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A Bi-criterion Approach

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in

Mechanical Engineering

by

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Dedicated to
my husband, Xuejun Hu,
and
my son, Ke Hu,
for their unlimited patience and understanding

Abstract

The selection of right process, use of optimal machining parameters and specification of best tolerance parameters have been recognized by industry as key issues to ensure product quality and reduce production cost. The three issues have thus attracted a great deal of attention over last several decades. However, they are often addressed separately in existing publications. In reality, the three issues are closely interrelated. Analyzing the three issues in isolation will inevitably lead to inconsistent, infeasible, or conflicting decisions. To avoid the drawbacks, an integrated approach is proposed to jointly solve process selection, machining parameter optimization, and tolerance design problems. The integrated problem is formulated as a bi-criterion model to handle both tangible and intangible costs. The model is solved using a modified Chebyshev goal programming method to achieve a preferred compromise between the two conflicting criteria.

The application of the proposed bi-criterion approach has been demonstrated by first using the single component single part feature case. The integrated approach is then extended to the multiple components multiple part features case (the assembly case).

Examples are provided to illustrate the application of the two models and the solution procedure. The results have shown that the decisions on process selection, machining parameter selection and tolerance design can be made simultaneously using the models.

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Nomenclature

Indices

h	part feature in an engineering drawing, $h = 1, \dots, H$
i	machining stage (different part feature h may require different number of machining stages according to its design tolerance requirement), $i = 1, \dots, I_h$
j	machining process code (different part feature h may require different number of machining processes), $j = 1, \dots, J_h$
n	rough turning pass for machining part feature h , $n = 1, \dots, N_h$
q	quality characteristic, $q = 1, \dots, Q$
s	rough milling pass for machining part feature h , $s = 1, \dots, S_h$

Parameters

BHN	workpiece hardness (Brinell hardness number)
$C_{hij}, C_{Ahij}, C_{Rhij}, C_{Shij}$	machining cost, inspection cost, rework cost, and scrap cost (\$/piece) of process j in stage i for part feature h
$C_j, C_{mj}, C_{tj}, C_{rj}$	machining cost (\$/piece), operation cost (\$/min), tool cost (\$/tool), and tool replacement cost (\$/min) of process j
$C_{Mhij}, C_{MChij}, C_{MRhij}, C_{MShij}$	expected cost per feature, per feature that conforms to specifications, per reworked feature, and per scrapped feature incurred in process (i, j) for part feature h (\$/piece)
C_{Uh}	raw material cost of part feature h (\$/piece)
C_{UMhij}	expected <i>cumulative</i> cost per acceptable part feature h after process (i, j)

	(\$/piece)
D_c	diameter of milling cutter (mm)
d_e	milling width
D_h	nominal design target of part feature h
D_0	initial diameter of the rotating workpiece (mm)
d_{Tolh}	machining allowance of milling process for part feature h
D_{Yq}, T_{Yq}	nominal value and tolerance of quality characteristic function Y_q
e	overtravel of milling cutter on the workpiece (mm)
E	machine tool efficiency
$E_C(L(x))_h$	expected value of $L(x)$ of part feature h
$F_{\max j}, P_{\max j}, R_{a\max j}$	maximum force, power, surface roughness for process j
$f(x)$	probability density function of quality characteristic x
K_{1h}, K_{2h}	coefficients in the loss function of part feature h
K_G, α_G, A	constants for grinding wheel life
$K_{G*1}, K_{G*2}, K_{G*3}, K_{G*4}$	Constants for grinding force ($* = F$) and power ($* = P$)
$K_{*j}, X_{*j}, Y_{*j}, Z_{*j}$	constants for drilling and reaming thrust force ($* = F$) and torque ($* = M$)
$K_{M*1}, K_{M*2}, K_{M*3}, K_{M*4}, K_{M*5}$	Constants for milling force ($* = F$) and power ($* = P$)
K_{Pj}	constants for drilling and reaming power
$K_{T*}, X_{T*}, U_{T*}, Y_{T*}, Z_{T*}, V_{T*}, S_{T*}, W_{T*}$	constants in the tool life equations for drilling ($* = D$), boring ($* = B$), reaming ($* = R$), and milling ($* = M$)
l_1	overtravel of milling cutter on the workpiece before cutting (mm)
l_2	overtravel of milling cutter on the workpiece after cutting (mm)

L_h	cutting length of the workpiece for machining part feature h (mm)
$L(x)$	quality loss function of quality characteristic x
N_t	number of teeth per milling cutter body
$P(x)$	probability of quality characteristic x
$R_3, K_{R1}, K_{R2}, K_{R3}$	constants for grinding surface roughness
T_j, T_{Lj}, T_{rj}	machining time, tool life, and tool replacement time of process j (min)
T_{Uh}, T_{Lh}	upper and lower design tolerances of part feature h (mm)
t_{hij}^l, t_{hij}^u	tolerance limits of process (i, j) for part feature h (mm)
v_*, f_*, d_*, D_*	Cutting speed (m/min), feed (mm/rev), depth of cut (mm), and diameter of work (or hole) (mm) of turning ($*$ = T), drilling ($*$ = D), boring ($*$ = B), reaming ($*$ = R), milling ($*$ = M) (for milling, D_* does not exist) and grinding ($*$ = G) (for grinding, $V_* = V_{Gw}$ surface speed of the rotating workpiece, and $D_* = D_{Gw}$ diameter of workpiece)
$v_{Gs}, B, D_s, D_{Gw}, D_G$	grinding wheel surface cutting speed (m/min), wheel width (mm), wheel diameter (mm), workpiece diameters before and after grinding (mm)
$v_{hij}^l, v_{hij}^u, f_{hij}^l, f_{hij}^u, d_{hij}^l, d_{hij}^u$	lower and upper limits of cutting speed (m/min), feed rate (mm/rev), and depth of cut (mm) in process (i, j) for part feature h
Y_q	function of quality characteristic q , $Y_q = f(D_1, \dots, D_h)$
α	percentage of rejected features after the finish stage
$\alpha_*, \beta_*, \gamma_*, B_*$	empirical parameters in the tool life ($*$ = T), cutting force ($*$ = F), and power consumption ($*$ = P) equations of turning processes
$\alpha_R, \beta_R, \eta, \theta, B_R$	empirical parameters in the surface roughness equation of turning

	process
σ_{mhj}, C_{phj}	process variance and capability index of process (i, j) for part feature h
$\Phi(x)$	standard normal distribution function of quality characteristic x

Variables

D_{hj}	Diameter after process (i, j) of part feature h (mm)
m_{hj}	Integer variable ($m_{hj}=1$, if process j in stage i is chosen to machine part feature h , 0, otherwise)
N_h	The number of rough turning passes used for machining part feature h
S_h	The number of rough milling passes used for machining part feature h
T_h	Design tolerance of part feature h
$v_{hij}, f_{hij}, d_{hij}, t_{hij}$	Cutting speed (for grinding passes, workpiece surface cutting speed) (m/min), feed rate (mm/rev), depth of cut and tolerance (mm) of process j in machining stage i for machining part feature h

Note: In the model of the single part feature case, the above parameters are used by simply dropping subscript h .

Chapter 1

Introduction

1.1 Overview

The increasingly intensified global competition has forced manufacturing companies to look for all possible means to reduce production costs and improve product quality. Selection of appropriate machining parameters such as cutting speed, feed rate and depth of cut has been a common practice for cost reduction in the metal cutting industry. Related literature has been substantially enriched by considerable research ranging from the early studies (i.e., Petropoulos, 1973) for turning, their extension to various multi-pass turning (i.e., Tan and Greese, 1995), and further to the recent work in grinding (Xiao and Malkin, 1996, Capello and Semeraro, 2002) as well as drilling (Shunmugam *et al.*, 2000). Tolerance related parameters including process mean and process tolerance have been mainly used as the variables for quality improvement. Notable contributions in this direction include, among others, a series of publications by Feng and Kusiak (1997, 2000), Jeang (1995, 2002) and Li (2000, 2002). It is important to note that most of the previous studies on cost and quality issues have been carried out separately. However, it is well known that the two are closely related. Optimizing one in isolation may be detrimental to the other and may lead to conflicting decisions. Though quality loss as a decision criterion has gained popularity in tolerance design, it represents only part of the total cost in a product life cycle. The lack of synergy between the cost and quality related studies inevitably undermines the significance of the outcome.

Analyzing the two issues in isolation also leads to the segregation in handling planning parameters that are interrelated by nature. When cost, in particular, the machining cost, is to be minimized, machining parameters are traditionally considered, whereas tolerance related parameters such as process mean and process tolerance are often neglected. Similarly, when dealing with quality issues, process mean and process tolerance are routinely used as the sole decision variables. In reality, the machining parameter decision is directly affected by, and in turn strongly influences tolerance decisions. For instance, the process tolerance cannot be specified without considering the process capability. By the same token, machining parameters, especially the ones for the finish machining process, should not be selected without knowledge of the resulting tolerance. The two sets of parameters jointly affect both production cost and product quality due to their mutual interactions. It is therefore inadequate to consider one and ignore the other for either cost reduction or quality improvement.

Another issue affecting both cost and quality is process selection. The process to be used has often been either assumed to be given or specified subjectively without detailed quantitative analysis. For example, most studies on machining parameter optimization deal with only one given process, e.g., turning, drilling, boring, grinding, or reaming. The fact is, however, that there often exist several alternative processes for the same job and it is not always clear which of the alternative processes is predominant over the others. For instance, a hole-making job can be done by drilling followed by either boring, reaming, grinding or their combinations, rendering different cost and quality outcomes. Hence it is irrelevant to “optimize” machining parameters without knowing which process is to be selected beforehand. Furthermore, most products require several machining stages. “Optimizing” the machining parameters for a single machining stage without considering other stages and their interactions may lead to inconsistent, infeasible or conflicting decisions.

Figure 1.1. Relationships among planning objectives, planning parameters and machining process

The interactions among the planning parameters (machining parameters, tolerance parameters and processes) and their combined effects on planning objectives (cost and quality) are illustrated in Fig. 1.1. To avoid the problems revealed above, all the planning parameters should be simultaneously considered and both planning objectives should be incorporated in the planning process. For this reason, an integrated approach is proposed in this thesis.

1.2 Problem Description and Objectives

As revealed earlier, process selection, machining parameter selection and tolerance specification are often carried out separately, thereby causing inconsistent, infeasible or conflicting decisions. This problem can be avoided only when an integrated approach is adopted. Due to the complexity of the integrated solution, the focus of this thesis is firstly on applying an integrated approach to the single component single part feature case. In manufacturing practice, many part features need to be machined to meet their

specifications, such as shafts, sleeves, and screws. In this study, analysis is focused on three part features, namely, two round shaped features including both external rotational surfaces (such as shafts, spindles, handles, and pistons) and internal rotational surfaces (e.g., holes and internal surfaces of cylinders), and one planar shaped feature (planar surfaces). From the manufacturing viewpoint, a part needs to go through several manufacturing processes. This may be due either to the complex geometry of the feature or to tight tolerance and strict surface finish requirements for the feature. For example, the process chain of drilling -- rough boring -- semi-finish boring -- grinding can be used to enlarge a hole with relatively high accuracy and surface finish requirements. It is well known that different combinations of manufacturing processes can be used to machine different part features. The external round surfaces are typically done by turning and grinding. The machining of internal rotational shapes generally involves drilling, boring, reaming, grinding and honing. Milling is required by machining the planar surfaces.

In this thesis, the machining tasks of the round shaped feature groups are classified into four stages: primary, rough, semi-finish and finish machining (Fig. 1.2) and the planar surface feature group into two stages: rough and finish machining (Fig. 1.3). Each stage is accomplished by a process selected from several alternative processes. Each process may require several passes. This mostly occurs in primary machining stage when a large volume of material needs to be removed. For external round surfaces, the primary stage includes one or several rough turning passes, the rough machining stage is normally done by a finish turning process, the semi-finish stage involves a rough grinding process, and finally the finish stage contains a finish grinding process (Fig. 1.2a). The four stages for internal rotational surfaces and the two machining stages for planar surfaces are shown in Fig. 1.2b and Fig. 1.3 respectively.

It should be pointed out that not all the machining stages are required for a particular product and not all the processes of a machining stage are necessary. The

Figure 1.2. Typical machining stages and processes for external rotational surfaces (a) and internal rotational surfaces (b)

Figure 1.3. Typical machining stages and processes for planar surfaces

decisions on selecting a process for each machining stage, specifying number of passes for a process, selecting machining parameters, and determining process tolerance and process mean depend not only on the product specifications but also on the tangible and intangible cost concerns. As mentioned earlier in this study, some cost components such as machining costs and quality loss can be quantified but other factors may not be directly analyzed in monetary values. For example, if the tolerance is tight, the operator has to pay more attention to the work. It would be more difficult and time consuming to set up the machine, tooling, fixture and workpiece. The degree of “attention” and the level of “difficulty” will eventually contribute to cost and production time. It has been suggested that the relation between tolerance t and manufacturing cost C be “quantified” using a quadratic function $C = A + (B/t^2)$ (Spotts, 1973) or linear function $C = A + (B/t)$ (Chase *et al.*, 1990) but it is not easy to accurately estimate parameters A and B . Furthermore, the interactions between tolerance and other factors such as machining parameters and their combined effect on the final cost cannot possibly be described by such overly simplified models.

For the above reasons, a bi-criterion approach for simultaneously solving process selection, machining parameter selection and tolerance design problems for a part feature is proposed in this thesis. One of the criteria is the tangible cost, i.e., the expected unit cost consisting of raw material cost, operation cost, tool cost, tool replacement cost, inspection cost, and quality loss. The other criterion is the intangible cost due to excessively tight tolerance. All the factors such as process, machining and tolerance parameters will be incorporated in order to consider their interactions and joint effect.

Furthermore, the most common tolerance design problem encountered by engineering designers is the distribution of the specified assembly tolerance among the components of a product assembly. This problem is called “tolerance synthesis” or “tolerance allocation”, which has been the topic of a large number of research publications. Most of the reported approaches, i.e., Rao and Hati (1978), Deng and Deng (2002) in this direction are based on the optimization of the cost-tolerance function. The manufacturing cost is usually considered as a function of a sole variable--tolerance, and other parameters are assumed to be given and fixed. However, as stated earlier, most of the parameters such as machining speed and feed rate are closely related and jointly affect the total manufacturing cost. Furthermore considering these related factors separately will lead to inconsistent and conflicting solutions. This may explain why the reported models have not been well accepted by industry. To overcome these drawbacks, we extend the integrated approach to the assembly case.

In view of the above, the primary objective of our work in this thesis is to develop an integrated approach for jointly solving the process selection, machining parameter selection and tolerance design problems considering both tangible and intangible cost criteria. Generalized models a) single component single part feature case, and b) multiple component multiple part feature (product assembly case) will be developed. The models

will be solved using a goal programming approach to find preferred compromise between the two non-commensurable and possibly conflicting decision criteria.

1.3 Organization

The rest of the thesis is organized as follows. Chapter 2 provides a brief review of previous work in process selection, machining parameter optimization and tolerance design. Chapter 3 presents the two generalized models for single component single part feature case and multiple component multiple part feature (product assembly case). The illustrative examples and the computational results are reported in Chapter 4. Chapter 5 concludes the thesis and suggests possible future work.

Chapter 2

Literature Review

As the major effort of this thesis is directed towards an integrated approach for solving process selection, machining parameter optimization and tolerance design problems simultaneously to avoid inconsistent and infeasible decisions, it is necessary to review the most prominent studies on the three topics in this chapter.

2.1 Process Selection (Process Sequence Determination)

In the modern manufacturing environment, alternative sets of machining processes can normally be generated for machining one feature of a part. Each set of manufacturing processes results in a specific manufacturing cost and requires a specific set of manufacturing resources, such as machines, fixtures/jigs and cutting tools. Therefore, process selection is a key factor governing the product cost in manufacturing. In the literature, the process selection problem has often been discussed related to tolerance design rather than machining parameter optimization since most studies on machining parameter optimization deal with either one given process or one given process sequence.

The study by Zhang *et al.* (1992) was probably among the first that simultaneously address process selection and tolerance issues. Stemming from this early work, Zhang and Wang (1993a) proposed a mathematical model to simultaneously allocate design and machining process tolerances for a given set of machining processes by using simulated annealing. They took the piston-cylinder hole assembly as an example, and predefined the two process sequences for machining the two parts. Although design and machining

process tolerances were obtained simultaneously, the process selection issue was missing from their analysis.

Later, Zhang and Wang (1993b) suggested a mathematical model to identify the optimal process sequence and determine the processing parameters (dimensions and tolerances) for all intermediate processes, by minimizing the manufacturing cost based on cost-tolerance functions for a rotational part. They classified all the machining processes into four sets of manufacturing processes: rough, semi-finish, finish, and extra-high precision, and searched for the optimal process sequence by choosing a process from each set of manufacturing processes. However, it should be pointed out that they did not optimize all the parameters simultaneously. Their method consisted of three steps. First, they used a goal-directed tree search approach to searching all possible process sequences. Secondly, they calculated the total manufacturing cost of each feasible process sequence through the optimization modeling procedure and chose the optimal process sequence with the minimum cost. Finally, after the process sequence of a feature had been determined, the manufacturing dimensions for all intermediate processes were calculated.

Besides the studies of Zhang and Wang (1993a, 1993b), Yeo *et al.* (1996) developed a methodology to optimise the process sequences and tolerance allocation for all intermediate processes using an expert system approach based on minimum production cost.

Recently, Ming and Mak (2001) proposed intelligent approaches to tolerance allocation and manufacturing process selection in process planning. Their approaches included two steps. The optimal tolerances were first determined for each process comprising a set of sequenced manufacturing processes, which was then used to machine a feature in such a way that the resulting manufacturing cost was minimized. Then the optimal sets of manufacturing processes required to machine all the features of a part

were identified. It is noteworthy that their approaches were achieved by minimizing the sum of the manufacturing costs of the selected sets of manufacturing processes and the dissimilarities in their manufacturing resource requirements. Like other approaches, the manufacturing cost of each process was also calculated based on a certain cost-tolerance function.

The above literature review on process selection indicates that the existing approaches have the following shortcomings:

- 1) The processes are often selected based on overly simplified cost-tolerance functions. More detailed and comprehensive relationships among process, machining parameters, cost and tolerance are overlooked.
- 2) Only partial, and sometimes inconsistent, infeasible and conflicting solutions are provided. Such solutions can rarely be implemented for shop floor decisions.

To avoid these drawbacks, a more comprehensive approach to the process selection needs to be developed.

2.2 Machining Parameter Optimization

In a competitive production environment, the selection of appropriate process variables is an essential step in the process planning of machining operations. For example, low speeds and feeds result in low production rates, whereas excessively high speeds and feeds may reduce tool life to the point where the cost of tool change outweighs the value of increased productivity, and production rates drop because of the time required for tool changes. In this study, six different processes (milling, turning, grinding, drilling, boring, and reaming) for machining three types of features: planar surfaces, external rotational surfaces, and internal rotational surfaces are to be considered. The optimal parameter selection in turning has been investigated extensively. However,

the optimal conditions in milling, grinding, drilling, boring and reaming have not received much attention. Related literature is reviewed in the following.

2.2.1 The Milling Process

Milling is the most versatile and widely used manufacturing process. Substantial volumes of material can be removed by rough milling with relatively low cutting speed, large feed rates and depths of cut. Good surface finish and small dimensional tolerances can be achieved by finish milling with high cutting speed, low feed rates and depths of cut.

As early as in the 1970s, Rao and Hati (1978) explained how to optimize milling process parameters. They solved the problem using the minimum production cost per piece, the maximum production rate and the maximum profit as the objectives, with cutting speed, feed rate, and spindle torque constraints. It should be mentioned that they actually optimized only two process parameters: cutting speed and feed rate since they adopted simplified tool life equation and machining time equations with only cutting speed and feed rate as variables.

Recent work in milling parameter selection and optimization ranges from off-line optimization based on various objectives to on-line adjustment. Liu and Wang (1999) designed an adaptive controller with optimization feature based on two kinds of neural networks. Their method is very innovative and can be used to adjust the machining parameters in real-time so as to ensure the quality, reduce the machining cost and increase the machining efficiency. It is noticed that milling depth and cutting speed were preset at different levels, and only the feed rate was adjusted.

Juan *et al.* (2003) constructed a machining model based on a polynomial network to optimize cutting parameters in terms of production cost. However, this method limited its application to the rough machining of high speed milling operation.

2.2.2 The Turning and Boring Processes

The selection of optimal machining parameters (cutting speed, feed rate, and depth of cut) for turning has been investigated extensively in the literature. Optimization efforts can be classified into two types: single-pass and multi-pass. For single-pass problem, Gopalakrishnan and Al-khayyal (1991) developed an optimization model to determine machine cutting speed and feed rate that minimize the total machining cost using geometric programming. In addition, the surface finish and the machine power constraints were considered in their modelling. Similarly, an optimization CAM software package for the selection of economic cutting conditions in single pass turning operations was developed by Wang *et al.* (2002). Their optimization was based on the maximum production rate subject to several practical constraints such as maximum cutting force and torque.

In the above studies, only one decision criterion was considered. Several researchers have addressed this issue involving multiple criteria. For example, Agapiou (1992) determined the optimum machining conditions for single-pass turning based on an objective function incorporating both production cost and production time. The two criteria were prioritized through their weight coefficients.

Some researchers have incorporated other factors into their optimization models, such as the stochastic nature of tool life and the cost of quality. Koulams (1991) considered the influence of tool chatter failure in the optimization model. In his study, a tool failure probability function was constructed and used as a penalty cost function added to the objective function. Hui *et al.* (2001) developed a time-dynamic economic model for single-pass turning. The model incorporated considerations on quality cost, the stochastic nature of tool-life and tool maintenance activities such as tool replacement and regrinding.

It should be noted that most of the approaches to the optimization of single-pass operation either deal with two variables (cutting speed and feed rate), i.e., optimize cutting speed and feed rate for a given depth of cut.

Ermer and Kromodi (1981) developed a multi-step mathematical model to solve a constrained multi-pass machining problem. In the conclusion, they pointed out that a multi-pass operation may be more economical than a single-pass operation with certain constant total depth-of-cut. Actually, the multi-pass machining problem that Ermer and Kromodi solved consisted of only two passes. The depth of cut in finish and rough passes must be assumed before the optimization problems could be solved.

To avoid those drawbacks, a tandem approach to selection of machinability data was developed by Yeo *et al.* (1995). They formulated the problem based on a constant depth of cut for the roughing operation to determine the optimal depth of cut, feed rate, speed and number of passes. The optimum depth of cut, feed rate, and speed for one finishing pass after several roughing passes were also determined simultaneously.

