Dynamics and Control of
Fast Automated Guided Vehicles
for High Load Applications

by
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In the Name of God
the Compassionate, the Merciful

For their patience and prayers
dedicated to my parents
Zahra Ardehali and Ali Asghar Naraghi

and

for her sacrifice and support
dedicated to my wife
Razieh Movahhedin
Abstract

Automated Guided Vehicles (AGV) are important components of modern automated transport systems. Increasing the system efficiency and throughput requires the use of heavy vehicles travelling at high speeds. As the AGV’s payload capacity and travelling speed increases, the ensuing increase in lateral acceleration requires thorough dynamic modelling and more sophisticated controller design.

To establish the sufficient level of model complexity necessary for this work, a 3-DOF nonlinear dynamic model comprising yaw, lateral, and roll motions is developed. The suspension, lateral and longitudinal load transfer, nonlinear behaviour of tires, and steering dynamics are included in this model. The model also comprises the effect of actuators, differential gear box, steering and tractive gear boxes. The model is validated through simulations and comparison with other models.

A dynamic-based approach to the control of a typical transport interfactory AGV in a semi structured environment is studied. The 2-DOF side slippage free dynamic model comprising steering, and actuators dynamics is used to design the controller. The input-output feedback linearization technique is employed to linearize the nonlinear dynamics of the vehicle. To improve robustness in the presence of parameter uncertainty, modelling errors and disturbance, a Boundary Layer Sliding Mode (BLSM) controller is adopted. The BLSM controller is later modified to improve performance and enhance robustness, using simultaneous variable boundary layer and multiple sliding surfaces strategies.

Simulations based on the 3-DOF nonlinear model show the satisfactory results for the Modified Boundary Layer Sliding Mode (MBLSM) controller.
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Nomenclature

\(A, B, D, E\)  
nonlinear tire model coefficients

\(a\)  
distance from c.g. to front axle

\(a_\text{abs}\)  
vehicle acceleration

\((a_\text{abs})_x\)  
acceleration component along the \(x\) axis

\((a_\text{abs})_y\)  
acceleration component along the \(y\) axis

\((a_\text{abs})_z\)  
acceleration component along the \(z\) axis

\(a_o\)  
acceleration of moving frame’s origin

\(a_y\)  
lateral acceleration

\(b\)  
distance from c.g. to rear axle

\(C_s, C_t\)  
equivalent gains of steering and tractive motors respectively

\(\overline{C}_s, \overline{C}_t\)  
viscous damping coefficient of steering and tractive motors respectively

\(C_a\)  
cornering stiffness of tire

\(C_\gamma\)  
camber stiffness of tire

\(c(s)\)  
path curvature

\(c_f, c_r\)  
front and rear suspension roll damping

\(c_{\text{tot}}\)  
total suspension roll damping

\(E\)  
decoupling matrix

\(e_i\)  
tracking error of the \(i\)th output

\(e_s, e_t\)  
unit vectors in path dependent coordinate

\(F(S)\)  
vector of attractiveness functions \(F_i(S)\)

\(F_x\)  
vector of longitudinal forces

\(F_y\)  
vector of cornering forces

\(F_c\)  
 sprung mass inertia force
\( F_{cf}, F_{cr} \) \( F_{r} \) front and rear sprung mass inertia forces respectively rolling resistance of tire
\( F_{cu}, F_{cur} \) \( F_t \) front and rear unsprung mass inertia forces respectively combined tractive forces of rear tires
\( (F_x)^{rr}, (F_x)^{rl} \) \( (F_x)^{rr}, (F_x)^{rl} \) tractive forces of rear-right and rear-left tires respectively
\( F_x, F_y, F_z \) \( F_{x}, F_{y}, F_{z} \) force component along the \( x \), \( y \), and \( z \) axes respectively
\( F_{x}, F_{y}, F_{z} \) \( F_{x}, F_{y}, F_{z} \) combined longitudinal forces of front and rear tires respectively
\( F_{x}, F_{y}, F_{z} \) \( F_{x}, F_{y}, F_{z} \) longitudinal forces of front-right and front-left tires respectively
\( F_{x}, F_{y}, F_{z} \) \( F_{x}, F_{y}, F_{z} \) longitudinal forces of rear-right and rear-left tires respectively
\( F_{x}, F_{y}, F_{z} \) \( F_{x}, F_{y}, F_{z} \) combined cornering forces of front and rear tires respectively
\( F_{ya} \) \( F_{ya} \) cornering force of tire
\( F_{yi} \) \( F_{yi} \) cornering force of the \( i^{th} \) tire
\( F_{yi} \) \( F_{yi} \) camber thrust of tire
\( f(\chi) \) smooth vector field of states
\( G(\chi) \) smooth matrix of \( g \), vector fields
\( g \) gravitational acceleration
\( H_{1s}, H_{2s} \) steering dynamics gains
\( h \) \( h \) height of unsprung mass cg
\( h(\chi) \) smooth vector field of \( h \), functions
\( h_s \) height of sprung mass cg
\( h_i \) height of vehicle cg
\( h_f, h_r \) \( h_f, h_r \) front and rear roll centre heights respectively
\( h_{ra} \) distance from sprung mass cg to roll axis
\( h_{uf}, h_{ur} \) \( h_{uf}, h_{ur} \) front and rear unsprung mass cg heights respectively
\( I \) vehicle moment of inertia about the \( z \) axis
\( I_x, I_y, I_z \) \( I_x, I_y, I_z \) moment of inertia about the \( x \), \( y \), and \( z \) axes
\( I_{xy}, I_{yz}, I_{zx} \) mass product of inertias about the \( xy \), \( yz \), and \( zx \) planes respectively
\( I_{zf}, I_{zr} \) moment of inertia of the front and rear unsprung masses about the \( z \) axis respectively

\( \bar{I}_{xx}, \bar{I}_{zz} \) moment of inertia of the sprung mass about the \( x \), and \( y \) centroidal axes respectively

\( J_s, J_t \) equivalent mass moment of inertia of steering and tractive motors respectively

\( \bar{J}_s, \bar{J}_t \) mass moment of inertia of steering and tractive motors respectively

\( K_s, K_t \) equivalent gains of steering and tractive motors respectively

\( \bar{K}_s, \bar{K}_t \) torque constant of steering and tractive motors respectively

\( k_f, k_r \) front and rear suspension roll stiffness

\( k_{1s}, k_{2s} \) steering gains

\( k_{1r}, k_{2r} \) front and rear suspension roll stiffnesses

\( L \) wheelbase

\( L_{spring} \) moment due to suspension roll stiffness

\( L_{damper} \) moment due to suspension roll damping

\( M \) Inertia matrix

\( M \) total mass of vehicle

\( M_r \) moment of sprung mass inertia force about the roll axis

\( \bar{M}_{cf} \) front moment of sprung mass inertia force about the roll axis

\( \bar{M}_{cr} \) rear moment of sprung mass inertia force about the roll axis

\( M_x, M_y, M_z \) moment components about the \( x \), \( y \), and \( z \) axes respectively

\( m_s \) sprung mass

\( m_{sf}, m_{sr} \) front and rear unsprung masses

\( n \) order of system

\( n_s, n_t \) gear ratio of steering and tractive motors respectively
\( R_{as}, R_{at} \) armature resistance of steering and tractive motors respectively
\( R_w \) radius of rear tires
\( r, p, q \) angular velocities about the \( x, y, \) and \( z \) axes respectively
\( r_i \) total relative degree of the system
\( r_i \) relative degree of the \( i^{th} \) output

\( S_i \) the \( i^{th} \) sliding surface
\( S \) vector of \( S_i \) sliding surfaces
\( s, n \) path dependent coordinates
\( sgn(S_i) \) sign function of the \( i^{th} \) sliding surface
\( sat\left(\frac{S_i}{n_i}\right) \) saturation function of the \( i^{th} \) sliding surface

\( t \) track width
\( t_f, t_r \) front and rear track width
\( t_p \) pneumatic trail of tire

\( U, V, W \) vehicle velocity components along \( x, y, \) and \( z \) axes respectively
\( U_{rel} \) velocity relative to moving frame along the \( x \) axis
\( U_{xrr}, U_{xrl} \) linear velocity of rear-right and rear-left wheels respectively

\( u \) control input vector

\( V_{rel} \) velocity relative to moving frame along the \( y \) axis
\( V_s, V_t \) terminal voltage of steering and tractive motors respectively
\( v \) vehicle velocity expressed in moving frame
\( v_O \) velocity of moving frame's origin
\( v_{NO} \) velocity of point \( N \) relative to moving frame's origin \( O \)
\( v_{abs} \) vehicle velocity
\( (v_{abs})_x \) velocity component along the \( x \) axis
\( (v_{abr})_y \) velocity component along the \( y \) axis
\( (v_{abr})_z \) velocity component along the \( x \) axis

\( W_b \) lateral load transfer due to body roll
\( W_{bf}, W_{br} \) front and rear lateral load transfers due to body roll respectively
\( W_{long} \) longitudinal load transfer
\( W_r \) lateral load transfer due to roll centre hight
\( W_{rf}, W_{rr} \) front and rear lateral load transfers due to roll centre hight respectively
\( W_{rel} \) velocity relative to moving frame along the \( z \) axis
\( W_s \) lateral load transfer due to change in the cg's hight
\( W_{sf}, W_{sr} \) front and rear lateral load transfers due to change in the cg's hight respectively
\( W_{sf}, W_{sr} \) total front and rear lateral load transfers respectively
\( W_u \) lateral load transfer due to unsprung weight
\( W_{uf}, W_{ur} \) front and rear lateral load transfers due to unsprung weights respectively

\( X, Y, Z \) inertial coordinate system
\( x, y, z \) coordinate system fixed to the vehicle at c.g.
\( \bar{x}, \bar{y}, \bar{z} \) coordinates of vehicle c.g. with respect to moving frame

\( y \) vector of system outputs
\( y_i \) the \( i^{th} \) system output
\( y_{id} \) the \( i^{th} \) desired output

\( z \) states vector in transformed coordinate

\( \alpha \) slip angle of tire
\( \alpha_i \) slip angle of the \( i^{th} \) tire
\( \alpha_{y_k}(x) \) scalar functions
\( \alpha_f, \alpha_r \) front and rear tire slip angles respectively
\( \gamma \)  
\[ \text{camber angle of tire} \]

\( \delta \)  
\[ \text{vehicle steer angle} \]

\( \delta_{fr}, \delta_{fl} \)  
\[ \text{steer angle of front-right and front-left tires respectively} \]

\( \delta_{rr}, \delta_{rl} \)  
\[ \text{steer angle of rear-right and rear-left tires respectively} \]

\( \zeta \)  
\[ \text{vector of system states in path dependent coordinates} \]

\( \lambda_i \)  
\[ \text{positive coefficients of the } i^{th} \text{ sliding surface} \]

\( K_i \)  
\[ \text{control gain of the } i^{th} \text{ sliding surface of the BLSM controller} \]

\( \bar{K}_i \)  
\[ \text{control gain of the } i^{th} \text{ sliding surface of the SM controller} \]

\( \kappa_i \)  
\[ \text{PD control gain vector of the } i^{th} \text{ output} \]

\( \hat{\eta}_i \)  
\[ \text{positive constant of } i^{th} \text{ sliding condition} \]

\( \theta \)  
\[ \text{yaw angle} \]

\( \theta_d \)  
\[ \text{desired yaw angle} \]

\( \dot{\theta}_{ms}, \dot{\theta}_{ml} \)  
\[ \text{angular speed referred to motor shaft for steering and tractive motors respectively} \]

\( \dot{\theta}_s, \dot{\theta}_t \)  
\[ \text{angular speed referred to gear box output for steering and tractive motors respectively} \]

\( \dot{\theta}_{rr}, \dot{\theta}_{rl} \)  
\[ \text{angular speed of the rear-right and rear-left wheels respectively} \]

\( \hat{\theta} \)  
\[ \text{yaw angle deviation from the desired path} \]

\( \nu \)  
\[ \text{decoupling control law} \]

\( \nu_i \)  
\[ \text{the } i^{th} \text{ output decoupling control law} \]

\( \xi_i, \eta_i \)  
\[ \text{normal coordinates} \]

xx
\( \rho \) relative position

\( \theta \) instantaneous radius of rotation

\( \tau_s, \tau_t \) torque referred to output shaft of steering and tractive motors respectively

\( \tau_{rr}, \tau_{rl} \) torque on rear-right and rear-left wheels respectively

\( \Phi(\chi) \) diffeomorphism function

\( \phi \) roll angle of vehicle

\( \chi \) vector of system states

\( \Psi_i \) the \( i^{th} \) boundary layer proportional constant

\( \varphi_i \) thickness of the \( i^{th} \) boundary layer

\( \varphi_{i0} \) desired thickness of the \( i^{th} \) boundary layer near origin

\( \omega \) vehicle angular velocity

\( \omega_w \) angular velocity of rear wheels
Chapter 1

Introduction

1.1. Background and State of the Art

Whether it is for a warehousing operation, manufacturing or assembly operation, or even food and supplies delivery in a hospital, material handling is reported to be a major cost of most industries at the present time. Different strategies for transfer of materials have been suggested and used in the past. Among these are hoists, cranes, conveyor belt systems, and automated or manual controlled carriers. However, demand for higher productivity, and increased emphasis on automation, have led to the use of Automated Guided Vehicles (AGVs) as a key element in transportation systems in recent years (Hammond, 1987; Tsumura, 1986).

Although, industrial adoption of AGVs may appear to be an expensive alternative, they offer several advantages over other systems. These include increased control over material movement, better floor-space utilization, increased flexibility in laying new routes, and the ability to operate in hazardous environments. In fact, the full scope of benefits offered by AGVs is yet to be perceived, and there are many that are task specific.

Sometimes called Wheeled Mobile Robots (WMR), Mobile Robots (MR) or Autonomous Vehicles (AV), AGVs are recognized by their extensive use in industry. The Materials Handling Institute (Pittsburgh) defines AGVs as battery powered driverless
vehicles that can be programmed for path selection and positioning and are equipped to
to follow a flexible guideway, which can be easily modified and expanded (Bose, 1986).

1.1.1. Historical Development

The first AGVs were developed around 1950's in the USA by Barrett Electronics. Four years later the first Automated Guided Vehicle System (AGVS) was installed at Mercury Motor Freight in Columbia, South Carolina. The controllers in these vehicles were based on vacuum tube technology, and they were guided around their environments by wires buried under the floor. In these types of systems the AGV follows the wire by monitoring the voltage induced between two coils on the vehicle. This information is used to determine the steering action. These early systems were all tugger systems, and used in warehousing applications.

In the next two decades, with the rapid advances in electronic technology the controllers were first transistorized and later replaced with Integrated Circuit (IC) technology. This permitted higher circuit integration and more functional and powerful controllers, and led to a faster improvement of AGV technology. However, it was the European automotive industry this time that used AGVs in manufacturing, 23 years after their inception, and demonstrated their benefits. The first application in manufacturing was in 1974 when Volvo's automated system was installed in Kalmar, Sweden. The system had 260 carriers. Since then, many other companies adopted this technology. In 1985, more than 10,000 vehicles were produced and serviced approximately 450 plants in Europe (Muller, 1987). Based on another survey published in 1986 (Tsumura, 1986), more than 30 companies were researching this field in Japan, and over 15 companies manufactured AGVs.

After the successful implementation of AGV systems in Europe, North American companies gave this technology another chance. From less than twenty-five hundred, within two years the number of AGVs operating in the US increased to four thousand in 1986. Today, GM's operation in Oshawa, Ontario, houses one of the world's largest concentrations of AGVs.
1.1.2. AGV Types and Components

Automated guided vehicles are generally available in five physical forms, towing, pallet trucks, unit-load, fork trucks, and assembly vehicles. All other vehicles in use are either modification of the above five, or application specific. Among different categories the "unit load" types are the most widely used. In fact, they account for 90 percent of the total number of AGVs in Japan (Takahashi, 1988). These vehicles are designed to carry single or multiple loads, and they can interface with conveyors, workstations, machine tools, and a variety of automated systems.

The essential groups of components of an AGV are as follows:

1) Mechanical structure, driving and steering assembly: The mechanical structure embodies all components of the vehicle and provides the space for loading the workpieces. The drive and steering mechanisms are basic components of the vehicle’s mechanical system. The drive mechanism causes movement of the vehicle at a desired speed and the steering ensures accurate tracking of the guidepath, usually within a fraction of an inch.

2) Electrical components, electronic components and interfaces, on board power and safety features: Electrical components such as relays, switches and LEDs provide trouble shooting and ease of carrier operation. Electronic components and interfaces such as sensors, servo amplifiers, and digital to analog converters are integral parts of the vehicle control system. Onboard power for computing facilities, steering and drive actuators are normally provided by lead-acid batteries. Safety aspect is also an essential feature for automated guided vehicles. A loaded AGV can weigh several tons and have a maximum speed of about 3 m/s. Therefore, adequate precautions should be taken to provide safety. Some common safety features are, contact and non-contact sensors, warning lights and perhaps horn, software and hardware brakes.

3) Guidance, and communication systems: Besides design variations, what really differentiates vehicles is the means of guidance systems they use. Until very recently, the choice of guidance was limited to inductive (imbedded wire) guidepath, and chemical/optical guidepaths. Although wire guidepath is still the most widespread, they suffer from a major drawback of lack of flexibility. This leads to limited ability to change the guide path, and
high installation cost. Chemical and optical guidance technologies are commonly used in light manufacturing and office environments. Although, they offer more flexibility, the maintenance cost due to wear and tear of the guidepath is high, and vehicles’ management at junctions is difficult (Premi and Besant, 1983; Boegli, 1985).

In an effort to make AGVs less dependant on fixed guidepaths, and to provide them with greater autonomy, virtual guidepaths are used (Boegli, 1985). In this generation of AGVs, the vehicles’ posture (position and orientation) is obtained from sensory information, and is processed using an onboard computer. Recent advances in techniques for obtaining positional information are, dead reckoning, sonic or laser beacons, optical or ultrasonic imaging, inertial navigation, corner-cube and laser scanning (Tsumura, 1986). Often, one technique alone is not enough to obtain satisfactory results, and two, or combination of some should be used.

4) Onboard controller and computing systems: The advances in the AGV technology have been very closely tied to the development of their control system. In fact, the issue is so important, that experts believe, it was only after the developments of sophisticated control systems that AGVs gained new interest in the US (Bose, 1986). The AGV control system usually comprises four levels, factory host, central controller, floor controller, and on board vehicle controls. The vehicle control is managed by a microprocessor and on board controllers. Basic functions involved are; steering, drive, and load handling tasks. The trend in on board controllers is toward smaller physical packages, and more flexible path programming.

1.1.3. AGV Applications

Recent technological advances in guidance systems, and developments of sophisticated control algorithms have increased tremendously the potential application domain of AGVs. Current growth is mainly in the three general areas of distribution, assembly, and manufacturing. The distribution application involves carrying material to and from production process, or within warehouses. The advantages offered in this regard are similar to those provided by a truck over a freight train, that is, independent control and
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flexibility. AGVs in manufacturing are mainly used for material movements between manufacturing cells in a Flexible Manufacturing System (FMS). The most impressive growth made in AGVs use has been in support of assembly. At present more than 40 percent of all AGVs work in automotive assemblies.

AGVs have also been used as moving platforms for robots. An example of this type is an AGV from Mentor Products Inc., equipped with a 7.5-ton, 25-ft-high, six axis Cybotech robot for painting aircrafts (Bose, 1986). AGVs are preferred to human operators for use in electronic productions and clean rooms. A “class 100” clean room, contains less than 100 particles of a maximum size of 0.5 μin. per 1 cu ft. A human operator typically emits 150,000 of such particles per minute. Nowadays vehicles are available to meet “class 10” clean room operations (Takahashi, 1988; Hammond, 1987). Although most AGVs have load capacities of 2000-5000 kg, some are specifically produced for heavy duty applications. AGVs with load capacities of 68 tons are being used to automatically load and unload press dies (Hammond, 1987).

Considerable interest has also been shown in using automated guided vehicles in non-manufacturing applications. These applications have a wide range, varying from food, and laundry transfer in hospitals, to carrying mail, messages, and packages in offices (Rajaram, 1988; Klafter, 1988). AGVS are currently being used as a means of transportation in places such as airports, freight stations, and ports where no manufacturing takes place (Evans, 1989). Hostile environments applications form another category where AGVs have potential use. Applications such as nuclear and explosive handling, and mining are contemplated as natural candidates for the AGV technology (Stone and Edmonds, 1992; Anderson and Donna, 1989). Military, security, space, and under water applications where the vehicle has to operate in almost entirely unstructured environment are also active research areas in the United States (Aviles et al., 1991; Schultz et al., 1991; Wilcox and Gennery, 1987; Rodseth and Hallset, 1991). AGVs and mobile robots are also developed for disabled mobility (Treherne, 1990), and patient care (Borenstein and Koren, 1987).

Given the range of applications and advantages offered by AGVs, there still remains many engineering problems to be addressed, and many potential applications to be investigated. The installation and maintenance costs are still high, and demand is for systems
to operate in structured environments, with reliable guidance and control, and reasonable path redesign costs.

1.2. Objectives of Research

Advanced automated factories require an unmanned transport system that assures efficient and flexible transfer of material in a semi-structured environment. Automated guided vehicles are important components of this modern automated transport system. Increasing the system efficiency and throughput requires the use of heavy vehicles travelling at high speeds. The main objective of this research is to contribute to the fundamental understanding of the dynamics and control of automated guided vehicles for high speed and high load applications.

A survey of pertinent literature reveals that experimental AGVs and WMRs with maximum weight of 120 kg and very low speeds have been the focus of research in the past (Petrov, 1991; Borenstein and Koren, 1987). Wheelbase designs of these vehicles are either simple differential drives or steerable wheels. However, a typical industrial AGV weighs around 350 kg and is capable of carrying 500 kg loads at its centre. These vehicles can typically travel at the maximum speed of 1 m/sec. In this research a transport interfactory AGV with one ton capacity is considered. The unloaded vehicle including the lifter weighs 700 kg, and is provided with pneumatic tires and suspension for high load applications. The vehicle has a maximum speed of 3 m/s.

Dynamics

To validate the controller performance, a suitable vehicle model that can accurately describe the behaviour of the AGV is required. The models used in the surveyed literature are mainly kinematic models (Sordalen and Canudas, 1992; Kanayama et al., 1990; Sung et al., 1989). This is due to the imposed simplification of low speed and light load conditions under which these vehicles operate. However, for practical AGVs designed to transport heavy loads and travel at high speeds, dynamic modelling of the vehicle is essential. A few researchers have derived dynamic models for AGVs (Hamdy and Badreddin, 1992; Saha and
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Angeles, 1989). These models however, are not valid for heavily loaded vehicles due to simplifying assumptions, and inaccurate tire modelling. In this work, a 3-DOF nonlinear dynamic model that represents the behaviour of the real vehicle under imposed condition is developed. The tire model is also considered. The effects of roll degree of freedom, suspension, lateral and longitudinal load transfer are also taken into account. In order to investigate the relevance of the dynamics in the AGV model, simulations of the performance of a kinematic model are compared to those of the dynamic model. Also, 3-DOF and 2-DOF dynamic models are compared for a 90-degree left turn.

Control

The main problem in controlling AGVs lies in the fact that these systems possess less control inputs than system outputs. A review of literature reveals that dynamic models of AGVs have scarcely been investigated in connection with control design. Previous works on the controller design of such systems are generally based on the kinematic models. However, dynamic-based control of these vehicles is the subject of recent research (d’Andrea et al., 1992; Hermami et al., 1992). Linearization by feedback is investigated for the simple wheel base configurations with solid tire models. However, the problem is not completely resolved for AGV’s with complicated wheel base configurations and realistic models. In a different approach, linearized dynamic model of Automated Transient Vehicles (ATV) with pneumatic tires is used for control design (Smith and Starkey, 1994; Jurie et al., 1994). Though, in this approach, the problem of suitable dynamic-based controller for emergency manoeuvres and curved routes with small radii is still the subject of more research.

