
(B.34)

\[
\nu_l = \nu_u = \frac{E(\Omega|\Delta) + E(z|\Delta) - \lambda \Delta^2 + \mu_u E((\Delta - l)^2|\Delta)}{\Delta - E(l|\Delta)} 
\]  
(B.35)

\[
\epsilon_u = \sup_{1 \leq d \leq \Delta} (\Phi(d)) 
\]  
(B.36)

\[
\epsilon_l = \inf_{1 \leq d \leq \Delta} (\Phi(d)) 
\]  
(B.37)
where

$$
\Phi(d) = \frac{E(\Omega|d) + E(z|d) + \mu_u[E(l^2|d) - d^2 - e^{-\lambda d}] - \nu_u[d - E(l|d) + e^{-\lambda d}]}{e^{-\lambda d}}
$$

(B.38)

Finally for $d = 1$ we have

$$
\mu_l + \nu_l + \varepsilon_l \leq W'_{1,l} \leq W_1 \leq W'_{1,u} \leq \mu_u + \nu_u + \varepsilon_u
$$

(B.39)

with

$$
W'_{1,u} = E(\Omega|1) + E(z|1) + \mu_u[E(l^2|1) - e^{-\lambda}] + \nu_u[E(l|1) - e^{-\lambda}] + \varepsilon_u[1 - e^{-\lambda}]
$$

(B.40)

and

$$
W'_{1,l} = W'_{1,u} - (\varepsilon_u - \varepsilon_l)(1 - e^{-\lambda})
$$

(B.41)

To compute $W'_{1,l}$ and $W'_{1,u}$ it is remaining for us to determine $E(\Omega|d)$, $E(z|d)$ and $E(l^2|d)$. Before doing that we can say that the expected delay from the regenerative theorem in [104] is given by

$$
D^i = \frac{W'_{1,l}}{\lambda H_{1,u}} \leq D \leq \frac{W'_{1,u}}{\lambda H_{1,l}} = D^u
$$

(B.42)

In this appendix we also show that the conditional expectations of the form $E(X|d)$ can be computed with high accuracy. Let us define $E(X|d, k)$: the conditional expectation of the random variable $X$, given that the arrival interval contains $k$ packets, and has length $d$. Then,

$$
E(X|d) = \sum_{k=0}^{\infty} E(X|d, k) \frac{(\lambda d)^k}{k!} e^{-\lambda d}
$$

(B.43)

The quantities $E(X|d, k)$ depend only on $k$. 

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B.3.1 Computation of $E(l^2|d)$

To compute $E(l^2|d)$ we need to compute first $E(l^2|d, k)$, then substitute the latter into (B.43). By following the same procedure as in section 5.4, the quantities $Y_k = E(l^2|d, k)$ can be computed recursively as:

$$Y_0 = 1$$  \hspace{2cm} (B.44)

$$Y_k = \frac{1}{1 - A_k^{(0)}} \left( F_k + \sum_{i=1}^{k} A_k^{(i)} Y_{k-i} \right) \text{ for } k \geq 1$$  \hspace{2cm} (B.45)

where $A_k^{(i)}, 1 \leq i \leq k$, are given by (5.21), and $F_k$ is given by:

$$F_k = \begin{cases} 
0 & k = 0 \\
2L_k - 1 + 2^{-k} P_{M,k}(0) \left( 1 + 2L_k \right) + 2^{-k} P_{M,k}(0) \sum_{j=0}^{k-1} \binom{k}{j} F_j \\
1 - 2^{-k} P_{M,k}(0) & k \geq 1 
\end{cases}$$  \hspace{2cm} (B.46)

B.3.2 Computation of $E(z|d)$

To compute $E(z|d)$ we need to compute first $E(z|d, k)$, then substitute the latter into (B.43). By following the same procedure as in section 5.4, the quantities $Z_k = E(z|d, k)$ can be computed recursively as:

$$Z_0 = 0$$  \hspace{2cm} (B.47)

$$Z_k = \frac{1}{1 - A_k^{(0)}} \left( I_{k,k} + \sum_{i=1}^{k} A_k^{(i)} Z_{k-i} \right) \text{ for } k \geq 1$$  \hspace{2cm} (B.48)

where $A_k^{(i)}, 0 \leq i \leq k$, are given by (5.21), and $I_{n,k}$ is given by:

$$I_{n,k} = \begin{cases} 
\frac{k + 2^{-n} k P_{M,n}(0)}{1 - 2^{-n} P_{M,n}(0)} & n = 1, k \geq 1 \\
\frac{k + 2^{-n} k P_{M,n}(0)}{1 - 2^{-n} P_{M,n}(0)} + \frac{2^{-n} P_{M,n}(0)}{1 - 2^{-n} P_{M,n}(0)} \sum_{j=1}^{n-1} \binom{n}{j} I_{j,k} & n > 1, k \geq 1 
\end{cases}$$  \hspace{2cm} (B.49)
B.3.3 Computation of $E(\Omega|d)$

We assume that the instant at which the packet arrive is an independent random variable distributed identically and uniformly over the examined interval. If the examined interval has a length $d$, thus the packet has to wait on the average $d/2$ before being served. Thus we conclude that

$$E(\Omega|d) = E(J|d) \frac{d}{2} = (\lambda d) \frac{d}{2} = \frac{\lambda d^2}{2}$$  \hspace{1cm} (B.50)
Appendix C

Accuracy of the Improved Gaussian Model for the MAI

As mentioned in Chapter 3, previous work on the computation of the probability of bit error for DS-CDMA system involved the use of moment spaces by many researchers [34], [37] to bound the effect of multiple access interference on a single data bit. And the use of Gaussian density approximation after determining the mean and variance of the MAI statistic. In [35], bounds on the bit error probability for deterministic sequences were developed from the convexity properties of the error probability function, and the characteristic function of the MAI component is integrated in [36] to find approximation to the data bit error.

In 1987, Lehnert and Pursley [38] developed upper and lower bounds on the probability of bit error for asynchronous DS-CDMA system by constructing the actual density function for the MAI. However, a tremendous computations are required for the evaluation of these bounds.

Later in 1989 and 1992 [41], [42], Morrow and Lehnert developed an approximation on the probability of bit error for asynchronous DS-CDMA system over AWGN channel by modeling the MAI as an improved Gaussian process. This approximation was compared to the bounds developed in [38] and it was found that a good accuracy
was obtained.

Figure C.1 shows a comparison of the probability of bit error for asynchronous DS-CDMA system over AWGN channel, based on the bounds developed in [38] and the improved Gaussian process as well as the gaussian process for the MAI developed in [41]. From this figure, it is evident that the results obtained from the improved Gaussian approximation provide greater accuracy than that of the standard Gaussian approximation for all number of simultaneous users. Finally as stated in [41], the improved Gaussian approximation not only provides accurate values for the probability of bit error for asynchronous DS-CDMA system for any number of simultaneous users, but it also allows us to incorporate the effects of bit-to-bit error dependence into the calculations.
Figure C.1: Comparison of the probability of bit error for asynchronous DS-CDMA system over AWGN channel using different methods. i) Bounds obtained in [38] by constructing the actual density function for the MAI. ii) By modeling the MAI as a standard Gaussian process. iii) By modeling the MAI as an improved Gaussian process.
Bibliography


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