Furthermore, Tan and Creese (1995) developed a more general multi-pass machining model and used an approximation solution approach for selecting the optimal number of passes, as well as the depth of cut, feed rate, and cutting speed for each individual pass. In their model, the depth of cut, feed rate, and cutting speed could be different for different rough passes, and there may be more than one finish pass.

Some researchers also attempted to include more factors into their optimization models for multi-pass turning. For example, Liang *et al.* (2001) presented an integrated approach for simultaneous optimisation of machining parameters, including machining speed, feed rate and depth of cut, number of passes, tool adjustment interval, and the amount of adjustment.

In the existing literature, very few papers dealt with the optimization problem for the boring process. Because the mechanism of a boring process is similar to that of a

turning process, most researchers (i.e., Zhao 1992, Agapiou 1992) selected the optimal parameters for the boring process using an approach similar to that used in the turning process.

2.2.3 The Grinding Process

Grinding is a machining process which utilizes a grinding wheel consisting of abrasive grains held together with a binder material. The actual cutting points on abrasive grains at the wheel surface are micro-cutting tools which interact with the workpiece material. Due to the complexity of grinding process and the numerous factors which affect the grinding performance, it is very difficult to optimize grinding parameters.

The selection of operating parameters for most metal cutting processes is usually based on the well-known Taylor tool life equation (Malkin, 1976). Unfortunately, no such relationship has been found for grinding processes because grinding wheel wear is an extremely complex process. There is no consensus even on the definition of tool life. Various definitions of a tool life criterion, such as deterioration of surface finish with grinding time, loss of wheel form, machine vibrations, surface cracking and a metallurgical criterion, are used in the existing optimization methods for grinding.

Malkin (1976) provided an analytical method to select operating parameters for grinding of steels when the grinding wheel tool life was limited by burning of the workpiece. The optimal workpiece speed and feed rate were chosen based on the effect of two parameters: the volume removed per wheel dressing and the production time needed to grind away a unit volume of material.

On the other hand, some studies showed that wheel wear behaviour can be empirically correlated to some extent with some parameters of the grinding process. For example, Shirai (1982) developed an equation of wheel life, which showed that the grinding wheel life was proportional to wheel speed, and inversely proportional to

workpiece speed and grinding depth of cut. If the diameter of grinding wheel exceeded a constant, the wheel should be dressed. Shirai's equation is adopted in this thesis.

A more comprehensive on-line optimization system was developed by Xiao *et al.* (1996) for cylindrical plunge grinding to minimize production time while ensuring quality requirement. This system incorporated more realistic constraints (i.e., thermal damage, surface roughness, out of roundness, taper, and wheel wear), considered time dependent behaviour, and also optimized the dressing interval. In this study, wheel wear was empirically related to the normal stress at the grinding zone and the rotational wheel speed.

It is noticed that most existing studies focused on cylindrical plunge grinding and very few on traverse cylindrical grinding. Capello and Semeraro (2002) is one of the few dealing with the traverse cylindrical grinding processes. In their experimental study, they suggested that the optimal grinding parameters (i.e. workpiece speed, feed rate and depth of cut) be selected based on minimum residual stresses.

2.2.4 The Drilling and Reaming Processes

Drilling and reaming are often used together in hole-making operations. If the diameter of hole is relatively large, a pilot hole may have to be drilled using a tool of smaller diameter and then be reamed to its final size with a larger tool.

As mentioned earlier, the optimization of process parameters in drilling and reaming has received little attention. Most approaches used in the two processes are very similar to those used in turning. Rao and Hati (1978), and Agapiou (1992), are among the few who addressed the optimization of process parameters for drilling. Their models used the production cost as the objective function. The constraints include allowable maximum cutting force, cutting temperature, depth of cut, cutting speed, feed rate, machining power limits and surface finish requirements.

Similarly, Shunmugam (2000) considered the optimal selection of parameters in multi-tool drilling. The minimum production cost was taken as the objective with the constraints of the speed and feed rate ranges, strength of the drill, maximum axial thrust allowed by the feed mechanism and cutting power. The model also included the preventive tool replacement strategy adopted in industries. It should be noted that, in this study, the hole-making operations were carried out by drilling with a single drill or pilot-drilling of one or more holes followed by enlargement operations.

As for reaming, Bezerra *et al.* (2001) proposed an experimental procedure to obtain the optimal reaming parameters based on two criteria: the dimensional stability (diameter, roundness and cylindricity) and surface roughness. However, this method did not optimize all the cutting parameters simultaneously.

2.2.5 Multiple Processes (Multi-stage Machining System)

Very often, a product needs to go through several different processes to meet surface finish and dimensional requirements. Therefore, it is more realistic to select optimal machining parameters for all the processes to be performed on the product. Surprisingly, the optimization problem for multiple processes has not received much attention.

Rao and Hati (1978) developed a procedure for optimizing cutting conditions (cutting speed and feed rate) for turning, drilling and milling with pre-specified process sequence.

Koulamas (1991) proposed an analytical model for the simultaneous optimization of cutting speed and feed rate, and optimal tool replacement policy for a two-stage machining system, which was predefined as a sequence of two processes (e.g. turning followed by drilling, drilling followed by reaming, etc.). Agapiou (1992) solved the optimization problem for a more complex multi-stage system consisting of five processes

(turning, boring, drilling, milling, and tapping). Like most approaches, however, the optimization model dealt with only two variables (cutting speed and feed rate).

As part of their effort in jointly solving tool travel scheduling, tool switch scheduling, tool selection and machining speed selection, Kolahan and Liang (2000) reported a tabu-search approach to minimize the total processing cost for a hole-making operations (drilling, reaming and tapping) by optimizing the cutting speed.

In summary, a variety of optimization models have been developed to select the optimal machining parameters for the six commonly used processes (milling, turning, grinding, drilling, boring, and reaming). Most of them considered three different objective functions, namely, minimization of unit production cost, maximization of production rate and profit. The common limitations of previous studies include:

- 1) Machining parameters are selected independent of tolerance decisions.
- 2) The interactions between current, up and down stream processes are overlooked and thus the machining parameters are selected for each process separately.
- 3) The depth of cut is left out as a decision variable.

The above situation calls for further investigation of this problem.

2.3 Tolerance Design

Tolerance design is essential part of design and manufacturing. Proper selection of tolerance parameters has a significant impact on product cost and quality. In the past decades, researchers have developed a variety of models to solve tolerance design problem. In this section, a brief review of three important topics in tolerance design, i.e., cost-tolerance functions, tolerance allocation methods, and tolerance design and process setting, is presented.

2.3.1 Cost-Tolerance Functions

Considerable research has been carried out for optimal tolerance design based on manufacturing cost consideration. It is well known that the smaller the tolerance of a manufacturing process, the higher the manufacturing cost. The five commonly used cost-tolerance functions are (Wu *et al.* 1988)

- 1) The Sutherland function: $C = bt^{-a}$
- 2) Reciprocal square function: $C = a/t^2$
- 3) Reciprocal function: $C = a/t$
- 4) Exponential function: $C = a \exp(-bt)$
- 5) Michael-Siddall function: $C = at^{-b} \exp(-dt)$

The coefficients in these cost models were determined from the empirical tolerance-cost data by a curve fitting. Based on their experimental work, Wu *et al.* (1988) concluded that none of the five functions fit the field data well and large errors were observed. The authors further pointed out, it is often difficult to estimate those coefficients using the field data.

In view of this, Dong and Hu (1991) introduced and tested combined linear and exponential, cubic polynomial, fourth-order polynomial, and fifth-order polynomial cost-tolerance models. The polynomial models are found to give a more accurate fit than others but if the data points do not exhibit a “polynomial nature”, the resulting curve may exhibit large oscillations.

In order to fit cost-tolerance empirical data more accurately, the natural spline model based on a numerical method was proposed by Yeo *et al.* (1996). In addition, for the same reason, a new analytical cost-tolerance model was developed by Diplaris and Sfantsikopoulos (2000). This model is more inclusive than the previous ones because it takes into account the size of the tolerated dimension, the tolerance size, the size of the initial tolerance before machining starts and the size of the workpiece surface that has to be machined to the specified tolerance.

Although many researchers have made great efforts to improve cost-estimating accuracy, the errors due to cost-tolerance functions presently in use seem to be unavoidable. This inaccuracy will be carried over to the optimization models for tolerance design, leading to more errors in later planning decisions.

2.3.2 Tolerance Allocation Methods (Tolerance Synthesis)

Tolerance synthesis has been the topic of many research publications. It is a method to assign tolerance to each dimension so that the desired quality characteristics of the assembled product are achieved while the manufacturing cost is minimized. In this review, the existing tolerance synthesis models are classified into two categories: optimization and robust design methods.

Most of the reported approaches, i.e., Zhang and Wang (1993), Diplaris and Sfantsikopoulos (2000), in this direction are based on the optimization of the cost-tolerance function. They usually attempt to optimize the tolerances based on manufacturing cost as a function of tolerances. Other factors are not considered. As early as in 1970s, Speckhart (1972) and Spotts (1973) started using the cost-tolerance functions to solve the tolerance allocation problem. Later, many researchers developed optimization models to deal with the tolerance allocation problem. For example, Ostwald and Huang (1977) proposed a zero-one linear programming method to allocate tolerances to a series of dimensions, and suggested the Balas 0-1 algorithm to solve large-scale design problems. It should be noted that a single cost-tolerance curve has assumed to describe the relationship between the total machining cost and design tolerance in the abovementioned studies. In other words, they just considered only one process or a combined process.

A more practical cost-tolerance modelling method was proposed by Zhang and Wang (1993) to simultaneously optimize design and machining tolerances for an

assembly. In this study, a number of machining processes in a feasible sequence were predefined to machine a surface feature to meet its specification. A different cost-tolerance curve was employed for each machining process. The problem was formulated as a non-linear model and solved using a simulated annealing algorithm.

Inspired by the above works, many researchers have recently attempted to assign the optimal component tolerance, and determine the optimal process and/or their sequences simultaneously. The representative studies include the work by Zhang and Wang (1993a, b), Ming and Mak (2001), and Yeo *et al.* (1996).

Robust design is another approach for tolerance allocation. In this case, the quality value (quality characteristic) of a product/process that is insensitive to process variation is selected. The variation of quality value can be reduced by two procedures: parameter design and tolerance design. The cost-tolerance functions and Taguchi's loss function are broadly used in most applications of quality engineering to tolerance synthesis. For example, Feng and Kusiak (1997) solved the tolerance allocation problem by optimizing the manufacturing cost in addition to the quality cost subject to a tolerance stackup and other constraints. In addition, other approaches, such as the design of experiment (DOE) (Feng and Kusiak, 2000), and the response surface methodology (RSM) (Jordaan and Ungerer, 2002), have also been applied.

2.3.3 Tolerance Design and Process Setting

As stated earlier, tolerance design is an essential part of design and manufacturing. Integration of tolerance design and process setting has been recognized as the key to achieving low cost and high quality. Tolerance related process setting parameters, including process mean and process tolerance, have been chosen by many researchers as the variables for quality improvement and cost reduction. In many existing publications, it is usually assumed that the process mean and process tolerance are equal to the design

target and the design tolerance, respectively (e.g. Feng and Kusiak, 1997). The design problem is simply to find the optimal tolerance to minimize cost and improve quality. However, in many cases, as the process mean may shift from the design target, this will reduce the range of process variation due to the tight design tolerance limits. In addition, Li (2000, 2001) pointed out that, when unbalanced tolerance design is applied, the process mean should be drifted away from the target value and be set at the optimal point to minimize the expected loss of quality.

Furthermore, Wen and Mergen proposed a method for setting the optimum process mean when a process is not capable of meeting specifications in the short term. Chen *et al.* (2002) modified Wen and Mergen's cost model to take into account the linear and quadratic asymmetrical quality loss of a product within specification for determining the optimum process mean.

Besides above efforts, an approach for integrating the product and process design was developed by Jeang (2002). Through the integration, the optimal process mean and process tolerance were determined to meet a specified design target and design tolerance with the possibility for further quality improvement and cost reduction.

It should be pointed out that, although the initial intention of the above-mentioned studies was to simultaneously determine the design target, design tolerance, optimal process mean and process tolerance, in fact only the optimal process mean and process tolerance were determined. The other values were pre-specified. All these studies focused on the single process problems and the results may not be directly applied to multiple process problems.

The above literature review on tolerance design indicates that the existing approaches suffer from at least one of the following shortcomings:

- 1) Large decision errors due to the use of cost-tolerance functions.
- 2) Lack of methodologies to optimize process mean.

- 3) Neglect of the interactions between tolerance and other factors such as machining parameters and their combined effect on cost and quality.
- 4) Lack of approaches to handle multiple process problems.

In view of the above, a more comprehensive approach free from all of these deficiencies needs to be developed.

2.4 Motivation

Process selection identifies the optimal set of machining processes to machine a part. Machining parameter selection determines all the machining parameters for all the chosen processes. Tolerance design specifies the process mean, process tolerance and design tolerance for the each part feature. All the planning parameters are closely related. The above literature review reveals that the process selection, machining parameter optimization and tolerance design are mostly carried out separately, thereby causing inconsistent and conflicting decisions. Therefore, it is highly desirable to develop a model and solution method capable of obtaining all the planning parameters simultaneously. This thesis will be directed toward the development of an integrated approach to solve the three problems jointly to enhance decision quality and coherency.

Chapter 3

Cost Analysis and Model Development

In this chapter, several major cost elements are analyzed. Integrated models for both single component single part feature and multiple components multiple part features (it will be called product assembly hereafter to simplify discussion) are then formulated based on the cost analysis. The models incorporated not only tangible cost elements but intangible cost as well. The details are described in the following sections.

3.1 Cost Analysis

As mentioned in Chapter 1, one of the criteria is the tangible cost, i.e., the expected unit cost consisting of raw material cost, operation cost, tool cost, tool replacement cost, inspection cost, and quality loss. The machining cost and quality loss are analyzed in the next two sections and the other cost components are directly included in section 3.1.3.

3.1.1 Machining Cost

The unit machining cost for any of the machining processes shown in Fig. 1.2 and Fig. 1.3 contains three cost components, i.e., operation cost, tool replacement cost and tool cost, and can be expressed in the following general form

$$C_j = C_{mj}T_j + (C_{tj} + C_{rj}T_{rj})\frac{T_j}{T_{Lj}} \quad (3.1)$$

where j indicates a particular process (milling, turning, grinding, drilling, boring or reaming), C_{mj} the machining operation cost, T_{η} tool replacement time, T_j machining time, T_{Lj} tool life, C_{η} and C_{η} tool replacement and tool costs respectively. It should be noted that other costs independent of machining time and machining parameters are not included in the above equation. The detailed analyses for different processes are presented in the following.

3.1.1.1 Milling Cost

End milling is the main focus in this thesis (Fig. 3.1). The following equations can also be used for other milling processes such as face milling with minor modifications. The rough milling process may require several passes in order to remove a large volume of material. The unit machining cost of a milling pass s is

$$C_{Ms} = C_{mM} T_{Ms} + (C_{tM} + C_{rM} T_{rM}) \frac{T_{Ms}}{T_{LMs}} \quad (3.2)$$

with machining time $T_{Ms} = \frac{\pi D_c (L+e)}{1000 N_t v_{Ms} f_{Ms}}$ (Rao and Hati, 1978), and tool-life of an end

milling cutter $T_{LMs} = \left(\frac{K_{MT} D_c^{X_{MT}}}{v_{Ms}^{U_{MT}} d_{Ms}^{Y_{MT}} f_{Ms}^{Z_{MT}} d_e^{V_{MT}} N_t^{S_{MT}}} \right)^{W_{MT}}$. Substituting T_{Ms} and T_{LMs} expressions

into Eq. (3.2), we obtain

$$C_{Ms} = \frac{\pi D_c (L+e)}{1000 N_t v_{Ms} f_{Ms}} (C_{mM} + (C_{tM} + C_{rM} t_{rM}) \left(\frac{v_{Ms}^{U_{MT}} d_{Ms}^{Y_{MT}} f_{Ms}^{Z_{MT}} d_e^{V_{MT}} N_t^{S_{MT}}}{K_{MT} D_c^{X_{MT}}} \right)^{W_{MT}}) \quad (3.3)$$

where, $e = l_1 + l_2$, and $l_1 = \sqrt{d_{Ms}(D_C - d_{Ms})}$ (Zhao, 1992)

Figure 3.1. Schematic diagram of end milling process
(* note: n is the rotational speed of the cutting tool - RPM)

To simplify the analysis, we assume that the same cutting speed, feed rate and depth of cut are used for all the S rough milling passes and they are denoted as v_{RM} , f_{RM} and d_{RM} respectively. Therefore, the total cost of the rough milling process is

$$C_{RM} = \frac{S\pi D_C(L+e)}{1000N_t v_{RM} f_{RM}} (C_{mM} + (C_{iM} + C_{rM} t_{rM}) \left(\frac{v_{RM}^{U_{MT}} d_{RM}^{Y_{MT}} f_{RM}^{Z_{MT}} d_e^{V_{MT}} N_t^{S_{MT}}}{K_{MT} D_C^{X_{MT}}} \right)^{W_{MT}}) \quad (3.4)$$

The machining cost for finish milling can be calculated using Eq. (3.3) by simply dropping subscript s .

3.1.1.2 Turning Cost

Like milling process, the rough turning process may require several passes in order to remove a large volume of material (Fig 3.2). The unit machining cost of a turning pass n is

$$C_{Tn} = C_{mT}T_{Tn} + (C_{iT} + C_{rT}T_{rT}) \frac{T_{Tn}}{T_{LTn}} \quad (3.5)$$

with machining time $T_{Tn} = \frac{\pi D_{n-1}L}{1000v_{Tn}f_{Tn}}$ and tool-life $T_{LTn} = \frac{B_T}{v_{Tn}^{\alpha_r} f_{Tn}^{\beta_r} d_{Tn}^{\gamma_r}}$. Substituting T_{Tn}

and T_{LTn} expressions into Eq. (3.2), we obtain

$$C_{Tn} = \frac{\pi D_{n-1}L}{1000B_T} \left(\frac{B_T C_{mT}}{v_{Tn} f_{Tn}} + v_{Tn}^{\alpha_r-1} f_{Tn}^{\beta_r-1} d_{Tn}^{\gamma_r} (C_{iT} + C_{rT}T_{rT}) \right) \quad (3.6)$$

Figure 3.2. Schematic diagram of turning process
(* note: n is the rotational speed of the cutting tool - RPM)

To simplify the analysis, we assume that the same cutting speed, feed rate and depth of cut are used for all the N rough turning passes and they are denoted as v_{RT} , f_{RT} and d_{RT} respectively. Therefore, the total cost of the rough turning process is

$$C_{RT} = \frac{\pi L}{1000 B_T} \left(\frac{B_T C_{mT}}{v_{RT} f_{RT}} + v_{RT}^{\alpha_T-1} f_{RT}^{\beta_T-1} d_{RT}^{\gamma_T} (C_{iT} + C_{rT} T_{rT}) \right) \sum_{n=1}^N D_{n-1} \quad (3.7)$$

where

$$D_{N-1} = D_0 - 2 \sum_{n=1}^{N-1} d_{RT} \quad (n = 1, \dots, N) \quad (3.8)$$

$$\begin{aligned} \sum_{n=1}^N D_{n-1} &= D_0 + D_1 + D_2 + \dots + D_{N-1} \\ &= ND_0 - N(N-1)d_{RT} \end{aligned} \quad (3.9)$$

Substituting Eq. (3.9) into Eq. (3.7), the total machining cost for the N rough turning passes of a component can be expressed as

$$C_{RT} = \frac{\pi L}{1000 B_T} \left(\frac{B_T C_{mT}}{v_{RT} f_{RT}} + v_{RT}^{\alpha_T-1} f_{RT}^{\beta_T-1} d_{RT}^{\gamma_T} (C_{iT} + C_{rT} T_{rT}) \right) (ND_0 - N(N-1)d_{RT}) \quad (3.10)$$

The machining cost for finish turning can be calculated using Eq. (3.5) by simply dropping subscript n .

3.1.1.3 Grinding Cost

Based on Eq. (3.1), the unit grinding cost can be written as

$$C_G = C_{mG}T_G + (C_{tG} + C_{rG}T_{rG})\frac{T_G}{T_{LG}} \quad (3.11)$$

where C_{mG} is the grinding operation cost, T_G is the grinding time, T_{LG} the grinding wheel life, C_{tG} the grinding wheel dressing cost, and C_{rG} the wheel replacement cost. As compared to the turning operations, grinding is much more complex and requires more detailed analysis.

In this thesis, we consider only traverse cylindrical grinding (Fig. 3.3). The grinding time for workpiece with diameter D_{Gw} and length L is $T_G = Vol$ (volume removed) / MRR (material remove rate). The material removal rate for cylindrical grinding is $MRR = 1000 \cdot f_G \cdot d_G \cdot v_{Gw}$ (Capello and Semeraro, 2002), with f_G being the grinding feedrate, d_G grinding depth, and v_{Gw} surface speed of the rotating workpiece. Because $D_{Gw} \gg d_G$, the volume removed can be approximated by $Vol \cong \pi D_{Gw} d_G L$. Therefore, the grinding time is

$$T_G = \frac{Vol}{MRR} \cong \frac{\pi D_{Gw} L}{1000 v_{Gw} f_G} \quad (3.12)$$

Grinding is a machining process which utilizes a grinding wheel consisting of abrasive grains held together with a binder material. The actual cutting points on abrasive grains at the wheel surface are micro-cutting tools which interact with the workpiece. Except for some heavy-duty and cut-off operations, the grinding wheel topography and the macroscopic wheel shape are initially generated by a dressing process prior to grinding or periodically between grinding operations. Therefore, the grinding wheel life T_{LG} is defined as the grinding time between two dressing processes. A comprehensive

grinding wheel life equation incorporating wheel speed, workpiece speed and grinding depth is suggested by Shirai (1982) and adopted in this study:

(a)

(b)

Figure 3.3. Illustration of the external cylindrical grinding process (a) and the internal cylindrical grinding process (b)

$$T_{GL} = \frac{\pi D_s}{K_G v_{Gw} d_G} \left(\frac{v_{Gs}}{v_{Gw} d_G} \right)^{a_g} A \quad (3.13)$$

where K_G and A are constants, v_{Gs} the surface speed of the grinding wheel, and D_s the diameter of the grinding wheel. Generally, if the diameter of grinding grit is less than A , the wheel should be dressed. A is normally set to 50% of the dressing depth d_d (Xiao and Malkin, 1996).

Substituting Eqs. (3.12) and (3.13) into Eq. (3.11), the unit grinding cost is

$$C_G = C_{Gm} \frac{\pi D_{Gw} L}{1000 v_{Gw} f_G} + C_d \frac{K_G L D_{Gw} d_G^{a_g+1} v_{Gw}^{a_g}}{1000 A D_s f_G v_{Gs}^{a_g}} \quad (3.14)$$

where $D_{Gw} = D_G + 2d_G$ for external grinding, and $D_{Gw} = D_G - 2d_G$ for internal grinding, and D_G is the work (or hole) diameter after grinding.