Limitations of proportional controllers under different conditions are shown in this thesis. Clearly, a proportional controller based on mere information of heading and orientation errors is not adequate for the particular conditions under study. A dynamic-based approach for a car-like AGV has been sought. For the controller design, a slippage free dynamic model comprising the steering dynamics and the effect of actuators is considered. The multi-input multi-output feedback linearization technique is employed to transform the given nonlinear system into two subsystems. The choice of different output functions is
studied, and a number of controllers for the linearized control subsystem are designed. The sliding mode control structure is adopted to improve the robustness of system in the presence of parameter uncertainty, modelling errors, and disturbance. To eliminate undesirable chattering, the boundary layer technique is employed. The robustness of the boundary layer sliding mode controller is further enhanced by utilizing simultaneous variable boundary layer and multiple sliding surfaces strategies.

1.3. Thesis Outline

This thesis consists of seven chapters. A review of the general concept of automated guided vehicles, and historical advances made in this field is presented in this chapter. Related issues like classification of types, comprising components and their functions, and different application areas are also discussed. The objective of the proposed research is also included.

Chapter 2 is devoted to the review of existing literature. A particular emphasis is given to the areas most relevant to the proposed study. In this respect, recent works on kinematic and dynamic modelling are presented. The Control problem of these vehicles is presented. Different approaches to the design of the control law for vehicle’s motion about a geometrical path, a time-index trajectory, or fixed points in posture space are also discussed. Kinematic and dynamic-based approaches to control design are reviewed separately.

Some general notation usually used in vehicle dynamics, and description of mechanisms of force and moment generation by pneumatic tires are discussed in Chapter 3.

The general equations of motion are presented in Chapter 4. Assumptions applicable to the particular case under study are discussed. Two models are presented: the first is a 3-DOF nonlinear model comprising yaw, lateral, and roll degrees of freedom, considering a nonlinear tire model, and load transfer. The second is a 2-DOF model comprising yaw and lateral degrees of freedom, and linear tire model. The models also incorporate steering dynamics, the effect of actuators, differential gear box, steering and tractive gear boxes.
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Dynamic-based control of AGVs is presented in Chapter 5. The 2-DOF nonlinear model is used to design a new control strategy for 4-wheel front steering AGVs. Exact input-output feedback linearization is applied to linearize the slippage free vehicle model. The boundary layer sliding mode technique is used to guarantee the robustness, and account for disturbances and modelling errors. This controller is later modified to enhance its performance and robustness by applying simultaneous variable boundary layer and multiple sliding surfaces strategies.

Chapter 6 deals with the computer simulations and discussion of results. The kinematic model, 2-DOF, and 3-DOF dynamic models are compared to demonstrate the necessity of proper dynamic modelling. The path tracking performance of the vehicle under different control schemes has been studied. By comparing proportional derivative, sliding mode and modified sliding mode controllers it is illustrated that by implementing the new control strategy significant improvement in tracking performance can be achieved.

Finally, Chapter 7 concludes with a summary and general discussion of results. An outlook for future work is also presented in this chapter.
Chapter 2

Literature Survey

2.1. Introduction

The studies of Automated Guided Vehicles and Wheeled Mobile Robots are fundamentally multi disciplinary, incorporating technologies from different disciplines like: mechanical engineering, electrical engineering, control theory, computer vision, robotics, estimation theory, artificial intelligence, operation research, programming languages, and physics (Cox and Wilfonge, 1990). Consequently a vast scope of knowledge is directly or indirectly relevant to this area of research. A classification of research works on AGVs follows:

- Task level planning, decision making and control
- Navigation
  - Map making and real world modelling
  - Path planning, and motion planning
  - Kinematics, dynamics and control
  - Intelligent AGVs and WMRs
- Sensors and sensing strategies
- Applications
  - Flexible manufacturing system
  - Systems and successful applications
Chapter 2  Literature Survey

A survey of papers discussing relevant approaches and techniques in areas that contribute most to kinematics, dynamics and control of AGVs and WMRs is presented in this chapter.

2.2. Wheelbase Configuration

This section emphasises on mechanical configurations mostly used in industry. Wheelbase design of most wheeled land vehicles falls into one of two categories, steered wheeled vehicles or differential drive (powered wheel steering) vehicles (Cox and Wilfonge, 1990).

The configuration of the first class typically consists of a fixed rear axle and steered (or sometimes driven) front wheels. The tricycle wheelbase, with only one front steered wheel, is a common example in mobile robots and automated guided vehicles. A general four wheeled vehicle is another example of this category, where steering is done by an Ackerman steering linkage. While four wheel vehicles offer more stability and more traction when all wheels are driven, three wheels offer less complex steering, and the possibility of suspension elimination.

The familiar example of a differentially driven vehicle is a military tank, in which steering is performed by differences in the two tracked wheels velocities or torques. Differentially driven AGVs, are commonly equipped with one or more castors in order to provide stability. An advantage of a differential drive vehicle is the ability to turn with zero turn radius. This is a marked advantage since physical limits on the steering angle of a steered wheeled vehicle can cause a large turning radius, and make manoeuvring more difficult in a cluttered environment.

2.3. Kinematics and Dynamics

The study of kinematic and dynamic modelling of automated guided vehicles and wheeled mobile robots is very recent, though research on such vehicles dates back to the late 1960’s. In this section the issue of kinematic and dynamic modelling is considered briefly.
2.3.1. Kinematic Modelling

The kinematic modelling of automated guided vehicles differs from that of manipulators in several aspects. Among these: a) AGVs are closed-chain mechanisms, b) for AGVs, only some degrees of freedom are actuated, c) both lower and higher joints (surface and point joints) are present in an automated guided vehicle, whereas a stationary robot arm has only lower joints (McKerrow, 1991). Consequently, AGVs cannot be modelled using Denavit-Hartenberg convention, which is commonly used to model robot arms.

Muir and Neuman (1986, 1987) presented a kinematic modelling methodology for wheeled mobile robots using the transformation approach. Sheth-Uicker (1971) convention is used to model the kinematics. This method can be thought of as a complement to the transformation modelling technique employed for robot manipulators. Although, the proposed method may increase complexity in the modelling for some cases (Saha and Angeles, 1989), the merit lays in its ability to be generalized into any wheelbase pattern, with different number of wheels and various combination of driving/driven, steerable/nonsteerable wheels.

Alexander and Maddocks (1989) examined the questions of what planar rigid body trajectories are accessible for a given wheel pattern, and what steering and drive rates are required to reach these trajectories, assuming an ideal rolling of all wheels. The authors also addressed the analysis of slippage, which may occur for incompatible steering and wheel velocities.

2.3.2. Dynamic Modelling

Although, much work has been done on some aspects of AGVs such as navigation (Tsumura et al., 1981; Fujiwara and Kawashima, 1981; Koren and Borenstein, 1991), and path planning (Graettinger and Krogh, 1989; Barraquand et al., 1990; Shiller and Gwo, 1991), very little attention is being given to the dynamic modelling of AGVs and WMRs (Cyril et al., 1989; Boyden and Velinsky, 1994).
Muir and Neuman (1988) presented a modelling scheme that is based on Newtonian dynamics, force/torque propagation notion, frictional coupling, and the kinematic transformation approach. This modelling technique can be applied to multibody robotic mechanisms having the following characteristics: (1) closed-chains, (2) friction, (3) higher pair joints, (4) unactuated joints and (5) unsensed joints.

In another approach, Saha and Angeles (1989), and Cyril et al. (1989) presented a general methodology for dynamic modelling of wheeled mobile robots and AGVs of arbitrary wheelbase configurations. The method is based on the Hybrid Newton-Euler, Euler-Lagrangian and Natural Complement methods.

Using Lagrangian formalism and differential geometry approach, d’Andrea-Novel et al. (1991) derived a general dynamical model for three wheeled mobile robots with nonholonomic constraints. The authors demonstrated that static feedback allows reduction of the system to the stage that stabilizing input-output linearization is possible.

Agullo et al. (1989) considered the dynamics of AGVs with directionally sliding wheels. Such wheels can freely slide along a direction at a definite angle from wheel plane. The virtual work method is used to derive the dynamic equations of motion.

Hamdy and Badreddin (1992) presented the dynamic modelling of a WMR to estimate the occurrence of slip, and slip magnitude. Once this is done, the results are used to correct sensed forward navigation.

All these approaches are based on solid tire assumptions. However, dynamic modelling can be accomplished using the classical techniques based on D’Alembert’s principal. This approach is usually used to model road and off-road vehicle dynamics considering the pneumatic tire model (Ellis, 1994; Wong, 1993). The simulation package of Huang (1991) employs a dynamic model for a 2-DOF AGV which is based on classical methods. However, this approach is not general in nature.

Boyden and Velinsky (1994) discussed the necessity of dynamic modelling of AGVs and WMRs for high load applications. The authors show that the use of kinematic models must be limited to vehicles that operate under very low speeds, very low accelerations and under lightly loaded conditions.
2.4. Control

Based on the type of constraints, one can classify mechanical systems as holonomic or nonholonomic systems. A mechanical system in which all joints are the result of holonomic constraints is called holonomic mechanical system. On the other hand, if the mechanical system has at least one nonholonomic constraint, it is known as a nonholonomic mechanical system. Wheeled vehicles are typical examples of mechanical systems with nonholonomic constraints. Though, the theory of mechanical systems with nonholonomic constraints has been developed at the end of the last century by authors such as Appell and Hamel (Campion et al., 1990), the control of these systems has only been studied very recently.

2.4.1. Controllability

Similar to robot manipulators, commonly used wheeled vehicles subject to nonholonomic constraints are usually completely controllable in their configuration space. This is true when the number of actuators is equal to the number of degrees of freedom. (Bloch and McClamroch, 1989; Samson, 1991b; d'Andrea-Novel et al., 1992; Zhao and BeMent, 1992). However, unlike manipulators, the controllability of these systems does not imply the existence of stabilizing feedback. (Bloch and McClamroch, 1989; Samson and Ait-Abderrahim, 1991c). This negative result can be explained by a theorem in the nonlinear control theory due to Irockett (1983).

Bloch and McClamroch (1989) first demonstrated that smooth feedback cannot stabilize a nonholonomic system to a single equilibrium point. Bloch et al. (1990) also showed that these systems are small-time locally controllable at the origin. In later publications, the authors explain a general procedure for constructing piecewise analytic state feedback to stabilize the systems about a point (Bloch et al., 1992).

Campion et al. (1990) showed that nonholonomic systems are strongly accessible despite the structure of the constraints. Although, smooth pure feedback cannot asymptotically stabilize these systems, it can globally marginally stabilize them.
2.4.2. Proportional Controller

Sometimes referred to as sensor-based controllers, in this approach usually some information about position and heading errors are analysed to decide the steering action of AGVs.

Rajagopalan et al. (1992), proposes a dual camera guidance for an AGV with pneumatic tires, where accurate path tracking, and faster alignment of the front and rear ends of the vehicle with the path is required. Both front and rear position and orientation information are used to obtain different control schemes for straight and curved paths. The controller gains are changed on-line, based on vehicle location relative to the track. Simulation is used to compare the results with the case of single camera guidance.

Usually a proportional gain is used to feedback the sensory information, though nonlinear control laws are also reported. Hemami et al. (1990) present a nonlinear controller to adjust the steering of a front drive/front steering vehicle. The dynamics of the vehicle with pneumatic tires is simulated to compare the tracking capability of the system with the one using linear proportional feedback of orientation and distance errors. Based on the simulation results, improvement in performance is reported. However, practical difficulty to measure the yaw rate term included in the nonlinear controller is not addressed.

2.4.3. Kinematic-based Control

The problem of designing a control law to stabilize motion about a desired trajectory, \( P_d(t) \), where \( t \) is the time, is specified as trajectory tracking. Samson and Ait-Abderrahim (1991a) investigated trajectory tracking of a powered wheel steering mobile robot. The authors show that by introducing a virtual reference cart, which has a predefined trajectory, feedback stabilization in both position and orientation becomes possible as long as the reference cart is moving.

Kanayama et al. (1990) developed a stable control law for precise trajectory tracking of autonomous vehicles. The method is suitable for vehicles capable of dead reckoning and distinct specification of the reference path and its position. Lyapunov function is used to
prove the stability of the control law. A condition for critical damping has been obtained through linearizing the systems differential equations. Then, this condition is used to find appropriate parameters for the specific controller. In order to handle non-smooth paths and avoid slippage, a velocity/acceleration limiting scheme is introduced. Simulation and experiments of the Yamabico-11 mobile robot are carried out. Close agreement between theory and experimental results is reported.

The design of a control law to stabilize motion about a desired geometric path, $P_s(s)$, where $s$ is any convenient path parameter, is referred to as path following or path tracking in the literature. Among existing path tracking schemes, some approaches obtain the steering angle based on vehicle orientation and position errors. Usually, gains for these errors are chosen by trial and error (Kanayama et al., 1988; Nelson and Cox, 1988). Other methods continuously generate paths that converge to the reference path from the deviated vehicle position, then using simple kinematics the generated paths are converted into steering angles and wheel velocities (Shin et al. 1992; Pears and Bumby, 1991).

Nelson and Cox (1988) described a path control system for Blanch, an autonomous robot cart. The on-board navigation system comprises a reference state generator, an error feedback controller, and odometers to sense cart’s location. The cart controller consists of path and motor controllers. At the path control level, a feedback controller based on normal and heading errors is used for steering control, and one based on tangential and speed error is used for drive control. Simulation and experimental results are reported.

Kim (1987) presented a theoretical optimal (and suboptimal) steering control of a wire-guided AGV. Since for tracking a circular reference path steady state error cannot be eliminated by simple proportional control law, an optimal proportional plus integral (PI) controller is employed. Also, the effect of design parameters of the AGV, such as forward velocity, wheel radius, and sensor position is studied.

Pears and Bumby (1991) formulated a steering control method for steering control of a differential drive AGV. The control law is based on generating a local demand heading pointing toward the vehicle’s current reference path, with a gradient proportional to the local normal distance error. This demand heading is used to generate a command turning curvature in proportion to the heading error. To have the vehicle’s performance independent
Chapter 2 Literature Survey

of velocity, controller parameters are derived as functions of velocity. Both, simulation and experimental tests for tracking straight and circular paths are done. The steady state offset error observed for a curved path, is compensated for by treating the reference curvature as an input.

Work on designing a control law for stabilizing the vehicle about a fixed point in the configuration space is another issue which has been addressed recently (Canudas de Wit et al., 1993). As explained earlier, the study of this class of problems show that the kinematic and dynamic model properties such as: controllability, linearization, decoupling, and stabilization, depend on the selected coordinates. Also, the kinematic and dynamic model of nonholonomic wheeled vehicles derived in the posture space cannot be stabilized by smooth pure state space feedback laws. To solve this problem, the following three approaches have attracted more attention, time-varying feedback laws (Samson, 1990b), discontinuous or piecewise smooth controllers (Bloch et al., 1990; Canudas de Wit and Sordalen, 1992), and hybrid strategy (Pomet et al., 1992).

2.4.4. Dynamic-based Control

A survey of the literature published on AGVs and WMRs shows a significant amount of research is being done on their kinematic modelling and control. Whereas the dynamics and control of these systems are subject of more recent investigations, and are not fully studied (Sarkar et al., 1994; Canudas de Wit and Roskam, 1991). Dynamic-based control is not only a natural extension of the kinematic approach, but it is also essential for the proper control of these systems (Deng and Brady, 1993a, Mehrabi et al., 1993). This is true specially in practical applications where load, required drive power, and dynamic constraints for feasible trajectories have to be considered (Boyden and Velinsky, 1994; Canudas de Wit and Roskam, 1991; Fraichard, 1993).

In the following sections, the dynamic-based control is considered. Among existing schemes, some approaches design controllers based on rigid vehicle dynamics. The tire model is usually ignored in these methods. Other approaches are based on the vehicle
dynamics considering the pneumatic tire model and characteristics. These two strategies are separately discussed.

2.4.4.1. Controllers Based on Rigid Vehicle Dynamics

Trajectory tracking of a three wheeled vehicle is considered by d’Andrea-Novel et al. (1991). Using Lagrangian dynamics, the authors show that a static feedback can reduce system dynamics to a form for which stabilizing input-output feedback linearization is possible. No simulation of a practical application is presented.

Iida and Yuta (1991) proposed a controller structure for steered wheeled mobile robots. A feed forward compensator comprising the inverse dynamics of the vehicle, is used to cancel vehicle dynamics, and control the wheels’ angular velocities. Simulations with 10 percent errors between controller parameters and vehicle dynamics, and experiment on the Yamabico mobile robot has been performed. Improved results for a feedforward controller are reported.

Deng and Brady (1993a, 1993b) applied Lagrange formalism and the feed back linearization method for trajectory tracking of a wheeled vehicle. The model is a 2-DOF, front driven and steered tricycle. Kinematic constraints are treated as a part of the control model. However, no simulation or comparison of results is reported to validate the method.

When the input output linearization is used to design controllers for nonlinear dynamic systems, the effectiveness of the control design based on the reduced order model depends on the stability of the internal dynamics (Slotine and Li, 1991). Yun and Yamamoto (1993) addressed the stability of internal dynamics and zero dynamics of a differential drive mobile robot. Using a novel Lyapunov function, the authors prove that the zero dynamics is always stable. However, the internal dynamics of the system is asymptotically stable when the reference point is commanded to move straight forward, and unstable for backward motion of the reference point. Both simulations, and experimental tests on a TRC LABMATE mobile platform were done to verify the results.

The input-output linearization and zero dynamics of a differential drive mobile robot is discussed by Sarkar et al. (1994). It is shown that the zero dynamic of the system is
Lagrange stable. Using simulations, control algorithms for trajectory tracking and path following are compared. It is concluded that, while trajectory tracking is preferred for situations where the vehicle must follow a curve in space-time coordinates, path following is a more realistic control strategy for vehicle control applications. However, the approach to controller design of path following is not general, and it depends on the tracking path.

Path tracking of a carlike mobile robot is considered by DeSantis (1993). Both kinematics and Newtonian dynamics are considered for planar motion with no side slippage. It is shown that a controller capable of tracking an assigned path may be computed in terms of the lateral, heading, and velocity offsets. A linearized, time invariant model is developed for the special case where the path is straight or circular curve, and the tracking velocity is constant. The author suggests a decoupled structure controller based on classical techniques such as PID or pole placement for the case where the offset is kept small. Nevertheless, no simulation or practical application is addressed to support the results. In more recent works, a similar approach is applied to controller design of double steered car-like robots (DeSantis, 1995a), and differential drive WMRs (DeSantis, 1995b).

When a nonlinear system cannot be stabilized at an equilibrium point by smooth state feedback, one alternative is to consider discontinuous feedback, as explained earlier. However, another possibility consists of considering smooth time-varying feedback was first introduced by Samson (1990b). Assuming the inputs to the system are wheels torque, Samson and Ait-Abderrahim (1991e) show that time-varying feedback can be applied to the dynamic model of a 2-DOF differential drive mobile robot. However, no simulation or experimental application of torque control is reported.

Some works on dynamic-based control design are motivated by the need for planning velocity profiles along the path so that these profiles comply with torque limitations, or verification of wheel-ground contact forces. Canudas de Wit and Roskam (1991), consider path following of a 2-DOF WMR, taking into account path and input torque constraints. Dahl and Nielsen (1989) on-line reference profile generation method is employed to reshape a nominal velocity profile to comply with the input torque constraints. Dynamic-based control is used for the motion control problem. The dynamic model has been transformed into a two-dimensional sub space (tangential velocity and orientation), then used to design
a computed torque type feedback law. Improvement of performance under input torque saturation is reported based on experiments with micro-WMR KITBORD. However, when the cart slips the feedback law cannot correct the induced path error. Also, the problem of path admissibility remains to be studied. This study is restricted only to path generation by straight segments.

To verify wheel-terrain contact forces and avoid slippage, Neculescu et al. (1994) use Newtonian dynamics for a 2-DOF, three wheeled mobile robot. Motion control is done by the input-output linearization for the dynamic model of the system in two-dimensional operational space. The system is linearized with respect to the third differentiation of outputs \((x, y)\). However, problems of practical measurement of acceleration, and stability of internal dynamics have not been addressed.

Since the exact model of the nonlinear vehicle system is not available when doing input-output linearizations, these approaches suffer form the fact that “no robustness is guaranteed in the presence of parameter uncertainty and unmodeled dynamics” (Slotine and Li, 1991).

2.4.4.2. Controllers Based on Vehicle Dynamics with Tire Model

The lateral and yaw motion dynamics of a vehicle are significantly governed by parameters such as vehicle speed and road tire interactions. These parameters are not constant during vehicle operation and must be considered in controller design for a more realistic speed and load ranges (Matsumoto and Tomizuka, 1992).

An approach to obtain optimal gains for a controller based on feedback of the position and orientation is proposed by Hemami et al. (1992). A linearized vehicle dynamic model is considered to derive the optimal control law. Minimization is based on a quadratic performance index of the offset, orientation errors, and the steering angle. Simulations for path tracking of an AGV with front wheel steering are depicted. However, performance of the designed controller is limited to straight line trajectories, and compared with other works, the effectiveness of the proposed method is not investigated.
Chapter 2  Literature Survey

Mehrabi et al. (1991), considered path following of a double steered AGV with pneumatic tires. A control law based on position and orientation errors of the mass centre is implemented. Simulations for straight line path tracking were conducted. In a more recent paper (Mehrabi et al., 1993) the authors report results on the CONCIC III experimental vehicle. Using dead reckoning reasonably good performance is reported except for the case of curvatures with sharp changes.

Makino (1993) studies hunting reduction of an automated guided vehicle. The three wheeled AGV has a total weight of 1700 kg with a maximum speed of 60 m/min. Navigation is based on guiding tapes along a straight line. Linearized dynamic model of the vehicle was used to design a control system. Controller gains were obtained based on simulation and experiments. Reduction in hunting motion is observed, but the problem of motion control along curved routes is not addressed.

To decide between 2-DOF model and 3-DOF models of WMRs, and Automated Transit Vehicles (ATVs) for control purpose, Cheng and Mehrabi (1992) proposed a dimensionless number called roll number. Two degrees of freedom models incorporates lateral and yaw motions, whereas 3-DOF includes roll motion also. The linearized dynamic model is used for this study; however, the suspension dynamics and the lateral load transfer are not considered.

Other studies available in the literature focus on the handling characteristics and directional stability of the Automated Highway Vehicles (AHV), and automated transit vehicles (Jurie et al., 1994; Matsumoto and Tomizuka, 1992; Shladover et al., 1991; Fenton and Mayhan, 1991). Vehicle dynamics becomes very important in designing controllers for AHVs, due to the higher speeds and accelerations at which they operate (Smith, 1991). However, unlike AGVs, a linearized dynamic model is often used to describe the structure of AHV controllers. This is mainly due to their operational environment where only large curvature routes or straight paths are encountered.

Among model-based approaches to controller design of AHVs, two have attracted more attention in the literature, one involving classical control theory (Fenton et al., 1976; Cormier and Fenton, 1980), and the other optimal control theory (Shladover, 1978; Johnston and Assefi, 1979; Hatwal and Mikulcik, 1986).
Chapter 3

Vehicle Dynamics

3.1. Introduction

Among different approaches to model the dynamics of automated vehicles, two have been considered more frequently, the Lagrangian formalism for nonholonomic systems (d’Andrea-Novel et al., 1991; Saha and Angeles, 1989; Cyril et al., 1989), and the Newtonian dynamics (Jurie et al., 1994; Muir and Neuman, 1989; Nisonger and Wormley, 1979). While the first method is based on rigid body assumption of the vehicle and wheels, the second is commonly used to model the vehicle incorporating pneumatic tires. Throughout this study the second approach has been implemented for the purpose of motion analysis, then simplified to allow controller synthesis.

Due to reliance of the adopted dynamic technique upon the understanding of the pneumatic tire and ground interactions, this chapter is devoted to review some general notation usually used in vehicle dynamics, and description of mechanisms of force and moment generation by pneumatic tires.

3.2. Vehicle Models and Degrees of Freedom

As a rigid body a vehicle has six degrees of freedom, translation along the $x$, $y$, and $z$ axes and rotation about these axes as represented in Figure 3.1. By inclusion of these motions in the analysis, complex dynamic models can be developed for simulation purposes.
Chapter 3 Vehicle Dynamics

Fortunately, the literature in this regard indicate that dealing with the whole complete model is not usually required (Jurie et al., 1994; Wong, 1993). Simplified versions of dynamic models are being proposed for certain conditions that are reasonably accurate. Within the scope of present study it is well accepted that a dynamic model that takes into account the lateral, yaw, and roll motions of the vehicle is quite adequate to represent the vehicle’s motion (Xia and Law, 1992; Shladover et al., 1978). Even simpler models that consider yaw and lateral motion are reported to give satisfactory results for controller design purposes under certain condition (Jurie et al. 1994; Fenton and Selim, 1988).

3.3. Mechanics of Pneumatic Tires

It has often been said that “the critical control forces that determine how a vehicle turns, brakes, and accelerates are developed in four contact patches no bigger than a man’s hand” (Gillespie, 1992). Apart from gravitational and aerodynamic forces, all other major forces acting on a vehicle result from tire-ground interaction. Extensive research has been devoted to analyse the mechanism of this interaction (Pacejka and Bakker, 1993; Palkovics and Elgindy, 1993; Bakker et al., 1987; Dugoff et al., 1977). Thus, an understanding of the tire mechanics on hard surfaces is essential to study the dynamic behaviour of ground vehicles (Wong, 1993). In what follows some basic properties of pneumatic tires, and the resulting forces and moments developed at the contact patches are discussed.

3.3.1. Tire Forces and Moments

To assist precise description of tire characteristics, forces, and moments, a set of axes with the origin located at the centre of the tire to road contact is adopted. The $X$ axis is pointing in the forward direction, and the $Z$ axis perpendicular to the road pointing upward.
Figure 3.1  The ISO vehicle dynamics axis system

Figure 3.2  Tire axis system
The ground reactions on the tire are described by three forces and three moments as shown in Figure 3.2, and are defined as:

- **Longitudinal Force** ($F_x$): The components of the resultant force acting on the tire by the road, in the $X$-direction. The force component in the direction of wheel travel is called Tractive force. Tractive forces are generated during acceleration and braking.

- **Lateral Force** ($F_y$): The components of the force acting on the tire by the road, in the $Y$-direction. Lateral forces are developed when the wheel is steered at an angle, and they have a dominant role on vehicle control.

- **Normal Force** ($F_z$): The components of the force acting on the tire by the road, in the $Z$-direction.

- **Overturning Moment** ($M_x$): Moment acting on the tire by the road, about the $X$-axis.

- **Rolling Resistance Moment** ($M_y$): Moment acting on the tire by the road, about the $Y$-axis.

- **Aligning Moment** ($M_z$): Moment acting on the tire by the road, about the $Z$-axis.

Also, two important angles are associated with the rolling tire, defined as:

- **Slip Angle** ($\alpha$): The angle formed between the direction of wheel heading and the direction of travel.

- **Camber Angle** ($\gamma$): The angle formed between the Wheel plane and $XZ$ plane.

More detailed description of these forces is given in the following sections.

### 3.3.2. Rolling Resistance

One of the major resistance forces acting on a vehicle is the rolling resistance of tires. In fact at low speeds, on level and hard pavements the rolling resistance is the primary motion resistance force. At least, seven mechanisms are responsible for the rolling resistance, these are energy losses due to deflection of the tire side walls and tread elements,
Figure 3.3  Tire behaviour when a driving torque is applied (Clark, 1971).

Figure 3.4  Typical variation of rolling resistance with inflation pressure for different surface conditions (Taborek, 1975).
Chapter 3  Vehicle Dynamics

scrubbing in the contact area, air drag on the inside and outside of the tire, tire slips in longitudinal and lateral directions, and energy loss on bumps (Gillespie, 1992).

When a tire is rolling, the front tire treads in ground contact are deflected. This results in a higher normal pressure at the front half of the contact patch than in rear half as shown in Figure 3.3. Thus, the centre of normal pressure is shifted to the front, causing a moment about the axis of rotation called the rolling resistance moment. In a free-rolling tire, to satisfy equilibrium condition, this moment should be balanced with a force known as the rolling resistance. The ratio of rolling resistance to the normal load is a dimensionless factor, and defined to be the coefficient of rolling resistance.

Several factors affect the rolling resistance of a tire (Wong, 1993): a) The tire structure, that is, material and design, and b) tire operating conditions such as tire temperature, inflation pressure, velocity, and diameter. For instance, variation of rolling resistance coefficient with inflation pressure is shown in Figure 3.4.

The complex relationship between rolling resistance, structural, and operational parameters of a tire makes it difficult to obtain an analytical method for predicting the rolling resistance. However, based on experimental results many empirical formulas have been proposed. At lower speeds this relationship can be represented as

\[
F_r = 0.01(1 + U/223.7)
\]

(3.1)

where \( U \) is speed expressed in m/s.

For the AGVs at the speed range of this study the effect of speed may be ignored, and the average value of \( F_r \) for a specific operating condition may be used. Table 3.1 lists some typical values for rolling coefficient (Wong, 1993; Gillespie, 1992).

<table>
<thead>
<tr>
<th>Tire Type</th>
<th>Surface</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete</td>
<td>Medium Hard Surface</td>
<td>Sand</td>
</tr>
<tr>
<td>Passenger Car</td>
<td>0.015</td>
<td>0.08</td>
<td>0.30</td>
</tr>
<tr>
<td>Truck</td>
<td>0.010</td>
<td>0.06</td>
<td>0.25</td>
</tr>
<tr>
<td>Tractor</td>
<td>0.020</td>
<td>0.04</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 3.1  Coefficient of rolling resistance (Taborek, 1975).
Figure 3.5  Variation of braking effort with normal load for a truck tire (Ervin, 1975).

Figure 3.6  Effect of surface condition on braking effort (Gillespie, 1992, notations are modified for consistency).
3.3.3. Tractive/Braking Effort

When a driving torque is applied to a tire and the vehicle accelerates, a tractive force is developed at the tire-ground patch. The tread elements are compressed as they enter the contact area, resulting in a distance travelled by the tire less than that in free rolling Figure 3.3. This is usually referred to as longitudinal slip. Whereas in braking, the tread elements stretch before entering the contact area causing a distance travel by the tire greater than that of a free wheel. This phenomenon is called the skid. The severity of tractive and braking effort are measured by the longitudinal slip and skid. Several parameters affect the tractive and braking effort of a tire these are: road characteristics, surface condition, normal load, and vehicle speed (Wong, 1993). The variation of braking effort with the normal load and surface condition are shown in Figure 3.5-3.6.

3.3.4. Cornering Properties of Tires

Vehicle handling characteristics mostly depend on the cornering properties of the tire. In fact, the tire should develop the lateral forces necessary to control the vehicle direction during turns and lane change manoeuvre.

3.3.4.1. Cornering Force

When a rolling tire is not subjected to any side force, it will move in a direction coinciding with the wheel plane. However, if a lateral force is applied to the tire, it drifts to the side, and moves along a path at angle \( \alpha \) with the wheel plane. This angle is known as slip angle. The side slip of a tire is mainly due to the elastic properties of the tire, and the mechanism responsible for that is better perceived considering the Figure 3.7.

For a rolling tire the tread elements that are not in contact with the ground are undeflected, and they have the same direction as the heading. However, as the tire advances the tread elements reach the contact surface, and deflect toward the direction of travel
Figure 3.7  Deformation of a rolling tire due to lateral force (Gillespie, 1992, notations are modified for consistency).

Figure 3.8  Typical characteristics of the cornering force (Wong, 1993).
producing a lateral force. Further advancement of the tire generates larger forces, up to a point where the lateral force overcomes the friction available and a slip occurs. (The slip region is in the rear edge of the contact patch). The point of action of the resultant force, \( F_{y_{a}} \) is behind the contact patch at a distance called the pneumatic trail, \( t_{p} \). The resultant force at zero camber angle is called the cornering force (Wong, 1993; Gillespie, 1992).

Extensive studies have been done to explain the relationship between the cornering force and slip angle for various tires. Most analyses suggest a typical characteristic as shown in Figure 3.8. It can be seen that for slip angles less than 4 degrees the cornering force is proportional to the slip angle, beyond that, the cornering force reaches a maximum value where the tire begins sliding laterally. The slope of this curve at zero slip angle is called cornering stiffness \( C_{\alpha} \), and is a measure for comparing the cornering behaviour of different tires.

The effect of normal loads on the cornering ability of tires is evaluated using the cornering coefficient. The cornering coefficient is defined as the ratio of the cornering stiffness to the normal load.

Different factors affect the cornering behaviour of pneumatic tires; and these are, the type of tire, the normal load, the inflation pressure, and the tread design (Wong 1993). Typical variations of the cornering force with the normal load and inflation pressure are shown in Figure 3.9-3.10.

3.3.4.2. Aligning Torque

As a result of uneven force distribution in the contact patch, the cornering force acts toward the rear of the contact patch. This produces a moment called the self aligning torque, which tends to align the wheel plane with the direction of motion. The aligning torque is given as

\[
M_{z} = t_{p} F_{y_{a}}
\]

(3.2)

The contribution of this moment to the overall yaw moments is small. It does, however, contribute to the reactions in the steering system. A carpet plot of the aligning torque versus
Figure 3.9  Typical variation of cornering force with slip angle (Bakker et al., 1987).

Figure 3.10  Variation of cornering force with inflation pressure at constant normal load. Note over inflation does not increase the cornering Force due to contact area reduction (Ellis, 1969).
the slip angle and normal load is shown in Figure 3.11. Path curvature, normal load, and inflation pressure of tires are some factors affecting the aligning torque (Gillespie, 1992).

3.3.4.3. Camber Thrust

Another factor contributing to the lateral force in a tire is the camber thrust. Camber thrust, $F_{\gamma}$, is produced as a result of non-vertical tire orientation. As shown in Figure 3.2, the inclination angle is known as the camber angle. The variation of camber thrust with the camber angle is shown in Figure 3.12. The initial slope of this curve is called camber stiffness, $C_{\gamma}$, and is typically in the range of 10 to 20 percent of the cornering stiffness. Similar to the cornering force, the camber thrust has linear properties for small camber angles and it is affected by a few of parameters such as type of tire, normal load, and inflation pressure (Wong, 1993; Gillespie, 1992).
Figure 3.11  Typical variation of aligning torque with normal force and slip angle (Ervin, 1975).

Figure 3.12  Common variation of camber thrust with camber angle and normal load (Gough, 1956).
Chapter 4

AGV Dynamic Model

4.1. Introduction

A common tendency in the modelling and control of Automated Guided Vehicles and Autonomous Vehicles is to develop the required kinematic equations, but to ignore a precise description of the vehicle dynamics. Other works exist, which compensate for the dynamics using "fudge factors" (Singh et al., 1991). However, having a suitable dynamic model is important for the following reasons:

1) An accurate dynamic model of the vehicle is required for simulation purposes. This model can be used to verify the control algorithms, before having to experiment directly on the vehicle.

2) Dynamic modelling is particularly important when the inertia forces, and load transfer due to heavy load and/or high velocities are significant.

3) Whenever wheel-ground forces are needed to confirm the dynamic constraints, a dynamic model of the vehicle should be used.

4) Dynamics is required when the input torque constraints due to power limitations in the motor actuators are to be verified.

Various dynamic models with different ranges of complexities have been introduced in the literature to investigate the handling characteristics and directional stability of autonomous vehicles (Yu and Moskwa, 1994; Xia and Law, 1992; Allen et al., 1987). The
focus of this chapter is to develop a model that accurately represents the behaviour of the vehicle, taking into account various parameters such as pneumatic tire model and load transfer. However, care should be taken to avoid an unduly complicated model by introducing only necessary characteristics relevant to the particular AGV under study. First the general equations of motion are developed, employing an approach similar to Ellis (1994). The equations are then simplified for the 3-DOF vehicle model of interest. Finally, the external forces and moments resulting from tire-ground interactions are derived.

![Coordinate System](image)

**Figure 4.1** The coordinate system.

### 4.2. Axis System and General Equations of Motion

Common practice in vehicle dynamic analysis is to use two sets of coordinate axes. One set is attached to the vehicle with the origin of the axes usually fixed at the mass centre of the total vehicle (moving frame), and the second set fixed to a stationary inertial frame (fixed frame).

With reference to Figure 4.1, consider a point $N(x, y, z)$ on the vehicle body having linear velocities $U_{rel}$, $V_{rel}$, $W_{rel}$ relative to the moving origin $O$, along the axes $x$, $y$, $z$, respectively, as well as rotational velocities $p$, $q$, $r$ about $x$, $y$, $z$ axes, respectively. The
positive sense of rotation is defined by the right-hand rule. The absolute velocity of point \( N \) with respect to the fixed frame is given by:

\[
\mathbf{v}_{\text{abs}} = \mathbf{v}_O + \mathbf{v}_{N\text{IO}} + \mathbf{\omega} \times \mathbf{\rho}
\]  

(4.1)

where

\[
\begin{align*}
\mathbf{v}_O &= \begin{bmatrix} U \\ V \\ W \end{bmatrix} \\
\mathbf{v}_{N\text{IO}} &= \begin{bmatrix} U_{\text{rel}} \\ V_{\text{rel}} \\ W_{\text{rel}} \end{bmatrix} \\
\mathbf{\omega} &= \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\
\mathbf{\rho} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\end{align*}
\]

the velocity components of point \( N \) parallel to the \( x, y, z \) axes, are given by \( (v_{\text{abs}})_x \), \( (v_{\text{abs}})_y \), \( (v_{\text{abs}})_z \) respectively. These are given by

\[
\begin{align*}
(v_{\text{abs}})_x &= U + U_{\text{rel}} - ry + qz \\
(v_{\text{abs}})_y &= V + V_{\text{rel}} - px + rx \\
(v_{\text{abs}})_z &= W + W_{\text{rel}} - qx + py
\end{align*}
\]  

(4.2)

For a rigid body vehicle in which the axes' origin is fixed relative to the body, \( v_{N\text{IO}} = 0 \) (i.e., \( U_{\text{rel}}=V_{\text{rel}}=W_{\text{rel}}=0 \)). Thus, Equation 4.1 reduces to,

\[
\mathbf{v}_{\text{abs}} = \mathbf{v}_O + \mathbf{\omega} \times \mathbf{\rho}
\]  

(4.3)

The acceleration of point \( N \) is expressed as

\[
\frac{d}{dt}(\mathbf{v}_{\text{abs}}) = \frac{d}{dt}(\mathbf{v}_O) + \frac{d}{dt}(\mathbf{\omega} \times \mathbf{\rho})
\]

\[
\mathbf{a}_{\text{abs}} = \mathbf{a}_O + \mathbf{\omega} \times \mathbf{v}_O + \dot{\mathbf{\omega}} \times \mathbf{\rho} + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{\rho})
\]  

(4.4)

where

\[
\mathbf{a}_O = [ \dot{U} \quad \dot{V} \quad \dot{W} ]^T
\]

It should be noted that \( \mathbf{v}_O \) is expressed in terms of moving frame axes. Therefore, the first term on the right-hand side of Equation 4.4 is due to change in magnitude of \( \mathbf{v}_O \), and the second term is due to change in direction of \( \mathbf{v}_O \). \( (a_{\text{abs}})_x \), \( (a_{\text{abs}})_y \), and \( (a_{\text{abs}})_z \) are the acceleration components of point \( N \) parallel to the \( x, y, z \) axes, respectively, and are given by
Chapter 4  AGV Dynamic Model

\[
\begin{align*}
(a_{abs})_x &= \dot{U} - rV + qW - (q^2 + r^2)x + (pq - r)y + (pr + q)z \\
(a_{abs})_y &= \dot{V} - pW + rU - (p^2 + r^2)y + (qr - p)z + (pq + r)x \\
(a_{abs})_z &= \dot{W} - qU + pV - (p^2 + q^2)z + (pr - q)x + (qr + p)y
\end{align*}
\] (4.5)

Applying D'Alembert's principle, the sum of all external forces and moments must be in equilibrium with the sum of the inertial forces, i.e.,

\[
\begin{align*}
\sum X &= \sum \delta m (a_{abs})_x \\
\sum Y &= \sum \delta m (a_{abs})_y \\
\sum Z &= \sum \delta m (a_{abs})_z \\
\sum L &= \sum \delta m [y (a_{abs})_z - z (a_{abs})_y] \\
\sum M &= \sum \delta m [z (a_{abs})_x - x (a_{abs})_z] \\
\sum N &= \sum \delta m [x (a_{abs})_y - y (a_{abs})_x]
\end{align*}
\] (4.6)

Substituting the accelerations from Equation 4.5 into Equation 4.6 gives

\[
\begin{align*}
\sum X &= M(\dot{U} - rV + qW) - M\ddot{x}(q^2 + r^2) + M\ddot{y}(pq - r) + M\ddot{z}(rp + q) \\
\sum Y &= M(\dot{V} - pW + rU) + M\ddot{x}(pq + r) - M\ddot{y}(p^2 + r^2) + M\ddot{z}(qr - p) \\
\sum Z &= M(\dot{W} - qU + pV) + M\ddot{x}(pr - q) + M\ddot{y}(q r + p) - M\ddot{z}(p^2 + q^2) \\
\sum L &= I_x \ddot{r} - (I_y - I_z) qr + I_{yz} (r^2 - q^2) - I_{zx} (pq + r) + I_{xy} (pr + q) \\
&\quad + M\ddot{y}(\dot{W} - qU + pV) - M\ddot{z}(\dot{V} - pW + r U) \\
\sum M &= I_y \ddot{q} - (I_z - I_x) pr + I_{xz} (p^2 - r^2) - I_{xy} (qr + p) + I_{zx} (pq + r) \\
&\quad + M\ddot{x}(\dot{U} - rV + qW) - M\ddot{z}(\dot{W} - qU + pV) \\
\sum N &= I_z \ddot{p} - (I_x - I_y) pq + I_{xy} (q^2 - p^2) - I_{yx} (pr + q) + I_{xx} (qr - p) \\
&\quad + M\ddot{x}(\dot{V} - pW + r U) - M\ddot{y}(\dot{U} - rV + qW)
\end{align*}
\] (4.7)

where

\[
\begin{align*}
\sum \delta m &= M \\
\sum \delta m x &= M \ddot{x} \\
\sum \delta m y &= M \ddot{y} \\
\sum \delta m z &= M \ddot{z}
\end{align*}
\] (4.8)

and moments of inertias are defined as,
\[
I_x = \sum \delta m \ (y^2 + z^2) \\
I_y = \sum \delta m \ (x^2 + z^2) \\
I_z = \sum \delta m \ (x^2 + y^2) \\
I_{xy} = \sum \delta m \ (xy) \\
I_{yz} = \sum \delta m \ (yz) \\
I_{xz} = \sum \delta m \ (xz) 
\] (4.9)

Equation 4.7, is given in the most general form, later the salient assumptions for the AGV under study will be used to simplify the equation.

4.3. Assumed Vehicle Model

The complexity of the dynamic model is influenced by the AGV model, system’s degrees of freedom, and the simplifying assumptions considered. In this section these issues will be examined.

4.3.1. Degrees of Freedom

Equation 4.7 is used to develop the dynamic model of vehicles with varying complexities. Considering certain conditions that are not far from reality (Nalecz and Bindemann, 1989; Nisonger and Wormley, 1979; Allen et al., 1987), the simplified versions of these equations are often used for simulation and steering control studies of automated transit vehicles. Among the different models, the 2-DOF and 3-DOF models have been the focus of more attention. The 2-DOF bicycle model combines left and right wheels at the front and rear of the vehicle and includes lateral and yaw degrees of freedom. On the other hand, the 3-DOF models using lateral, yaw, and roll degrees of freedom has shown to be quite accurate for manoeuvres where the lateral acceleration is less than 0.3 g (Nisonger and Wormley, 1979). For this study the three degrees of freedom AGV model is considered. The model consists of a sprung mass and the two unsprung masses of front and rear with the centre of masses located as shown in Figure 4.2.
4.3.2. Assumptions

To derive the dynamic model for the AGV under study, the following assumptions are made:

1) The AGV is moving on a horizontal plane surface.
2) Pitch and bounce degrees of freedom are not considered due to the above assumption.
3) External disturbances like road irregularities, and aerodynamics are not considered.
4) Only steering and speed command inputs are allowed.
5) The sprung mass and the two unsprung masses are positioned such that the sum of their moment about the centre of gravity of the whole vehicle is zero.
6) The sprung mass will roll about the x axis (Segel, 1956). This axis is defined with respect to the unsprung masses, rather than the actual dynamic roll axis, which is generally defined using suspension deformations.
7) The effects of inertia properties of the wheels are considered negligible.
8) Front and rear track widths are equal.
4.4. Three DOF Dynamic Model

Unsymmetrical load distribution about the xz plane will slightly deform the wheels on one side more than the other. This will cause the vehicle to tend to travel along an arc rather than along a straight line (Borenstein and Koren, 1987). Therefore, it is a common practice in vehicle modelling to consider symmetry about xz plane. This results in $\tilde{M} = 0$, $I_{xy} = 0$, and $I_{yz} = 0$. Also, in the operating environment where the plane surface assumption is valid, vertical movements of the vehicle and pitch motions are ignored. In the absence of movement along the z axis, $W = 0$, and rotation about the y axis $q = 0$. Thus for the centre of the sprung mass at $\tilde{x} = 0$, $\tilde{y} = 0$, and $\tilde{z} = 0$ Equation 4.7 simplifies to:

$$
\sum X = M(\tilde{U} - rV) + M\tilde{z} pr
$$

$$
\sum Y = M(\tilde{V} + rU) - M\tilde{z} \tilde{p}
$$

$$
\sum L = I_{x}\ddot{p} - I_{xz} \dot{r} - M\tilde{z}(\tilde{V} + rU)
$$

$$
\sum N = I_{x} \dot{r} - I_{xz} \tilde{p}
$$

(4.10)

Similar equations can be derived for the case where the centre of mass is at $\tilde{x} = 0$, $\tilde{y} = 0$, and $\tilde{z} = 0$ (unsprung masses). Again, ignoring pitch and bounce movements, and considering symmetry about the xz plane Equation 4.7 reduces to

$$
\sum X = M(\tilde{U} - rV) - M\tilde{z} r^2
$$

$$
\sum Y = M(\tilde{V} + rU) + M\tilde{z} \dot{r}
$$

$$
\sum L = I_{x}\ddot{p} - I_{xz} \dot{r}
$$

$$
\sum N = I_{x} \dot{r} - I_{xz} \tilde{p} + M\tilde{z}(\tilde{V} + rU)
$$

(4.11)