3.1.1.4 Drilling Cost

The unit drilling cost also consists of three major components, i.e., operation cost, tool replacement cost and tool cost and is written as

$$C_D = C_{mD} T_D + (C_{wD} + C_{rD} T_{rD}) \frac{T_D}{T_{LD}} \quad (3.15)$$

where the tool-life is given as $T_{LD} = \left(\frac{K_{Dr} D_D^{x_{Dr}}}{v_D^{u_{Dr}} f_D^{z_{Dr}}} \right)^{w_{Dr}}$ (Meng, 1996) and the drilling time is

$$T_D = \frac{\pi D_D L}{1000 v_D f_D} \text{ with } D_D \text{ being the hole diameter (Fig 3.4a). Substituting } T_{LD} \text{ and } T_D$$

expressions into Eq. (3.15) leads to

$$C_D = \frac{\pi D_D L}{1000 v_D f_D} (C_{mD} + (C_{tD} + C_{rD} T_{rD}) \left(\frac{v_D^{U_{Dr}} f_D^{Z_{Dr}}}{K_{DT} D_D^{X_{Dr}}} \right)^{W_{Dr}}) \quad (3.16)$$

3.1.1.5 Boring Cost

Because the mechanism of a boring process (Fig 3.4b) is similar to that of a turning process, the same tool-life equation can be used with modified notations

$$T_{LB} = \left(\frac{K_{BT}}{v_B^{U_{BT}} d_B^{Y_{BT}} f_B^{Z_{BT}}} \right)^{W_{BT}} \quad (3.17)$$

The machining time can also be calculated similarly and hence the machining cost for a boring process is

$$C_B = \frac{\pi D_B L}{1000 v_B f_B} (C_{mB} + (C_{tB} + C_{rB} T_{rB}) \left(\frac{v_B^{U_{BT}} d_B^{Y_{BT}} f_B^{Z_{BT}}}{K_{BT}} \right)^{W_{BT}}) \quad (3.18)$$

3.1.1.6 Reaming Cost

The tool life equation for reaming is (Meng, 1996)

$$T_{LR} = \left(\frac{K_{RT} D_R^{X_{RT}}}{v_R^{U_{RT}} d_R^{Y_{RT}} f_R^{Z_{RT}}} \right)^{W_{RT}} \quad (3.19)$$

where D_R is the diameter of the reaming tool (Fig 3.4c). The machining cost for a reaming process is therefore

Figure 3.4. Schematic diagram of drilling (a) boring (b) and reaming (c) processes
 (* note: n is the rotational speed of the cutting tool - RPM)

$$C_{RE} = \frac{\pi D_R L}{1000 v_R f_R} (C_{mR} + (C_{tR} + C_{tR} T_{tR}) (\frac{v_R^{U_{RT}} d_R^{Y_{RT}} f_R^{Z_{RT}}}{K_{RT} D_R^{X_{RT}}})^{W_{RT}}) \quad (3.20)$$

3.1.2 Quality loss

Quality loss is the cost (such as maintenance cost, lost use because of malfunction, pollution, etc.) incurred after product release due to the deviation of a quality characteristic from its target value. In this study, the asymmetric loss function is employed since the rework cost and scrap cost are not necessarily identical. According to Taguchi *et al.* (1989), the quality loss function can be expressed as a quadratic function of the quality deviation from the target dimension. If the quality loss is different in both directions around the target, the quality loss function for the asymmetric case is

$$L(X) = \begin{cases} K_1(x-D)^2 & (x < D) \\ K_2(x-D)^2 & (x \geq D) \end{cases} \quad (3.21)$$

where T_L and T_U are respectively the lower and upper tolerance limits, x the quality characteristic of the product, and D the quality target value, $K_1 = \frac{C_S}{T_L^2}$ and $K_2 = \frac{C_R}{T_U^2}$ for external rotational and planar surfaces machining, and $K_1 = \frac{C_R}{T_L^2}$ and $K_2 = \frac{C_S}{T_U^2}$ for internal rotational surface machining.

The loss function is used only for the finish machining stage. For the other machining stages, there will be no loss function applied if the dimension falls within the

process tolerance range and the cost will be C_R or C_S if the dimension is below or above the process tolerance limits. These scenarios are shown in Fig. 3.5.

(a)

(b)

Figure 3.5. Scrap cost, rework cost and quality loss for (a) non-finish stages ($1 \leq i \leq I$), (b) finish stage ($i = I$)

3.1.3 Expected Unit Cost

Expected unit cost will be estimated in this section for external, internal rotational surfaces, and planar surfaces based on the following assumptions.

- a) There are I manufacturing stages and J available machining processes for machining a type of components. Only one process is selected in each stage.

- b) 100% inspection will be performed after each manufacturing stage.
- c) For external rotational surface and planar surface machining, the machined components with dimensions above the specification will be reworked and those with dimensions below the specification will be scrapped. For internal rotational surface machining, the surface will be re-machined if its dimension is below the specification and scrapped if above the specification.
- d) All the reworked components conform to specification.
- e) Only the conforming components can be sent to next manufacturing stage.
- f) Dimension measurement x of the machined components follows a normal distribution with known process mean and process variance.

The expected unit costs in different stages are analyzed in next section.

3.1.3.1 *Expected unit cost after stage ($i = 1$)*

If process j ($j = 1, \dots, J$) in stage 1 is selected, all the raw materials are machined to their working dimension D_{1j} with tolerance t_{1j} by process j of stage 1. All the machined components are inspected and classified into three groups: a) those conforming to the specification ($D_{1j} - t_{1j} \leq x \leq D_{1j} + t_{1j}$), b) those above the specification ($x \geq D_{1j} + t_{1j}$) and c) those below the specification ($x \leq D_{1j} - t_{1j}$). The cost analysis is given below for each of the three groups of the components.

a) *When $D_{1j} - t_{1j} \leq x \leq D_{1j} + t_{1j}$*

When the dimension measurement x of the finished part falls into an acceptable range, its manufacturing cost is the sum of unit material cost C_U , unit machining cost

C_{1j} , and unit inspection cost C_{A1j} . Hence, the unit cost of a conforming component that has been machined by process j in stage 1 is

$$C_{MC1j} = \int_{D_{1j}-t_{1j}}^{D_{1j}} (C_U + C_{1j} + C_{A1j}) f(x) dx + \int_{D_{1j}}^{D_{1j}+t_{1j}} (C_U + C_{1j} + C_{A1j}) f(x) dx \quad (3.22)$$

b) When $x \geq D_{1j} + t_{1j}$

For external rotational and planar surfaces machining, if the dimension of a component falls above the specification, it needs to be reworked. In this case, the unit cost is

$$C_{MR1j} = \int_{D_{1j}+t_{1j}}^{+\infty} (C_U + C_{1j} + C_{A1j} + C_{R1j}) f(x) dx \quad (3.23)$$

Here, we assume that the component can be reworked by process j with machining cost C_{1j} , and needs to be inspected after rework. Therefore, the rework cost is $C_{R1j} = C_{1j} + C_{A1j}$.

For internal rotational surface machining, if the dimension of finished component falls above the specification, it needs to be scraped and the unit cost is

$$C_{MS1j} = \int_{D_{1j}+t_{1j}}^{+\infty} C_{S1j} f(x) dx \quad (3.24)$$

where the unit scrap cost is the sum of unit material cost C_U , unit machining cost C_{1j} , and unit inspection cost C_{A1j} , and is written as $C_{S1j} = C_U + C_{1j} + C_{A1j}$.

c) When $x \leq D_{1j} - t_{1j}$

For external rotational and planar surfaces machining, when the dimension of a part falls below the lower limits, it will be scrapped. As mentioned before, scrap cost is given by $C_{S1j} = C_U + C_{1j} + C_{A1j}$. Hence the unit cost is

$$C_{MS1j} = \int_{-\infty}^{D_{1j}-t_{1j}} (C_U + C_{1j} + C_{A1j})f(x)dx \quad (3.25)$$

For internal rotational surface machining, when the dimension of finished component falls below the specification, it has to be reworked and the unit cost is

$$C_{MR1j} = \int_{-\infty}^{D_{1j}-t_{1j}} (C_U + 2C_{1j} + 2C_{A1j})f(x)dx \quad (3.26)$$

d) The expected unit cost of per acceptable component

For external rotational and planar surfaces machining, the expected unit cost is obtained by combining Eqs. (3.22), (3.33) and (3.35) with Eqs. (A.1) to (A.3) of Appendix A as follows:

$$C_{M1j} = C_{MC1j} + C_{MR1j} + C_{MS1j} = C_U + C_{1j}(2 - \Phi(Z_{21j})) + C_{A1j}(2 - \Phi(Z_{21j})) \quad (3.27)$$

Similarly, for internal rotational surface machining, we can obtain the expected unit cost by summing up Eqs. (3.22), (3.34), (3.36), (A.4) to (A.7):

$$C_{M1j} = C_{MC1j} + C_{MR1j} + C_{MS1j} = C_U + C_{1j}(1 + \Phi(Z_{11j})) + C_{A1j}(1 + \Phi(Z_{11j})) \quad (3.28)$$

Because all the un-repairable components are scrapped and all the reworked components conform to the specification, the fraction of conforming components that enter the next

manufacturing stage is $1 - \Phi(Z_{11j})$ after an external or planar surface machining process or $\Phi(Z_{21j})$ after an internal rotational surface machining process. The expected unit cost per acceptable component after process j of the first stage is therefore:

For external rotational or planar surface machining:

$$C_{UM1j} = \frac{C_U + (2 - \Phi(Z_{21j}))C_{1j} + (2 - \Phi(Z_{21j}))C_{A1j}}{1 - \Phi(Z_{11j})} \quad (3.29)$$

For internal rotational surface machining:

$$C_{UM1j} = \frac{C_U + (1 + \Phi(Z_{11j}))C_{1j} + (1 + \Phi(Z_{11j}))C_{A1j}}{\Phi(Z_{21j})} \quad (3.30)$$

3.1.3.2 The Expected unit cost after process j of an intermediate stage

$(1 < i < I)$

The expected unit cost per acceptable component after process j of an intermediate stage i , C_{UMij} is a sum of the cost incurred in the current process and the cost of all the previously selected processes. Hence, it is written as in an accumulative format by substituting C_U with $\sum_{j=1}^J m_{(i-1)j} C_{UM(i-1)j}$ in Eqs. (3.29) and (3.30):

For external rotational or planar surface machining:

$$C_{UMij} = \frac{\sum_{j=1}^J m_{(i-1)j} C_{UM(i-1)j} + (2 - \Phi(Z_{2ij}))C_{ij} + (2 - \Phi(Z_{2ij}))C_{Aij}}{1 - \Phi(Z_{1ij})} \quad (3.31)$$

For internal rotational surface machining:

$$C_{UMij} = \frac{\sum_{j=1}^J m_{(i-1)j} C_{UM(i-1)j} + (1 + \Phi(Z_{1ij})) C_{ij} + (1 + \Phi(Z_{1ij})) C_{Aij}}{\Phi(Z_{2ij})} \quad (3.32)$$

3.1.3.3 The Expected unit cost after process j of the last stage ($i = I$)

After finishing the last stage I , all components are again classified into three groups: the components that conform to the design specification ($D - T_L \leq x \leq D + T_U$), those above the design specification ($x \geq D + T_U$), and those below the design specification ($x \leq D - T_L$). The asymmetric design tolerance is used in this case. For the symmetric design tolerance case, T_L and T_U can be simply replaced with T . The expected unit cost per acceptable component after stage I is calculated as follows

For external rotational or planar surface machining:

$$C_{UMIj} = \frac{\sum_{j=1}^J m_{(I-1)j} C_{UM(I-1)j} + (2 - \Phi(Z_{2Ij})) C_{Ij} + (2 - \Phi(Z_{2Ij})) C_{AIj} + E_c(L(x))_{Ij}}{1 - \Phi(Z_{1Ij})} \quad (3.33)$$

For internal rotational surface machining:

$$C_{UMIj} = \frac{\sum_{j=1}^J m_{(I-1)j} C_{UM(I-1)j} + (1 + \Phi(Z_{1Ij})) C_{Ij} + (1 + \Phi(Z_{1Ij})) C_{AIj} + E_c(L(x))_{Ij}}{\Phi(Z_{2Ij})} \quad (3.34)$$

3.2 Model Formulation

In the above cost analysis, processes, machining parameters and tolerance limits have all been included to explicitly consider their mutual effects on tangible costs. Minimizing the expected unit cost will provide outputs for process, machining parameters and tolerance decisions. However, the intangible cost due to tight tolerance has not been taken into account. To alleviate the cost burden due to the tight tolerance, we suggest that the tolerance allocated to each chosen process be increased or relaxed as much as possible so as to reduce the intangible cost to the minimum. However, simply maximizing tolerance will increase quality loss, and then lead to a large increase in the tangible cost, i.e., the expected unit cost. For this reason, a bi-criterion model is proposed which will be applied to both models presented in next two sub-sections.

3.2.1 The single component single part feature case (Model 1)

A surface feature needs to go through several processes to meet its specification requirement. We consider its tangible cost (C_{umj} the expected unit cost after process j of the last stage I) and intangible cost as two objectives to be minimized. It should be pointed out that the latter is implemented by maximizing the sum of the tolerances of those surfaces created by intermediate manufacturing processes t_{ij} . A number of constraints such as the relationship between two adjacent processes, the physical constraints of cutting power, cutting force, and surface finish, and total depth of cut requirement are considered in the model. The model is given as follows:

Model 1

$$\text{Min } \text{sum_cost} = \sum_{j=1}^J m_{ij} C_{UMj} \quad (3.35)$$

$$\text{Min } \text{sum_tol} = \sum_{i=1}^I \sum_{j=1}^J m_{ij} t_{ij} \quad (3.36)$$

Subject to

(a1) Machining process selection constraints

$$\sum_{j=1}^J m_{ij} = 1 \quad \forall i \quad (3.37)$$

(b1) Design specification

$$D - T_L \leq \sum_{j=1}^J m_{ij} (D_{ij} \pm t_{ij}) \leq D + T_U \quad (3.38)$$

(c1) Dimension allowances

For external rotational surface machining:

$$d_{ij}^l \leq \left(\sum_{j=1}^J m_{i-1j} D_{i-1j} - \sum_{j=1}^J m_{ij} D_{ij} \pm \left(\sum_{j=1}^J m_{i-1j} t_{i-1j} + \sum_{j=1}^J m_{ij} t_{ij} \right) \right) / 2 \leq d_{ij}^u \quad i = 2, \dots, I \quad (3.39)$$

For internal rotational surface machining:

$$d_{ij}^l \leq \left(\sum_{j=1}^J m_{ij} D_{ij} - \sum_{j=1}^J m_{i-1j} D_{i-1j} \pm \left(\sum_{j=1}^J m_{i-1j} t_{i-1j} + \sum_{j=1}^J m_{ij} t_{ij} \right) \right) / 2 \leq d_{ij}^u \quad i = 2, \dots, I \quad (3.40)$$

For planar surface machining:

$$d_{ij}^l \leq \sum_{j=1}^J m_{ij} d_{ij} \pm \left(\sum_{j=1}^J m_{i-1j} t_{i-1j} + \sum_{j=1}^J m_{ij} t_{ij} \right) \leq d_{ij}^u \quad i = 2, \dots, I \quad (3.41)$$

(d1) Total depth of cut

For external rotational surface machining:

$$\sum_{j=1}^J m_{ij} D_{ij} + 2 \sum_{i=2}^I \sum_{j=1}^J m_{ij} d_{ij} + 2m_{11} N d_{11} = D_0 \quad (3.42)$$

For internal rotational surface machining:

$$D_{11} + 2 \sum_{i=2}^I \sum_{j=2}^J m_{ij} d_{ij} = \sum_{j=1}^J m_{ij} D_{ij} \quad (3.43)$$

For planar surface machining:

$$\sum_{i=2}^I \sum_{j=1}^J m_{ij} d_{ij} + m_{11} S d_{11} = d_{Tot} \quad (3.44)$$

(e1) Consistency of workpiece or hole diameters between consecutive processes

For external rotational surface machining:

$$\sum_{j=1}^J m_{i-1,j} D_{i-1,j} = \sum_{j=1}^J m_{ij} (D_{ij} + 2d_{ij}) \quad i = 2, \dots, I \quad (3.45)$$

For internal rotational surface machining:

$$\sum_{j=1}^J m_{i-1,j} D_{i-1,j} = \sum_{j=1}^J m_{ij} (D_{ij} - 2d_{ij}) \quad i = 2, \dots, I \quad (3.46)$$

(f1) Process capability constraints

$$t_{ij}^l \leq t_{ij} \leq t_{ij}^u \quad \{i, j \mid m_{ij} = 1\} \quad (3.47)$$

(g1) Product quality requirements

$$\Phi(Z_{2lj}) - \Phi(Z_{1lj}) \geq 1 - \alpha \quad \{j | m_{lj} = 1\} \quad (3.48)$$

(h1) Restriction of machining parameters

$$v_{ij}^l \leq v_{ij} \leq v_{ij}^u \quad \{i, j | m_{ij} = 1\} \quad (3.49)$$

$$f_{ij}^l \leq f_{ij} \leq f_{ij}^u \quad \{i, j | m_{ij} = 1\} \quad (3.50)$$

$$d_{ij}^l \leq d_{ij} \leq d_{ij}^u \quad \{i, j | m_{ij} = 1\} \quad (3.51)$$

(i1) Constraints of cutting force, power and surface roughness for milling (Zhao, 1992)

$$K_{MP1} D_C^{K_{MP2}} v_{ij}^{K_{MP3}} f_{ij}^{K_{MP4}} d_{ij}^{K_{MP5}} d_e N_t \leq P_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.52)$$

From $P = \frac{Fv}{60E}$, we can obtain cutting force constraint

$$K_{MF1} D_C^{K_{MF2}} v_{ij}^{K_{MF3}} f_{ij}^{K_{MF4}} d_{ij}^{K_{MF5}} d_e N_t E \leq F_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.53)$$

(j1) Constraints of cutting force, power and surface roughness for turning and boring (Meng, 1996)

$$B_F v_{ij}^{\alpha_F} f_{ij}^{\beta_F} d_{ij}^{\gamma_F} \leq F_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.54)$$

$$B_P v_{ij}^{\alpha_P} f_{ij}^{\beta_P} d_{ij}^{\gamma_P} \leq P_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.55)$$

$$B_R r^\eta BHN^\theta v_{ij}^{\alpha_R} f_{ij}^{\beta_R} \leq R_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.56)$$

(k1) Constraints of cutting force, power, and surface roughness for grinding (Meng, 1996, and Malkin, 1989)

$$K_{GF1j} E (v_{ij} f_{ij} d_{ij})^{K_{GF2j}} k_{GF3j} k_{GF4j} / v_{Gsj} \leq F_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.57)$$

$$K_{GP1j}(v_{ij}f_{ij}d_{ij})^{K_{GP2j}}k_{GP3j}k_{GP4j} \leq P_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.58)$$

$$R_{3j}s_{dj}^{K_{R1j}}d_{dj}^{K_{R2j}}\left(\frac{v_{ij}d_{ij}}{v_{Gsj}}\right)^{K_{R3j}} \leq R_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.59)$$

(11) Constraints of thrust force of drilling and reaming processes (Zhao, 1992)

$$K_{Fj}D_{ij}^{X_{\eta}}d_{ij}^{Y_{\eta}}f_{ij}^{Z_{\eta}} \leq F_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.60)$$

(m1) Constraints of torque of drilling and reaming processes (Zhao, 1992)

$$K_{Mj}D_{ij}^{X_{\mathcal{M}}}d_{ij}^{Y_{\mathcal{M}}}f_{ij}^{Z_{\mathcal{M}}} \leq M_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.61)$$

(n1) Constraints of power of drilling and reaming processes (Zhao, 1992)

$$F_{ij}M_{ij} / K_{GPj} \leq P_{\max j} \quad \{i, j | m_{ij} = 1\} \quad (3.62)$$

The process selection, machining parameter selection and tolerance design decisions can be made simultaneously based on the solution of the above bi-criterion model. Constraint (a1) specifies that exactly one process is selected for each manufacturing stage. Constraint (b1) ensures that the resultant deviation of dimensions and tolerances are within design specifications. Constraint (c1) limits the machining allowances to permissible stock removal ranges of the selected process. Constraint (d1) states that the final diameter and total depth of cut should add up to the original workpiece diameter. The diameter consistency between consecutive processes was guaranteed by constraint (e1). Constraint (f1) ensures that the selected tolerance will not exceed the process capability. It is also necessary to meet the requirement of the specified percentage of acceptable components coming off the last process. This is imposed by constraint (g1). The machining parameter selection ranges are specified using constraint

(h1). The cutting force, power, and surface quality restrictions for various processes are written as constraints (i1) to (n1). In constraints (k1), the value of R_3 can be identified or estimated from the measured roughness, d_d is the dressing depth, S_d is the dressing lead for single point diamond dressing, and typically $K_{R3} \approx 0.5$.

3.2.2 The assembled product case (Model 2)

This sub-section presents a model for an assembly which involves several components. In this case, the distribution of the specified assembly tolerance among different components will also be considered.