Referring to Figure 4.2, and using the x components of Equation 4.10 for sprung mass ($m_s$), and Equation of 4.11 for the unsprung masses ($m_{uf}$ and $m_{ur}$) the following set of equations will result,

$$
(\sum X)_s = m_s(\tilde{U} - rV) + m_s h_{rs} pr
$$

$$
(\sum X)_{uf} = m_{uf}(\tilde{U} - rV) - m_{uf} ar^2
$$

$$
(\sum X)_{ur} = m_{ur}(\tilde{U} - rV) + m_{ur} br^2
$$

Summing these equations gives,

$$
\sum X = (m_s + m_{uf} + m_{ur})(\tilde{U} - rV) + m_s h_{rs} rp + r^2(m_{ur} b - m_{uf} a)
$$
Chapter 4  AGV Dynamic Model

where \( a, b \) and \( h_{ra} \) are positive constants. Using the 5th assumption in Section 4.3.2, which stipulates that \( m_{wr} b = m_{yr} a \), the above equation reduces to

\[
\sum X = M(\dot{U} - r\dot{V}) + m_z h_{ra} \dot{p} r
\]

The other components of the equation of motion for the AGV are derived similarly. The entire equation of motion for the AGV is given by:

\[
\begin{align*}
\sum X &= M(\dot{U} - r\dot{V}) + m_z h_{ra} \dot{p} r \\
\sum Y &= M(\dot{V} + r U) - m_z h_{ra} \ddot{p} \\
\sum L &= I_x \ddot{\dot{p}} - I_{xx} \dot{\dot{p}} - m_z h_{ra} (\dot{V} + r U) \\
\sum N &= I_z \ddot{p} - I_{zz} \dot{p}
\end{align*}
\]  

(4.12)

The moments of inertias in Equation 4.12, are given by

\[
\begin{align*}
I_x &= \Gamma_{xx} + m_z h_{ra}^2 \\
I_z &= \Gamma_{zz} + I_{z,fr} + I_{z,wr} \\
I_{xx} &= 0
\end{align*}
\]

(4.13)

where

\( \Gamma_{xx} \) is the moment of inertia of the sprung mass about the \( x \) centroidal axis

\( \Gamma_{zz} \) is the moment of inertia of the sprung mass about the \( z \) centroidal axis

\( I_{z,fr} \) is the moment of inertia of the front unsprung mass about the \( z \) axis

\( I_{z,wr} \) is the moment of inertia of the rear unsprung mass about the \( z \) axis

Substituting for the moments of inertias, the final form of the equation of motion for the 3-DOF AGV model follows

\[
\begin{align*}
\sum X &= M(\dot{U} - r\dot{V}) + m_z h_{ra} \dot{p} r \\
\sum Y &= M(\dot{V} + r U) - m_z h_{ra} \ddot{p} \\
\sum L &= I_x \ddot{\dot{p}} - m_z h_{ra} (\dot{V} + r U) \\
\sum N &= I_z \ddot{p}
\end{align*}
\]

(4.14)

The expressions for the external forces in the left-hand side equation 4.14 are derived in the next section.
Two DOF Dynamic Model

The 2-DOF model is derived by ignoring the terms related to roll motion in Equation 4.14. This model will be used for controller design and comparison purposes in coming chapters, and is given by

\[
\begin{align*}
\sum X &= M(\ddot{U} - rV) \\
\sum Y &= M(\dot{V} + rU) \\
\sum N &= I_\zeta \dot{\zeta}
\end{align*}
\]  

(4.15)

4.5. External Forces and Moments Acting on the AGV

![Diagram of AGV and tire forces](image)

Figure 4.3  Plan view of AGV and tire forces

The desired mathematical model is obtained by equating the inertia reactions to their respective external forces and moments. External longitudinal and side forces, and yaw moments acting on the AGV are created in the ground plane and are derived from the tire-ground force interactions. The moment about the $x$ axis, on the other hand, results from the suspension springs, the dampers, and the gravitational forces. Referring to Figure 4.3, the force summation in the $x$, and the $y$ directions are
\[
\begin{align*}
\sum F_x &= F_{xfl} \cos(\delta_f) + F_{xfr} \cos(\delta_f) + F_{xrl} \cos(\delta_r) + F_{xrr} \cos(\delta_r) \\
&- F_{yfl} \sin(\delta_f) - F_{yfr} \sin(\delta_f) - F_{yrl} \sin(\delta_r) - F_{yrr} \sin(\delta_r) \\
\sum F_y &= F_{yfl} \cos(\delta_f) + F_{yfr} \cos(\delta_f) + F_{yrl} \cos(\delta_r) + F_{yrr} \cos(\delta_r) \\
&+ F_{xfl} \sin(\delta_f) + F_{xfr} \sin(\delta_f) + F_{xrl} \sin(\delta_r) + F_{xrr} \sin(\delta_r)
\end{align*}
\] (4.16)

In the above equations \( \delta_i \) is the vehicle steering angle, where the first subscript denotes the front/rear steering angle, and the second subscript denotes the right/left wheels. Also, \( F_{ik} \) represents wheel ground forces, where the subscripts denote the force directions, front/rear wheels, and right/left wheels, respectively. The longitudinal wheel forces are \( F_{xfl}, F_{xfr}, F_{xrl}, \) and \( F_{xrr} \) and are given by

\[
\begin{align*}
F_{xfl} &= (-F_r + F_t)_{xfl} \\
F_{xfr} &= (-F_r + F_t)_{xfr} \\
F_{xrl} &= (-F_r + F_t)_{xrl} \\
F_{xrr} &= (-F_r + F_t)_{xrr}
\end{align*}
\] (4.17)

where

- \( F_r \) is the rolling resistance
- \( F_t \) is the tractive force

The moment summation about the \( x \) and \( z \) axes are

\[
\begin{align*}
\sum L &= m_2 g \overline{h} - L_{spring} - L_{damper} \\
\sum N &= a \left[ F_{xfl} \cos(\delta_f) + F_{xfr} \cos(\delta_f) + F_{xfl} \sin(\delta_f) + F_{xfr} \sin(\delta_f) \right] \\
&- b \left[ F_{yrl} \cos(\delta_r) + F_{yrr} \cos(\delta_r) + F_{xrl} \sin(\delta_r) + F_{xrr} \sin(\delta_r) \right] \\
&+ \frac{t}{2} \left[ F_{xfl} \sin(\delta_f) - F_{xfr} \sin(\delta_f) - F_{xfr} \cos(\delta_f) + F_{xfr} \cos(\delta_f) \right] \\
&+ \frac{t}{2} \left[ F_{yrl} \sin(\delta_r) - F_{yrr} \sin(\delta_r) - F_{xrl} \cos(\delta_r) + F_{xrr} \cos(\delta_r) \right]
\end{align*}
\] (4.18)

where, referring to Figure 4.4 for small angles \( \phi \)

\[
\begin{align*}
\overline{h} &= h_{ra} \phi \\
L_{spring} &= (k_f + k_r)\phi = k_{tot} \phi \\
L_{damper} &= [c_f + c_r]p = c_{tot} p
\end{align*}
\]
Simplification for the Two Wheel Steering AGV

For the rear drive, front wheel steering AGV, assuming the following,

\[
\begin{align*}
\delta_{rl} &= \delta_{rr} = 0 \\
\delta_{fl} &= \delta_{fr} = \delta \\
t_f &= t_r = t \\
(F_{v_{fl}}) &= 0 \\
(F_{v_{fr}}) &= 0
\end{align*}
\]  

(4.19)

then, Equations 4.16 and 4.18 simplifies to

\[
\begin{align*}
\sum F_x &= (F_{x_{fl}} + F_{x_{fr}}) \cos \delta + F_{xrl} + F_{xrr} - (F_{y_{fl}} + F_{y_{fr}}) \sin \delta \\
\sum F_y &= (F_{y_{fl}} + F_{y_{fr}}) \cos \delta + F_{yrl} + F_{yrr} + (F_{y_{fl}} + F_{y_{fr}}) \sin \delta \\
\sum L &= (m_{x}g h_{ra} - k_{tot}) \phi - c_{tot} P \\
\sum N &= a [(F_{y_{fl}} + F_{y_{fr}}) \cos \delta + (F_{y_{fl}} + F_{y_{fr}}) \sin \delta] - b (F_{yrl} + F_{yrr}) \\
&+ \frac{1}{2} [(F_{y_{fl}} - F_{y_{fr}}) \sin \delta + (F_{y_{fr}} - F_{y_{fl}}) \cos \delta - F_{xrl} + F_{xrr}]
\end{align*}
\]  

(4.20)

Equation 4.20, is the final form of the relationships for the external forces and moments acting on the AGV. The lateral tire forces in this equation will be determined based on the tire model.
4.6. Tire Model

For the dynamic model to be reliable, an accurate representation of the wheel-ground interaction is necessary. The following tire models are used in the present work for dynamic modelling and simulation purposes.

4.6.1. Linear Tire Model

The simplest tire model that gives good results in many applications is the linear model. This model stipulates that the side forces do vary linearly with side slip angle and the camber angle and is valid if these angles are restricted to small magnitudes of about 4 degrees (Wong, 1993). The model assumes that the cornering stiffness is constant within this range, and is given as

\[ F_{yl} = C_a \alpha_l \]  \hspace{1cm} (4.21)

where

- \( F_{yl} \) is the cornering (lateral) force (N)
- \( C_a \) is the cornering stiffness (N/rad)
- \( \alpha_l \) is the slip angle (rad)

The subscript \( l \) denotes the wheel number as illustrated in Figure 4.3. This tire model is used for the 2-DOF AGV. The load transfer effect is not considered in this model.

4.6.2. Nonlinear Tire Model

Many publications exist on the subject of nonlinear modelling of tire mechanics. Bakker et al. (Bakker, 1987) approach contains equations that are not unduly complicated, and can be programmed to provide tire forces and moments. In addition the results agree quite well with experimental data (Brach, 1991). The following set of equations is a simplification of the model presented by Bakker.
\[ \begin{align*}
A_i &= (-0.0221 \times 10^{-3} F_{zi} + 1.011) F_{zi} \\
B_i &= -0.354 \times 10^{-3} F_{zi} + 0.707 \\
D_i &= \frac{C_s}{(1.30 A_i)} \\
E_i &= (1 - B_i) \alpha_i + \left( \frac{B_i}{D_i} \right) \arctan(\alpha_i D_i) \\
F_{yi} &= A_i \sin(1.30 \arctan(D_i E_i))
\end{align*} \]

where

\[ F_n \] is the normal force on each tire, (N)
\[ F_{yi} \] is the cornering force on each tire (N)

Similar to the linear model, the subscript \( i \) denotes the wheel number. Also, units for \( C_s \) and \( \alpha_i \) are N/deg and deg respectively. Cornering stiffness, \( C_s \), is constant, and the effect of tractive and braking forces are ignored.

### 4.7. Tire Slip Angles

Due to the elastic deformation of the tire, the velocity vector of the wheel does not lie in the wheel plane. The angle between the velocity vector and the wheel plane is called the slip angle. With reference to Figure 4.3, the slip angles are given by

\[ \begin{align*}
\alpha_{fr} &= \delta_{fr} - \arctan\left( \frac{V \cdot \omega_f}{U_{\frac{L}{2}} f} \right) \\
\alpha_{fl} &= \delta_{fl} - \arctan\left( \frac{V \cdot \omega_f}{U_{\frac{L}{2}} l} \right) \\
\alpha_{rl} &= \delta_{rl} - \arctan\left( \frac{V \cdot \omega_r}{U_{\frac{L}{2}} r} \right) \\
\alpha_{rr} &= \delta_{rr} - \arctan\left( \frac{V \cdot \omega_r}{U_{\frac{L}{2}} r} \right)
\end{align*} \]

For a front wheel steering AGV, the slip angles of the front and the rear tires are given by
\[ \alpha_p = \delta - \arctan\left(\frac{v \cdot x_p}{U - \frac{1}{2}r}\right) \]
\[ \alpha_l = \delta - \arctan\left(\frac{v \cdot x_l}{U - \frac{1}{2}r}\right) \]
\[ \alpha_r = -\arctan\left(\frac{v \cdot br}{U + \frac{1}{2}r}\right) \]
\[ \alpha_r = -\arctan\left(\frac{v \cdot br}{U + \frac{1}{2}r}\right) \]

4.8. Load Transfer

When the vehicle is turning, or changing speed, the effect of load transfer is to change the distribution of the normal load on the tires, which in turn changes the cornering forces. This will affect the vehicle directional stability (Wong, 1993). The change in the wheel normal load caused by lateral acceleration is referred to as lateral load transfer, while the change in the wheel normal load caused by longitudinal accelerations is known as longitudinal load transfer.

The total lateral load transfer on the front and rear tires of the vehicle are

\[ W_f = W_{bf} + W_{df} + W_{af} + W_{gf} \]
\[ W_r = W_{br} + W_{nr} + W_{ar} + W_{gr} \]

(4.25)

where the four components of lateral load transfer are

- \( W_b \) is the lateral load transfer due to body roll
- \( W_r \) is the lateral load transfer due to roll centre height
- \( W_u \) is the lateral load transfer due to unsprung weight
- \( W_s \) is the lateral load transfer due to change in the centre of gravity's height
A quasi-static approximation of the dynamic load transfer effect is employed in this work. As shown in Figure 4.5, the total lateral inertia force acting on the vehicle during a turn consists of the sprung mass inertia force, $F_c$, and the unsprung mass inertia forces of front and rear suspensions represented by $F_{caf}$ and $F_{cur}$, given by

$$F_c = m_g a_y$$
$$F_{caf} = m_{af} a_y$$
$$F_{cur} = m_{ar} a_y$$

The inertia force of sprung mass acting on its centre of gravity, can be transferred to the vehicle roll axis with the addition of a roll couple of magnitude

$$M_r = F_c h_{ra}$$

This roll couple causes the sprung mass to rotate about the roll axis. This motion is resisted by the roll stiffness of the vehicles suspension. The portions of the roll couple resisted by the front and rear suspensions are proportional to the their suspension's roll stiffness, and given by
\[ M_{df} = M_r \frac{k_f}{k_{tot}} \]

\[ M_{rr} = M_r \frac{k_r}{k_{tot}} \]

The load transfer due to the vehicle body roll is given by

\[ W_{bf} = \frac{M_{df}}{t_f} \]

\[ W_{br} = \frac{M_{rr}}{t_r} \]

The vehicle’s parameters and mass distribution for a general roll axis are shown in Figure 4.6, however, as explained earlier for the AGV under consideration the roll axis is considered to coincide with the x axis.

![Diagram of AGV mass distribution](image)

**Figure 4.6** Roll axis and AGV mass distribution

The second elements of the total load transfer are due to the portions of sprung mass inertia force carried by each axle, and given by

\[ W_{df} = F_{df} \frac{h_f}{t_f} \]

\[ W_{rr} = F_{rr} \frac{h_r}{t_r} \]

where
\[ F_{cf} = F_c \frac{b}{L} \]
\[ F_{cr} = F_c \frac{a}{L} \]

The third components of the total lateral weight transfer result from inertia forces of the unsprung masses, and are given by
\[ W_{w_f} = F_{w_f} \frac{h_{w_f}}{t_f} \]
\[ W_{w_r} = F_{w_r} \frac{h_{w_r}}{t_r} \]

Again, with reference to Figure 4.4, the last terms of the total load transfer expression are due to the change in location of the centre of mass of the sprung mass
\[ W_{s_f} = m_s g h_{sa} \phi \frac{b}{L t_f} \]
\[ W_{s_r} = m_s g h_{sa} \phi \frac{a}{L t_r} \]

With reference to assumptions 6 and 8 in Section 4.3.2, the following is true:
\[ h_{w_f} = h_{w_r} = h \]
\[ h_f = h_r = h \]
\[ t_f = t_r = t \]

using the above set of equations the terms of the total lateral load transfer in Equation 4.25, reduce to
\[ W_{bf} = m_s h_{ra} (\dot{V} + Ur) \frac{k_r}{tk_{ax}} \]
\[ W_{br} = m_s h_{ra} (\dot{V} + Ur) \frac{k_r}{tk_{ax}} \]
\[ W_{cf} = m_s h (\dot{V} + Ur) \frac{b}{Lt} \]
\[ W_{cr} = m_s h (\dot{V} + Ur) \frac{a}{Lt} \]
\[ W_{wf} = h (\dot{V} + Ur) \frac{m_w}{t} \]
\[ W_{wr} = h (\dot{V} + Ur) \frac{m_w}{t} \]
\[ W_{rf} = m_s g h_{ra} \phi \frac{b}{Lt} \]
\[ W_{rr} = m_s g h_{ra} \phi \frac{b}{Lt} \]  \hspace{1cm} (4.26)

The magnitude of the longitudinal load transfer due to speed change is computed using the following relation
\[ W_{long} = \frac{m_i \dot{U}_h}{2L} + \frac{m_w \dot{U}_h}{2L} + \frac{m_w \dot{U}_h}{2L} \]  \hspace{1cm} (4.27)

The normal ground reactions on the four tires required in Equation 4.22, are hence given by
\[ F_{sfl} = \frac{Mb}{2L} + W_{long} - W_{df} \]
\[ F_{sfr} = \frac{Mb}{2L} + W_{long} + W_{df} \]
\[ F_{srl} = \frac{Ma}{2L} - W_{long} - W_{dr} \]
\[ F_{srr} = \frac{Ma}{2L} - W_{long} + W_{dr} \]  \hspace{1cm} (4.28)

4.9. Tractive Force

It is assumed that the entire effort of the motion is provided by the forces between the driving wheels and the working surface. For the case where the wheels roll without slipping, and the rear wheels are driven by an electric motor through a gear box and differential gear, the total tractive forces of rear tires are given by (Appendix A1),
\[ F_i = K_i V_i - J_i \ddot{U} - C_i U \]  

(4.29)

where

\[
    K_i = \frac{n_i \bar{K}_i}{R_w R_{at}}
\]

\[
    J_i = \frac{n_i^2 \bar{J}_i}{R_w^2}
\]

\[
    C_i = \frac{n_i}{R_w^2} \left( \frac{\bar{K}_i^2}{R_{at}} + n_i \bar{C}_i \right)
\]

Using Equation 4.17, and substituting for \( F_i = F_{xrl} + F_{xrp} \) in Equation 4.20, will result in the AGV dynamic equations, including the effect of tractive motor dynamics, gear boxes.

### 4.10. Steering System

Different models are suggested to represent the steering dynamics of AGVs, WMRs, and ATVs. Some authors use a second order system to model the dynamics of the steering system (Deng and Brady, 1993a; Makino, 1993), others, argue that the effect of steering inertia is negligible, and have used a simple proportional model, \( \tau_s = \delta \) (DeSantis, 1995a; Yu and Moskwa, 1994). However, for all practical purposes of this work a first order lag system (damper and spring) is used to represent the dynamics of steering system. This later model is often practically implemented for simulation and design controllers for ATVs, AGVs, and WMRs (Smith and Starkey, 1994; Shin et al., 1992; Fenton et al., 1988; Shladover et al., 1978). The steering dynamics is represented by

\[
    \tau_s = H_{1s}(\delta + H_{2s}\delta)
\]  

(4.30)

Referring to Appendix A2, for the case where steering system is actuated by a DC motor the steering torque is given by

\[
    \tau_s = K_s V_s - C_s \dot{\delta}
\]  

(4.31)
where

\[ K_z = \frac{n_s \bar{K}_s}{R_{ar}} \]

\[ C_z = n_s \left( \frac{\bar{K}_s^2}{R_{ar}} + n_s \bar{C}_s \right) \]

Substituting for \( \tau_z \) into Equation 4.30 results in the steering dynamics

\[ \delta = \frac{1}{k_{1z}} V_z - k_{2z} \delta \]  

(4.32)

where

\[ k_{1z} = \frac{H_{1z} + C_z}{K_z} \]

\[ k_{2z} = \frac{H_{1z} H_{2z}}{H_{1z} + C_z} \]
Chapter 5

AGV Control

5.1. Introduction

In order to obtain satisfactory performance for an automated guided vehicle, an efficient control scheme is required to generate the control signals. This controller should be capable of correcting the errors in a stable manner, in a reasonable time, and without oscillations about the path (hunting). Controllers based on mere feedback information of position and orientation errors are shown to be incapable of proper tracking. Even the kinematic-based controllers are not adequate for this purpose, as payload and travelling speed increases. In this chapter other alternatives are being investigated, and in particular a dynamic-based controller that takes into account the vehicle's dynamic behaviour is explored. A 2-DOF nonlinear model is used to design the controller. The vehicle model is first transformed into a Side Slippage Free (SSF) model, then exact input-output feedback linearization is used to linearize the nonlinear system. A sliding mode controller is applied to guarantee the robustness, and deal with disturbances and unmodeled dynamics.

5.2. Conversion to 2-DOF Side Slippage Free Model

2-DOF Bicycle Model Dynamics

While in practice using a more complicated model for simulation studies is quite acceptable (Allen et al., 1987; Elgindy and Ilosvai, 1983; Brach, 1991), a simpler model is
preferred for control purposes. For non-emergency operations, where lateral accelerations are less than 0.3 g, 2-DOF models are often used to develop steering controllers (Smith and Starkey, 1994; Jurie et al., 1994; Fenton and Selim, 1988). Since, the AGV's under normal industrial operating conditions experience accelerations that are usually less than 0.3 g, a 2-DOF bicycle model that neglects the vehicle body roll is used in this study to develop the AGV controller. Furthermore, the motion of the vehicle is assumed to be free of side slippage. The 3-DOF model developed in Chapter 4 will ultimately be used for simulation purposes (Chapter 6).

![Figure 5.1 Plane view of bicycle model tire forces.](image)

With reference to Figure 5.1, and Equations 4.15, 4.17, and 4.20 the motion equations of the 2-DOF bicycle model can be written as,

\[
\begin{align*}
M \ddot{U} &= MVr + F_{xf} \cos \delta - F_{xf} \sin \delta + F_t + F_{x r} \\
M \ddot{V} &= -MVr + F_{xf} \sin \delta + F_{xf} \cos \delta + F_{y r} \\
I \dot{\gamma} &= a F_{xf} \sin \delta + a F_{yf} \cos \delta - b F_{yr} 
\end{align*}
\]  

(5.1)

where

- \( F_{xf} \) is the combined rolling resistance force of front tires
- \( F_{xr} \) is the combined rolling resistance force of rear tires
- \( F_{xf} \) is the combined lateral force of front tires
- \( F_{yr} \) is the combined lateral force of rear tires
- \( F_t \) is the combined tractive force of rear tires
- \( I \) is \( I_z \), the total moment of inertia about z axis
Substituting for the relevant terms related to the effect of tractive motor dynamics, gear box, and the differential gear box given by Equation 4.29, Equation 5.1 can be rewritten in matrix notation as

\[ M \ddot{\gamma} = MG_0 + G_x F_x + G_y F_y + G_t V_t \]  \hspace{1cm} (5.2)

where the parameters are given by

\[
G_x = \begin{bmatrix}
\cos \delta & 1 \\
\sin \delta & 0 \\
a \sin \delta & 0
\end{bmatrix} \quad G_y = \begin{bmatrix}
-\sin \delta & 0 \\
\cos \delta & 1 \\
a \cos \delta & -b
\end{bmatrix} \quad G_0 = \begin{bmatrix}
V_r - \frac{C_u}{M} \\
-U_r \\
0
\end{bmatrix} \quad G_t = \begin{bmatrix}
K_t \\
0
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
M+J_t & 0 & 0 \\
0 & M & 0 \\
0 & 0 & I
\end{bmatrix} \quad \dot{\gamma} = \begin{bmatrix}
\dot{U} \\
\dot{r} \\
\dot{r}
\end{bmatrix} \quad F_x = \begin{bmatrix}
F_{\gamma x} \\
F_x \\
F_{\gamma x}
\end{bmatrix} \quad F_y = \begin{bmatrix}
F_{\gamma y} \\
F_y \\
F_{\gamma y}
\end{bmatrix}
\]

Recalling the steering actuator dynamics given in Equation 4.32, the dynamics of 2-DOF model is formulated as

\[
\dot{\gamma} = M^{-1} \left[ MG_0 + G_x F_x + G_y F_y + G_t V_t \right]
\]

\[
\dot{\delta} = \frac{1}{k_{\delta}} V_x - k_{\delta} \delta
\]

\[
\dot{\theta} = r
\]

\[
\dot{X} = U \cos \theta - V \sin \theta
\]

\[
\dot{Y} = U \sin \theta + V \cos \theta
\]

### Side Slippage Free Motion

For the purposes of controller design a motion free from side slippage, which corresponds to setting front and rear slip angles equal to zero, is assumed. Although, this assumption may not be true for highway applications, it has been proven to be quite realistic for AGV applications where the maximum speed is usually small (DeSantis 1995a, 1995b).
In the absence of slippage the angular and lateral velocities can be expressed in terms of the steering angle and forward velocity. This interdependency greatly simplifies the dynamic equations of the vehicle.