We consider an assembly with H part features to meet requirement of Q quality characteristics. Since there are multiple features, the sum of the tangible cost and intangible cost of these features in the assembly are used as the two objectives to be optimized. The latter is achieved by maximizing the sum of the all H part features' process tolerances t_{hij} . In addition to the constraints used in Model 1, the worst case (WC) and root sum square tolerance (RSS) stackup constraints are also used to deal with the tolerance distribution problem. The proposed model for the assembled product case can be formulated as follows:

Model 2

$$\text{Min } \text{Sum_cost}_A = \sum_{h=1}^H \sum_{j=1}^{J_h} m_{h,j} C_{UMh,j} \quad (3.63)$$

$$\text{Max } \text{Sum_tol}_A = \sum_{h=1}^H \sum_{i=1}^{I_h} \sum_{j=1}^{J_h} m_{hij} t_{hij} \quad (3.64)$$

Subject to

(a2) Machining process selection constraints

$$\sum_{j=1}^{J_h} m_{hij} = 1 \quad \forall h, i \quad (3.65)$$

(b2) Design target constraints

$$\sum_{j=1}^{J_h} m_{hij} D_{hij} = D_h \quad \forall h, i \quad (3.66)$$

(c2) Dimension allowances

For external rotational machining

$$d_{hij}^l \leq \left(\sum_{j=1}^{J_h} m_{hi-1j} D_{hi-1j} - \sum_{j=1}^{J_h} m_{hij} D_{hij} \pm \left(\sum_{j=1}^{J_h} m_{hi-1j} t_{hi-1j} + \sum_{j=1}^{J_h} m_{hij} t_{hij} \right) \right) / 2 \leq d_{hij}^u \quad i = 2, \dots, I_h \quad \forall h \quad (3.67)$$

For internal rotational machining

$$d_{hij}^l \leq \left(\sum_{j=1}^{J_h} m_{hij} D_{hij} - \sum_{j=1}^{J_h} m_{hi-1j} D_{hi-1j} \pm \left(\sum_{j=1}^{J_h} m_{hi-1j} t_{hi-1j} + \sum_{j=1}^{J_h} m_{hij} t_{hij} \right) \right) / 2 \leq d_{hij}^u \quad i = 2, \dots, I_h \quad \forall h \quad (3.68)$$

For planar surface machining

$$d_{hij}^l \leq \sum_{j=1}^{J_h} m_{hij} d_{hij} \pm \left(\sum_{j=1}^{J_h} m_{hi-1j} t_{hi-1j} + \sum_{j=1}^{J_h} m_{hij} t_{hij} \right) \leq d_{hij}^u \quad i = 2, \dots, I_h \quad \forall h \quad (3.69)$$

(d2) Total depth of cut

For external rotational surface machining

$$\sum_{j=1}^{J_h} m_{hij} D_{hij} + 2 \sum_{i=2}^{I_h} \sum_{j=1}^{J_h} m_{hij} d_{hij} + 2m_{h11} N_h d_{h11} = D_{0h} \quad \forall h \quad (3.70)$$

For internal rotational surface machining

$$D_{h11} + 2 \sum_{i=2}^{I_h} \sum_{j=2}^{J_h} m_{hij} d_{hij} = \sum_{j=1}^{J_h} m_{h1j} D_{h1j} \quad \forall h \quad (3.71)$$

For planar surface machining

$$\sum_{i=2}^{I_h} \sum_{j=1}^{J_h} m_{hij} d_{hij} + m_{h11} S_h d_{h11} = d_{Tolh} \quad \forall h \quad (3.72)$$

(e2) Consistency of workpiece dimensions between consecutive processes

For external rotational surface machining

$$\sum_{j=1}^{J_h} m_{hi-1j} D_{hi-1j} = \sum_{j=1}^{J_h} m_{hij} (D_{hij} + 2d_{hij}) \quad i = 2, \dots, I_h \quad (3.73)$$

For internal rotational surface machining

$$\sum_{j=1}^{J_h} m_{hi-1j} D_{hi-1j} = \sum_{j=1}^{J_h} m_{hij} (D_{hij} - 2d_{hij}) \quad i = 2, \dots, I_h \quad (3.74)$$

For planar surface machining

$$\sum_{j=1}^{J_h} m_{hi-1j} D_{hi-1j} = \sum_{j=1}^{J_h} m_{hij} (D_{hij} + d_{hij}) \quad i = 2, \dots, I_h \quad (3.75)$$

(f2) Process capability constraints

$$t_{hij}^l \leq t_{hij} \leq t_{hij}^u \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.76)$$

(g2) Product quality requirements

$$\Phi(Z_{2h1j}) - \Phi(Z_{1h1j}) \geq 1 - \alpha_h \quad \{j \mid m_{h1j} = 1\} \quad (3.77)$$

(h2) Restriction of machining parameters

$$v_{hij}^l \leq v_{hij} \leq v_{hij}^u \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.78)$$

$$f_{hij}^l \leq f_{hij} \leq f_{hij}^u \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.79)$$

$$d_{hij}^l \leq d_{hij} \leq d_{hij}^u \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.80)$$

(i2) Constraints of cutting force, power, and surface roughness for milling

$$K_{MP1} D_C^{K_{MP2}} v_{hij}^{K_{MP3}} f_{hij}^{K_{MP4}} d_{hij}^{K_{MP5}} d_e N_t \leq P_{\max j} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.81)$$

$$K_{MF1} D_C^{K_{MF2}} v_{hij}^{K_{MF3}} f_{hij}^{K_{MF4}} d_{hij}^{K_{MF5}} d_e N_t E \leq F_{\max j} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.82)$$

(j2) Constraints of cutting force, power, and surface roughness for turning and boring

$$B_F v_{hij}^{\alpha_F} f_{hij}^{\beta_F} d_{hij}^{\gamma_F} \leq F_{\max hj} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.83)$$

$$B_P v_{hij}^{\alpha_P} f_{hij}^{\beta_P} d_{hij}^{\gamma_P} \leq P_{\max hj} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.84)$$

$$B_R r^\eta BHN^\theta v_{hij}^{\alpha_R} f_{hij}^{\beta_R} \leq R_{\max hj} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.85)$$

(k2) Constraints of cutting force, power, and surface roughness for grinding

$$K_{GF1j} E (v_{hij} f_{hij} d_{hij})^{K_{GF2j}} k_{GF3j} k_{GF4j} / v_{Gsj} \leq F_{\max j} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.86)$$

$$K_{GP1j} (v_{hij} f_{hij} d_{hij})^{K_{GP2j}} k_{GP3j} k_{GP4j} \leq P_{\max j} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.87)$$

$$R_{3j} s_{dj}^{K_{R1j}} d_{dj}^{K_{R2j}} \left(\frac{v_{hij} d_{hij}}{v_{Gsj}} \right)^{K_{R3j}} \leq R_{\max j} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.88)$$

(l2) Constraints of thrust force of drilling and reaming processes

$$K_{Fj} D_{hij}^{X_{Fj}} d_{hij}^{Y_{Fj}} f_{hij}^{Z_{Fj}} \leq F_{\max hj} \quad \{h, i, j \mid m_{hij} = 1\} \quad (3.89)$$

(m2) Constraints of torque of drilling and reaming processes

$$K_{Mj} D_{hij}^{x_M} d_{hij}^{y_M} f_{hij}^{z_M} \leq M_{\max hj} \quad \{h, i, j | m_{hij} = 1\} \quad (3.90)$$

(n2) Constraints of power of drilling and reaming processes

$$F_{hij} M_{hij} / K_{GPhj} \leq P_{\max hj} \quad \{h, i, j | m_{hij} = 1\} \quad (3.91)$$

(o2) Design tolerance constraints

$$T_h = \sum_{j=1}^{J_h} m_{hij} t_{hij} \quad \forall h \quad (3.92)$$

(p2) Tolerance stackup constraints

For the worst case (WC) constraint

$$\sum_{h=1}^H \left(\left| \frac{\partial Y_q}{\partial D_h} \right| T_h \right) \leq T_{Yq} \quad \forall q \quad (3.93)$$

For the root sum square (RSS) constraint

$$\sqrt{\sum_{h=1}^H \left(\left(\frac{\partial Y_q}{\partial D_h} \right)^2 T_h^2 \right)} \leq T_{Yq} \quad \forall q \quad (3.94)$$

The process selection, machining parameter selection and tolerance design and distribution decisions for the assembled product case can be made simultaneously based on the solution of the above Model 2. Constraints (a2) to (n2) impose restrictions similar to those specified by constraints (a1) through (n1) in Model 1. The new constraint (o2) requires that the design tolerance T_h of part feature h be equal to its last process

tolerance $t_{h,j}$. Constraint (p2) ensures that the cumulative tolerance of components for a quality characteristic does not exceed the associated design tolerance. It should be noted that only one of the two tolerance stackup constraints is to be used depending on the situation.

Chapter 4

Solution Procedure and Examples

This chapter consists of two sections. The first section illustrates the solution procedure of Model 1 developed in Chapter 3. Two examples are given and solved to illustrate the application of the Model 1 for two types of surface features (external and internal rotational surfaces) respectively. Two classical examples taken from existing publications are used in the second section for Model 2. The first example contains a single quality characteristic and the second is a multi-quality-characteristic problem. Three types of surface features are involved in the two examples.

4.1 Examples for model 1

Figure 4.1 shows a shaft and a hole. The specified diameter of the shaft and the diameter of the hole are respectively $50.8^{+0.007}_{-0.017}$ mm and $50.856^{+0.019}_{-0.007}$ mm. Four external machining stages are needed for the shaft. The first stage contains several rough turning passes and the other three stages require only one pass. The associated machining stages and processes are listed in Table 4.1a.

Table 4.1a: Machining stages and processes for the shaft

<i>Machining processes</i>	<i>Machining process</i>	
	<i>j = 1</i> turning pass	<i>j = 2</i> grinding pass
<i>i = 1</i> Primary	<i>N</i> rough turning passes	
<i>i = 2</i> Rough	Finish turning	
<i>i = 3</i> Semi-finish		Rough grinding
<i>i = 4</i> Finish		Finish grinding

(a) A rotational stock and a finished shaft

(b) A rotational stock and a finished hole

Figure 4.1. Transformation of stocks to finished parts
(Note: all dimensions are in mm)

The internal surface machining of the hole is also to be carried out in four stages. There are two alternative processes for stage 3 and three alternatives for stage 4. The machining stages and processes are shown in Table 4.1b.

Table 4.1 b: Machining stages and processes for the hole

<i>Machining stage</i>	<i>Machining process</i>			
	<i>j</i> = 1 drilling	<i>j</i> = 2 boring	<i>j</i> = 3 reaming	<i>j</i> = 4 grinding
<i>i</i> = 1 Primary	Drilling			
<i>i</i> = 2 Rough		Rough boring		
<i>i</i> = 3 Semi-finish		Semi-finish boring	Semi-finish reaming	
<i>i</i> = 4 Finish		Finish boring	Finish reaming	Grinding

4.1.1 Example 1: the shaft (external rotational surface machining)

The diameter and length of the initial stock are 68mm and 60mm, respectively. The diameter of the shaft is to be machined to 50.8mm at a length of 60mm. The tolerance level is specified as $T_L = 0.017$ mm and $T_U = 0.007$ mm. The cast iron shaft is first to be turned using carbide tools from a rotational stock. The traverse cylindrical grinding is then performed in this example. A silicon carbide grinding wheel (37C46-L-VK) is chosen from the Industrial Catalogue of Norton Ltd for grinding the cast iron shaft. The grinding wheel is 25.4 mm wide with grit of diameter 0.35 mm (grit size 46) and the maximum grinding wheel surface speed is 2040 m/min. The single-point diamond dresser is used in this example. The parameters in the tool life, surface roughness, cutting force, cutting power equations and other input data are given in Table 4.2.

Table 4.2. Input data for shaft machining

<i>Parameter</i>	<i>Value</i>
Unit raw material cost C_U	\$10
Process capability index $C_{p11}, C_{p21}, C_{p32}, C_{p42}$	1.2, 1.1, 1.0, 0.85
Percentage of allowable rejected components α	5.3%
Unit inspection cost $C_{A11}, C_{A21}, C_{A32}, C_{A42}$	\$1.0, \$1.5, \$2.0, \$2.5
Operation cost of C_{m1}, C_{m2}	\$1/min, \$5/min
Tool replacement cost and time $C_{r1}, C_{r1}, T_{r1}, T_{r1}$	\$1/min, \$1/min; 1 min, 1 min
Tool cost C_{t1}, C_{t2}	\$3/edge, \$10/dressing
Grinding wheel diameter D_s	355.6 mm (14")
Grinding wheel surface speed v_{Gs}	1800 m min ⁻¹
Dressing depth d_d	0.02 mm
Dressing lead s_d	0.15 mm/rev
Cutting speed limits $v'_{11}, v''_{11}, v'_{21}, v''_{21}, v'_{32}, v''_{32}, v'_{42}, v''_{42}$	50, 130, 140, 200, 10, 25, 20, 40 m/min
Feed rate limits $f'_{11}, f''_{11}, f'_{21}, f''_{21}, f'_{32}, f''_{32}, f'_{42}, f''_{42}$	0.5, 1.0, 0.1, 0.4, 6, 12, 1, 6 mm rev ⁻¹
Depth of cut limits $d'_{11}, d''_{11}, d'_{21}, d''_{21}, d'_{32}, d''_{32}, d'_{42}, d''_{42}$	1.5, 5.0, 0.5, 1.5, 0.05, 0.5, 0.005, 0.05 mm

Machine tool efficiency E	75 %
Constants for turning tool life $\alpha_T, \beta_T, \gamma_T, B_T$	5.0, 2.0, 0.75, 847288609443
Constants for grinding wheel life K_G, α_G, A	2.4, 0.2, 0.01 mm
Constants for turning force $\alpha_F, \beta_F, \gamma_F, B_F$	0, 0.75, 1.0, 0.9
Constants for grinding force $K_{GF1}, K_{GF2}, K_{GF3}, K_{GF4},$	86.4, 0.7, 0.8, 1.05
Constants for turning surface roughness $BHN, r, \alpha_R, \beta_R, \gamma_R, \eta, \theta, B_R$	195, 1, -1.52, 1.004, 0, -0.714, -0.323, 122
Constants for grinding surface roughness $R_3, K_{R1}, K_{R2}, K_{R3}$	0.2, 0.5, 0.25, 0.5
Constants for turning power $\alpha_P, \beta_P, \gamma_P, B_P$	1.0, 0.75, 1, 0.02
Constants for grinding power $K_{GP1}, K_{GP2}, K_{GP3}, K_{GP4}$	1.44, 0.7, 0.8, 1.05
Maximum force, power $F_{\max 1}, F_{\max 2}, P_{\max 1}, P_{\max 2}$	4.5 kN, 1.5 kN, 10 kW, 12 kW
Surface roughness $R_{amax21}, R_{amax32}, R_{amax42}$	0.0032, 0.0025, 0.00125 mm
Tolerance limits $t_{11}^l, t_{11}^u, t_{21}^l, t_{21}^u, t_{32}^l, t_{32}^u, t_{42}^l, t_{42}^u$	0.508, 0.127, 0.305, 0.05, 0.076, 0.013, 0.025, 0.005 mm

The bi-criterion model is solved using the modified Chebyshev goal programming approach (also called fuzzy programming) (Ignizio and Vavalier, 1994) and Lingo software. The procedure is detailed as follows

Step 1. Formulate the bi-criterion model

$$\text{Min } \text{sum_cost} = \sum_{j=1}^2 m_{4j} C_{UM4j}$$

$$\text{Max } \text{sum_tol} = \sum_{i=1}^4 \sum_{j=1}^2 m_{ij} t_{ij}$$

Subject to: all the constraints (as listed in Model1, Chapter 3)

Step 2. Solve the problem with one objective at a time

Solving Min sum_cost with all the constraints, we obtain

$$\text{sum_cost}_1^* = 18.21315 \quad \text{and} \quad \text{sum_tol}_1^* = 0.195$$

Other outputs are omitted due to space limit. Solving Max sum_tol with all the constraints, we obtain

$$sum_cost_2^* = 23.98925 \quad \text{and} \quad sum_tol_2^* = 0.858$$

The objective values of the above solutions are summarized below:

	Min sum_cost	Max Sum_tol	Difference
Value of sum_cost	18.21315	23.98925	5.77610
Value of sum_tol	0.195	0.858	0.663

As we can see, the two objectives conflict with each other. Minimizing sum_cost leads to a lower sum_tol and maximizing sum_tol causes increase in sum_cost . Neither situation is desirable particularly when the differences are as substantial as shown in the above table. This calls for a compromised solution by solving the modified Chebyshev goal programming problem in the next step.

Step 3. Formulate and solve the modified Chebyshev goal programming problem

Max δ

Subject to

$$\delta \leq \frac{(sum_tol - sum_tol_1^*)}{(sum_tol_2^* - sum_tol_1^*)} = (sum_tol - 0.195) / 0.663$$

$$\delta \leq \frac{(sum_cost_2^* - sum_cost)}{(sum_cost_2^* - sum_cost_1^*)} = (23.98925 - sum_cost) / 5.77610$$

and all the other constraints.

The notion is that the best deviation from the two worst values of two objectives is obtained by maximizing δ . Solving this problem, we obtain

$$\delta = 0.9510511, \text{ sum_cost}^* = 18.49588, \text{ and } \text{sum_tol}^* = 0.826$$

The unit cost is now \$18.50 which is very close to \$18.21 obtained by minimizing cost as the only objective and is substantially lower than \$23.99, the cost resulted by considering the tolerance objective alone. The tolerance objective value is now 0.826, 4.23 times higher than that yielded by considering cost as a single objective and is only 3.73% lower than 0.858, the optimal value obtained by solely maximizing the total tolerance. The other outputs are summarized in Table 4.3.

Table 4.3. Output of the shaft problem

<i>Parameters</i>	<i>Symbols</i>	<i>Outputs</i>			
Stage index	<i>i</i>	1	2	3	4
Index of the selected process	<i>j</i>	1	1	2	2
Number of rough turning	<i>N</i>	2			
Cutting speed	v_{ij}	114.50000	200.00000	10.70000	40.00000
Feed rate	f_{ij}	1.000000	0.400000	12.000000	6.000000
Depth of cut	d_{ij}	3.737973	0.903277	0.205777	0.0144963
Process diameter	D_{ij}	53.04811	51.24155	50.83000	50.80101
Process tolerance	t_{ij}	0.50800	0.30500	0.01300	0.00599

¹Note: all the data in this table are obtained directly from Lingo outputs. They have not been rounded since rounding would cause inaccuracy in the final calculation.

Table 4.3 shows that two rough passes are needed with a depth of cut of 3.738 mm for each pass, feed rate of 1 mm/rev and cutting speed of 114.5 m/min. For the second stage, i.e., the finish turning, the cutting speed, feed rate and depth of cut are respectively 200.0 m/min, 0.4 mm/rev., and 0.903 mm. The machining parameters for the rough and finish grinding processes are also provided in the table. The process means (diameters of the components) after the four processes are 53.04811 mm, 51.24155 mm, 50.83000 mm,

and 50.80101 mm respectively. The decision on process tolerances can also be made simultaneously. The process tolerances for the four processes are respectively 0.50800 mm, 0.30500 mm, 0.01300 mm, and 0.00599 mm.

4.1.2 Example 2: the hole (internal rotational surface machining)

The length and required diameter of the hole are 100 mm and 50.856 mm, respectively. The material of the rotational stock is cast iron. The diameter of the hole is machined to 50.856 mm at a length of 100 mm by a combination of drilling, boring, reaming, and grinding passes. The tolerance level is specified as $T_L = 0.007$ mm and $T_L = 0.019$ mm. Carbide tools are to be used for drilling and boring processes, and high speed steel tools for reaming the hole.

The cost of a drilling tool is proportioned to its diameter. The costs of solid carbide tool costs of five different diameters are obtained from (Shunmugam *et al.*, 2000) and listed in Table 4.4.

Table 4.4. The drilling tool cost data

Tool diameter, D (mm)	Cost C_d (\$)
10	4.38
20	14.42
30	33.6
40	80.36
50	123.62

The data in Table 4.4 are used to obtain following cost-diameter relationship by curve fitting:

$$C_d(D) = -66.48 + 13.467D - 0.8633D^2 + 0.0237D^3 - 0.000206D^4 \quad (4.1)$$

where $C_t(D)$ is the tool cost as a function of tool diameter D .

The internal cylindrical traverse grinding pass is used following the drilling, and silicon carbide grinding wheel (37C46-L-VK) with grit of size 46 (grit diameter is 0.35 mm) is chosen from the Industrial Catalogue of Norton Ltd. The width of the wheel is 10 mm, and the maximum grinding wheel surface speed is 2700 m/min. The single-point diamond dresser is used. The grinding force equation, power consumption equation, surface roughness equation, and grinding wheel life equation are the same as those used for the shaft example. For drilling, boring, reaming, and grinding processes, the parameters in the tool life, force, torque, power consumption equations, and other input data are given in Table 4.5.

Table 4.5: Input data for the hole

<i>Parameters</i>	<i>Values</i>
Unit raw material cost C_U	\$10
Process capability index $C_{p11}, C_{p22}, C_{p32}, C_{p33}, C_{p42}, C_{p43}, C_{p44}$	1.2, 1.1, 1.0, 1.0, 0.9, 0.9, 0.9
Percentage of allowable rejected components α	5.3%
Inspection cost $C_{A11}, C_{A22}, C_{A23}, C_{A32}, C_{A33}, C_{A42}, C_{A43}, C_{A44}$	\$1, \$1.5, \$1.8, \$1.5, \$1.9, \$1.8, \$2
Operation cost of the four processes $C_{m1}, C_{m2}, C_{m3}, C_{m4}$	\$2/min, \$3/min, \$3/min, \$5/min
Tool cost C_{t2}, C_{t3}, C_{t4}^*	\$ 7/insert, \$282/tool, \$10/dressing
Tool replacement cost and time $C_{r1}, C_{r2}, C_{r3}, C_{r4};$ $T_{r1}, T_{r2}, T_{r3}, T_{r4}$	\$1/min, \$1/min, \$1/min, \$1/min; 1min, 1min, 1min, 1min,
Grinding wheel diameter D_s	35 mm
Wheel surface speed v_{Gs}	2400 m min ⁻¹
Machine tool efficiency E	75%
Cutting speed limits $v_{11}^l, v_{11}^u, v_{22}^l, v_{22}^u, v_{32}^l, v_{32}^u, v_{33}^l, v_{33}^u, v_{42}^l, v_{42}^u, v_{43}^l, v_{43}^u, v_{44}^l, v_{44}^u$	70, 110, 110, 140, 130, 160, 6, 12, 140, 170, 12, 20, 20, 45 m/min
Feed rate limits $f_{11}^l, f_{11}^u, f_{22}^l, f_{22}^u, f_{32}^l, f_{32}^u, f_{33}^l, f_{33}^u, f_{42}^l, f_{42}^u, f_{43}^l, f_{43}^u, f_{44}^l, f_{44}^u$	0.2, 0.6, 0.5, 1.0, 0.1, 0.4, 0.8, 2.0, 0.06, 0.2, 0.5, 1.0, 2.0, 8.0 mm/rev

Depth of cut limits $d_{22}^l, d_{22}^u, d_{32}^l, d_{32}^u, d_{33}^l, d_{33}^u, d_{42}^l, d_{42}^u, d_{43}^l, d_{43}^u, d_{44}^l, d_{44}^u$	1.0, 6.0, 0.1, 1.0, 0.1, 1, 0.007, 0.1, 0.007, 0.1, 0.005, 0.1 mm
Constants for drilling tool life $K_{T1}, X_{T1}, U_{T1}, Z_{T1}, W_{T1}$	42.3, 0.5, 1, 0.5, 2.5
Constants for boring tool life $K_{T2}, X_{T2}, U_{T2}, Y_{T2}, Z_{T2}, W_{T2}$	292, 0, 1, 0.15, 0.2, 5
Constants for reaming tool life $K_{T3}, X_{T3}, U_{T3}, Y_{T3}, Z_{T3}, W_{T3}$	15.1, 0.2, 1, 0.1, 0.5, 3.33
Constants for grinding wheel life K_G, α_G, A	2.4, 0.2, 0.005
Dressing depth d_d	0.01 mm
Dressing lead s_d	0.15 mm rev ⁻¹
Constants for drilling thrust force $K_{F11}, X_{F11}, Y_{F11}, Z_{F11}$	418, 1.2, 0, 0.75
Constants for boring cutting force $K_{F2}, X_{F2}, Y_{F2}, Z_{F2}$	900, 0, 1.0, 0.75
Constants for reaming thrust force $K_{F3}, X_{F3}, Y_{F3}, Z_{F3}$	233, 0, 1.2, 0.4
Constants for grinding force $K_{F1}, K_{F2}, K_{F3}, K_{F4},$	58.8, 0.7, 0.8, 1.25
Constants for drilling torque $K_{M11}, X_{M11}, Y_{M11}, Z_{M11}$	120, 2.2, 0, 0.8
Constants for cutting power $\alpha_P, \beta_P, \gamma_P, B_P$	1.0, 0.75, 1, 0.02
Constants for grinding power $K_{GP1}, K_{GP2}, K_{GP3}, K_{GP4}$	0.98, 0.7, 0.8, 1.25
Constants for reaming torque $K_{Mi3}, X_{Mi3}, Y_{Mi3}, Z_{Mi3} (i = 2, 3)$	846, 1, 0.75, 0.8
Constants for drilling and reaming power K_{P1}, K_{P1}	9545513, 9545513
Constants for grinding surface roughness $R_3, K_{R1}, K_{R2}, K_{R3}$	0.2, 0.5, 0.25, 0.5
Surface roughness $R_{G\max 4}$	0.00125 mm
Maximum cutting force, power $F_{\max 1}, P_{\max 1}, F_{\max 2}, P_{\max 2}, F_{\max 3}, P_{\max 3}, F_{\max 4}, P_{\max 4}$	2000 N, 10 kW, 4.5 kN, 10 kW, 2000 N, 10 kW, 1.5 kN, 12 kW
Tolerance limits $t_{11}^l, t_{11}^u, t_{22}^l, t_{22}^u, t_{32}^l, t_{32}^u, t_{33}^l, t_{33}^u, t_{42}^l, t_{42}^u, t_{43}^l, t_{43}^u, t_{44}^l, t_{44}^u$	0.178, 0.508, 0.076, 0.305, 0.015, 0.127, 0.014, 0.126, 0.01, 0.07, 0.009, 0.06, 0.008, 0.05 mm

* Note: the drilling tool cost C_{t1} is determined using Eq. (4.1)

This problem is again solved using the modified Chebyshev goal programming problem as detailed below.