Recalling Equation 4.24, the tire slip angles for the bicycle model are given by

\[ \alpha_f = \delta - \arctan\left(\frac{v_x - v_y}{U}\right) \]
\[ \alpha_r = \arctan\left(\frac{v_x + Ub}{U}\right) \]  \hspace{1cm} (5.4)

Setting \( \alpha_f = \alpha_r = 0 \), leads to the nonholonomic constraints

\[ V = \frac{U b \tan \delta}{L} \]
\[ r = \frac{U \tan \delta}{L} \]  \hspace{1cm} (5.5)

Differentiating Equation 5.5 gives

\[ \dot{V} = \frac{k \tan \delta}{L} \dot{U} + \frac{Ub}{L \cos^2 \delta} \dot{\delta} \]
\[ \dot{r} = \frac{\tan \delta}{L} \dot{U} + \frac{U}{L \cos^2 \delta} \dot{\delta} \]  \hspace{1cm} (5.6)

In matrix notation, Equations 5.5 and 5.6 are represented as

\[ \mathbf{v} = J_1 \dot{U} \]
\[ \mathbf{\dot{v}} = J_1 \ddot{U} + J_2 \dot{\delta} \]  \hspace{1cm} (5.7)

where

\[ J_1 = \begin{pmatrix} 1 \\ \frac{b \tan \delta}{L} \\ \frac{\tan \delta}{L} \end{pmatrix} ; \quad J_2 = \begin{pmatrix} 0 \\ \frac{bU}{L \cos^2 \delta} \\ \frac{U}{L \cos^2 \delta} \end{pmatrix} \]

Using an approach similar to that of DeSantis (1995b), both sides of the first equality in Equation 5.3 are pre-multiplied by \( J_1^T M \). Since \( J_1^T G_y = 0 \) the following results

\[ J_1^T M \mathbf{v} = M J_1^T G_0 + J_1^T G_x F_x + J_1^T G_y V \]  \hspace{1cm} (5.8)

Substituting for \( \dot{v} \) and \( \dot{\delta} \) from Equations 5.7 and 5.3 respectively it follows that
\[ \dot{U} = M (J_1^T M J_1)^{-1} J_1^T G_0 + (J_1^T M J_1)^{-1} J_1^T G_x F_x + (J_1^T M J_1)^{-1} J_1^T G_t V_t \\
- k_{i_1}^{-1} (J_1^T M J_1)^{-1} J_1^T M J_2 V_s + k_{2_1} (J_1^T M J_1)^{-1} J_1^T M J_2 \delta \] (5.9)

Equation 5.9 can be written in a general form as follows:
\[ \dot{U} = g_x F_x + g_t V_t + g_s V_s + g_0 \]

where the \( g \) coefficients are defined as follows:
\[
\begin{align*}
g_p &= (J_1^T M J_1)^{-1} \\
g_x &= (J_1^T M J_1)^{-1} J_1^T G_x \\
g_t &= (J_1^T M J_1)^{-1} J_1^T G_t \\
g_s &= -k_{i_1}^{-1} (J_1^T M J_1)^{-1} J_1^T \\
g_0 &= M(J_1^T M J_1)^{-1} J_1^T G_0 + k_{2_1} (J_1^T M J_1)^{-1} J_1^T M J_2 \delta
\end{align*}
\]

Substituting for the matrices \( M, J_1, J_2, G_x, G_t, \) and \( G_0 \) in the above equation, the complete dynamics of the vehicle under side slip free condition is given by
\[
\begin{align*}
\dot{U} &= g_x F_x + g_t V_t + g_s V_s + g_0 \\
\dot{\delta} &= \frac{1}{L_s} \dot{V}_s - k_{2_1} \delta \\
\dot{\theta} &= \frac{U \tan \delta}{L} \\
\dot{X} &= \frac{U}{\cos \delta} \left( \cos \theta \cos \delta - \frac{b}{L} \sin \theta \sin \delta \right) \\
\dot{Y} &= \frac{U}{\cos \delta} \left( \sin \theta \cos \delta + \frac{b}{L} \cos \theta \sin \delta \right)
\end{align*}
\] (5.12)

where
\[ g_p = \left[ M + J_t + \frac{m a^2 \delta}{I^2} (M b^2 + I) \right]^{-1} \]

\[ g_x = \begin{bmatrix} 1 \\ \cos \delta \end{bmatrix} g_p \]

\[ g_t = g_p K_t \]

\[ g_s = \left[ -\frac{U \tan \delta}{k_s \cos^2 \delta} (M b^2 + I) \right] g_p \]

\[ g_0 = -g_p C_t U - k_{1s} k_{2s} g_s \delta \]

The nonlinear dynamic system in Equation 5.12 can finally be described in the general state space form as

\[ \dot{\chi} = f(\chi) + g(\chi) u \quad (5.13) \]

where

\[ \chi = [ U \delta \theta X Y ]^T \]

\[ u = [ V_t \ V_s ]^T \]

\[ f(\chi) = \begin{bmatrix} g_x F_x + g_0 & -k_{2s} \delta & \frac{U \tan \delta}{L} & \frac{U D_1}{\cos \delta} & \frac{U D_2}{\cos \delta} \end{bmatrix}^T \]

\[ g(\chi) = \begin{bmatrix} g_t & 0 & 0 & 0 & 0 \\ g_s & \frac{1}{k_{ts}} & 0 & 0 & 0 \end{bmatrix}^T \]

the parameters \( D_1 \) and \( D_2 \) are defined as

\[ D_1 = (\cos \theta \cos \delta - \frac{b}{L} \sin \theta \sin \delta) \]

\[ D_2 = (\sin \theta \cos \delta + \frac{b}{L} \cos \theta \sin \delta) \]

### 5.3. Feedback Linearization

Feedback linearization is an approach for the controller design of a nonlinear system which has received considerable attention in recent years in areas such as high performance aircraft and industrial robots (Hedrick, 1993; Jagannathan et al. 1994). The central idea in
this approach is to algebraically transform the nonlinear system into a linear or partially linear system by exact state transformation and feedback. Linear control techniques are then used to design the controller for the linearized system. This method is entirely different from approximate linearization approach known as the Jacobian linearization, where Taylor expansion is used to linearize the system about equilibrium point (Slotine and Li, 1991). The remainder of this chapter deals with the linearization and the synthesis of the feedback control laws for the nonlinear system given in Equation 5.13.

5.3.1. Definitions

This section introduces some useful definitions for understanding the methodology used in this chapter. The presentation is primary based on lisidori (1995, 1989), Slotine and Li (1991), Khalil (1992) and Vidyasagar (1993).

In what follows the Multi-Input Multi-Output (MIMO) nonlinear control systems with $n$ states $\chi$, $m$ inputs $u$, and $m$ outputs $y$ (i.e., square systems) are studied. The MIMO systems are described in the state space form as

$$\begin{align*}
\dot{\chi} &= f(\chi) + \sum_{k=1}^{m} g_k(\chi) u_k \\
y_i &= h_i(\chi) \quad 1 \leq i \leq m
\end{align*} \tag{5.14}$$

in which $f, g_1, g_2, \ldots, g_m$ are smooth vector fields, and $h_1, h_2, \ldots, h_m$ are smooth functions defined on an open set of $\mathbb{R}^n$. Whenever possible and convenient, these equations will be written in the short form

$$\begin{align*}
\dot{\chi} &= f(\chi) + g(\chi) u \\
y &= h(\chi)
\end{align*} \tag{5.15}$$

where

- $\chi$ is the $n \times 1$ state vector
- $u$ is the $m \times 1$ control input vector of components $u_i$
- $y$ is the $m \times 1$ vector of system outputs of components $y_i$
- $f, h$ are smooth vector fields
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\[ g \] is a \( n \times m \) matrix whose columns are smooth vector fields \( g \).

**Definition 1** Smooth Vector Field: Based on differential geometry terminology a vector function \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is called a smooth vector field, if \( f(\chi) \) has continuous partial differential of any order. Also, a scalar function \( h : \mathbb{R}^n \rightarrow \mathbb{R} \) having continuous partial differential of any order is called a smooth scalar function.

**Definition 2** Lie Derivative: Suppose \( h : \mathbb{R}^n \rightarrow \mathbb{R} \) and that \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) denote smooth scalar and vector functions on \( \mathbb{R}^n \), respectively. Then the Lie derivative of \( h \) with respect to \( f \) is also a scalar function and is defined as

\[ L_f h = \nabla h \cdot f \]

In other words, the resulting function is the directional derivative of \( h \) along \( f \). Of course repeated use of this operation is possible and defined recursively as

\[ L_f^0 h = h \]
\[ L_f^i h = L_f (L_f^{i-1} h) = \nabla (L_f^{i-1} h) \cdot f \quad i=1,2,... \]

**Definition 3** Lie Bracket: Let two vector fields \( f \) and \( g \) be defined on \( \mathbb{R}^n \). The Lie bracket (or Lie product) of \( f \) and \( g \) is a new smooth vector field defined by

\[ [f, g] = \nabla g \cdot f - \nabla f \cdot g \]

Repeated bracketing of a vector field \( g \) with the same vector field \( f \) is also possible. Whenever this is needed, it is a common practice to show the Lie bracket of \([f, g]\) with \( \text{ad}_f g \), where \( \text{ad} \) stands for “adjoint.”

\[ \text{ad}_f^0 g = g \]
\[ \text{ad}_f^i g = [f, \text{ad}_f^{i-1} g] \quad i=1,2,... \]

**Definition 4** Involutivity: Suppose \( \{f_1, f_2, \ldots, f_m\} \) denotes a set of linearly independent vector fields. The set is involutive if, and only if, there are scalar functions \( \alpha_{yk} : \mathbb{R}^n \rightarrow \mathbb{R} \) such that
Thus, the Lie bracket of any pairs of vector fields $f_i$ and $f_j$ from an involutive set can be expressed as a linear combination of the original set of vector fields.

**Definition 5** Diffeomorphism: A function $\Phi: \mathbb{R}^n \to \mathbb{R}^n$, defined in a region $\Omega$, is called a diffeomorphism if its inverse exists, and if both $\Phi$ and $\Phi^{-1}$ are smooth. A transformation of this type is called global diffeomorphism, if $\Omega$ is the whole space of $\mathbb{R}^n$. Sometimes, to find a transformation defined for all $\chi$ is difficult. Thus, in most cases one rather looks for local diffeomorphisms that are defined only in a neighbourhood of a given point.

**Definition 6** Relative Degree: A multi variable nonlinear system of the form given in Equation 5.15 has a (vector) relative degree $\{r_1, r_2, ..., r_m\}$ at $\chi_0$, and the scalar $r_i = r_1 + r_2 + ... + r_m$ is called the total relative degree of the system at $\chi_0$. Where, $r_i$ is the number of times one has to differentiate the $i^{th}$ output $y_i$ in order to have at least one component of the input vector $u$ to explicitly appear.

**Definition 7** Input-State Linearization: Given a set of vector fields $f$ and $g_1, g_2, ..., g_m$ being smooth in $\Omega = \mathbb{R}^n$. The MIMO system in Equation 5.15 (without output) is said to be input-state linearizable in a neighbourhood $\Omega$ of $\chi_0$, if there exists a diffeomorphism $\phi: \Omega \to \mathbb{R}^n$ and, a nonlinear feedback control law

$$u = \alpha(\chi) + \beta(\chi)v$$

such that the new variable $z = \phi(\chi)$ and the new input $v$ satisfy a linear time invariant relation

$$\dot{z} = Az + Bv$$

where $A$ and $B$ are controllable.

**Definition 8** Input-Output Linearization: The system in Equation 5.15 is said to be input-output linearizable if it is possible to generate a linear differential relation between the output $y$, and inputs $u_i$ such that
\[ y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{k=1}^{m} (L_{i_k} L_f^{r_i-1} h_i) u_k \] 

(5.16)

with \( r \), being the minimum order of derivative of \( y \), where the coefficient of at least one \( u_k \) is not zero.

Performing the above procedure for each output \( y_i \) yields

\[ \Upsilon = b(\chi) + E(\chi) u \] 

(5.17)

where

\[ \Upsilon = \begin{bmatrix} y_1^{(r_1)} & \ldots & y_m^{(r_m)} \end{bmatrix}^T \]

\[ b(\chi) = \begin{bmatrix} L_1 h_1(\chi) & \ldots & L_m h_m(\chi) \end{bmatrix}^T \]

Also the \( m \times m \) matrix \( E(\chi) \) is called the decoupling matrix of the system and defined as

\[ E(\chi) = \begin{bmatrix} L_{i_1} L_f^{r_1-1} h_1 & L_{i_2} L_f^{r_1-1} h_1 & \ldots & L_{i_m} L_f^{r_1-1} h_1 \\ L_{i_1} L_f^{r_2-1} h_2 & L_{i_2} L_f^{r_2-1} h_2 & \ldots & L_{i_m} L_f^{r_2-1} h_2 \\ \vdots & \vdots & \ddots & \vdots \\ L_{i_1} L_f^{r_m-1} h_m & L_{i_2} L_f^{r_m-1} h_m & \ldots & L_{i_m} L_f^{r_m-1} h_m \end{bmatrix} \]

Using transformations similar to that of the Single Input Single Output (SISO) case yields \( m \) equations of the simple form

\[ y_i^{(r_i)} = u_i \] 

(5.18)

**Definition 9** Normal form: Let a system has a relative degree of

\[ r_1 + r_2 + \ldots + r_m \leq n \]

and

\[ \xi_i = \begin{bmatrix} \xi_1^i & \xi_2^i & \ldots & \xi_{r_i}^i \end{bmatrix}^T = \begin{bmatrix} h_1(\chi) & L_f h_1(\chi) & \ldots & L_f^{r_i-1} h_1(\chi) \end{bmatrix}^T \]

Then the normal form of the system in a neighbourhood \( \Omega \) of a point \( \chi_0 \), can be written as
\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\vdots \\
\dot{\xi}_r &= \xi_{r+1} \\
\dot{\xi}_{r+1} &= b_i(\xi, \eta) + \sum_{j=1}^{m} a_{ij}(\xi, \eta) u_j \\
y_i &= \xi_i \\
\dot{\eta} &= q(\xi, \eta)
\end{align*}
\]

The $\xi_i$ and $\eta_i$ are referred to as normal coordinates or normal states in $\Omega$ (or at $\chi_a$).

**Definition 10** *Internal Dynamics*: The part of the system dynamics rendered unobservable by the process of input-output Linearization. This is called internal dynamics because it cannot be seen from external input-output relation.

**Definition 11** *Zero Dynamics*: The zero dynamics is defined to be the internal dynamics of the system when the system outputs are kept at zero by the inputs. The zero dynamics is an intrinsic feature of the nonlinear system which does not depend on the choice of the control law or the desired trajectories. Also, examining the stability of the zero dynamics is much easier than examining the stability of the internal dynamics.

### 5.3.2. Input-Output feedback Linearization

The development of a linear differential relation that describes the input-output relationship of the dynamic model under consideration follows.

#### 5.3.2.1. Choice of Outputs

While the state Equations 5.13 are determined by the dynamic characteristics of the vehicle, the output functions $y_i = h_i(\chi)$ are chosen in such a way to achieve a desired task performance and would result in a convenient controller design. Moreover, different choices of output functions will result in different internal dynamics. Thus, it is quite possible to have stable internal dynamics for a certain choice of output variables, while another choice
of outputs leads to an unstable one. In this section some possible choices of output variables are presented and discussed separately.

For a system having two inputs, any choice of two functions or variables is possible. The following options are considered in this work.

Type I: \( y = h(\chi) = [X \ Y]^T \)

Type II: \( y = h(\chi) = [Y \ \theta]^T \)

Type III: \( y = h(\chi) = [U \ \delta]^T \)

Type IV: \( y = h(\chi) = [U \ Y]^T \)

**Type I Output**

The input-output linearization can be achieved only when the decoupling matrix \( E \) is nonsingular in the region \( \Omega \). For the case under study the outputs are a function of state variables only, and the decoupling matrix for the system is \( 2 \times 2 \). The mathematical derivation of \( E \) for the type I output is presented in Appendix B1, and the final result is given by

\[
E(\chi) = \begin{bmatrix} g_1D_1 & g_2D_1 & U\sin\theta \\ \cos\delta & \cos\delta & Lk_{12}\cos^2\delta \\ g_1D_2 & g_2D_2 & Ubcos\theta \\ \cos\delta & \cos\delta & Lk_{12}\cos^2\delta \end{bmatrix}
\]

Substituting for \( g_1, D_1, \) and \( D_2 \), the determinant of decoupling matrix simplifies to

\[
\det E = \frac{g_1Ub}{k_{12}L\cos^2\delta}
\]

\[
= \left\{ M + J_{\text{r}} + \sin^2\delta \left[ I + M(b^2 - L^2) \right] \right\}^{-1} \frac{K_{12}bU}{k_{12}L}
\]  \( (5.19) \)

This system has a total relative degree of four at any point \( \chi_0 \) provided that \( E^{-1} \) is not singular. The determinant of Equation 5.19 is singular if and only if \( b=0 \), or \( U=0 \). The first condition indicates that the trajectory tracking of a representative point (in this case the centre of gravity) on the rear wheel axle is not possible. This is due to the presence of the nonholonomic constraints and has already been pointed out by Samson and Ait-Abderrahim
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(1990a). Where as the presence of $U$ in the determinant is a result of the particular wheel base configuration under study. Had the AGV been powered by a differential drive, for the same outputs, the decoupling matrix would have been independent from $U$ (Jagannathan et al. 1994.)

**Type II Outputs**

Here the desired outputs to be controlled are $Y$, and $\theta$. Following the same procedure as for type I, the decoupling matrix determinant is obtained as shown in Appendix B2 and is given by

$$\det E = \frac{g_1 Ub \sin \theta}{k_{1s} L \cos^2 \delta}$$

The system has a relative degree of four. The matrix $E$ is singular if $b=0$, $U=0$, or $\sin \theta=0$. Thus, the system is linearizable only in portions of the entire state space region. Due to this limitation type II outputs are not of practical value, further, it is difficult to specify control tasks having these outputs.

**Type III Outputs**

Here instead of controlling the positional variables, the forward velocity $U$, and the steering angle $\delta$, are controlled. The decoupling matrix determinant derived in Appendix B3 is given by

$$\det E = \frac{g_1}{k_{1s}}$$

This determinant is nonsingular for the entire region of state space. However, the total relative degree of the system is two for this choice of outputs, resulting in a complex nonlinear internal dynamics of order three. Moreover, due to selecting $\delta$ as one of the outputs, it is not convenient to compensate for the tracking error in vehicle position.
Type IV Outputs

When a human driver controls a vehicle along a road, the two primary considerations are:

1) To maintain a desired forward velocity.

2) To follow the road by staying as close as possible to a reference path.

Based on these observations, and considering straight routes for the time being, it is desired to define the outputs as the forward velocity \( U \), and the lateral error from the reference line \( Y \). This is sometimes referred to as dynamic path following (Shin et al., 1992; Sarkar et al., 1994; DeSantis, 1994), and differs from trajectory tracking, where the longitudinal error along the path, and time history of position are of paramount concern. The determinant of the decoupling matrix for this case is derived in Appendix B4 and given by

\[
\det E = \frac{g_1 U b \cos \theta}{k_1 L \cos^2 \delta}
\]

(5.20)

This system has a total relative degree of three, leading to a more involved internal dynamics compared to Type I outputs. The determinant in Equation 5.20 is singular if and only if \( b=0 \), \( U=0 \), and \( \cos \theta = \frac{\pi}{2} \). The first condition implies that the trajectory tracking of a representative point on the rear wheel axis is not possible, which is due to the presence of the nonholonomic constraints. Whereas the presence of \( U \) in the determinant is a result of the particular wheel base configuration under study. The third condition occurs when the vehicle is normal to the path. This is because the direction along the path is not explicitly specified in the function \( h_i(\chi) = Y \).

5.3.2.2. Choice of Coordinates

The nonlinear state equations of the AGV, Equation 5.13, can be described in different coordinate systems. One can express these equations in inertial coordinates, vehicle’s fixed coordinates, or path dependent coordinates. Often, for a particular choice of output equations it is more meaningful to deal with one coordinate system rather than the others. In what follows Equation 5.13 is first transformed into the path dependent
coordinates, then the input output (I/O) linearization for different choice of outputs is carried out.

Path Dependent Coordinates

\[ v = V_t \hat{e}_t + V_n \hat{e}_n \]
\[ v = (U \cos \bar{\theta} - V \sin \bar{\theta}) \hat{e}_t + (V \sin \bar{\theta} + V \cos \bar{\theta}) \hat{e}_n \]

Figure 5.2  AGV's motion with respect to path dependent coordinates.

Referring to Figure 5.2, the two components of velocity in the AGV's fixed coordinates are depicted as forward, \( U \) and lateral, \( V \). Thus, the total vehicle velocity \( v \) in the path dependent coordinates is given by

\[ \dot{s} = \left[ \begin{array}{c} \theta \\ \bar{\theta} - n \end{array} \right] \quad \dot{\theta} = \frac{V_\theta}{1 - c(s)} \]
\[ \dot{n} = V_n \]
\[ \dot{\theta}_d = \frac{\dot{s}}{\bar{\theta}} = \frac{c(s) V_\theta}{1 - c(s)} \]
where
\[ q \] is the instantaneous radius of rotation
\[ c(s) \] is the curvature of path
\[ n \] is the normal distance from the path

Using Equation 5.21 and 5.22, the dynamics of the vehicle in the path dependent coordinates can be represented as
\[ \dot{\zeta} = f(\zeta) + g(\zeta) u \]

where
\[ \zeta = [\omega_x, \delta, \delta, s, n]^T \]
\[ u = [V_t, V_s]^T \]
\[ f(\zeta) = \begin{bmatrix}
    \frac{1}{R_w} (g_x F_x + g_0) \\
    -k_{22} \delta \\
    \frac{R_w \omega_{w}}{\cos \delta} \left[ \sin \delta \right] \\
    \frac{R_w \omega_{w} D_1}{\cos \delta} \\
    \frac{R_w \omega_{w} D_2}{\cos \delta}
\end{bmatrix} \]
\[ g(\zeta) = \begin{bmatrix}
    \frac{g_x}{R_w} \\
    \frac{g_y}{R_w} \\
    0 \\
    0 \\
    0
\end{bmatrix} \]

\( R_w \) and \( \omega_w \) are the radius and angular velocity of the rear wheels respectively. Also, the values for \( g_x, g_y, g_0, g_{00}, D_1 \), and \( D_2 \) are obtained using a similar approach to that used for the inertial coordinate case and are given in Appendix C.

**I/O Linearization for Path Dependent Coordinates**

The output choices considered are

**Type V Outputs**

The desired outputs in this case are the path coordinates \( s \) and \( n \). Following the same procedure of differentiating the variables will result in decoupling matrix determinant derived in Appendix C and is given by
\[ \det E = \frac{g_1 R_w \omega_w b}{k_{11} L \cos^2 \delta} \]

The result is very similar to the decoupling matrix in Equation 5.19, for the type I outputs. The total relative degree of the linearized subsystem system is four, and the order internal dynamics is one.

Type VI Outputs

The wheels angular velocity \( \omega_w \), and normal distance from the path, \( n \), are chosen as the desired outputs. The decoupling matrix determinant for this case is derived in Appendix C and is given by

\[ \det E = \frac{g_1 \omega_w b \cos \delta}{k_{11} L \cos^2 \delta} \]

Comparable to the type IV outputs, the linearized subsystem has a total relative degree of three, and results in a second order internal dynamics.

5.3.3. Internal Dynamics

Since the total relative degree of the system \( r_n \), is less than the system's order \( n \), the input output linearization technique decomposes the AGV’s nonlinear dynamics into two parts, external and internal. The external part results in a linear differential relation between \( y_i \) and \( u_i \), where a suitable design of \( u_i \) yields the desired behaviour of \( y_i \). However, the internal behaviour or unobservable part of the system has to be investigated to assure the internal states remain bounded. Referring to Definition 10, in the normal coordinates the internal dynamics corresponds to the last \( n-r \), equations of the normal form.

---

1 Analogous result is obtained if the forward linear velocity, \( U \), is selected instead of angular velocity \( \omega_w \).
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For a discussion of the internal dynamics and the zero dynamics, first, a
diffeomorphism is constructed to transform the whole system into the normal form of the
nonlinear systems, then the stability issue is discussed.

Since the total relative degree of the system is four for this choice of output, the
dimension of internal dynamics is one, that is

\[
\text{Dim(Internal Dyn)} = n - \text{Dim(I/O)}
\]

where \(n=5\) is the order of original system. To transform the system into normal form, the
first four components of the required diffeomorphism are constructed as

\[
\begin{align*}
\xi_1^1 &= h_1(\chi) = X \\
\xi_1^2 &= L_f h_1(\chi) = \frac{ud_1}{\cos \delta} \\
\xi_2^1 &= h_2(\chi) = Y \\
\xi_2^2 &= L_f h_2(\chi) = \frac{ud_2}{\cos \delta}
\end{align*}
\]

Since the distribution spanned by the vector fields \(g_i(\chi)\) and \(g_4(\chi)\) is involutive, it is possible
to choose the remaining new coordinate in such a way that

\[
L_{\xi_j} \eta_i = 0 \quad \forall \ 1 \leq i \leq n-r, \quad 1 \leq j \leq m
\]

thus having \(L_{\xi_1} \eta_1 = 0\) and \(L_{\xi_2} \eta_1 = 0\) results in

\[
\begin{align*}
\frac{\partial \eta_1}{\partial U} g_i &= 0 \\
\frac{\partial \eta_1}{\partial U} g_2 + \frac{\partial \eta_1}{\partial \delta} \frac{1}{k_{1s}} &= 0
\end{align*}
\]

(5.24)

To satisfy the partial differential Equations 5.24, one possible choice is \(\eta_1 = 0\). Thus, the
proposed diffeomorphic transformation would be \(\Phi(\chi)\), where for simplicity new notations
\(z_i\)'s are being adopted\(^1\)

\[
\Phi(\chi) = \begin{bmatrix} \xi_1^1 & \xi_1^2 & \xi_2^1 & \xi_2^2 & \eta_1 \end{bmatrix}^T = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}^T
\]

\(^1\)If the distribution is not involutive then one can always look for \(\eta_i\)'s, such that the
Jacobian matrix of \(\Phi(\chi)\) is nonsingular.
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To verify that $\Phi(\chi)$ is indeed a diffeomorphism, one has to check that $\nabla \Phi(\chi)$ has full rank (Appendix D). The inverse transformation is given by

\[ U = z_2 \cos z_5 + z_4 \sin z_5 \]

\[ \delta = \arctan \left( \frac{z_5 \cos z_5 - z_2 \sin z_5}{\delta} \right) \frac{1}{\delta} (z_2 \cos z_5 + z_4 \sin z_5) \]

\[ \theta = z_5 \]

\[ X = z_1 \]

\[ Y = z_3 \]

Therefore, the internal dynamics in the normal form is given by

\[ z_5' = \frac{1}{\delta} (z_4 \cos z_5 - z_2 \sin z_5) \]

(5.25)

5.3.4. Stability of Zero Dynamics

The zero dynamics of a control system is defined as the dynamics of the system when the outputs are identically zero. If the outputs are identically zero, for this case this implies that

\[ z_5' = 0 \]

(5.26)

The zero dynamics in this case is stable, but not asymptotically stable.

5.4. Sliding Mode Control

A variety of control laws can be designed for the linearized control subsystem given in Equation 5.18 to track the desired outputs $X_d$ and $Y_d$. However, due to the model imprecision, parameter uncertainty, and the disturbances inherited in the vehicle systems, a robust controller is preferred. In what follows the attractive features of variable structure control system are being used to achieve this purpose. This technique is shown to be successful in other applications (Richards and Ready, 1991; Ro and Kim, 1994; Yu and Moskwa, 1994). First, some preliminary concepts are explained. Later on, the boundary layer sliding mode technique is used to design a suitable controller.
5.4.1. Basic Concepts

The basic idea in the sliding mode technique is to first describe the desired closed loop behaviour of the system by a hyper surface, called the sliding surface. Then, to force the dynamic system to restrict its motion to this user defined surface. This is achieved by altering the control structure in such a way to direct the system trajectories toward the sliding surface. The main advantages of Sliding Mode (SM) control are its invariant properties and the ability to replace \( n \)th dimensional system by an equivalent lower order problem.

Sliding Surface

Define \( S_i \), the \( i \)th sliding surface element of the vector \( S \) as (Fernandez and Hedrick, 1987)

\[
S_i = \sum_{j=0}^{r_i-1} \lambda_{ij} \frac{d^j e_i}{dt^j} = \sum_{j=0}^{r_i-1} \lambda_{ij} e_i^{(j)}
\]

where \( r_i \) is the relative order of the \( i \)th output and

\[
e_i = y_i - y_{id}
\]

The \( m \) surfaces \( S_i \), of Equation 5.27 are defined such that they are decoupled from one another, that is \( S_i \) is only function of \( e_i \). Then, differentiating \( S_i \), will result in

\[
\dot{S}_i = \sum_{j=1}^{r_i} \lambda_{i(j-1)} e_i^{(j)}
\]

\[
= \lambda_{i(r_i-1)} e_i^{(r_i)} + \sum_{j=1}^{r_i-1} \lambda_{i(j-1)} e_i^{(j)}
\]

Without loss of generality \( \lambda_{i(r_i-1)} \) can be set to unity. Substituting 5.16 into Equation 5.28 gives

\[
\dot{S}_i = -\bar{Y}_i + L^T_j h_i + \sum_{k=1}^{m} (L_k L_j^T h_i) u_k
\]

where
\[
\bar{Y}_t^{(r)} = y_{id} - \sum_{j=1}^{r-1} \lambda_{(j-1)} e_i^{(j)}
\]

Equation 5.29 can be rewritten in vector form as follows
\[
\dot{S} = -\bar{Y} + b(\chi) + E(\chi)u \tag{5.30}
\]

where
\[
\bar{Y} = \begin{bmatrix}
\bar{Y}_1 & \bar{Y}_2 & \ldots & \bar{Y}_m
\end{bmatrix}^T
\]

**Sliding Condition**

To guarantee that all the surfaces \( S_i = 0 \), will be reached in a finite time, and that sliding will occur on all surfaces, the sliding condition is defined as
\[
\frac{1}{2} \frac{d}{dt} S_i^2 \leq -\eta_i |S_i| \tag{5.31}
\]

Where \( \eta_i \) is a strictly positive constant. Satisfying Equation 5.31 implies that the squared distance to the sliding surfaces, quantified by \( S_i^2 \), will decrease with movement along all system trajectories (Slotine and Li, 1991).

The attraction condition to the sliding surface as defined by Equation 5.31 is assured by a suitable choice of
\[
\dot{S}_i = F_i(S) \tag{5.32}
\]

One obvious choice for \( F_i(S) \) is
\[
F_i(S) = -\bar{K}_i \text{sgn}(S_i)
\]

Substituting Equation 5.32 into 5.30 yields the control input as
\[
u = E^{-1}(\chi) \left[ \bar{Y} + F(S) - b(\chi) \right] \tag{5.33}
\]

where
\[
F(S) = \begin{bmatrix}
F_1(S) & F_2(S) & \ldots & F_m(S)
\end{bmatrix}
\]
5.5. Controller Design

In this section a suitable controller for the linearized system with the type 1 outputs is designed. The decoupling matrix and internal dynamics for this case are given in Equations 5.19 and 5.25 respectively. Differentiating the outputs twice will result in

$$\dot{Y} = b(\chi) + \dot{E}(\chi)u$$  \hspace{1cm} (5.34)

where

$$Y = \begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \end{bmatrix}^T$$

$$u = \begin{bmatrix} V_t \\ V_s \end{bmatrix}^T$$

$$b(\chi) = \begin{bmatrix} \frac{D_1}{\cos \delta} (g_x F_x + g_0) + \frac{u}{L \cos^2 \delta} (b \delta k_2 \sin \theta - UD_2 \sin \delta) \\ \frac{D_2}{\cos \delta} (g_x F_x + g_0) - \frac{u}{L \cos^2 \delta} (b \delta k_2 \cos \theta - UD_1 \sin \delta) \end{bmatrix}$$

Identifying the following nonlinear feedback

$$u = E^{-1}(\chi)[u - b(\chi)]$$ \hspace{1cm} (5.35)

Substituting Equation 5.35 into Equation 5.34 yields the simple linear relation

$$\dot{Y} = u$$ \hspace{1cm} (5.36)

5.5.1. PD Controller

For the linear system thus obtained and described by Equation 5.36, $u$ can be designed in different ways to achieve a desirable performance for $y_i$ outputs. For instance, one can impose a new feedback control laws like

$$u_i = -\kappa_i e_i - \kappa_i \dot{e}_i - ... - \kappa_i(r_{i-1}) e_i^{(r_{i-1})} + y_{id}(r_i) \quad 1 \leq i \leq m$$ \hspace{1cm} (5.37)

Where

$$e_i = y_i - y_{id} \quad 1 \leq i \leq m$$

and
Chapter 5  AGV Control

\[ \kappa_i = [\kappa_{1i}, \kappa_{2i}, \ldots, \kappa_{(r-1)i}] \quad 1 \leq i \leq m \]

can be chosen in such a way to assign a specific set of eigenvalues, or to satisfy an optimality criterion. The schematic diagram of a PD controller for the linearized control subsystem is shown in Figure 5.3.

![PD controller schematic diagram]

**Figure 5.3**  PD controller for linearized control subsystem.

### 5.5.2. Sliding Mode Controller

For the case where \( X \) and \( Y \) are the desired outputs (i.e., type I outputs), using Equation 5.33, the controller outputs are given by

\[
\begin{pmatrix}
\dot{y}_d - \lambda_1 \dot{e}_1 - K_1 \text{sgn}(S_1) \\
\dot{y}_d - \lambda_2 \dot{e}_2 - K_2 \text{sgn}(S_2)
\end{pmatrix} = -E^{-1}(\chi)b(\chi) + E^{-1}
\]

\[ (5.38) \]

### 5.5.3. Boundary Layer Technique

The control laws that satisfy Equation 5.31 are discontinuous across \( S_i(t) \) and lead to "perfect" tracking. However, this is obtained at the cost of high control activity (i.e., control chattering). In general, chattering is undesirable and should be eliminated. As proposed by Slotine (1984), this can be achieved using a thin boundary layer \( \varphi \), and having the \( \text{sgn}(S_i) \) replaced by the function \( \text{sat}(S_i) \). Where

\[
\text{sat}(S_i) = \begin{cases} 
\text{sgn}(S_i), & |S_i| \geq \varphi_i \\
\frac{S_i}{\varphi_i}, & |S_i| < \varphi_i 
\end{cases} \quad 1 \leq i \leq m
\]

\[ (5.39) \]
modified sliding condition given by

\[
\frac{1}{2} \frac{d}{dt} S_i^2 \leq (\hat{\varphi}_i - \hat{\eta}_i) |S_i|
\]  
(5.40)

Inside the boundary layer the continuous control gives smooth \( S_i \) dynamics. This leads to tracking within a precision zone rather than perfect tracking. The controller output \( u \) for this case is given by

\[
\begin{bmatrix}
\dot{y}_{id} - \lambda_{10} \dot{e}_1 - K_1 \text{sat} \left( \frac{s_1}{\chi_1} \right) \\
\dot{y}_{2d} - \lambda_{20} \dot{e}_2 - K_2 \text{sat} \left( \frac{s_2}{\chi_2} \right)
\end{bmatrix} = -E^{-1}(\chi)b(\chi) + E^{-1}
\]  
(5.41)

### 5.5.4. Multiple Sliding Surfaces

Since the desirable properties of variable structure systems exist only in the sliding mode, it is required to converge to this mode as quickly as possible. If a high value of \( \lambda_{id} \) is used, the system may not be in sliding mode for a long portion of transient time. This makes the system susceptible to parameter variations and disturbance. Although, the robustness could be improved using high feedback gains, this could be impractical to implement (Richards and Reay, 1991; El-Sharkawi and Huang, 1989). Another alternative is to use non-stationary sliding surface, or Multiple Sliding Surfaces (MSS). The system initially slides on the surface with small value of \( \lambda_{id} \) then switches to the next surface which can be incremented as a function of time. The concept is useful where a system is subjected to large variations in parameters (Kaynak et al., 1982). Improved results are reported applying this strategy to a dc motor (Alasty and Naraghi, 1991; El-Sharkawi and Hung, 1989).

### 5.5.5. Variable Boundary Layer Thickness

Due to the \( \text{sat}(\dot{S}_i) \) function given in Equation 5.38, the system trajectory is directed toward the sliding surface for the condition of \( |S_i| > \varphi_i \), however, for \( |S_i| \leq \varphi_i \) the attraction to the sliding surface is not guaranteed by the proposed method. Thus, the introduction of a saturation function restricts the system trajectory to remain in a
neighbourhood of sliding surface. The larger the boundary layer thickness the less chattering is expected. However, if the boundary layer thickness is too large, besides large tracking error, the system's trajectory may never reach the equilibrium point (Yeung and Chen, 1988), and in some cases even non-smooth responses are expected. To benefit from the advantages of a boundary layer and the same time reduce the boundary layer thickness specially near the equilibrium point (i.e., is the origin of phase plane), a Variable Boundary Layer (VBL) strategy is adopted. The boundary layer is set to be proportional to the error but has a minimum desired thickness at the origin to avoid chattering at the equilibrium point, i.e.,

$$\varphi_t = \varphi_{t0} + \psi_t |e_t|$$

Where $\varphi_{t0}$ is the desired boundary layer thickness at origin, and $\psi_t$ is boundary layer proportional constant. A constant reduction of boundary layer thickness does not have this advantage, and will result in chattering.

5.5.6. Modified Boundary Layer Sliding Mode

In this case, the MSS and the VBL strategies are simultaneously implemented to improve the BLSM controller. In the proposed controller, robustness in the presence of parameter changes and modelling errors is further enhanced by applying the MSS. Whereas, using the VBL scheme restricts the system trajectories within the boundary layer to stay in the vicinity of sliding regime. The reduction of the boundary layer thickness in proportion to the tracking error results in a diminished tracking error, improved tracking responses, and reduced chattering. This new controller is referred to as Modified Boundary Layer Sliding Mode (MBLSM) for the remaining part of this thesis.
Chapter 6

Simulations and Results

6.1. Introduction

In this chapter the controllers designed in the preceding chapter will be evaluated through simulations. The 3-DOF dynamic model is used as a simulation testbed for this purpose. Since, it is important that the model accurately represents the vehicle behaviour for practical conditions. The first part of this chapter is devoted to the validation of the dynamic model. Then a thorough study of the AGV control schemes is presented. The numerical integration routine used to solve the equations of motion is based on the fourth order Runge-Kutta method. MATLAB has been used for the different simulation programs and the input data for the vehicle are presented in Appendix E.

6.2. Validation of the Dynamic Model

In order to emphasize the significance of dynamic modelling for the problem under study, the performance of the kinematic model of the vehicle is compared to the dynamic model through computer simulations. Also, to assess the level of sophistication of the required dynamic model, the 3-DOF model is compared with the 2-DOF dynamic model. For the sake of comparison with other studies, the electrical motors' equations are ignored only for the simulations in this section.
6.2.1. Analysis of the Kinematic Model

In an approach similar to Boyden (Boyden and Velinsky, 1994), the open loop model of the AGV is simulated for a 90-degree left turn. In the first set of simulations the kinematic model and dynamic model are compared, in order to demonstrate the limitations of the kinematic model. Forward speed is maintained constant. The steering angle is increased sinusoidally to 30 degrees, and maintained constant for a certain period, then returned to zero. This experiment is repeated for forward speeds of 0.3, 0.5, 1, 2 m/sec.

Results of this test are shown in Figure 6.1. With reference to this figure the kinematic model is evidently independent of speed and always predicts the perfect 90-degree left turn. However, the response of the dynamic model is highly speed dependent, and at 2 m/s the difference is significant. The difference increases considerably with speed. The deviation rate from the path due to speed increase is shown in Figure 6.2.

6.2.2. Analysis of the Dynamic Model

A similar strategy is used to compare the 2-DOF and 3-DOF dynamic models. The rear wheels’ tractive forces are controlled to obtain the desired constant forward speed. The steering angle is increased sinusoidally to 30 degrees, and maintained constant for a certain period, then returned to zero.

Simulation results for comparing the path trace of the 2-DOF and the 3-DOF dynamic models are presented in Figure 6.3. The difference in the path trace for a left turn manoeuvre is due to including the roll degree of freedom, and the lateral load transfer in the dynamic model.

The effect of weight on the path trace for the unloaded AGV (700 Kg), and when it is fully loaded (1700 Kg) is shown in Figure 6.4. The vehicle’s path traces differ considerably under unloaded and loaded conditions.

The pneumatic tires and their characteristics are often neglected in the reported dynamic modelling of AGVS and WMRs. Figure 6.5 shows the effect of tire cornering stiffness $C_\alpha$ on the AGV performance. As expected from vehicle dynamic theory, the
increase in cornering stiffness results in a more damped system. Referring to Figure 6.6, the deviation rate from the path due to the change in cornering stiffness is shown.

6.2.3. AGV Performance Using A Proportional Controller

The ability of the 2-DOF and 3-DOF models to accurately track a specific path is studied in this section. A proportional controller based on position and distance errors is used for this purpose. Although this control strategy is often used in practice, no systematic analysis exists on how to obtain the controller’s gains. This control strategy has also other limitations that will be elaborated on in Section 6.2.3.3.

6.2.3.1. Roll DOF and Load Transfer Effect

The tracking ability of the 3-DOF dynamic model comprising yaw, lateral and roll motions is compared in Figure 6.7 with the 2-DOF model using a proportional controller based on position and orientation errors. The test track consists of a “line-circle-line” path. The load transfer across the wheels, which was not considered in the AGV dynamic models reported in the literature, has also been included. The maximum tracking error for the two, and the three degrees of freedom model in Figure 6.7 are shown in Figure 6.8. The results show this maximum error to be 22 cm and 27 cm for the 2-DOF and 3-DOF models respectively.

The tire normal forces as the AGV navigates the line-circle-line path, are shown in Figure 6.9, for a front-right tire. The 2-DOF model has a constant normal force throughout the path, while the normal force increases as the 3-DOF model enters the curve and it is governed by the vehicle’s load transfer.

Based on the tire models, the cornering forces vary for the two and three degrees of freedom models. Figure 6.10 shows the increase in front-right tire cornering force for the 3-DOF model due to the load transfer. An increase in cornering force can initiate earlier saturation of the tire force in the presence of braking, acceleration, or slippery floor (a track with lower coefficient of friction).
6.2.3.2. Disturbance

The ability of a vehicle to converge to its path after being subjected to disturbances is generally referred to as a vehicle's directional stability. A directionally stable vehicle will return to a steady state condition upon removal of the disturbances. This is not the case for a directionally unstable vehicle (Wong, 1993). In practice, the disturbances to a control system are not known ahead of time but are random in nature. However, suitable test signals are frequently used to compare the performance of different control systems. This is of particular interest for analysing and designing a suitable controller. Many design criteria are based on performance of systems to these test inputs (Ogata, 1990).

The response of the AGV due to a step input in the steering angle is evaluated. To create such an input a 0.3 m shift in the path is considered. Performances of the 2-DOF and 3-DOF models travelling at a constant speed of 2 m/sec are illustrated in Figure 6.11 and Figure 6.12. The lateral deviation due to step input is higher for a 3-DOF model, indicating the effect of terms included in this model.

6.2.3.3. Limitations of the Proportional Controller

Trial and error methods are generally used to obtain suitable gains for proportional controllers. Hemami et al. (1992) proposed a systematic approach for gain adjustment of such controllers. However, the proposed approach is limited to a vehicle with linearized model travelling on a straight paths.

Although proportional controllers are simple to implement, they have numerous drawbacks. These controllers are highly speed, load, and path dependent, and suffer from steady state error in tracking circular paths. Referring to Figure 6.13, the performance of the 3-DOF AGV model using the proportional controller is shown for various load and speed conditions. The gains that are tuned for mid load and speed ranges are not suitable for other conditions and result in overdamped response for lower loads and speeds and under damped response for higher loads and speeds. Figure 6.14, shows the performance of the controller used in the case shown in Figure 6.13 when tracking a circular path. Using gains tuned for step response the distance error is notable (Figure 6.15).
Figure 6.1  Comparison of kinematic and dynamic models.

Figure 6.2  Deviation rate from the path versus speed for dynamic model.
Figure 6.3  Comparison of path trace for 2-DOF and 3-DOF AGV models.

Figure 6.4  The effect of load on the AGV path trace.
Figure 6.5  The effect of cornering stiffness on the AGV path trace.

Figure 6.6  The effect of cornering stiffness on deviation rate from the path.
Figure 6.7  Path tracking performance of 2-DOF and 3-DOF models.

Figure 6.8  Distance error for tracking a line-circle-line path.
Figure 6.9  The normal force of front-right tire for tracking a line-circle-line path.

Figure 6.10  Cornering force of the front-right tire for tracking a line-circle-line path.
Figure 6.11  The effect of step disturbance, Step = 0.3 m.

Figure 6.12  The effect of step disturbance, Step = -0.3 m.
Figure 6.13  Step response of 3-DOF model using proportional controller for various loading and speed conditions.

Figure 6.14  The 3-DOF AGV model tracking a circular path.
6.3. Performance Evaluation of the Control Schemes

In this section, the tracking capability of the AGV while utilizing the control schemes developed in Chapter 5 is studied. The vehicle parameters given in Appendix E will be used. The motors specifications are given in Appendix F. The emphasis here will be on controllers based on the Type I outputs. Some preliminary results of a controller design based on the Type VI outputs will also be presented.