Step 1. Formulated the bi-criterion problem

$$\text{Min } \text{sum_cost} = \sum_{j=1}^4 m_{4j} C_{UM4j}$$

$$\text{Max } \text{sum_tol} = \sum_{i=1}^4 \sum_{j=1}^4 m_{ij} t_{ij}$$

Subject to: all the constraints (as listed in Model 1, Chapter 3)

Solving the problem with one objective at a time yields the following results:

	Min <i>sum_cost</i>	Max <i>sum_tol</i>	Difference
Value of <i>sum_cost</i>	19.77889	28.78405	9.00516
Value of <i>Sum_tol</i>	0.277	0.908	0.631

Step 2. Formulated and solve the modified Chebyshev problem

Max δ

Subject to

$$\delta \leq (\text{sum_tol} - 0.277) / 0.631$$

$$\delta \leq (28.78405 - \text{sum_cost}) / 9.00516$$

and all the other constraints.

Solving this model gives: $\delta = 0.9442014$, $\text{sum_cost}^* = 20.28137$, and $\text{sum_tol}^* = 0.873$. The relative difference between the Chebyshev solution and the single objective solutions are 2.54% ($=|(19.77889-20.28137)/19.77889|$) and 3.85% ($=|(0.908-0.873)/0.908|$) respectively. The outputs are given in Table 4.6.

Table 4.6. Output of the hole problem

<i>Parameters</i>	<i>Symbols</i>	<i>Outputs</i>			
Stage index	<i>i</i>	1	2	3	4
Index of selected process	<i>j</i>	1	2	2	4
Cutting speed	v_{ij}	74.35692	110.00000	60.00000	45.00000
Feed rate	f_{ij}	0.50748	0.79732	0.40000	8.00000

Depth of cut	d_{ij}	0 [*]	5.387051	0.8216045	0.03489554
Process diameter	D_{ij}	38.36990	49.14400	50.78721	50.85700
Process tolerance	t_{ij}	0.508000	0.305000	0.051791	0.008000

[†] Note: all the data in this table are obtained directly from Lingo outputs. They have not been rounded since rounding would cause inaccuracy in the final calculation.

^{*} The depth of cut the first process is 0 since this is a drilling operation. In this case the diameters of the hole and the drill bit are the same and so the “depth of cut” is 0.

Now the decisions on process selection, machining parameter selection and process means and tolerances for all the selected processes can be made simultaneously based on the outputs. There exist two and three alternative processes for stages 3 and 4 respectively. Table 4.6 shows that semi-finish boring instead of semi-finish reaming is selected for the semi-finish stage and grinding is chosen from the three alternatives for the finish stage. The machining parameters are provided for all the selected processes. The process means and tolerances are also specified for all the selected processes.

4.2 Examples for model 2

4.2.1 Example 3: Single quality characteristic

Suppose that a piston and a cylinder bore in an engine assembly taken from (Zhang and Wang, 1993) have the following design specifications:

The design dimensions (diameters) for the piston and cylinder bore are: 50.8mm and 50.856mm, respectively. The length and diameter of the raw stocks of the piston and cylinder are 100 mm, 70 mm, 60 mm and 68 mm, respectively.

There is one design characteristic and two part feature dimensions for this assembly. The characteristic Y is the clearance between the piston and cylinder bore (see Figure 4.2)

The linear design function is

$$Y = f(D_C, D_P) = D_C - D_P \quad (4.2)$$

Figure 4.2. Illustration of the Piston-cylinder assembly

The nominal value and tolerance of Y are 0.056 ± 0.025 mm. The unit material costs of piston and cylinder are \$16 and \$32 respectively. The required manufacturing processes and other input data in this example are the same as used for the shaft and hole examples in section 4.1.

The solution procedure is as follows:

Step 1. Formulate the bi-criterion model

$$\text{Min } \text{Sum_cost}_A = \sum_{j=1}^2 m_{14j} C_{UM14j} + \sum_{j=1}^4 m_{24j} C_{UM24j}$$

$$\text{Max } \text{Sum_tol}_A = \sum_{i=1}^4 \sum_{j=1}^2 m_{1ij} t_{1ij} + \sum_{i=1}^4 \sum_{j=1}^4 m_{2ij} t_{2ij}$$

Subject to: all the constraints (as listed in Model 2, Chapter 3)

Solving the problem with one objective at a time yields the following results:

a) the WC constraint in the constraint set (p2) is used

	Min sum_cost_A	Max sum_tol_A	Difference
Value of sum_cost_A	70.9201	72.91754	1.99744
Value of sum_tol_A	0.472	1.766	1.294

b) the RSS constraint in the constraint set (p2) is used

	Min sum_cost_A	Max sum_tol_A	Difference
Value of sum_cost_A	70.9201	72.97039	2.05029
Value of sum_tol_A	0.472	1.766	1.294

Step 2. Formulate and solve the modified Chebyshev problem

a. The WC constraint

$$\max \delta$$

Subject to

$$\delta \leq (sum_tol_A - 0.472)/1.294$$

$$\delta \leq (72.91754 - sum_cost_A)/1.99744$$

and all other constraints

Solving this model gives: $\delta = 0.8676048$, $sum_cost_A^* = 71.18455$, and $sum_tol_A^* = 1.594681$. The relative difference between the Chebyshev solution and the single objective solutions are 0.373% ($=|(70.9201-71.18455)/70.9201|$) and 9.8% ($=|(1.766-1.594681)/1.766|$) respectively. The outputs of the Chebyshev model are given in Table 4.7a.

Table 4.7a. Output of the piston and cylinder assembly problem with the WC constraint

<i>Parameters</i>	<i>Symbols</i>	<i>Outputs</i>			
Part feature index	h	1 [*]			
Stage index	i	1	2	3	4
Index of the selected process	j	1	1	2	2
Number of rough turning	N	2			
Cutting speed	v_{hij}	114.3604	200.00000	12.98235	40.00000
Feed rate	f_{hij}	1.000000	0.400000	12.000000	6.000000
Depth of cut	d_{hij}	3.773231	0.867752	0.170252	0.01553379
Process diameter	D_{hij}	52.90708	51.17157	50.83107	50.80000
Process tolerance	t_{hij}	0.5080000	0.227504	0.0130000	0.0080676
Design tolerance	T_h	0.0080676			
Part feature index	h	2 [*]			
Stage index	i	1	2	3	4
Index of the selected process	j	1	2	2	4
Cutting speed	v_{hij}	74.35515	110.00000	160.00000	45.00000
Feed rate	f_{hij}	0.5075376	0.797367	0.4000000	8.0000000
Depth of cut	d_{hij}	0 [#]	5.3868310	0.8400000	0.0175541
Process diameter	D_{hij}	38.36723	49.14089	50.82089	50.85600
Process tolerance	t_{hij}	0.5080000	0.3050000	0.0150000	0.0101081
Design tolerance	T_h	0.0101081			

[†] Note: all the data in this table are obtained directly from Lingo outputs. They have not been rounded since rounding would cause inaccuracy in the final calculation.

* Part feature 1 belongs to the piston, and part feature 2 belongs to the cylinder.

[#] The depth of cut is 0 here since this is a drilling operation. In this case the diameters of the hole and the drill bit are the same and so the “depth of cut” is 0.

The machining parameters (the number of rough turning passes, cutting speed, feed rate, and depth of cut), process means and process tolerances for all the selected processes that are needed to machine the two part features are provided in Table 4.7a. Meanwhile, design tolerances for the two part features are also given in the table. This example illustrates that, for a given assembly quality characteristic, the decisions on

machining parameters, process means, process tolerances, and design tolerances for involved part features can be made concurrently using the bi-criterion model.

b. The RSS constraint

max δ

Subject to

$$\delta \leq (sum_tol_A - 0.472)/1.294$$

$$\delta \leq (72.97039 - sum_cost_A)/2.05029$$

and all other constraints

The Lingo outputs yield: $\delta = 0.8579569$, $sum_cost_A^* = 71.21133$, and $Sum_tol_A^* = 1.582196$. The relative difference between the Chebyshev solution and the single objective solutions are 0.41% ($=|(70.9201-71.21133)/70.9201|$) and 10.4% ($=|(1.766-1.582196)/1.766|$) respectively. The other output data are summarized in Table 4.7b.

Table 4.7b. Output of the piston and cylinder assembly problem with the RSS constraint

<i>Parameters</i>	<i>Symbols</i>	<i>Outputs</i>			
Part feature index	<i>h</i>	1			
Stage index	<i>i</i>	1	2	3	4
Index of selected process	<i>j</i>	1	1	2	2
The number of rough turning	<i>N</i>	2			
Cutting speed	<i>v_{hij}</i>	114.31810	200.00000	13.83585	40.00000
Feed rate	<i>f_{hij}</i>	1.000000	0.400000	12.000000	6.000000
Depth of cut	<i>d_{hij}</i>	3.782549	0.8572500	0.1597500	0.01790219
Process diameter	<i>D_{hij}</i>	52.86980	51.15530	50.83580	50.800000
Process tolerance	<i>t_{hij}</i>	0.5080000	0.2065000	0.0130000	0.0128044
Design tolerance	<i>T_h</i>	0.0128044			

Part feature index	h	2^*			
Stage index	i	1	2	3	4
Index of selected process	j	1	2	2	4
Cutting speed	v_{hij}	74.37213	110.00000	160.00000	45.00000
Feed rate	f_{hij}	0.506503	0.802604	0.400000	8.000000
Depth of cut	d_{hij}	0 [#]	5.3604490	0.8400000	0.01944593
Process diameter	D_{hij}	38.41621	49.13711	50.81711	50.85600
Process tolerance	t_{hij}	0.5080000	0.3050000	0.0150000	0.0138919
Design tolerance	T_h	0.0138919			

[†] Note: all the data in this table are obtained directly from Lingo outputs. They have not been rounded since rounding would cause inaccuracy in the final calculation.

* Part feature 1 belongs to the piston, and part feature 2 belongs to the cylinder.

[#] The depth of cut is 0 here since this is a drilling operation. In this case the diameters of the hole and the drill bit are the same and so the “depth of cut” is 0.

Compared with the computing results in Table 4.7a and in Table 4.7b, it is found that the values of the number of rough turning, cutting speed, feed rate, depth of cut, process diameter and process tolerance (except the last one) are either the same or very close. However, there are some differences in design tolerances (the last process tolerances) in the two tables. The design tolerances for the two features are 0.0080676 mm and 0.0101081 mm when the WC constraint is used (Table 4.7a) and they increase to 0.0128044 mm and 0.0138915 mm when the RSS constraint is used (Table 4.7b). Hence the WC constraint leads to tighter design tolerances as compared to the RSS constraint.

4.2.2 Example 4: multiple quality characteristics

The example with linear design functions is taken from the work reported in (Deng and Deng, 2002). There are four design characteristics and eight part feature dimensions for this assembly. The width of the base part is 127.127 mm. The first characteristic Y_1 is the machining allowance of the right surface of the base part. The other three

characteristics Y_2 , Y_3 and Y_4 specify the clearance conditions between the two parts. The design characteristics of Y_1 , Y_2 , Y_3 and Y_4 , and the eight feature dimensions are shown in Figure 4.3.

The nominal dimensions D_1 to D_8 in Figure 4.3 are respectively 25.4, 50.8, 76.2, 101.6, 25.4, 25.3492, 50.8 and 76.1492 mm. The four linear design functions are

$$Y_1 = f(D_4, D_5) = -D_4 - D_5 + 127.127 \quad (4.3)$$

$$Y_2 = f(D_1, D_2, D_7, D_8) = D_2 - D_1 + D_7 - D_8 - 0.00762 \quad (4.4)$$

$$Y_3 = f(D_2, D_3, D_6, D_7) = D_7 - D_6 - D_3 + D_2 - 0.0254 \quad (4.5)$$

$$Y_4 = f(D_3, D_4, D_6) = D_4 - D_6 - D_3 - 0.00762 \quad (4.6)$$

Figure 4.3. The base and top parts of the assembly

Substituting the nominal dimensions D_1 to D_8 into Equations (4.3) to (4.6) gives the nominal values of the four characteristics. Therefore, the nominal values and tolerances of Y_1 , Y_2 , Y_3 and Y_4 are assumed to be 0.127 ± 0.58 , 0.04318 ± 0.28 , 0.0254 ± 0.17 , and 0.04318 ± 0.23 mm, respectively.

(a)

(b)

Figure 4.4. Initial stocks of the top part (a) and the base part (b)
(Note: All dimensions are in mm and the shaded areas are to be removed)

The required manufacturing processes in this example are die casting, rough milling, and finish milling. After die casting, the main dimensions of the two mild steel stocks are illustrated in Figure 4.4 (the shaded areas are to be removed by milling operations). The first machining stage contains several rough milling passes and the second stage requires only one pass. The associated machining stages and processes are listed in Table 4.8.

Table 4.8. Machining stages and processes for Example 2

<i>Machining stages</i>	<i>Machining processes</i>
	$j = 1$ milling pass
$i = 1$ Rough	S rough milling passes
$i = 2$ Finish	Finish milling

An HSS end milling cutter is used to machine the shaded areas. The parameters in the tool life, cutting force, cutting power equations and other input data are given in Table 4.9.

Table 4.9. Input data for the base and top parts

Parameters	Values
Unit material cost of the base and the top C_{uB}, C_{uT}	\$30, \$25
Process capability index C_{p11}, C_{p21}	1.2, 0.9
Percentage of allowable rejected components α	5.3%
Unit inspection cost for pass C_{A11}, C_{A21}	\$1.0, \$1.5
Operation cost of C_{m1}	\$3/min
Tool replacement cost and time C_{r1}, T_{r1}	\$1/min, 1min
The cost of end milling cutter C_{uM}	\$18/tool
Cutting speed limits $v_{11}^l, v_{11}^u, v_{21}^l, v_{21}^u$	50, 90, 90, 140 m/min
Feed rate limits $f_{11}^l, f_{11}^u, f_{21}^l, f_{21}^u$	0.05, 0.09, 0.02, 0.06 mm/rev
Depth of cut limits $d_{11}^l, d_{11}^u, d_{21}^l, d_{21}^u$	0.8, 3.0, 0.08, 0.8 mm
The diameter of milling cutter D_c	16 mm

Number of teeth in the milling cutter N_i	3
Milling length $L_1, \dots, L_8$	40, 40, 40, 40, 40, 40, 40, 40 mm
Width of cut $d_{e1}, \dots, d_{e8}$	30, 20, 20, 30, 50, 20, 20, 40 mm
Overtravel before cutting l_2	1 mm
Machining allowance $d_{Tol1}, \dots, d_{Tol8}$	5, 5, 5, 5, 5, 5, 5, 5 mm
Machine tool efficiency E	0.75
Constants for milling tool life $K_{T1}, X_{T1}, U_{T1}, Y_{T1}, Z_{T1}, V_{T1}, S_{T1}, W_{T1}$	53, 0.45, 1, 0.5, 0.5, 0.1, 0.1, 3.03
Constants for milling power $K_{MP1}, K_{MP2}, K_{MP3}, K_{MP4}, K_{MP5}$	0.0107, -0.86, 1, 0.72, 0.86
Constants for milling force $K_{MF1}, K_{MF2}, K_{MF3}, K_{MF4}, K_{MF5}$	0.642, -0.86, 0, 0.72, 0.86
Maximum force, power $F_{\max1}, P_{\max1}$	0.8 kN, 1.5 kW
Tolerance limits $t_{11}^l, t_{11}^u, t_{21}^l, t_{21}^u$	0.08, 0.5, 0.01, 0.1mm

The problem is solved following the same procedure as illustrated in example section 4.2.1 and is explained as follows:

Step 1. Formulate and solve the bi-criterion model

$$\text{Min } \text{sum_cost}_A = \sum_{h=1}^8 C_{UMh21}$$

$$\text{Max } \text{sum_tol}_A = \sum_{h=1}^8 \sum_{i=1}^2 m_{hi1} t_{hi1}$$

Subject to: all the constraints (as listed in Model 2, Chapter 3)

Solving the problem separately with one objective at a time gives the following results:

a) The WC constraint in the constraint set (p2) is used

	Min $sum\ cost_A$	Max $sum\ tol_A$	Difference
Value of $sum\ cost_A$	270.0896	273.734	3.6444
Value of $sum\ tol_A$	1.12	2.74	1.62

b) The RSS constraint in the constraint set (p2) is used

	Min $sum\ cost_A$	Max $sum\ tol_A$	Difference
Value of $sum\ cost_A$	270.0896	277.4795	7.3899
Value of $sum\ tol_A$	1.12	2.88956	1.76956

Step 2. Formulate and solve the modified Chebyshev problem

a. The model with the WC constraint

$\max \delta$

Subject to

$$\delta \leq (sum_tol_A - 1.12)/1.62$$

$$\delta \leq (273.734 - sum_cost_A)/3.6444$$

and all other constraints

Solving this model gives $\delta = 0.9212978$, $sum_cost_A^* = 270.3764$, and $sum_tol_A^* = 2.612502$. The relative difference between the Chebyshev solution and the single objective solutions are 0.1% ($=|(270.0896-270.3764)/270.0896|$) and 4.65% ($=|(2.74-2.612502)/2.74|$) respectively. The outputs are summarized in Table 4.10a.

Table 4.10a. Output of the base and the top parts using the WC constraint

Parameters	Symbols	Outputs			
Part feature index	h	1*		2*	
Stage index	i	1	2	1	2
Index of selected process	j	1	1	1	1
Number of rough milling	S_h	4		2	

Cutting speed	v_{hij}	85.50970	140.0000	84.37500	140.0000
Feed rate	f_{hij}	0.0900000	0.0600000	0.0703571	0.0600000
Depth of cut	d_{hij}	1.1400000	0.4400000	2.2800000	0.4400000
Process diameter	D_{hij}	25.840000	25.400000	51.240000	50.800000
Process tolerance	t_{hij}	0.2600000	0.1000000	0.3482195	0.0117800
Design tolerance	T_h	0.1		0.01178	
Part feature index	h	3 [*]		4 [*]	
Number of rough milling	S_h	2		4	
Cutting speed	v_{hij}	84.37500	140.0000	84.37500	0 [#]
Feed rate	f_{hij}	0.0703571	0.0600000	0.0821327	0 [#]
Depth of cut	d_{hij}	2.2800000	0.4400000	1.2500000	0 [#]
Process diameter	D_{hij}	76.640000	76.200000	101.600000	0 [#]
Process tolerance	t_{hij}	0.3107321	0.049268	0.1130193	0 [#]
Design tolerance	T_h	0.0492680		0.1130193	
Part feature index	h	5 [*]		6 [*]	
Number of rough milling	S_h	6		2	
Cutting speed	v_{hij}	84.37500	0 [#]	84.37500	140.00000
Feed rate	f_{hij}	0.0655729	0 [#]	0.0703571	0.06
Depth of cut	d_{hij}	0.8333333	0 [#]	2.28	0.44
Process diameter	D_{hij}	25.400000	0 [#]	25.789200	25.349200
Process tolerance	t_{hij}	0.4669807	0 [#]	0.3211578	0.0388422
Design tolerance	T_h	0.4669807		0.0388400	
Part feature index	h	7 [*]		8 [*]	
Number of rough milling	S_h	2		5	
Cutting speed	v_{hij}	84.37500	140.00000	84.37500	140.00000
Feed rate	f_{hij}	0.0703571	0.0600000	0.0830306	0.6000000
Depth of cut	d_{hij}	2.2800000	0.4400000	0.8865005	0.5674976
Process mean	D_{hij}	51.240000	50.800000	76.7166976	76.14920
Process tolerance	t_{hij}	0.3484632	0.0115368	0.1325024	0.1000000
Design tolerance	T_h	0.0115368		0.1000000	

¹ Note: all the data in this table are obtained directly from Lingo outputs. They have not been rounded since rounding would cause inaccuracy in the final calculation.

* Part features 1, 2, 3, 4, and 5 belong to the base part; part features 6, 7 and 8 belong to the top part.

All the outputs are 0 in the column of process 1 in stage 2 (finish milling) for part features 4 and 5 since the finish milling process is not chosen to machine the two features.