6.3.1. PD Controller

When the PD controller developed in Section 5.5.1 is applied to a 2-DOF Side Slippage Free (SSF) model, it yields a critically damped response for the decoupled control position subsystem. This is accomplished by proper tuning of the controller gains in Equation 5.37. Figure 6.16 depicts the 2-DOF SSF model responses for speeds of 2 and 3 m/s. The vehicle is simulated when tracking a straight path with initial lateral offset of 0.5
m and zero initial heading angle \( \theta_i \). In general, the initial velocity and initial heading angle are the most important factors that affect the trajectory of the 2-DOF SSF model. For this particular heading angle, identical response is observed for all the speeds and almost all the loading conditions. However, for the more complete 3-DOF model of the AGV, as shown in Figure 6.17, the vehicle response is highly dependent on the particular speed and load conditions. This is due to the inclusion of the tire model as well as other terms (like load transfer, roll...etc) that have been ignored in the 2-DOF SSF model.

6.3.2. SM controller

Simulation results of comparing the vehicle tracking capability using the PD and the SM controllers are presented in Figures 6.16-6.17. When applied to the 2-DOF SSF model, the SM controller is tuned to obtain a performance similar to that of the PD controller. To attain this the MIMO sliding mode controller terms \( \lambda_{sp} \) and \( K_s \) are set to three and six, respectively. Referring to Figure 6.16, both controllers produce smooth exponential response for speeds of 2 and 3 m/s. However, for the 3-DOF model, considerably better tracking is realized when applying the SM controller, and the responses are almost identical with regard to variation in speed as shown in Figure 6.17. This is mainly due to the robust performance of the sliding mode controller which does not requires an accurate model of the system. For the remainder of this chapter, all the simulation will be based on the 3-DOF model.

The robustness in the presence of modelling imprecision and disturbance is obtained by applying discontinuous control law across the sliding surface. This results in chattering as the controller switches between the regimes across the sliding surface, and is shown in Figure 6.18 for output Y as an example. The resulting high frequency oscillations involve significant control activity and may excite unmodeled dynamic modes (Slotine and Li, 1991). Furthermore, for this study, chattering results in undesirable steering activity, as shown in Figure 6.19.
6.3.3. Boundary Layer Sliding Mode Controller

Introducing a thin boundary layer around the sliding surface as shown in Figure 6.20 results in reduced chattering with a considerably smoother steering, as shown in Figure 6.21. The chattering across the sliding surface is a function of the boundary layer thickness. As the thickness of the boundary layer decreases, the function \( \text{sat}(S) \) approaches the \( \text{sgn}(S) \). In the simulations the boundary layer thicknesses \( \varphi_1 \) and \( \varphi_2 \) are set to 0.1 m. The BLSM controller is studied in the remainder of this chapter, due to the chattering disadvantage of conventional SM controllers.

6.3.4. Comparison of PD and BLSM Controllers

The performance of PD and BLSM controllers are examined in this section under different operating conditions. The simulations presented here are carried out for an AGV having fixed parameters for the controllers. Since for all practical purposes the oscillatory response of the vehicle is not desirable, the PD controller gains of the linear decoupled control system are designed in such a way so as to obtain a non-oscillatory response even for the worst load-speed conditions. Consequently the response will be sluggish for other load-speed cases.

6.3.4.1. Effect of Speed

With Reference to Figure 6.22, the tracking performance of the PD controller is shown for the loaded vehicle at three different speeds of 1, 2, and 3 m/s. Under similar conditions results, for BLSM controller are shown in Figure 6.23. The Figures show that as the speed increases the effect of the terms not included in the design process of the controller becomes apparent.

6.3.4.2. Effect of Parameter Change and Model Uncertainty

Modelling uncertainties are the outcome of difficulty in measuring or estimating the kinematic or dynamic parameters, or in some cases lack of complete knowledge of the
system components. The modelling uncertainties in the payload and the tire cornering stiffness are simulated and the results are presented in this section. This is achieved by design of the controller for a given load and tire cornering stiffness, then carrying out simulations for different conditions.

**Effect of Payload**

One of the parameters that is being fed back to the plant in the feedback linearization scheme is the vehicle's mass $m$. In the following simulations this parameter is set to a fixed value of 1200 kg for the controller. The vehicle trajectories for PD and BLSM controllers when it is fully loaded ($m=1700$ kg) and unloaded ($m=700$ kg) are shown in Figure 6.24 and 6.25, respectively for a speed of 3 m/s. Both controllers show robust performances under given conditions. However, the vehicle response is faster and less sensitive to variation in payload when the BLSM controller is utilized.

**Effect of Cornering Stiffness**

The effect of tire characteristics on the vehicle's tracking performance is shown in Figure 6.26 and 6.27 for the PD and the BLSM controllers, respectively. The simulations for the present tire cornering stiffness $C_a$ is compared with the cases of $2C_a$ and $.6C_a$. The vehicle speed is maintained constant at 2 m/s. The response of PD controller tends to oscillate as the tire cornering stiffness decreases, while a robust performance is observed for the BLSM controller. Also, the SM controller reacts more quickly to converge the vehicle to the desired path.

**6.3.4.3. Effect of Disturbance**

The disturbance is introduced here in the form of a sudden displacement from the intended path. Figure 6.28 shows the effect of such a step disturbance of 0.3 m when the vehicle is moving along a straight track. For AGV speeds of 2 and 3 m/s the transient response of BLSM controller is fast, accurate, and insensitive to step disturbance, while the PD controller performance is sluggish, speed dependent, and oscillatory at higher speeds.
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The settling time for the SM controller is 4.2 seconds whereas, the PD controller does not settle during the first 7 seconds of simulation.

6.3.4.4. Effect of Path Geometry

In practice, the paths for routing automated guided vehicles within their workspace are generally composed of line and circular arc segments (Nelson, 1989). Also, it has been shown that for any given initial and final posture of the vehicle, a family of paths comprising only lines and circles segments can be found between the source and destination (Sarkar et al., 1994). Figure 6.29 depicts the tracking performance for the second basic path segment, i.e. circular arc segment. Both controllers are examined at speeds of 1.5 m/s and a fully loaded vehicle. The normal distance error is shown in Figure 6.30. The BLSM controller has a faster response with considerably smaller error when compared to the PD controller. Furthermore, the application of the PD controller results in a significant steady state error.

6.3.4.5. Effect of Initial Conditions

Among the factors which have a major effect on the convergence of the AGV to the required path of the vehicle is its initial heading. For tracking a straight path, the performance of the loaded vehicle for different initial headings is shown in Figure 6.31. Both controllers are tested for a speed of 2 m/s. Similar results for tracking a circular arc segment are shown in Figures 6.32-6.34. The vehicle speed is set to 1.5 m/s for circular path simulations. Examination of Figures 6.31 and 6.32 show that the BLSM controller reacts more quickly to bring the vehicle to the desired path. The steady state errors in Figure 6.33 and 6.34 shows the prominent results. The PD controller can not compensate for the steady state error because it does not have any integral compensator, where as the BLSM controller can compensate for the steady state error without any integral term.
6.3.5. Performance of Type VI Outputs

In this section the vehicle tracking capability with a PD controller implementation based on type VI outputs, PD-VI (i.e. vehicle speed and normal distance to the path) is compared with that based on type I outputs, PD-I. Figure 6.35, and 6.36 show the results for both controllers. The circular arc segment is used to simulate the performance of a loaded vehicle with a speed of 1.5 m/s. In both cases the trajectories converge toward the desired path. The PD-VI controller merges gradually with the intended path, while PD-I reacts more quickly and it has an overshoot. For most practical purposes a smoother response similar to that of the PD-VI is more desirable.

6.3.6. Modified BLSM Controller

Despite all the merits of the BLSM controller discussed so far, during the simulations this type of controller resulted in certain undesirable vehicle responses for particular cases. These cases are studied in detail in this section.

a) The robust performance of the BLSM controller was ensured through the high gain setting to the attractive surface. This however, causes the cornering force threshold to be reached for initial headings beyond π/3 rad, and would result in tire slippage. The resulting undesirable vehicle performance when tracking a circular path with initial heading of 1 rad at a speed of 1.5 m/s is shown as BLSM curve in Figures 6.37-6.38. The saturation of the cornering force was eliminated by reducing the gains $K_c$, this is shown as BLSM-1 curve in the figures. However, varying the gains $K_v$ results in a non-smooth response for other critical operating conditions and contradicts the idea of unique adjustment of gains for the whole operating range of the AGV. This is observed when gain value of $K_v = 3$ and speed of 3 m/s for a loaded vehicle moving on a straight path as shown in Figure 6.39.

b) Referring to Figure 6.23, some distortion is observed in the BLSM controller response for the speed of 3 m/s and gains of $K_v = 5$. The distortion occurs after the first second of motion and is more pronounced for the case shown in Figure 6.39, when the gains are reduced to $K_v = 3$. To examine the BLSM controller behaviour further, the lateral deviation phase diagram for both gain settings is plotted in Figure 6.40. For both cases, the
system trajectory hits the sliding surface and proceeds to pass the boundary layer. Consequently the system is not in the sliding regime for a long period of time. Also, due to boundary layer thickness the system trajectory is not forced to sliding surface, and is oscillatory in spite of being within the boundary layer. This is magnified as the gains $K_r$ decrease.

Two schemes may be used to deal with these shortcomings: 1) Increase the controller gains, $K_1$ and $K_2$ so as to strengthen the sliding surface attractiveness property. This approach, however, aggravates the problem described in part (a) above. 2) Reduce the boundary layer thickness. The comparison of vehicle response using the BLSM controllers with $\varphi_i = .1$ (BLSM curve) and $\varphi_i = 0.03$ (BLSM-1 curve) is shown in Figure 6.41. The corresponding system trajectories are also shown in Figures 6.40 and 6.42. Although this approach is helpful, it initiates the chattering as shown in Figure 6.42 (BLSM-2 curve).

In present work, the MBLSM controller developed in Chapter 5 is implemented to further enhance robustness and reduce the boundary layer thickness without elevating chattering. Application of VBL technique to reduce the chattering is shown in Figure 6.42 as VBL curve. Result of simultaneous application of MSS and VBL strategies to the vehicle are presented in Figure 6.43 and the corresponding phase diagram is shown in Figure 6.44. The vehicle speed is 3 m/s, the sliding surface slope varies from 1 to 3, the controller gains are reduced to $K_r = 3$, and boundary layer is defined as

$$\varphi_i = .03 + .1|e_i|$$

Noticeable improvements are observed. Compared to the BLSM, the MBLSM controller resulted in a smooth response with the same settling time as the BLSM controller. Results for tracking a circular path are also shown in Figures 6.45 and 6.46. The vehicle speed is 1.5 m/s. When compared to the PD and the BLSM controllers the MBLSM has a fast, and accurate response with no overshoot.

Although the MBLSM controller is designed for an AGV under the specified working condition given in Chapter 1, the controller’s performance is robust when the speed exceeds beyond the design value of 3 m/s. Figures 6.47 shows the vehicle responses when the speed is increased up to 40 percent of the design value. Using the same controller gains as before, the vehicle response tends to oscillate as the speed is increased beyond 40 percent.
Robustness to overload has not been evaluated since an AGV overload would have other adverse effects and should be avoided.

6.4. Discussion of Results

The AGV's kinematic model that is most often used for simulation studies is compared to the dynamic model. It is shown that for high load and speed conditions the kinematic model is entirely incapable of representing the real vehicle behaviour. The characteristics of the pneumatic tires are modelled using a simple tire model. Significant differences in the vehicle response, due to tire parameters indicate the necessity for proper tire modelling. The performance of the 2-DOF and 3-DOF dynamic models are compared to investigate the effect of the roll degree of freedom and the load transfer. Notable differences are observed for the vehicle under study. Limitations in the proportional controller based on position and orientation errors show that a more sophisticated controller is required.

The performance of the proposed PD, and SM controllers are compared for different operating conditions. Due to presence of chattering in the steering angle the SM controller was not a suitable choice, therefore the boundary layer sliding mode controller was studied instead. Almost for all the cases the BLSM controller showed a superior robustness and faster response compared to the PD controller. The dominant factors affecting the AGV's tracking capabilities are identified as speed, load and cornering stiffness of tires. Some shortcomings were observed in the BLSM. These were compensated by implementing the MBLSM controller with simultaneous application of VBL and MSS strategies. The results for Type VI outputs were promising when compared to Type I outputs. The test results show that using velocity and normal distance error as the outputs to develop a decoupled control system results in a smoother tracking of circular path.
Figure 6.16 The performance of the PD and the SM controllers for tracking a straight path, utilizing the 2-DOF SSF vehicle model.

Figure 6.17 The performance of the PD and the SM controllers for tracking a straight path, utilizing the 3-DOF vehicle model.
Figure 6.18  The phase portrait of vehicle's lateral position, utilizing the SM controller.

Figure 6.19  Chattering in the steering angle as a result of employing the SM controller.
Chapter 6  Simulations and Results

Figure 6.20  The phase portrait of vehicle's lateral position, utilizing the BLSM controller.

Figure 6.21  Improved steering activity due to introducing a thin boundary layer around the sliding surface.
Figure 6.22  The effect of speed on the tracking performance of the PD controller.

Figure 6.23  The effect of speed on the tracking performance of the BLSM controller.
Figure 6.24  The effect of change in the payload on the PD controller performance.

Figure 6.25  The effect of change in the payload on the BLSM controller performance.
Figure 6.26  The effect of change in the tire cornering stiffness on the PD controller performance.

Figure 6.27  The effect of change in the tire cornering stiffness on the BLSM controller performance.
Figure 6.28  Performance in the presence of step disturbance in the path.

Figure 6.29  The performance of the PD and the BLSM controllers in tracking a circular path.
Figure 6.30  Comparison of distance error of the PD and the BLSM controllers in tracking a circular path.

Figure 6.31  The effect of initial heading on the vehicle response utilizing the PD and the BLSM controllers.
Figure 6.32  The effect of initial heading on the vehicle response for the PD and the BLSM controllers.

Figure 6.33  Comparison of distance error due to different initial headings for the PD controller.
Figure 6.34  Comparison of distance error for the BLSM controller due to different initial conditions.

Figure 6.35  Comparison of vehicle's tracking capability implementing PD controllers based on type I and type VI outputs.
Figure 6.36  Distance error for PD controllers based on type I and type VI outputs.

Figure 6.37  The effect of reducing $K_1$ and $K_2$ on the performance of the BLSM controller.
Figure 6.38  The effect of reducing $K_1$ and $K_2$ on the distance error for the BLSM controller.

Figure 6.39  Non-smooth response of the BLSM controller for tracking a straight path at a speed of 3 m/s, due to reducing $K_1$ and $K_2$. 
Figure 6.40  The effect of reducing $K_1$ and $K_2$ on the system's trajectory (output $Y$) for the BLSM controller.

Figure 6.41  Performance of the BLSM controller using the VBL strategy.
Figure 6.42  Phase portrait of the VBL controller and the BLSM controller with reduced boundary layer thickness.

Figure 6.43  Performance of the BLSM and the MBLSM controllers for tracking a straight path.
Figure 6.44 The phase portrait of output Y for the BLSM and the MBLSM controllers.

Figure 6.45 The performance of the PD, the BLSM and the MBLSM controllers for tracking a circular path.
Figure 6.46 Comparison of distance error for the PD, the BLSM, and the MBLSM controllers.

Figure 6.47 Response of the MBLSM controller for speed increase of up to 40 percent of the design value.
Chapter 7

Conclusion and Future Work

7.1. Summary

The ultimate goal of this research is to improve the path tracking and steering performance of a high load transport interfactory AGV with maximum travelling speed of 3 m/sec. Steering control of AGVs has been the subject of substantial research in the past. However, the focus has been to develop control strategies based on kinematic models, mainly due to the low speed and light weight of the vehicles analysed. These operating conditions do not resemble the actual requirements of automated transport systems in industry, where fast transport of heavy materials are essential to increase the overall efficiency. Some recent works exist on dynamic-based trajectory control of tricycle mobile robots applying Lagrangian formulation and input-output feedback linearization. Nevertheless, almost no reported work exists on dynamic-based control of AGVs taking into consideration the realistic loads, speeds, and the pneumatic tires used in practice. Other works that are available in the field of vehicle dynamics usually utilize linearized dynamic models for control purposes. This approach is generally valid for ATVs due to the small steering angles necessary for lane changes and curve negotiations of these vehicles. However, for the AGVs working in a flexible manufacturing plants, where curved routes of a few metres exist, this is not a satisfactory solution. As the AGV’s load capacity and
Chapter 7  Conclusion and future work

travelling speed increases, the ensuing increase in lateral acceleration requires thorough
dynamic modelling and more sophisticated controller design.

7.1.1. Dynamic Modelling

In this study, a nonlinear 3-DOF dynamic model, comprising yaw, lateral, and roll
motions is developed. The suspension, lateral and longitudinal load transfer, nonlinear
behaviour of tires, and steering dynamics are included in this model. The model also
comprises the effect of actuators, differential gear box, steering and tractive gear boxes. To
establish the sufficient level of model complexity necessary for this work, the 3-DOF model
is compared with the commonly used kinematic model and a 2-DOF dynamic model. Also,
the responses to step disturbance and various speeds and loading conditions are presented.
Tracking performance of the 2-DOF and the 3-DOF models controlled by a conventional
proportional controller are demonstrated as well. Based on simulation results for the
particular AGV under study, the kinematic model is entirely incapable of representing the
real vehicle. A proper tire modelling is required as load and speed increase. The 3-DOF
model is adequate as a simulator for the design of the control system, however, a more
sophisticated controller is required.

7.1.2. Control Design

The 2-DOF bicycle model was exploited to develop a dynamic based controller for
the AGV. Assumption of a motion free of side slippage greatly simplified the kinematics and
dynamics of the AGV model and led to a concise dynamic model. MIMO feedback
linearization technique was employed to transform the given nonlinear system into two
subsystems. A linear controllable subsystem of dimension $r_t$, which is the only one
responsible for input-output behaviour, and a nonlinear subsystem of dimension $n-r_t$, whose
stability should be inspected\(^1\). Among the six different output functions that were studied two
were given more emphasise. These are designated as: a) type I outputs: based on vehicle

\(^1\)Recalling that $r_t$, and $n$ are total relative degree and order of system respectively.
position variables \( X \) and \( Y \), with the linearized decoupled subsystem of order four \( (r_1=4) \) and internal dynamics of order one \( (n-r_1=1) \) derived in the Cartesian coordinates. b) type VI outputs: based on the wheel angular speed \( \omega_w \) (or vehicle forward speed \( U \)) and normal distance \( n \). The order of the linearized decoupled subsystem in this case is three \( (r_1=3) \), and the internal dynamics is of order two \( (n-r_1=2) \) derived in path dependent coordinates. To analyse the internal dynamics and zero dynamics, both systems were transformed into normal form coordinates.

A number of controllers are designed for the decoupled linear subsystem and applied to the full 3-DOF system. Positive characteristics of variable structure system control were exploited to enhance robustness in the presence of disturbance, unmodeled dynamics and change in parameters. To achieve this a sliding mode controller was designed. Due to chattering the sliding mode controller was modified by introducing a thin boundary layer to the sliding surface. The boundary layer sliding mode controller was later modified for a better performance and enhance robustness using simultaneous variable boundary layer and multiple sliding surfaces strategies.

The proposed BLSM controller effectively suppressed disturbances and the control system is shown to be insensitive to the physical parameter variation such as payload and tire characteristics. The simulation results show good and robust performance against speed, path geometry and initial heading variations. The modified boundary layer sliding mode controller enhanced the robustness of the BLSM controller and improved its performance. The proposed control law is not complicated and is suitable for real time computer control.

7.2. Contributions

The major contributions of this research can be summarized as follows:

A complete dynamic model for a 4-wheel automated guided vehicle is developed. The 3-DOF nonlinear model comprises yaw, lateral, and roll motions. The suspension, lateral and longitudinal load transfer, nonlinear behaviour of tires, and steering dynamics are included in this model. The model also incorporates the effect of actuators, differential gear box, steering and tractive gear boxes.
Chapter 7  Conclusion and future work

A dynamic based approach to the control of AGVs for high load applications was introduced. Robustness of the controller was guaranteed by applying the sliding mode technique to the linearized model. Boundary Layers were introduced to reduce chattering. The BLSM controller was modified for enhanced robustness and better tracking performance. The formulation and implementation of a controller in this form for an AGV with complete 3-DOF dynamic model has not been addressed in the past.

Although input-output feedback linearization has been applied to mobile robots, and sliding mode technique is frequently used for robot manipulators control, the study of their simultaneous use for nonholonomic systems is a new subject.

Under side slippage free assumption a general approach to input output linearization of automated guided vehicle dynamics was studied. The choice of vehicle speed and normal distance as the system outputs, in the path dependent coordinate, resulted in a new controller for AGVs. Complete development of the theory and preliminary simulation results of vehicle control using this class of controllers are presented.

The boundary layer sliding mode controller was modified to enhance robustness in the presence of parameter change, modelling errors, and disturbance. In this regard both variable boundary layer and multiple sliding surfaces schemes were simultaneously implemented.

7.3. Future Work

The present work can serve as a basis for different areas of research in WMR's, AGV's and ATV's. This work can be extended in several directions. A few important directions of future research are:

1) Selection of type VI outputs based on vehicle speed and normal distance to the path, to develop a controller for the AGV in the path dependent coordinate system led to feasible results. Also, due to natural interpretation of outputs in task space this topic can be extrapolated for future work.

2) The sliding mode controller can be further enhanced using a methodology similar to the one proposed by Slotine(1984). In this regard, modelling uncertainties can be included
in the design of the sliding controller to obtain a time varying boundary layer for optimal tracking performance. However, this will be achieved at the cost of additional complexity of the controller.

3) The simulation results suggest that the performance of the proposed MBLSM controller is reasonably robust not only with respect to variation in vehicle parameters, but also with respect to assumption of side slippage free motion. This hints that this class of controllers can be recommended for control of AGV’s at higher speeds or ATV’s for low speed applications. A more detailed study of this subject is needed.

4) An experimental study of the proposed control system would be revealing and beneficial. This has to be done before the practical potentials of the proposed controller can be accurately assessed. Prospects for practical implementation are nevertheless promising.
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Appendix A1

Tractive Motor Dynamics

The tractive force is the force, which the motor can provide to the vehicle and depends on the motor characteristics only. It is assumed that the whole effort of the motion is provided by the forces between driving wheels and working surface.

\[
\tau_i = n_i \left[ \frac{K_t V_i}{R_{at}} - n_i J_i \frac{d^2 \theta_i}{dt^2} - \left( \frac{K_t^2}{R_{at}} + n_i C_i \right) \frac{d\theta_i}{dt} \right]
\]  

(A1.1)

Where

\( K_t \) is the motor torque constant

Figure A1.1 Model of motor and load system

Referring to Figure A1.1, the basic torque-voltage equation for the tractive dc motor is presented as (Ogata, 1990),
Appendix A1  Tractive Motor Dynamics

\[ \tau_t \] is the torque referred to output shaft
\[ V_t \] is the same as \( V_m \) and denotes terminal voltage
\[ R_m \] is armature resistance
\[ \dot{\theta}_m \] is the speed referred to motor shaft
\[ \dot{\theta}_t \] is the speed referred to output shaft (input to differential gear)
\[ J_t \] is the motor moment of inertia
\[ C_t \] is the motor viscous damping coefficient
\[ n_t \] is the tractive motor gear ratio

In the Equation A1.1, subscript \( t \) denotes tractive motor specifications, and armature inductance is assumed to be small and negligible.