Table 4.10a shows that two machining stages are needed for machining part feature 1. In the primary stage, four rough milling passes are needed with a depth of cut of 1.14 mm for each pass, feed rate of 0.09 mm/rev and cutting speed of 85.5097 m/min. For the second stage (finish milling), the cutting speed, feed rate and depth of cut are respectively 140 m/min, 0.06 mm/rev and 0.44 mm. The process means for the rough milling and finish milling processes are 25.84mm and 25.4 mm. The process tolerances for the two processes are 0.26 mm and 0.1 mm (design tolerance). All the detailed parameters for the other seven part features are also provided in the table. It is noted that process 1 in stage 2 (finish milling) for part features 4 and 5 is not chosen (all the outputs are 0 in this column), which means that only process 1 in stage 1 (rough milling) is required for part features 4 and 5 to achieve the first quality characteristic Y_1 . The simultaneous solutions to the process selection, machining parameter selection and tolerance design as summarized in Table 4.10a have effectively avoided decision conflict. The use of bi-criterion approach has helped to achieve preferred compromise between the two non-commensurable planning objectives.

b. The model with the RSS constraint

$$\max \delta$$

Subject to

$$\delta \leq (sum_tol_A - 1.12) / 1.769565$$

$$\delta \leq (277.4795 - sum_cost_A) / 7.3899$$

and all other constraints

Solving this model yields $\delta = 0.9973602$, $sum_cost_A^* = 270.1091$, and $Sum_tol_A^* = 2.884889$. The relative difference between the Chebyshev solution and the single objective solutions are 0.0072% ($=|(270.0896-270.1091)/270.0896|$) and 0.162% ($=|(2.88956-2.884889)/2.88956|$) respectively. The outputs are given in Table 4.10b.

Table 4.10b. Output of the base and top parts using the RSS constraint

<i>Parameters</i>	<i>Symbols</i>	<i>Outputs</i>			
Part feature index	h	1*		2*	
Stage index	i	1	2	1	2
Process index	j	1	1	1	1
Number of rough milling	S_h	4		2	
Cutting speed	v_{hij}	90.0000	130.11890	84.37500	111.17240
Feed rate	f_{hij}	0.0818318	0.03915162	0.0703571	0.0600000
Depth of cut	d_{hij}	1.140000	0.440000	2.280000	0.440000
Process diameter	D_{hij}	25.840000	25.400000	51.240000	50.800000
Process tolerance	t_{hij}	0.3499863	0.0100137	0.3192150	0.0407850
Design tolerance	T_h	0.0100137		0.0407850	
Part feature index	h	3*		4*	
Number of rough milling	S_h	2		4	
Cutting speed	v_{hij}	84.37500	140.00000	81.76004	0 [#]
Feed rate	f_{hij}	0.0703571	0.0600000	0.0625865	0 [#]
Depth of cut	d_{hij}	2.280000	0.440000	1.250000	0 [#]
Process diameter	D_{hij}	76.640000	76.200000	101.600000	0 [#]
Process tolerance	t_{hij}	0.3128297	0.0471703	0.2248888	0 [#]
Design tolerance	T_h	0.0471703		0.2248888	
Part feature index	h	5*		6*	
Number of rough milling	S_h	6		2	
Cutting speed	v_{hij}	84.37500	0 [#]	84.37500	140.00000
Feed rate	f_{hij}	0.0655729	0 [#]	0.0703571	0.0600000
Depth of cut	d_{hij}	0.8333333	0 [#]	2.2800000	0.4400000
Process diameter	D_{hij}	25.400000	0 [#]	25.789200	25.349200

Process tolerance	t_{hij}	0.5000000	0 [#]	0.3500000	0.0100000
Design tolerance	T_h	0.5000000		0.0100000	
Part feature index	h	7 [*]		8 [*]	
Number of rough milling	S_h	2		5	
Cutting speed	v_{hij}	84.37500	111.17240	84.37500	140.00000
Feed rate	f_{hij}	0.0703571	0.0600000	0.0802653	0.6000000
Depth of cut	d_{hij}	2.2800000	0.4400000	0.9120000	0.4400000
Process diameter	D_{hij}	51.240000	50.800000	76.589200	76.1492000
Process tolerance	t_{hij}	0.2627099	0.0972901	0.2600000	0.1000000
Design tolerance	T_h	0.0972901		0.1000000	

[†] Note: all the data in this table are obtained directly from Lingo outputs. They have not been rounded since rounding would cause inaccuracy in the final calculation.

^{*} Part features 1, 2, 3, 4, and 5 belong to the base part, while part features 6, 7 and 8 belong to the top part.

[#] All the outputs are 0 in the column of process 1 in stage 2 (finish milling) for part features 4 and 5 since the finish milling process is not chosen to machine the two features.

As stated in example 1, it is observed that the values of the number of rough milling, cutting speed, feed rate, depth of cut, and process diameter are not very sensitive to the tolerance stackup constraint. Table 4.10a shows that, when the WC stackup constraint is applied, the design tolerances for part features 1 to 8 are respectively 0.1 mm, 0.01178 mm, 0.049268 mm, 0.1130193 mm, 0.4669807 mm, 0.03884 mm, 0.0115368 mm, and 0.1 mm. The sum of those is 0.8914248 mm. When the RSS stackup constraint is chosen (Table 4.10b), the design tolerances for part features 1 to 8 are respectively 0.0100137 mm, 0.040785 mm, 0.0471703 mm, 0.2248888 mm, 0.5 mm, 0.01 mm, 0.0972901 mm, and 0.1 mm. The total is 1.0301479 mm. It is noted that, although the total design tolerance increases when the RSS constraint is used, design tolerances for part features 1, 3 and 6 are lower. Generally speaking, the WC constraint results in tighter design tolerances for most part features as compared to the RSS constraint.

Chapter 5

Conclusions

The conclusions of this study follow in two sections. The first section gives concluding remarks on the proposed bi-criterion approach, and the second section contains recommendations for future work.

5.1 Concluding Remarks

The objectives proposed in this study have been achieved. The accomplished work is summarized as follows:

- 1) An integrated approach has been proposed to simultaneously solve process selection, machining parameter optimization and tolerance design problems for single component, single quality characteristic case. The associated model has been developed and solved as illustrated using two examples in sections 4.1.1 and 4.1.2.
- 2) The above approach has been extended to the multi-component-multi-part-feature (assembly) case. The tolerance allocation problem has also been included. A more generalized model has been formulated and applied to two illustrative examples in sections 4.2.1 and 4.2.2.
- 3) A bi-criterion approach has been in cases above to incorporate tangible cost (expected unit cost) and intangible cost caused by different tolerance specifications.

- 4) A modified Chebyshev goal programming method has been applied to obtain a preferred compromise between the two non-commensurable and conflicting decision objectives.
- 5) Unlike most of the previous studies, the machining and tolerance parameters in this study are selected in the context of multiple machining stages with alternative processes. The interactions among machining stages, processes, machining conditions and tolerance parameters and their joint effect on planning objectives have all been taken into account and hence inconsistent, infeasible and conflicting decisions can be avoided.
- 6) In the assembly case, the design tolerance allocation (or synthesis) issue has been integrated with manufacturing planning problems. This will avoid the shortcomings of the conventional approaches in which the manufacturing aspects are often ignored.

5.2 Future Research

To further improve decision quality in manufacturing planning, the above work could be extended in the following directions:

1. *Single-component, multiple-quality-characteristics*. The first model presented in section 3.2.1 can only be applied to single-component, single-characteristic cases. To deal with products involving multiple quality characteristics, a new model needs to be developed.

2. *Multiple-components, multiple-quality-characteristics with non-linear design functions*. The second model developed in this study can effectively solve multi-components, multi-quality-characteristic problems with linear design functions. However, it cannot be directly applied to the problems with non-linear design functions.

Modification to the model is thus necessary for more complex assembly products where design functions are often non-linear.

3. *Probabilistic tool life and machining time.* The tool lives and machining times in the two models are assumed to be deterministic due to the lack of reliable data. It would be more realistic to treat them as probabilistic parameters providing that the data are available.

4. *The consideration of other planning objectives.* The two planning objectives adopted in this study are tangible and intangible costs. Other important planning criteria completion time, tardiness, and mean flow time could be considered in the future.

5. *Inclusion of other part features and processes.* Three part features (external and internal rotational surfaces and planar surfaces) and six processes (milling, turning, grinding, drilling, boring and reaming) are considered in the two models. Other features such as gears and threads and associated processes should receive similar attention.

6. *Solution method for large sized problems.* The two models for the four example problems are solved using a commercial software package, Lingo, in this thesis. Due to the large number of integer variables and the nonlinearity of the models, efficient solution algorithms should be developed to provide quick solutions for large scale problems.

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Appendix A

A derivation for the expected quality cost

Case 1. The expected quality cost for process j in stage i ($i < I$):

The specification for a selected process $\{i, j \mid m_{ij}=1\}$ is $D_{ij} \pm t_{ij}$, with D_{ij} being the process mean and t_{ij} the process tolerance. The raw stocks are machined according to their process mean D_{ij} and tolerance t_{ij} by pass j . Because the dimension measurement x of the products follows a normal distribution with the process mean and the process variance as parameters, the probability density function is

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

$$\text{Assumed } Z_{1ij} = \frac{D_{ij} - t_{ij} - D_{ij}}{\sigma_{mij}} = -\frac{t_{ij}}{\sigma_{mij}}, \quad Z_{2ij} = \frac{D_{ij} + t_{ij} - D_{ij}}{\sigma_{mij}} = \frac{t_{ij}}{\sigma_{mij}} \quad \text{and the}$$

manufacturing process is stable (i.e., the process capability index $C_{pij} = \frac{t_{ij}}{3\sigma_{mij}} = \text{constant}$),

the results are

$$Z_{1ij} = -3C_{pij}, \quad Z_{2ij} = 3C_{pij}, \quad \sigma_{mij} = t_{ij} / (3C_{pij})$$

The probabilities that a component's dimension is below, within, and above the specification range are respectively

$$P(x \leq D_{ij} - t_{ij}) = \int_{-\infty}^{D_{ij} - t_{ij}} f(x) dx = \Phi\left(-\frac{t_{ij}}{\sigma_m}\right) = \Phi(Z_{1ij}) \quad (\text{A.1})$$

$$P(x \geq D_{ij} + t_{ij}) = \int_{D_{ij} + t_{ij}}^{+\infty} f(x) dx = 1 - \Phi\left(\frac{t_{ij}}{\sigma_m}\right) = 1 - \Phi(Z_{2ij}) \quad (\text{A.2})$$

and

$$P(D_{ij} - t_{ij} \leq x \leq D_{ij} + t_{ij}) = \int_{D_{ij} - t_{ij}}^{D_{ij} + t_{ij}} f(x) dx = \Phi(Z_{2ij}) - \Phi(Z_{1ij}) \quad (\text{A.3})$$

Case 2. The expected quality cost for a process j of the finish stage $i = I$

If process j is selected for the last manufacturing stage (i.e., $m_{ij} = 1$), the components will be machined according to their process mean D_{ij} and tolerance t_{ij} by process j . After machining, all the components should be measured according to product design specification $D_{-T_L}^{+T_U}$. Assumed $Z_{1ij} = \frac{D - T_L - D_{ij}}{\sigma_{mij}}$, $Z_{2ij} = \frac{D + T_U - D_{ij}}{\sigma_{mij}}$, the results are

$$P(x \leq D - T_L) = \int_{-\infty}^{D - T_L} f(x) dx = \Phi(Z_{1ij}) \quad (\text{A.4})$$

$$P(x \geq D + T_U) = \int_{D + T_U}^{+\infty} f(x) dx = 1 - \Phi(Z_{2ij}) \quad (\text{A.5})$$

and

$$P(D - T_L \leq x \leq D + T_U) = \int_{D - T_L}^{D + T_U} f(x) dx = \Phi(Z_{2ij}) - \Phi(Z_{1ij}) \quad (\text{A.6})$$

The expected quality loss after the finish stage is then

$$\begin{aligned}
E_C(L(x))_{lj} &= \int_{D-T_L}^D K_{1lj}(x-D)^2 f(x)dx + \int_D^{D+T_U} K_{2lj}(x-D)^2 f(x)dx \\
&= K_{1lj}\sigma_m((D-D_{lj})\varphi(\frac{D-D_{lj}}{\sigma_{mlj}}) - (D+T_L-D_{lj})\varphi(Z_{1lj})) \\
&\quad + K_{1lj}(\sigma_{mlj}^2 + (D-D_{lj})^2)(\Phi(\frac{D-D_{lj}}{\sigma_{mlj}}) - \Phi(Z_{1lj})) \\
&\quad + K_{2lj}\sigma_{mlj}((D-T_U-D_{lj})\varphi(Z_{2lj}) - (D-D_{lj})\varphi(\frac{D-D_{lj}}{\sigma_{mlj}})) \\
&\quad + K_{2lj}(\sigma_{mlj}^2 + (D-D_{lj})^2)(\Phi(Z_{2lj}) - \Phi(\frac{D-D_{lj}}{\sigma_{mlj}}))
\end{aligned} \tag{A.7}$$

Assumed that the last process is also stable, the process variance σ_m can therefore be estimated from the known process capability index C_{plj} . If the process is normally distributed, C_{plj} can be defined as $C_{plj} = \min \left\{ \frac{USL-U}{3\sigma_m}, \frac{U-LSL}{3\sigma_m} \right\}$, which yields:

$$\sigma_m = \frac{\min((D-T_U)-D_{lj}, D_{lj}-(D-T_L))}{3C_{plj}}$$

Appendix B

Source code in Lingo language for Example 1

!This cost Model 1 for Example 1 (shaft) is presented.
The inputs are based on all practical data from handbooks
and reliable sources;

MODEL:

SETS:

STAGES /1..4/;

PROCESSES /1..2/: FMAX, PMAX;

LINKS (STAGES, PROCESSES):

m, t, V, F, D, Dia, Cost, tL, tU, FL, FU, VL, VU, DL, DU, Pz1, Pz2, z1, z2, z3
, E_loss,

C_UM, Cp, C_S, C_R, C_i, Yz1, Yz2;

ENDSETS

!----- Input the original data -----
-----;

DATA:

L=60;

!The dimension of the raw bar;

Do=68.0;

Design=50.8;

!Design target;

Tol_L = 0.017;

Tol_U = 0.007;

Cp=1.2 0

!The process capability index for each process;

1.1 0

0 1.0

0 0.85;

C_i = 1.0 0

!The inspection fee for each process;

1.5 0

0 2.0

0 2.50;

a1=0.053;

pi=3.1415926;

```

DL = 1.5 0          !Depth of cut lower limit for processes;
    0.5 0
    0 0.05
    0 0.005;

DU = 5.0 0          !Depth of cut upper limit for
processes;
    1.5 0
    0 0.5
    0 0.05;

tL = 0.127 0        !Tolerance lower limit for processes;
    0.050 0
    0 0.013
    0 0.005;

tU = 0.508 0        !Tolerance upper limit for processes;
    0.305 0
    0 0.076
    0 0.025;

VL = 50 0           !Speed lower limit for processes;
    140 0
    0 10
    0 20;

VU = 130 0          !Speed upper limit for processes;
    200 0
    0 25
    0 40;

FL = 0.5 0          !Feed rate lower limit for processes;
    0.1 0
    0 6.0
    0 1.0;

FU = 1.0 0          !Feed rate upper limit for processes;
    0.4 0
    0 12.0
    0 6.0;

C_u=10;              !The unit raw material cost($);
Cr=1.0;              !The tool replacement cost($/min);
tr=1;                !The tool replacement time(min);
Ct=3.0;              !The tool cost($/edge);
Cm = 1.0;            !The unit machining cost of turning
process($/min);

```

```

C_Gm = 5;                !The unit machining cost of grinding
process($/min);
C_d=9;                    !The unit dressing cost($/min);
D_s=355.6;                !The diameter of the grinding wheel (mm);
D_dr=0.02;                !The dressing depth;
S_d=0.15;                 !The grit diameter (mm);
V_Gs=1800;                !The speed of grinding whell surface (m/min);
B=25.4;                   !The width of grinding wheel;
K_G=2.4;                  !Constants for grinding wheel life;
A=0.01;
K_F1=0.8;                 !Costants for grinding force;
K_F2=1.05;                !Costants for grinding force;
K_P1=0.8;                 !Costants for grinding power;
K_P2=1.05;                !Costants for grinding power;
E=0.75;                   !Machine tool efficiency;

Bt=847288609443;         !Empirical turning parameters in the
tool life equation;
alphat=5.0;
betat=2.0;
gammamat=0.75;

Bf=0.9;                   !Empirical parameters in the turning
force equation;
alphaf=0.0;
betaf=0.75;
gammaf=1.0;

Bp=0.02;                  !Empirical parameters in the power
consumption equation;
alphap=1.0;
betap=0.75;
gammap=1;

!Empirical turning parameters in the surface roughness
equation;
Br=122;
alphar=-1.52;
betar=1.004;
gammar=0;
etar=-0.714;
thetar=-0.323;
BHN=195;
r=1;

m = 1 0
    1 0

```

```

0 1
0 1;

!Maximum turning force, power and surface roughness;

Fmax = 4.5 1.5;
Pmax = 10 12;
RGFmax = 0.00125;
RGRmax = 0.0025;
RTFmax = 0.0032;

ENDDATA

INIT:

t = 0.416 0
    0.092 0
    0 0.035
    0 0.010;
n = 2;
m = 1 0
    1 0
    0 1
    0 1;
F = 0.7 0
    0.3 0
    0 8.0
    0 4.0;
D = 3.9 0
    0.6 0
    0 0.15
    0 0.0495;
V = 90 0
    170 0
    0 18
    0 35;

ENDINIT

!----- here is the objection function-----;

!Min = P_cost;           !The first objective function;
Max = Sum_tol;           !The second objective function;
!Max = Dummy;            !Fuzzy GP;

Sum_tol = @SUM( Links(I,J): m(I,J)*t(I,J) );
P_cost = @SUM(Links(I,J) | I #EQ# 4: m(I,J)*C_UM(I,J) );

```

```

!DUMMY <= (Sum_tol-0.195)/0.663;          !Fuzzy GP;
!DUMMY <= (23.98925-P_cost)/5.7761;

!*****Judge that tolerance is in which stage's
range*****;

!(1)when the design tolerance is in the range of No.4
finish machining stage;

@FOR (STAGES(I):
    @SUM(PROCESSES(J):m(I,J)) = 1;
);

@FOR ( LINKS(I,J) | m(I,J) #EQ# 0:
    V(I,J)=0;
    F(I,J)=0;
    D(I,J)=0;
    t(I,J)=0;
);

!(2)when the design tolerance is in the range of No.3 semi-
finish machining stage;
!@FOR (STAGES(I) | I #EQ# 4:
    !    @SUM(PROCESSES(J):m(I,J)) = 0;
!);

!@FOR (STAGES(I) | I #LE# 3:
    !    @SUM(PROCESSES(J):m(I,J)) = 1;
!);

!(3)when the design tolerance is in the range of No.2 rough
machining stage;
!@FOR (STAGES(I) | I #GE# 3:
    !    @SUM(PROCESSES(J):m(I,J)) = 0;
!);
!@FOR (STAGES(I) | I #LE# 2:
    !    @SUM(PROCESSES(J):m(I,J)) = 1;
!);

!(4)when the design tolerance is in the range of No.1
primary machining stage;
!@FOR (STAGES(I) | I #GE# 2:
    !    @SUM(PROCESSES(J):m(I,J)) = 0;
!);

!@FOR (STAGES(I) | I #EQ# 1:

```



```

!      @SUM(PROCESSES(J):m(I,J)) = 1;
!);

!*****;

!calculate the average manufacturing cost of rough turning
pass;

@FOR (STAGES(I) | I #EQ# 1:      !if process 1 in stage
1 is chosen;
      @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(I,J) #EQ# 1:

          C_UM(I,J) =
(C_u+(Cost(I,J)+C_i(I,J))*(1+Pz2(I,J)))/(1-Pz1(I,J));

          Cost(I,J) = Pi*L*0.001*Bt^(-1)*(Cm*Bt*V(I,J)^(-
1)*F(I,J)^(-1)+
          V(I,J)^(alphan-1)*F(I,J)^(betat-
1)*D(I,J)^gammat*(Ct+Cr*tr))*(n*Do-n*(n-1)*D(I,J));

          C_S(I,J) = C_u+Cost(I,J)+C_i(I,J);
          C_R(I,J) = Cost(I,J)+C_i(I,J);

          Pz1(I,J) = @PSN(z1(I,J));
          Pz2(I,J) = 1-@PSN(z2(I,J));

          @FREE(z1(I,J));
          z1(I,J) = -3*Cp(I,J);
          z2(I,J) = 3*Cp(I,J);

      );
);

!Calculate the average manufacturing cost of finish turning
pass;

@FOR (STAGES(I) | I #EQ# 2:      !if process 1 in stage 2
is chosen;
      @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(I,J) #EQ# 1:

          C_UM(I,J) =
(C_UM(I,1)+(Cost(I,J)+C_i(I,J))*(1+Pz2(I,J)))/(1-Pz1(I,J));

          Cost(I,J) = Pi*L*0.001*Bt^(-1)*(Cm*Bt*V(I,J)^(-
1)*F(I,J)^(-1)
          +V(I,J)^(alphan-1)*F(I,J)^(betat-
1)*D(I,J)^gammat*(Ct+Cr*tr))*(Dia(I,J)+2*D(I,J));

```

```

C_S(I,J) = C_UM(1,1)+Cost(I,J)+C_i(I,J);
C_R(I,J) = Cost(I,J)+C_i(I,J);

Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));

@FREE(z1(I,J));
z1(I,J) = -3*Cp(I,J);
z2(I,J) = 3*Cp(I,J);

);
);

!Calculate the average manufacturing cost of rough grinding
passes;

@FOR (STAGES(I) | I #EQ# 3: !if process 2 in stage 3 is
chosen;
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(I,J) #EQ# 1:

        C_UM(I,J) =
(C_UM(2,1)+(Cost(I,J)+C_i(I,J))*(1+Pz2(I,J)))/(1-Pz1(I,J));

Cost(I,J)=C_Gm*Pi*(Dia(I,J)+2*D(I,J))*L*0.001*V(I,J)^(-
1)*F(I,J)^(-1)+(C_d+Cr*tr)*K_G*L

*(Dia(I,J)+2*D(I,J))*(D(I,J)^1.2)*(V(I,J)^0.2)/(1000*A*D_s*
F(I,J)*(V_Gs^0.2));

        C_S(I,J) = C_UM(2,1)+Cost(I,J)+C_i(I,J);
        C_R(I,J) = Cost(I,J)+C_i(I,J);

        Pz1(I,J) = @PSN(z1(I,J));
        Pz2(I,J) = 1-@PSN(z2(I,J));

        @FREE(z1(I,J));
        z1(I,J) = -3*Cp(I,J);
        z2(I,J) = 3*Cp(I,J);

    );
);

!Calculate the average manufacturing cost of rough grinding
passes;

```

```

@FOR (STAGES(I) | I #EQ# 4:      !if process 2 in stage 4
is chosen;
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(I,J) #EQ# 1:

        C_UM(I,J) =
(C_UM(3,2)+(Cost(I,J)+C_i(I,J))*(1+Pz2(I,J))+E_loss(I,J))/(
1-Pz1(I,J));

        Cost(I,J)=C_Gm*Pi*(Dia(I,J)+2*D(I,J))*L*0.001*V(I,J)^(-
1)*F(I,J)^(-1)+(C_d+Cr*tr)*K_G*L

*(Dia(I,J)+2*D(I,J))*(D(I,J)^1.2)*(V(I,J)^0.2)/(1000*A*D_s*
F(I,J)*(V_Gs^0.2));

        C_S(I,J) = C_UM(3,2)+Cost(I,J)+C_i(I,J);
        C_R(I,J) = Cost(I,J)+C_i(I,J);

!Here, calculate the expected loss;

E_loss(I,J) = K1*Sigma*((Design-Dia(I,J))*Yz3-
(Design+Tol_L-Dia(I,J))*Yz1(I,J))+K1*(Sigma^2+
(Design-Dia(I,J))^2)*(Pz3-
Pz1(I,J))+K2*Sigma*((Design-Tol_U-Dia(I,J))*Yz2(I,J)-
(Design-Dia(I,J))*Yz3)+K2*(Sigma^2+(Design-
Dia(I,J))^2)*(1-Pz2(I,J)-Pz3);

K1 = C_S(I,J)/(Tol_L^2);
K2 = C_R(I,J)/(Tol_U^2);

!Calculate the value of standard normal probability density
function;

Yz1(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z1(I,J)^2));
Yz2(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z2(I,J)^2));
Yz3 = (1/((2*Pi)^0.5))*@EXP(-0.5*(z3(I,J)^2));

!Calculate the value of cumulative standard normal
probability distribution;
Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));
Pz3 = @PSN(z3(I,J));

@FREE(z1(I,J));
@FREE(z3(I,J));
z1(I,J) = (Design-Tol_L-Dia(I,J))/Sigma;
z2(I,J) = (Design+Tol_U-Dia(I,J))/Sigma;
z3(I,J) = (Design-Dia(I,J))/Sigma;

```

```

!Calculate the Sigma; !Cpk=min[(USL-U)/(3*Sigma), (U-
LSL)/(3*Sigma)];

Sigma = @SMIN(Sigma_1,Sigma_2);
Sigma_1 = (Design+Tol_U-Dia(I,J))*(3*Cp(I,J))^(-1);
Sigma_2 = (Dia(I,J)-(Design-Tol_L))*(3*Cp(I,J))^(-1);

);
);

Dia(1,1) = Do-2*n*D(1,1);

!Limit the diameters relationship between two processes;

@FOR (STAGES(I) | I #GE# 2:

    @SUM (PROCESSES(J): m(I,J)*(Dia(I,J)+2*D(I,J)))
        = @SUM (PROCESSES(H): m(I-1,H)*Dia(I-1,H));

);

!Let the last process's diameter equal to the working
diameter;

@FOR (STAGES(I) | I #EQ# 4:

    @FOR (PROCESSES(J) | m(I,J) #EQ# 1:
        Dia(I,J) - U =0;

    );
);

@GIN(n);
@FOR( LINKS: @BIN(m));

!-----The following section is used to set
constraints-----;

!1)process parameter constraints;

@FOR ( LINKS(I,J)| m(I,J) #EQ# 1:

    @BND(VL(I,J),V(I,J),VU(I,J));      !process speed restriction;
    @BND(FL(I,J),F(I,J),FU(I,J));!process feed rate restriction;
    @BND(DL(I,J),D(I,J),DU(I,J));      !process depth restriction;
    @BND(tL(I,J),t(I,J),tU(I,J));      !process tolerance

```

```

restriction;

);

!2)machining allowance constraints;

@FOR (STAGES(I) | I #GE# 2:

    (@SUM (PROCESSES(J): m(I-1,J)*(Dia(I-1,J)-t(I-1,J)))-
     @SUM (PROCESSES(H):
m(I,H)*(Dia(I,H)+t(I,H)+2*DL(I,H))))/2 >= 0;

    (@SUM (PROCESSES(J): m(I-1,J)*(Dia(I-1,J)+t(I-1,J)))-
     @SUM (PROCESSES(H): m(I,H)*(Dia(I,H)-
t(I,H)+2*DU(I,H))))/2 <= 0;

);

!3)functionality requirement constraints;

@FOR (STAGES(I) | I #EQ# 4:

    @FOR (PROCESSES(J) | m(I,J) #EQ# 1:
    U >= Design-Tol_L+t(I,J);
    U <= Design+Tol_U-t(I,J);

);
);

!4)process total depth of cut constraints;

U+2*@SUM(LINKS(I,J) | I #GE# 2: m(I,J)*D(I,J))+2*n*D(1,1) =
Do;

!U+2*(D(2,1)+D(3,2)+D(4,2))+2*n*D(1,1)=Do;

!5)constraints for product quality requirement;

@FOR (STAGES(I) | I #EQ# 4:
    @FOR (PROCESSES(J) | m(I,J) #EQ# 1:

        1-Pz1(I,J)-Pz2(I,J) >= 1-a1;

);
);

!6)constraints of cutting force,power,and surface roughness;

```

```

@FOR ( STAGES(I) | I #LE# 2:
    @FOR ( PROCESSES(J) | J #EQ# 1:

        Bf*V(I,J)^alphaf*F(I,J)^betaf*D(I,J)^gammaf <= Fmax(J);
        Bp*V(I,J)^alphap*F(I,J)^betap*D(I,J)^gammap <= Pmax(J);

    );
);

@FOR ( STAGES(I) | I #EQ# 2:
    @FOR ( PROCESSES(J) | J #EQ# 1:

        Br*r^eta*BHN^(theta)*V(I,J)^alphan*F(I,J)^betar <=
RTFmax;

    );
);

@FOR ( STAGES(I) | I #EQ# 3:
    @FOR ( PROCESSES(J) | J #EQ# 2:

        86.4*E*(V(I,J)*F(I,J)*D(I,J))^0.7*K_F1*K_F2*V_Gs^(-1)
<= Fmax(J);
        1.44*(V(I,J)*F(I,J)*D(I,J))^0.7*K_P1*K_P2 <= Pmax(J);
        0.2*(V(I,J)*D(I,J)*V_Gs^(-
1))^0.5*D_dr^0.25*S_d^0.5+R_slack1 = RGRmax;

    );
);

@FOR ( STAGES(I) | I #EQ# 4:
    @FOR ( PROCESSES(J) | J #EQ# 2:

        86.4*E*(V(I,J)*F(I,J)*D(I,J))^0.7*K_F1*K_F2*V_Gs^(-1)
<= Fmax(J);
        1.44*(V(I,J)*F(I,J)*D(I,J))^0.7*K_P1*K_P2 <= Pmax(J);
        0.2*(V(I,J)*D(I,J)*V_Gs^(-
1))^0.5*D_dr^0.25*S_d^0.5+R_slack2 = RGFmax;

    );
);

END

```

Appendix C

Source code in Lingo language for Example 2

!This cost model 1 for Example 2 (the hole) is presented.
The inputs are based on all practical data from handbooks
and reliable sources;

MODEL:

SETS:

STAGES /1..4/;

PROCESSES /1..4/:

Cm,K_T,U_T,X_T,Y_T,Z_T,W_T,K_F,X_F,Y_F,Z_F,K_M,X_M,Y_M,Z_M,
Fmax,Pmax;

LINKS(STAGES,PROCESSES):

m,t,tL,tU,V,VL,VU,F,FL,FU,D,DL,DU,Dia,Cost,Force,Torque,pow
er,Pz1,Pz2,z1,z2,z3,E_loss,
C_UM,Cp,C_S,C_R,C_i,Yz1,Yz2;

ENDSETS

!----- Input the original data -----;
DATA:

L=100; !the dimension of the raw bar;

Design=50.856; !design target;

Tol_L=0.007;

Tol_U=0.019;

Cp = 1.2 0 0 0 !The process capability index

for each pass;

0 1.1 0 0

0 1.0 1.0 0

0 0.9 0.9 0.9;

C_i = 1 0 0 0

!The inspection fee for each pass;

0 1.5 0 0

0 1.8 1.5 0

0 1.9 1.8 2;

```

a1=0.053;

pi=3.1415926;

DL = 0 0 0 0          !depth of cut lower limit for passes;
    0 1.0 0 0
    0 0.1 0.1 0
    0 0.007 0.007 0.005;
DU = 0 0 0 0          !depth of cut upper limit for passes;
    0 6.0 0 0
    0 1 1 0
    0 0.1 0.1 0.1;
tL = 0.178 0 0 0      !tolerance lower limit for passes;
    0 0.076 0 0
    0 0.015 0.014 0
    0 0.01 0.009 0.008;
tU = 0.508 0 0 0      !tolerance upper lower
limit for passes;
    0 0.305 0 0
    0 0.127 0.126 0
    0 0.07 0.06 0.05;

VL = 70 0 0 0         !speed lower limit for passes;
    0 110 0 0
    0 130 6 0
    0 140 12 20;
VU = 110 0 0 0        !speed upper limit for passes;
    0 140 0 0
    0 160 12 0
    0 170 20 45;

FL = 0.2 0 0 0        !feed rate lower limit for
passes;
    0 0.5 0 0
    0 0.1 0.8 0
    0 0.06 0.5 2.0;
FU = 0.6 0 0 0        !feed rate upper limit for
passes;
    0 1.0 0 0
    0 0.4 2.0 0
    0 0.2 1.0 8.0;

C_u=10;               !The unit raw material cost($);
C_tB=7;               !The boring insert cost for
boring process($/each);
C_tR=282;             !The reamer cost for reaming
process;

```



```

Cm = 2, 3, 3, 5;           !The unit machining cost of four
processes($/min);
C_d=10;                    !The unit dressing cost($/min);
Cr=1.0;                    !The tool replacement cost($/min);
tr=1;                      !The tool replacement time(min);
D_s=35;                    !The diameter of the grinding wheel (mm);
D_dr=0.01;                !Dressing depth;
S_d=0.15;                 !The dressing lead;
V_Gs=2400;                !The speed of grinding wheel
surface (m/min);
B=10;                      !The width of grinding wheel;
K_G=2.4;                   !Constants for grinding wheel
life;
A=0.005;                  !Grinding wear;
E=0.75;                   !Machine tool efficiency;

K_T = 42.3, 292, 15.1, 0;   !empirical constants in the
tool life equation;
X_T = 0.5, 0, 0.2, 0;
U_T = 1, 1, 1, 0;
Y_T = 0, 0.15, 0.1, 0;
Z_T = 0.5, 0.2, 0.5, 0;
W_T = 2.5, 5, 3.33, 0;

K_F = 418, 900, 233, 0;    !empirical parameters in the
processes' force equation;
X_F = 1.2, 0, 0, 0;
Y_F = 0, 1.0, 1.2, 0;
Z_F = 0.75, 0.75, 0.4, 0;

K_M = 120, 0, 846, 0;      !empirical parameters in the
torque equation;
X_M = 2.2, 0, 1, 0;
Y_M = 0, 0, 0.75, 0;
Z_M = 0.8, 0, 0.8, 0;

m = 1 0 0 0               !enumerate all possible process
choice;
    0 1 0 0
    0 1 1 0
    0 1 1 1;

Bp=0.02;                  !empirical parameters in the boring power
consumption equation;
alphap=1.0;
betap=0.75;

```

```

gammap=1;

!maximum drilling, boring, reaming and grinding force,
power, torque roughness;

Fmax = 20000 4500 20000 1.5;
Pmax = 10 10 10 12;
RGmax = 0.00125;

ENDDATA

INIT:

U = 50.86;
t = 0.3 0 0 0
    0 0.1 0 0
    0 0.05 0 0
    0 0 0 0.008;
F = 0.4 0 0 0
    0 0.2 0 0
    0 0.1 0 0
    0 0 0 6.0;
D = 0 0 0 0
    0 3 0 0
    0 0.4 0 0
    0 0 0 0.03;
V = 90 0 0 0
    0 130 0 0
    0 150 0 0
    0 0 0 30;
m = 1 0 0 0
    0 1 0 0
    0 1 0 0
    0 0 0 1;

ENDINIT

!----- here is the objection function-----;

!Min = P_cost;           !The first objective function;
!Max = Sum_tol;          !The second objective function;
Max = Dummy;             !Fuzzy GP;

Sum_tol = @SUM( Links(I,J): m(I,J)*t(I,J) );
P_cost = @SUM(Links(I,J) | I #EQ# 4: m(I,J)*C_UM(I,J) );

DUMMY <= (Sum_tol-0.277)/0.631;           !Fuzzy GP;

```

```

DUMMY <= (28.78405-P_cost)/9.00516;

!*****Judge that tolerance is in which
stage's range*****;
!(1)when the design tolerance is in the range of No.4
finish machining stage;

@FOR (STAGES(I):
                                !Ensure each process
only choose one pass;
    @SUM(PROCESSES(J):m(I,J)) = 1;
);

!(2)when the design tolerance is in the range of No.3 semi-
finish machining stage;
!@FOR (STAGES(I) | I #EQ# 4:
    !    @SUM(PROCESSES(J):m(I,J)) = 0;
!);

!@FOR (STAGES(I) | I #LE# 3:
    !    @SUM(PROCESSES(J):m(I,J)) = 1;
!);

!(3)when the design tolerance is in the range of No.2 rough
machining stage;
!@FOR (STAGES(I) | I #GE# 3:
    !    @SUM(PROCESSES(J):m(I,J)) = 0;
!);
!@FOR (STAGES(I) | I #LE# 2:
    !    @SUM(PROCESSES(J):m(I,J)) = 1;
!);

!(3)when the design tolerance is in the range of No.1
primary machining stage;
!@FOR (STAGES(I) | I #GE# 2:
    !    @SUM(PROCESSES(J):m(I,J)) = 0;
!);

!@FOR (STAGES(I) | I #EQ# 1:
    !    @SUM(PROCESSES(J):m(I,J)) = 1;
!);
!*****
*****;

@FOR ( LINKS(I,J) | m(I,J) #EQ# 0:    !ensure the
unselected passes' parameter equal to 0;
    V(I,J)=0;
    F(I,J)=0;

```

```

        D(I,J)=0;
        t(I,J)=0;
    );

@FOR( LINKS: @BIN(m));          !ensure m(I,J) equal to 1, or
0;

!calculate the average manufacturing cost of drilling
process;

@FOR ( STAGES(I) | I #EQ# 1:          !if pass 1 in process
1 is chosed;
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(I,J) #EQ# 1:

        C_UM(I,J) =
(C_u+(Cost(I,J)+C_i(I,J))*(1+Pz1(I,J)))/(1-Pz2(I,J));

        Cost(I,J) = Pi*L*0.001*Dia(I,J)*V(I,J)^(-1)*F(I,J)^(-
1)*(Cm(J)+((-66.48+13.467
        *Dia(I,J)-
0.8633*Dia(I,J)^2+0.0237*Dia(I,J)^3-
0.000206*Dia(I,J)^4)+Cr*tr)
        *(V(I,J)^U_T(J)*F(I,J)^Z_T(J)*K_T(J)^(-
1)*Dia(I,J)^(-X_T(J)))^W_T(J));

        C_S(I,J) = C_u+Cost(I,J)+C_i(I,J);
        C_R(I,J) = Cost(I,J)+C_i(I,J);

        Pz1(I,J) = @PSN(z1(I,J));
        Pz2(I,J) = 1-@PSN(z2(I,J));

        @FREE(z1(I,J));
        z1(I,J) = -3*Cp(I,J);
        z2(I,J) = 3*Cp(I,J);

    );
);

!calculate the machining cost of rough, semi-finish and
finish boring process;

@FOR (STAGES(I) | I #EQ# 2:          !if pass 2 in
process 2 is chosed;
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(I,J) #EQ# 1:

        C_UM(I,J) =

```

```

(C_UM(1,1)+(Cost(I,J)+C_i(I,J))*(1+Pz1(I,J)))/(1-Pz2(I,J));

Cost(I,J) = Pi*L*0.001*Dia(I,J)*V(I,J)^(-1)*F(I,J)^(-
1)*(Cm(J)+(C_tB+Cr*tr)*(V(I,J)^U_T(J)
*D(I,J)^Y_T(J)*F(I,J)^Z_T(J)*K_T(J)^(-
1))^W_T(J));

C_S(I,J) = C_UM(1,1)+Cost(I,J)+C_i(I,J);
C_R(I,J) = Cost(I,J)+C_i(I,J);

Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));

@FREE(z1(I,J));
z1(I,J) = -3*Cp(I,J);
z2(I,J) = 3*Cp(I,J);

);
);

@FOR (STAGES(I) | I #EQ# 3: !if pass 2 in
process 3 is chosed;
@FOR (PROCESSES(J) | J #EQ# 2 #AND# m(I,J) #EQ# 1:

C_UM(I,J) =
(C_UM(2,2)+(Cost(I,J)+C_i(I,J))*(1+Pz1(I,J)))/(1-Pz2(I,J));

Cost(I,J) = Pi*L*0.001*Dia(I,J)*V(I,J)^(-1)*F(I,J)^(-
1)*(Cm(J)+(C_tB+Cr*tr)*(V(I,J)^U_T(J)
*D(I,J)^Y_T(J)*F(I,J)^Z_T(J)*K_T(J)^(-
1))^W_T(J));

C_S(I,J) = C_UM(2,2)+Cost(I,J)+C_i(I,J);
C_R(I,J) = Cost(I,J)+C_i(I,J);

Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));

@FREE(z1(I,J));
z1(I,J) = -3*Cp(I,J);
z2(I,J) = 3*Cp(I,J);

);
);

```

!calculate the machining cost of semi-finish and finish

reaming pass;

```
@FOR (STAGES(I) | I #EQ# 3 :                                !if pass 3 in
process 3 is chosed;
    @FOR (PROCESSES(J) | J #EQ# 3 #AND# m(I,J) #EQ# 1 :

        C_UM(I,J) =
        (C_UM(2,2)+(Cost(I,J)+C_i(I,J))*(1+Pz1(I,J)))/(1-Pz2(I,J));

        Cost(I,J) = Pi*L*0.001*Dia(I,J)*V(I,J)^(-1)*F(I,J)^(-
1)*(Cm(J)+(C_tr+Cr*tr)*(V(I,J)^U_T(J)
        *D(I,J)^Y_T(J)*F(I,J)^Z_T(J)*K_T(J)^(-
1)*Dia(I,J)^(-X_T(J)))^W_T(J));

        C_S(I,J) = C_UM(2,2)+Cost(I,J)+C_i(I,J);
        C_R(I,J) = Cost(I,J)+C_i(I,J);

        Pz1(I,J) = @PSN(z1(I,J));
        Pz2(I,J) = 1-@PSN(z2(I,J));

        @FREE(z1(I,J));
        z1(I,J) = -3*Cp(I,J);
        z2(I,J) = 3*Cp(I,J);

    );
);
```

!calculate the machining cost of grinding pass;

```
@FOR (STAGES(I) | I #EQ# 4:                                !if pass 4 in
process 4 is chosed;
    @FOR (PROCESSES(J) | J #EQ# 4 #AND# m(I,J) #EQ# 1:

        C_UM(I,J)=(m(3,2)*C_UM(3,2)+m(3,3)*C_UM(3,3)+(Cost(I,J)+
        C_i(I,J))*(1+Pz1(I,J))+E_loss(I,J))/(1-
        Pz2(I,J));

        C_S(I,J) =
        m(3,2)*C_UM(3,2)+m(3,3)*C_UM(3,3)+Cost(I,J)+C_i(I,J);
        C_R(I,J) = Cost(I,J)+C_i(I,J);

        Cost(I,J)=Cm(J)*Pi*(Dia(I,J)-
        2*D(I,J))*L*0.001*V(I,J)^(-1)*F(I,J)^(-1)+(C_d+Cr*tr)*K_G*L
        *(Dia(I,J)-
        2*D(I,J))*(D(I,J)^1.2)*(V(I,J)^0.2)/(1000*A*D_s*F(I,J)*(V_G
        s^0.2));
```

```

!here, calculate the expected loss;

E_loss(I,J) = K1*Sigma*((Design-Dia(I,J))*Yz3-
(Design+Tol_L-Dia(I,J))*Yz1(I,J))+K1*(Sigma^2+
(Design-Dia(I,J))^2)*(Pz3-
Pz1(I,J))+K2*Sigma*((Design-Tol_U-Dia(I,J))*Yz2(I,J)-
(Design-Dia(I,J))*Yz3)+K2*(Sigma^2+(Design-
Dia(I,J))^2)*(1-Pz2(I,J)-Pz3);

K1 = C_R(I,J)/(Tol_L^2);
K2 = C_S(I,J)/(Tol_U^2);

!Calculate the value of standard normal probability density
function;

Yz1(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z1(I,J)^2));
Yz2(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z2(I,J)^2));
Yz3 = (1/((2*Pi)^0.5))*@EXP(-0.5*(z3(I,J)^2));

!Calculate the value of cumulative standard normal
probability distribution;
Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));
Pz3 = @PSN(z3(I,J));

@FREE(z1(I,J));
@FREE(z3(I,J));
z1(I,J) = (Design-Tol_L-Dia(I,J))/Sigma;
z2(I,J) = (Design+Tol_U-Dia(I,J))/Sigma;
z3(I,J) = (Design-Dia(I,J))/Sigma;

!calculate the Sigma; !Cpk=min[(USL-U)/(3*Sigma),(U-
LSL)/(3*Sigma)];

Sigma = @SMIN(Sigma_1,Sigma_2);
Sigma_1 = (Design+Tol_U-Dia(I,J))*(3*Cp(I,J))^(-1);
Sigma_2 = (Dia(I,J)-(Design-Tol_L))*(3*Cp(I,J))^(-1);

);

!if pass 3 in process 4 is chosed;
  @FOR (PROCESSES(J) | J #EQ# 3 #AND# m(I,J) #EQ# 1:

C_UM(I,J)=(m(3,2)*C_UM(3,2)+m(3,3)*C_UM(3,3)+(Cost(I,J)+
C_i(I,J))*(1+Pz1(I,J))+E_loss(I,J))/(1-