Differential Gear

A differential gear is being used to divide the input torque from the gearbox equally between the two rear wheels, despite their relative speeds of rotation, therefore the following holds

\[ \dot{\theta}_{rl} + \dot{\theta}_{rr} = 2\dot{\theta}_t \]
\[ \tau_{rl} = \tau_{rr} = \frac{\tau_t}{2} \]  (A1.2)

where
\[ \dot{\theta}_{rl} \] is the angular velocity of the left wheel
\[ \dot{\theta}_{rr} \] is the angular velocity of the right wheel
\[ \tau_{rl} \] is the torque on the rear-left wheel
\[ \tau_{rr} \] is the torque on the rear-right wheel

Differentiating first equation of the A1.2 results in

\[ \ddot{\theta}_{rl} + \ddot{\theta}_{rr} = 2\ddot{\theta}_t \]  (A1.3)
Let \( U_{rl} \), \( U_{rr} \) be the linear velocity of left and right wheels respectively, then angular velocities are

\[
\dot{\theta}_{rl} = \frac{U_{rl}}{R_w} = \frac{1}{R_w} (U + tr)
\]

\[
\dot{\theta}_{rr} = \frac{U_{rr}}{R_w} = \frac{1}{R_w} (U - tr)
\]

\[
\dot{\theta}_t = \frac{U}{R_w}
\]

and the accelerations are

\[
\ddot{\theta}_{rl} = \frac{1}{R_w} \frac{dU_{rl}}{dr} = \frac{1}{R_w} (\dot{U} + tr)
\]

\[
\ddot{\theta}_{rr} = \frac{1}{R_w} \frac{dU_{rr}}{dt} = \frac{1}{R_w} (\ddot{U} - tr)
\]

\[
\ddot{\theta}_t = \frac{\ddot{U}}{R_w}
\]

Substituting A1.4, and A1.5 into A1.1 and considering A1.2 the tractive forces for rear wheels are given by

\[
(F_t)_{xrl} = (F_t)_{xrr} = \frac{n_t}{2R_w} \left[ \frac{\bar{K}_t V_t}{R_\text{at}} - \frac{n_r \bar{J}_t}{R_w} \dot{U} - \frac{1}{R_w} \left( \frac{\bar{K}_t^2}{R_\text{at}} + n_t C_t \right) U \right]
\]

\[
\text{(A1.6)}
\]

The total tractive force of rear wheels is given by

\[
F_t = 2(F_t)_{xrl} = 2(F_t)_{xrr}
\]

\[
\text{(A1.7)}
\]
Appendix A2

Steering Motor Dynamics

Referring to Appendix A1, the basic torque-voltage equation for the steering dc motor is given by

\[ \tau_s = n_s \left[ \frac{K_s V_s}{R_{ar}} - n_s J_s \frac{d^2 \theta_s}{dt^2} - \left( \frac{K_s^2}{R_{ar}^2} + n_s C_s \right) \frac{d \theta_s}{dt} \right] \]  

(B2.1)

Where
- \( K_s \) is the motor torque constant
- \( \tau_s \) is the torque referred to output shaft
- \( V_s \) is the same as \( V_{ar} \) and denotes terminal voltage
- \( R_{ar} \) is armature resistance
- \( \dot{\theta}_{ma} \) is the speed referred to motor shaft
- \( \dot{\theta}_s \) is the speed referred to output shaft
- \( J_s \) is the motor moment of inertia
- \( C_s \) is the motor viscous damping coefficient
- \( n_s \) is the steering gear ratio

In the Equation A2.1, subscript \( s \) denotes steering motor specifications, and armature inductance is assumed to be small and negligible. Also, the contribution of steering motor inertia to total steering torque is negligible, and can be ignored. The simplified equation is given by
\[ \tau_s = n_s \left[ \frac{K_s V_s}{R_{at}} - \left( \frac{K_s^2}{R_{at}} + n_s C_s \right) \frac{d\theta_s}{dt} \right] \] (B2.2)

Alternatively the steering angle \( \delta \) can be substituted for \( \theta_s \).
Appendix B1

Input-Output Linearization: Outputs X, Y

For this case the outputs are defined as

\[ y_1 = h_1(x) = X \]
\[ y_2 = h_2(x) = Y \]  \hspace{1cm} (B1.1)

and the desired decoupling matrix determinant is given by

\[ \det E(x) = \begin{vmatrix} L_{x_1} L_f h_1(x) & L_{x_2} L_f h_1(x) \\ L_{x_1} L_f h_2(x) & L_{x_2} L_f h_2(x) \end{vmatrix} \]  \hspace{1cm} (B1.2)

The elements of B1.2 are obtained based on the following procedure

\[ L_{x_1} h_1(x) = \nabla h_1 g_1(x) \]
\[ = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} g_1 & 0 & 0 & 0 \end{bmatrix}^T \]
\[ = 0 \]

\[ L_{x_2} h_1(x) = \nabla h_1 g_2(x) \]
\[ = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} g_2 & \frac{1}{k_s} & 0 & 0 & 0 \end{bmatrix}^T \]
\[ = 0 \]

and similarly

\[ L_{x_1} h_2(x) = \nabla h_2 g_1(x) = 0 \]
\[ L_{x_2} h_2(x) = \nabla h_2 g_2(x) = 0 \]

Thus
Appendix B1  Input-Output Linearization: Outputs X, Y

\[ L_f h_1(x) = \nabla h_1 f(x) = \frac{UD_1}{\cos \delta} \]

\[ L_f h_2(x) = \nabla h_2 f(x) = \frac{UD_2}{\cos \delta} \]

Moreover

\[ L_{g_{11}} L_f h_1(x) = \nabla(L_f h_1) g_1 \]

\[ = \left[ \frac{D_1}{\cos \delta} - \frac{Ub \sin \theta}{\cos^2 \delta} \right. \left. - \frac{UD_2}{\cos \delta} 0 0 \right] \begin{bmatrix} g_1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \]

\[ = \frac{g_1 D_1}{\cos \delta} \]

\[ L_{g_{12}} L_f h_1(x) = \nabla(L_f h_1) g_2 \]

\[ = \frac{g_2 D_1}{\cos \delta} \frac{Ub \sin \theta}{L k_{1s} \cos^2 \delta} \]

Similarly, the second row elements are obtained

\[ L_{g_{21}} L_f h_1(x) = \nabla(L_f h_2) g_1 \]

\[ = \left[ \frac{D_2}{\cos \delta} \frac{Ub \cos \theta}{\cos^2 \delta} \left. \frac{UD_1}{\cos \delta} 0 0 \right] \begin{bmatrix} g_1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \]

\[ = \frac{g_1 D_2}{\cos \delta} \]

\[ L_{g_{22}} L_f h_2(x) = \nabla(L_f h_2) g_2 \]

\[ = \frac{g_2 D_2}{\cos \delta} \frac{Ub \cos \theta}{L k_{1s} \cos^2 \delta} \]

Thus the decoupling matrix for this case is given by
Appendix B1  Input-Output Linearization: Outputs $X, Y$

\[
E(x) = \begin{bmatrix}
g_1 \frac{D_1}{\cos \delta} & g_2 \frac{P_1}{\cos \delta} & \frac{Ub \sin \theta}{Lk_1 \cos^2 \delta} \\
g_1 \frac{D_2}{\cos \delta} & g_2 \frac{P_2}{\cos \delta} & \frac{Ub \cos \theta}{Lk_1 \cos^2 \delta} 
\end{bmatrix}
\]  

(B1.3)

and the determinant of $E$ is obtained as

\[
\det E = \frac{g_1 Ub}{k_1 L \cos^2 \delta}
\]

(B1.4)
Appendix B2

Input-Output Linearization: Outputs $Y, \theta$

For this case the outputs are defined as

\[
\begin{align*}
  y_1 &= h_1(\chi) = Y \\
  y_2 &= h_2(\chi) = \theta 
\end{align*}
\]  \hspace{1cm} (B2.1)

and the desired decoupling matrix determinant is given by

\[
\det E(\chi) = \begin{vmatrix}
  L_{y_1} L_f h_1(\chi) & L_{y_1} L_f h_1(\chi) \\
  L_{y_2} L_f h_2(\chi) & L_{y_2} L_f h_2(\chi) 
\end{vmatrix}
\]  \hspace{1cm} (B2.2)

The elements of B2.2 are obtained based on the following procedure

\[
\begin{align*}
  L_{y_1} h_1(\chi) &= \nabla h_1 \beta_1(\chi) = 0 \\
  L_{y_2} h_1(\chi) &= \nabla h_1 \beta_2(\chi) = 0 \\
  L_{y_1} h_2(\chi) &= \nabla h_2 \beta_1(\chi) = 0 \\
  L_{y_2} h_2(\chi) &= \nabla h_2 \beta_2(\chi) = 0
\end{align*}
\]

and

\[
\begin{align*}
  L_{y_1} h_2(\chi) &= \nabla h_2 \beta_1(\chi) = 0 \\
  L_{y_2} h_2(\chi) &= \nabla h_2 \beta_2(\chi) = 0
\end{align*}
\]

thus

\[
\begin{align*}
  L_f h_1(\chi) &= \nabla h_1 \beta(\chi) = \frac{UD_2}{\cos \delta} \\
  L_f h_2(\chi) &= \nabla h_2 \beta(\chi) = \frac{Ut \tan \delta}{L}
\end{align*}
\]
moreover

\[ L_{g_1} L_f h_1(\chi) = \nabla(L_f h_1) \quad g_1 = \frac{g_1 D_2}{\cos \delta} \]

\[ L_{g_2} L_f h_1(\chi) = \nabla(L_f h_1) \quad g_2 = \frac{g_2 D_2}{\cos \delta} + \frac{U b \cos \theta}{L k_{1c} \cos^2 \delta} \]

Similarly the second row elements are obtained

\[ L_{g_1} L_f h_2(\chi) = \nabla(L_f h_2) \quad g_1 = \frac{g_1 \tan \delta}{L} \]

\[ L_{g_2} L_f h_2(\chi) = \nabla(L_f h_2) \quad g_2 = \frac{g_2 \tan \delta}{L} + \frac{U}{L k_{1c} \cos^2 \delta} \]

Thus the decoupling matrix for this case is given by

\[
E(\chi) = \begin{bmatrix}
\frac{g_1 D_2}{\cos \delta} & \frac{g_2 D_2}{\cos \delta} + \frac{U b \cos \theta}{L k_{1c} \cos^2 \delta} \\
\frac{g_1 \tan \delta}{L} & \frac{g_2 \tan \delta}{L} + \frac{U}{L k_{1c} \cos^2 \delta}
\end{bmatrix}
\]

(B2.3)

and the determinant of \( E \) is obtained as

\[
\det E = \frac{g_1 U \sin \theta}{k_{1c} L \cos^2 \delta}
\]

(B2.4)
Appendix B3

Input-Output Linearization: Outputs $U, \delta$

For this case the outputs are defined as

$$y_1 = h_1(\chi) = U$$
$$y_2 = h_2(\chi) = \delta$$  \hspace{1cm} (B3.1)

and the desired decoupling matrix determinant is given by

$$\det E(\chi) = \begin{vmatrix}
L_{g_1} h_1(\chi) & L_{g_1} h_1(\chi) \\
L_{g_2} h_2(\chi) & L_{g_2} h_2(\chi)
\end{vmatrix}$$  \hspace{1cm} (B3.2)

The elements of B3.2 are obtained based on the following procedure

$$L_{g_1} h_1(\chi) = \nabla h_1 g_1(\chi) = g_i$$
$$L_{g_1} h_1(\chi) = \nabla h_1 g_2(\chi) = g_s$$

similarly the second row elements are obtained

$$L_{g_2} h_2(\chi) = \nabla h_2 g_1(\chi) = 0$$
$$L_{g_2} h_2(\chi) = \nabla h_2 g_2(\chi) = \frac{1}{k_\mu}$$

and thus the decoupling matrix for this case is given by

$$E(\chi) = \begin{bmatrix}
g_i & g_s \\
0 & \frac{1}{k_\mu}
\end{bmatrix}$$  \hspace{1cm} (B3.3)
Appendix B3  Input-Output Linearization: Outputs \( U, \delta \)

and the determinant of \( E \) is obtained as

\[
\det E = \frac{g_t}{k_{1z}} \tag{B3.4}
\]
Appendix B4

**Input-Output Linearization: Outputs U, Y**

For this case the outputs are defined as

\[
\begin{align*}
    y_1 &= h_1(\chi) = U \\
    y_2 &= h_2(\chi) = Y
\end{align*}
\]  
(B4.1)

and the desired decoupling matrix determinant is given by

\[
\det E(\chi) = \begin{vmatrix}
    L_{g_1} h_1(\chi) & L_{g_2} h_1(\chi) \\
    L_{g_1} L_f h_2(\chi) & L_{g_2} L_f h_2(\chi)
\end{vmatrix}
\]  
(B4.2)

The elements of B4.2 are obtained based on the following procedure

\[
\begin{align*}
    L_{g_1} h_1(\chi) &= \nabla h_1 g_1(\chi) = g_t \\
    L_{g_2} h_1(\chi) &= \nabla h_1 g_2(\chi) = g_s
\end{align*}
\]

and

\[
\begin{align*}
    L_{g_1} h_2(\chi) &= \nabla h_2 g_1(\chi) = 0 \\
    L_{g_2} h_2(\chi) &= \nabla h_2 g_2(\chi) = 0
\end{align*}
\]

The second row elements are obtained as

\[
L_f h_2(\chi) = \nabla h_2 f(\chi) = \frac{UD_2}{\cos \delta}
\]

moreover
\[ L_2 \nabla (L_2 h_1(\chi)) = \nabla(L_2 h_2) \quad g_2 = \frac{g_2 D_2}{\cos \delta} + \frac{Ub \cos \theta}{Lk_{12} \cos^2 \delta} \]

Thus the decoupling matrix for this case is given by

\[
E(\chi) = \begin{bmatrix}
g_1 & g_2 \\
g_1 D_2 \cos \delta & g_2 D_2 \cos \delta + \frac{Ub \cos \theta}{Lk_{12} \cos^2 \delta}
\end{bmatrix}
\]  \hspace{1cm} (B4.3)

and the determinant of \( E \) is obtained as

\[
\text{det } E = \frac{g_1 Ub \cos \theta}{k_{12} L \cos^2 \delta}
\]  \hspace{1cm} (B4.4)
Appendix C

Path Dependent I/O Linearization

Type I Outputs

For this case the outputs are defined as

\[
\begin{align*}
  y_1 &= h_1(\zeta) = s \\
  y_2 &= h_2(\zeta) = n
\end{align*}
\]

(C.1)

and the desired decoupling matrix determinant is given by

\[
\det E(\zeta) = \begin{vmatrix}
  L_{s_1} L_f h_1(\zeta) & L_{s_2} L_f h_1(\zeta) \\
  L_{s_1} L_f h_2(\zeta) & L_{s_2} L_f h_2(\zeta)
\end{vmatrix}
\]

(C.2)

The elements of C.2 are obtained based on the following procedure

\[
L_{s_1} h_1(\zeta) = \nabla h_1 g_1(\zeta)
\]

\[
= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{R_w} \frac{1}{r_s} & 0 & 0 & 0 & 0 \end{bmatrix}^T
\]

\[
= 0
\]

\[
L_{s_2} h_1(\zeta) = \nabla h_1 g_2(\zeta)
\]

\[
= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{R_w} \frac{1}{r_s} & 0 & 0 & 0 & 0 \end{bmatrix}^T
\]

\[
= 0
\]

and similarly

\[
L_{s_1} h_2(\zeta) = \nabla h_2 g_1(\zeta) = 0
\]

\[
L_{s_2} h_2(\zeta) = \nabla h_2 g_2(\zeta) = 0
\]
Thus

\[ L_f h_1(\zeta) = \nabla h_1 f(\zeta) = \frac{R_w \omega_w D_1}{\cos \delta} \]

\[ L_f h_2(\zeta) = \nabla h_2 f(\zeta) = \frac{R_w \omega_w D_2}{\cos \delta} \]

moreover

\[ L_{g_1} L_f h_1(\zeta) = \nabla(L_f h_1) g_1 \]

\[ = \left[ \frac{R_w D_1}{\cos \delta} - \frac{R_w \omega_w b \sin \theta}{\cos^2 \delta} - \frac{R_w \omega_w D_2}{\cos \delta} 0 0 \right] \left[ \frac{1}{R_w} g_i 0 0 0 0 \right]^T \]

\[ = \frac{g_i D_1}{\cos \delta} \]

\[ L_{g_2} L_f h_1(\zeta) = \nabla(L_f h_1) g_2 \]

\[ = \frac{g_i D_1}{\cos \delta} - \frac{R_w \omega_w b \sin \theta}{L k_{Is} \cos^2 \delta} \]

Similarly the second row elements are obtained

\[ L_{g_1} L_f h_2(\zeta) = \nabla(L_f h_2) g_1 \]

\[ = \frac{g_i D_2}{\cos \delta} \]

\[ L_{g_2} L_f h_2(\zeta) = \nabla(L_f h_2) g_2 \]

\[ = \frac{g_i D_2}{\cos \delta} + \frac{R_w \omega_w b \cos \theta}{L k_{Is} \cos^2 \delta} \]

Thus, the determinant of \( E \) is obtained as

\[ \det E = \frac{g_i R_w \omega_w b}{k_{Is} L \cos^2 \delta} \] (C.3)
Type VI Outputs

The outputs are defined as

\[ y_1 = h_1(\zeta) = \omega_w \]
\[ y_2 = h_2(\zeta) = n \]  \hspace{1cm} (C.4)

and the desired decoupling matrix determinant is given by

\[
\det E(\chi) = \begin{vmatrix}
L_{g_1} h_1(\chi) & L_{g_1} h_1(\chi) \\
L_{g_1} L_f h_2(\chi) & L_{g_1} L_f h_2(\chi)
\end{vmatrix}
\]  \hspace{1cm} (C.5)

The elements of (C.5) are obtained based on the following procedure

\[ L_{g_1} h_1(\zeta) = \nabla h_1 g_1(\zeta) = \frac{1}{R_w} g_i \]
\[ L_{g_1} h_1(\zeta) = \nabla h_1 g_2(\zeta) = \frac{1}{R_w} g_s \]

and

\[ L_{g_1} h_2(\zeta) = \nabla h_2 g_1(\zeta) = 0 \]
\[ L_{g_1} h_2(\zeta) = \nabla h_2 g_2(\zeta) = 0 \]

The second row elements are obtained as

\[ L_f h_2(\zeta) = \nabla h_2 f(\zeta) = \frac{R_w \omega_w D_2}{\cos \delta} \]

moreover

\[ L_{g_1} L_f h_1(\zeta) = \nabla(L_f h_2) \cdot g_2 = \frac{g_s D_2}{\cos \delta} + \frac{R_w \omega_w b \cos \tilde{\theta}}{L k_{1s} \cos^2 \delta} \]

Thus, the determinant of \( E \) is obtained as

\[ \det E = \frac{g_i \omega_w b \cos \tilde{\theta}}{k_{1s} L \cos^2 \delta} \]  \hspace{1cm} (C.6)
Appendix D

Diffeomorphic Transformation $\Phi(\chi)$

To verify that $\Phi(\chi)$ is indeed a diffeomorphism, it is easy to check if the Jacobian of $\Phi$ has full rank.

\[
\nabla \Phi(\chi) = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 \\
\frac{D_1}{\cos \delta} & -\frac{Ub \sin \theta}{\cos^2 \delta} & -\frac{UD_2}{\cos \delta} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\frac{D_2}{\cos \delta} & \frac{Ub \cos \theta}{\cos^2 \delta} & \frac{UD_1}{\cos \delta} & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]  

(D.1)

Interchange the 3rd and 1st rows with the 5th and 4th rows respectively, will not change the determinant. The resulting matrix is given by

\[
\nabla \Phi(\chi) = \begin{bmatrix}
\frac{D_2}{\cos \delta} & \frac{Ub \cos \theta}{\cos^2 \delta} & \frac{UD_1}{\cos \delta} & 0 & 0 \\
\frac{D_1}{\cos \delta} & -\frac{Ub \sin \theta}{\cos^2 \delta} & -\frac{UD_2}{\cos \delta} & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]  

(D.2)

Thus, the determinant of $\nabla \Phi(\chi)$ is obtained as (Ogata, 1990)
\begin{equation}
\det [\nabla \Phi(\chi)] = |A||D| \tag{D.3}
\end{equation}

Where $A$ and $D$ are defined as follows

\[
\nabla \Phi(\chi) = \begin{vmatrix}
A_{(n \times n)} & B_{(n \times m)} \\
C_{(m \times n)} & D_{(m \times m)}
\end{vmatrix}
\]

An easy calculation of D.3 shows that

\[
\det [\nabla \Phi(\chi)] = (1) \left[ \frac{D_2}{\cos \delta} \left( \frac{-Ub \cos \delta}{L \cos^2 \delta} \right) - \frac{D_1}{\cos \delta} \left( \frac{Ub \cos \delta}{L \cos^2 \delta} \right) \right]
\]

\[
= \frac{Ub}{L \cos^2 \delta}
\]

\begin{equation}
\tag{D.4}
\end{equation}

Thus, $\nabla \Phi(\chi)$ has the full rank if $U$, and $b$ are non-zero which are the cases for this study.
## Appendix E

### AGV Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Loaded</th>
<th>Unloaded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass</td>
<td>Kg</td>
<td>1700</td>
<td>700</td>
</tr>
<tr>
<td>Sprung mass</td>
<td>Kg</td>
<td>1300</td>
<td>300</td>
</tr>
<tr>
<td>Front unsprung mass</td>
<td>Kg</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Rear unsprung mass</td>
<td>Kg</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Roll moment of inertia</td>
<td>Kg-m$^2$</td>
<td>1170</td>
<td>150</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>Kg-m$^2$</td>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td>Front roll axis stiffness</td>
<td>N-m/rad</td>
<td>45000</td>
<td>45000</td>
</tr>
<tr>
<td>Rear roll axis stiffness</td>
<td>N-m/rad</td>
<td>45000</td>
<td>45000</td>
</tr>
<tr>
<td>Front roll axis damping</td>
<td>N-m.s/rad</td>
<td>4500</td>
<td>4500</td>
</tr>
<tr>
<td>Rear roll axis damping</td>
<td>N-m.s/rad</td>
<td>4500</td>
<td>4500</td>
</tr>
<tr>
<td>Distance from front axle to C. G.</td>
<td>m</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Distance from rear axle to C. G.</td>
<td>m</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Vertical distance from sprung mass to C. G.</td>
<td>m</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Longitudinal distance from sprung mass to C. G.</td>
<td>m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Height of sprung mass from C. G.</td>
<td>m</td>
<td>1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Front unsprung mass height</td>
<td>m</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Rear unsprung mass height</td>
<td>m</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Front track width</td>
<td>m</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Rear track width</td>
<td>m</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Front tire cornering stiffness</td>
<td>N/rad</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>Rear tire cornering stiffness</td>
<td>N/rad</td>
<td>6000</td>
<td>6000</td>
</tr>
</tbody>
</table>
Appendix F

Motor Specifications

The specifications for tractive motor is given by

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque constant</td>
<td>$K_t$</td>
<td>.5085</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>$J_t$</td>
<td>$3.5 \times 10^3$</td>
</tr>
<tr>
<td>Motor armature resistance</td>
<td>$R_{ar}$</td>
<td>1.2</td>
</tr>
<tr>
<td>Motor damping coefficient</td>
<td>$C_t$</td>
<td>$.774 \times 10^3$</td>
</tr>
<tr>
<td>Motor gear ratio</td>
<td>$n_t$</td>
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</tr>
</tbody>
</table>

Also, for the steering motor the specifications are given as

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque constant</td>
<td>$K_s$</td>
<td>.339</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>$J_s$</td>
<td>$2.33 \times 10^3$</td>
</tr>
<tr>
<td>Motor armature resistance</td>
<td>$R_{ar}$</td>
<td>.8</td>
</tr>
<tr>
<td>Motor damping coefficient</td>
<td>$C_s$</td>
<td>$.516 \times 10^3$</td>
</tr>
<tr>
<td>Motor gear ratio</td>
<td>$n_s$</td>
<td>5</td>
</tr>
</tbody>
</table>