```

```

Pz2(I,J));

C_S(I,J) =
m(3,2)*C_UM(3,2)+m(3,3)*C_UM(3,3)+Cost(I,J)+C_i(I,J);
C_R(I,J) = Cost(I,J)+C_i(I,J);

Cost(I,J) = Pi*L*0.001*Dia(I,J)*V(I,J)^(-1)*F(I,J)^(-
1)*(Cm(J)+(C_tr+Cr*tr)*(V(I,J)^U_T(J)
*D(I,J)^Y_T(J)*F(I,J)^Z_T(J)*K_T(J)^(-1)*Dia(I,J)^(-
X_T(J)))^W_T(J));

E_loss(I,J) = K1*Sigma*((Design-Dia(I,J))*Yz3-
(Design+Tol_L-Dia(I,J))*Yz1(I,J))+K1*(Sigma^2+
(Design-Dia(I,J))^2)*(Pz3-
Pz1(I,J))+K2*Sigma*((Design-Tol_U-Dia(I,J))*Yz2(I,J)-
(Design-Dia(I,J))*Yz3)+K2*(Sigma^2+(Design-
Dia(I,J))^2)*(1-Pz2(I,J)-Pz3);

K1 = C_R(I,J)/(Tol_L^2);
K2 = C_S(I,J)/(Tol_U^2);

!Calculate the value of standard normal probability density
function;

Yz1(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z1(I,J)^2));
Yz2(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z2(I,J)^2));
Yz3 = (1/((2*Pi)^0.5))*@EXP(-0.5*(z3(I,J)^2));

!Calculate the value of cumulative standard normal
probability distribution;
Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));
Pz3 = @PSN(z3(I,J));

@FREE(z1(I,J));
@FREE(z3(I,J));
z1(I,J) = (Design-Tol_L-Dia(I,J))/Sigma;
z2(I,J) = (Design+Tol_U-Dia(I,J))/Sigma;
z3(I,J) = (Design-Dia(I,J))/Sigma;

!calculate the Sigma; !Cpk=min[(USL-U)/(3*Sigma),(U-
LSL)/(3*Sigma)];

Sigma = @SMIN(Sigma_1,Sigma_2);
Sigma_1 = (Design+Tol_U-Dia(I,J))*(3*Cp(I,J))^(-1);
Sigma_2 = (Dia(I,J)-(Design-Tol_L))*(3*Cp(I,J))^(-1);

```



```

);

!if pass 2 in process 4 is chosed;
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(I,J) #EQ# 1:

C_UM(I,J)=(m(3,2)*C_UM(3,2)+m(3,3)*C_UM(3,3)+(Cost(I,J)+
    C_i(I,J))*(1+Pz1(I,J))+E_loss(I,J))/(1-
Pz2(I,J));

    C_S(I,J) =
m(3,2)*C_UM(3,2)+m(3,3)*C_UM(3,3)+Cost(I,J)+C_i(I,J);
    C_R(I,J) = Cost(I,J)+C_i(I,J);

    Cost(I,J) = Pi*L*0.001*Dia(I,J)*V(I,J)^(-1)*F(I,J)^(-
1)*(Cm(J)+(C_tB+Cr*tr)*(V(I,J)^U_T(J)
    *D(I,J)^Y_T(J)*F(I,J)^Z_T(J)*K_T(J)^(-
1))^W_T(J));

E_loss(I,J) = K1*Sigma*((Design-Dia(I,J))*Yz3-
(Design+Tol_L-Dia(I,J))*Yz1(I,J))+K1*(Sigma^2+
    (Design-Dia(I,J))^2)*(Pz3-
Pz1(I,J))+K2*Sigma*((Design-Tol_U-Dia(I,J))*Yz2(I,J)-
    (Design-Dia(I,J))*Yz3)+K2*(Sigma^2+(Design-
Dia(I,J))^2)*(1-Pz2(I,J)-Pz3);

K1 = C_R(I,J)/(Tol_L^2);
K2 = C_S(I,J)/(Tol_U^2);

!Calculate the value of standard normal probability density
function;

Yz1(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z1(I,J)^2));
Yz2(I,J) = (1/((2*Pi)^0.5))*@EXP(-0.5*(z2(I,J)^2));
Yz3 = (1/((2*Pi)^0.5))*@EXP(-0.5*(z3(I,J)^2));

!Calculate the value of cumulative standard normal
probability distribution;

Pz1(I,J) = @PSN(z1(I,J));
Pz2(I,J) = 1-@PSN(z2(I,J));
Pz3 = @PSN(z3(I,J));

@FREE(z1(I,J));
@FREE(z3(I,J));
z1(I,J) = (Design-Tol_L-Dia(I,J))/Sigma;
z2(I,J) = (Design+Tol_U-Dia(I,J))/Sigma;

```

```

z3(I,J) = (Design-Dia(I,J))/Sigma;

!calculate the Sigma; !Cpk=min[(USL-U)/(3*Sigma), (U-
LSL)/(3*Sigma)];

Sigma = @SMIN(Sigma_1,Sigma_2);
Sigma_1 = (Design+Tol_U-Dia(I,J))*(3*Cp(I,J))^( -1);
Sigma_2 = (Dia(I,J)-(Design-Tol_L))*(3*Cp(I,J))^( -1);

);
);

!limit the diameters relationship between processes;

@FOR (STAGES(I) | I #GE# 2:

    @SUM (PROCESSES(J): m(I,J)*(Dia(I,J)-2*D(I,J)))
    = @SUM (PROCESSES(H): m(I-1,H)*Dia(I-1,H));

);

!let the last process's diameter equal to the working
diameter;

@FOR (STAGES(I) | I #EQ# 4:
    @FOR (PROCESSES(J) | m(I,J) #EQ# 1:

        Dia(I,J) - U =0;

    );
);

!-----The following section is used to set constraints-----;

!process parameter constraints;

@FOR ( LINKS(I,J) | m(I,J) #EQ# 1:

    @BND(VL(I,J),V(I,J),VU(I,J));                !process speed
restriction;
    @BND(FL(I,J),F(I,J),FU(I,J));                !process feed
rate restriction;
    @BND(DL(I,J),D(I,J),DU(I,J));                !process depth
restriction;
    @BND(tL(I,J),t(I,J),tU(I,J));                !process
tolerance restriction;

```

```

);

!machining allowance constraints;

@FOR (STAGES(I) | I #GE# 2:

    (@SUM (PROCESSES(J): m(I,J)*(Dia(I,J)-t(I,J)-
2*DL(I,J)))-
    @SUM (PROCESSES(H): m(I-1,H)*(Dia(I-1,H)+t(I-
1,H))))/2 >= 0;

    (@SUM (PROCESSES(J): m(I,J)*(Dia(I,J)+t(I,J)-
2*DU(I,J)))-
    @SUM (PROCESSES(H): m(I-1,H)*(Dia(I-1,H)-t(I-
1,H))))/2 <= 0;

);

!functionality requirement constraints;

@FOR (STAGES(I) | I #EQ# 4:
    @FOR (PROCESSES(J) | m(I,J) #EQ# 1:

        U >= Design-Tol_L+t(I,J);
        U <= Design+Tol_U-t(I,J);

    );
);

!process total depth of cut constraints;

2*@SUM(LINKS(I,J) | I #GE# 2: m(I,J)*D(I,J))+Dia(1,1) = U;

!constraints for product quality requirement;
@FOR (STAGES(I) | I #EQ# 4:
    @FOR (PROCESSES(J) | m(I,J) #EQ# 1:

        1-Pz1(I,J)-Pz2(I,J) >= 1-a1;

    );
);

!*****constraints of cutting force,power,torque and
surface roughness*****

!calculate the Cutting force,torque and power of drilling

```

```

process;

@FOR (STAGES(I) | I #EQ# 1:
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(I,J) #EQ# 1:

        K_F(J)*Dia(I,J)^X_F(J)*F(I,J)^Z_F(J) = Force(I,J);
        K_M(J)*Dia(I,J)^X_M(J)*F(I,J)^Z_M(J) = Torque(I,J);
        Force(I,J) <= Fmax(J);
        Power(I,J) =
Force(I,J)*Torque(I,J)/(7018760*1.36*1000);
        Power(I,J) <= Pmax(J);

    );
);

!calculate the Cutting force,power of boring processes;

@FOR (STAGES(I) | I #GE# 2:
@FOR ( PROCESSES(J) | J #EQ# 2 #AND# m(I,J) #EQ# 1:
    K_F(J)*Dia(I,J)^X_F(J)*D(I,J)^Y_F(J)*F(I,J)^Z_F(J) =
Force(I,J);
    Bp*V(I,J)^alphap*F(I,J)^betap*D(I,J)^gammap =
Power(I,J);
    Force(I,J) <= Fmax(J);
    Power(I,J) <= Pmax(J);
);
);

!calculate the Cutting force,turque and power of reaming
processes;

@FOR (STAGES(I) | I #GE# 3:
@FOR ( PROCESSES(J) | J #EQ# 3 #AND# m(I,J) #EQ# 1:
    K_F(J)*Dia(I,J)^X_F(J)*D(I,J)^Y_F(J)*F(I,J)^Z_F(J) =
Force(I,J);
    K_M(J)*Dia(I,J)^X_M(J)*D(I,J)^Y_M(J)*F(I,J)^Z_M(J) =
Torque(I,J);
    Force(I,J) <= Fmax(J);
    Power(I,J) = Force(I,J)*Torque(I,J)/(7018760*1.36);
    Power(I,J) <= Pmax(J);
);
);

!calculate the Cutting force,power and surface roughness of
grinding processes;

@FOR (STAGES(I) | I #GE# 4:

```

```

@FOR ( PROCESSES(J) | J #EQ# 4 #AND# m(I,J) #EQ# 1:
    58.8*E*(V(I,J)*F(I,J)*D(I,J))^0.7*0.8*1.25*V_Gs^(-1) =
Force(I,J);
    0.98*(V(I,J)*F(I,J)*D(I,J))^0.7*0.8*1.25 =
Power(I,J);
    0.2*(V(I,J)*D(I,J)*V_Gs^(-1))^0.5*D_dr^0.25*S_d^0.5 =
RG;
    Force(I,J) <= Fmax(J);
    Power(I,J) <= Pmax(J);
    RG <= RGmax;
);
);

END

```

Appendix D

Source code in Lingo language for Example 3

!This cost model 2 for an assembly including one single characteristic (piston and cylinder) is presented.
The inputs are based on all practical data from handbooks and reliable sources;

MODEL:

SETS:

COMPONENT /1..2/:

TOL, Sigma, L, D_s, D_dr, C_Tm, V_Gs, B, A, Design, K1, K2, a1, C_u;

STAGES /1..4/;

PROCESSES /1..4/:

K_T, U_T, X_T, Y_T, Z_T, W_T, K_F, X_F, Y_F, Z_F, K_M, X_M, Y_M, Z_M;

PLINKS (COMPONENT, PROCESSES): Cm, FMAX, PMAX, E_loss;

MLINKS (STAGES, PROCESSES): Force, Torque, power;

LINKS (COMPONENT, STAGES, PROCESSES): m, t, V, F, D, Dia, Cost, tL, tU,

FL, FU, VL, VU, DL, DU,

C_UM, Cp, C_i, Pz2, Pz1;

ENDSETS

!----- Input the original data -----
-----;

DATA:

T_A = 0.025;

L = 60 100;

!the dimension of the raw bar;

Do = 68.0;

Design = 50.8 50.856;

!design target;

a1 = 0.055 0.053;

pi = 3.1415926;

Cp = 1.2 0 0 0

!The process capability index of each

pass for piston;

```

1.1 0 0 0
0 1.0 0 0
0 0.85 0 0
1.2 0 0 0      !The process capability index of each
pass for cylinder;
0 1.1 0 0
0 1.0 1.0 0
0 0.9 0.9 0.85;

C_i = 1.0 0 0 0      !The inspection fee of each pass for
piston;
1.5 0 0 0
0 2.0 0 0
0 2.5 0 0
1 0 0 0      !The inspection fee of each pass for
cylinder;
0 1.5 0 0
0 1.8 1.5 0
0 1.9 1.8 2;

DL = 1.5 0 0 0      !depth of cut lower limit for component
1(piston);
0.5 0 0 0
0 0.05 0 0
0 0.005 0 0
0 0 0 0      !depth of cut lower limit for component
2(cylinder);
0 1.0 0 0
0 0.1 0.1 0
0 0.007 0.007 0.005;

DU = 5.0 0 0 0      !depth of cut upper limit for component
1(piston);
1.5 0 0 0
0 0.5 0 0
0 0.05 0 0
0 0 0 0      !depth of cut upper limit for component
2(cylinder);
0 6.0 0 0
0 1 1 0
0 0.1 0.1 0.1;

tL = 0.127 0 0 0      !tolerance lower limit for component
1(piston);
0.050 0 0 0
0 0.013 0 0
0 0.005 0 0

```

```

    0.178 0 0 0      !tolerance lower limit for component
2(cylinder);
    0 0.076 0 0
    0 0.015 0.014 0
    0 0.01 0.009 0.008;

tU = 0.508 0 0 0      !tolerance upper limit for component
1(piston);
    0.305 0 0 0
    0 0.076 0 0
    0 0.025 0 0
    0.508 0 0 0      !tolerance upper limit for component
2(cylinder);
    0 0.305 0 0
    0 0.127 0.126 0
    0 0.07 0.06 0.05;

VL = 50 0 0 0          !speed lower limit for component
1(piston);
    140 0 0 0
    0 10 0 0
    0 20 0 0
    70 0 0 0          !speed lower limit for component
2(cylinder);
    0 110 0 0
    0 130 6 0
    0 140 12 20;

VU = 130 0 0 0         !speed upper limit for component
1(piston);
    200 0 0 0
    0 25 0 0
    0 40 0 0
    110 0 0 0         !speed upper limit for component
2(cylinder);
    0 140 0 0
    0 160 12 0
    0 170 20 45;

FL = 0.5 0 0 0         !feed rate lower limit for component
1(piston);
    0.1 0 0 0
    0 6.0 0 0
    0 1.0 0 0
    0.2 0 0 0         !feed rate lower limit for component
2(cylinder);
    0 0.5 0 0

```



```

    0 0.1 0.8 0
    0 0.06 0.5 2.0;

FU = 1.0 0 0 0      !feed rate upper limit for component
1(piston);
    0.4 0 0 0
    0 12.0 0 0
    0 6.0 0 0
    0.6 0 0 0      !feed rate upper limit for component
2(cylinder);
    0 1.0 0 0
    0 0.4 2.0 0
    0 0.2 1.0 8.0;

C_u = 16 32;          !The unit raw material
cost($);
Cr = 1.0;             !The tool replacement cost($) for
turning process;
tr=1;                 !The tool replacement time(min);
Ct = 3.0;             !The tool cost for turning
process($/edge);
C_tB = 7;             !The boring insert cost for boring
process($/each);
C_tR = 282;           !The tool cost for reaming
process($/each);

Cm = 1.0 5 0 0      !unit machining cost for component
1(piston);
    2 3 3 5;        !unit machining cost for component
2(cylinder);
C_d = 10;           !The unit dressing cost($/min);

D_s = 355.6 35; !The diameter of the grinding wheels for
component 1,2(mm);
D_dr = 0.02 0.01; !Dressing depth for component 1,2(mm);
S_d = 0.15;        !The dressing lead;
V_Gs = 1800 2400; !The speed of grinding wheel surface
(m/min);
B = 25.4 10;       !The width of grinding wheel for
component 1,2;
K_G = 2.4;          !Constants for grinding wheel
life;
A = 0.01 0.005;    !Grinding wear for component 1,2;
E = 0.75;           !Machine tool efficiency;

Bt = 847288609443; !empirical turning parameters in the
tool life equation;

```

```

alphat = 5.0;
betat = 2.0;
gammat = 0.75;

Bf = 0.9;           !empirical parameters in the turning
force equation;
alphaf = 0.0;
betaf = 0.75;
gammaf = 1.0;

Bp = 0.02;          !empirical parameters in the power
consumption equation;
alphap = 1.0;
betap = 0.75;
gammap = 1;

Br = 122;           !empirical turning parameters in the surface
roughness equation;
alphar = -1.52;
betar = 1.004;
gammar = 0;
eta = -0.714;
theta = -0.323;
BHN = 195;
r = 1;

K_T = 42.3, 292, 15.1, 0;   !empirical constants in the
tool life equation;
X_T = 0.5, 0, 0.2, 0;
U_T = 1, 1, 1, 0;
Y_T = 0, 0.15, 0.1, 0;
Z_T = 0.5, 0.2, 0.5, 0;
W_T = 2.5, 5, 3.33, 0;

K_F = 418, 900, 233, 0;    !empirical parameters in the
processes' force equation;
X_F = 1.2, 0, 0, 0;
Y_F = 0, 1.0, 1.2, 0;
Z_F = 0.75, 0.75, 0.4, 0;

K_M = 120, 0, 846, 0;      !empirical parameters in the
torque equation;
X_M = 2.2, 0, 1, 0;
Y_M = 0, 0, 0.75, 0;
Z_M = 0.8, 0, 0.8, 0;

Fmax = 4.5 1.5 0 0

```

```

20000 4500 20000 1.5;

Pmax = 10 12 0 0
      10 10 10 12;

m = 1 0 0 0 !enumerate all possible process choice for
component 1(piston);
      1 0 0 0
      0 1 0 0
      0 1 0 0
      1 0 0 0 !enumerate all possible process choice for
component 2(cylinder);
      0 1 0 0
      0 1 1 0
      0 1 1 1;

RGFmax = 0.00125;
RGRmax = 0.0025;
RTFmax = 0.0032;
RGmax = 0.00125;

ENDDATA

INIT:

t = 0.416 0 0 0      !tolerance for component 1;
    0.092 0 0 0
    0 0.035 0 0
    0 0.010 0 0
    0.3 0 0 0      !tolerance for component 2;
    0 0.1 0 0
    0 0.05 0 0
    0 0 0 0.008;

n = 1;
m = 1 0 0 0
    1 0 0 0
    0 1 0 0
    0 1 0 0
    1 0 0 0
    0 1 0 0
    0 1 0 0
    0 0 0 1;

F = 0.7 0 0 0
    0.3 0 0 0
    0 8.0 0 0
    0 4.0 0 0
    0.4 0 0 0

```

```

      0 0.2 0 0
      0 0.1 0 0
      0 0 0 6.0;
D = 3.9 0 0 0
      0.6 0 0 0
      0 0.15 0 0
      0 0.0495 0 0
      0 0 0 0
      0 3 0 0
      0 0.4 0 0
      0 0 0 0.03;
V = 90 0 0 0
      170 0 0 0
      0 18 0 0
      0 35 0 0
      90 0 0 0
      0 130 0 0
      0 150 0 0
      0 0 0 30;

```

ENDINIT

!----- here is the objection function -----
 -----;

```

!MIN = P_cost;           !The first objective function;
!Max = Sum_tol;          !The second objective function;
Max = Dummy;             !Fuzzy GP;

```

```

Sum_tol = @SUM( LINKS(H,I,J): m(H,I,J)*t(H,I,J) );
P_cost = @SUM(LINKS(H,I,J) | I #EQ# 4:
m(H,I,J)*C_UM(H,I,J) );

```

```

!DUMMY <= (Sum_tol-0.472)/1.294;           !Fuzzy GP;
!DUMMY <= (72.97039-P_cost)/2.05029;

```

```

DUMMY <= (Sum_tol-0.472)/1.294;           !Fuzzy GP;
DUMMY <= (72.91754-P_cost)/1.99744;

```

!-----calculate the machining cost of component 1
 (piston)-----;

!1)calculate the machining cost of rough turning pass;

```

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I) | I #EQ# 1:
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(H,I,J) #EQ# 1:

```

```

C_UM(H, I, J) =
(C_u(H) + (Cost(H, I, J) + C_i(H, I, J)) * (1 + Pz2(H, I, J))) / (1 -
Pz1(H, I, J));

Cost(H, I, J) = Pi * L(H) * 0.001 * Bt ^ (-
1) * (Cm(H, J) * Bt * V(H, I, J) ^ (-1) * F(H, I, J) ^ (-
1) + V(H, I, J) ^ (alphat - 1) * F(H, I, J) ^ (betat -
1) * D(H, I, J) ^ gammat * (Ct + Cr * tr)) * (n * Do - n * (n - 1) * D(H, I, J));

Pz1(H, I, J) = @PSN(z1);
Pz2(H, I, J) = 1 - @PSN(z2);

@FREE(z1);
z1 = -3 * Cp(H, I, J);
z2 = 3 * Cp(H, I, J);

);
);
);

!2) Calculate the machining cost of finish turning pass;

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I) | I #EQ# 2:
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(H, I, J) #EQ# 1:

      C_UM(H, I, J) =
      (C_u(1, 1, 1) + (Cost(H, I, J) + C_i(H, I, J)) * (1 + Pz2(H, I, J))) / (1 -
      Pz1(H, I, J));

      Cost(H, I, J) = Pi * L(H) * 0.001 * Bt ^ (-
      1) * (Cm(H, J) * Bt * V(H, I, J) ^ (-1) * F(H, I, J) ^ (-
      1) + V(H, I, J) ^ (alphat - 1) * F(H, I, J) ^ (betat -
      1) * D(H, I, J) ^ gammat * (Ct + Cr * tr)) * (Dia(H, I, J) + 2 * D(H, I, J));

      Pz1(H, I, J) = @PSN(z11);
      Pz2(H, I, J) = 1 - @PSN(z21);

      @FREE(z11);
      z11 = -3 * Cp(H, I, J);
      z21 = 3 * Cp(H, I, J);

    );
  );
);

```

!3)Calculate the machining cost of grinding passes;

```

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I) | I #EQ# 3:
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:

      C_UM(H,I,J) =
        (C_UM(1,2,1)+(Cost(H,I,J)+C_i(H,I,J))*(1+Pz2(H,I,J)))/(1-
        Pz1(H,I,J));

      Cost(H,I,J)=Cm(H,J)*Pi*(Dia(H,I,J)+2*D(H,I,J))*L(H)*0.001*V
        (H,I,J)^(-1)*F(H,I,J)^(-
        1)+(C_d+Cr*tr)*K_G*L(H)*(Dia(H,I,J)+2*D(H,I,J))*(D(H,I,J)^1
        .2)*(V(H,I,J)^0.2)/(1000*A(H)*D_s(H)*F(H,I,J)*(V_Gs(H)^0.2)
        );

      Pz1(H,I,J) = @PSN(z12);
      Pz2(H,I,J) = 1-@PSN(z22);

      @FREE(z12);
      z12 = -3*Cp(H,I,J);
      z22 = 3*Cp(H,I,J);

    );
  );
);

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I) | I #EQ# 4:
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:

      C_UM(H,I,J) =
        (C_UM(1,3,2)+(Cost(H,I,J)+C_i(H,I,J))*(1+Pz2(H,I,J))+E_loss
        (H,J))/(1-Pz1(H,I,J));

      Cost(H,I,J)=Cm(H,J)*Pi*(Dia(H,I,J)+2*D(H,I,J))*L(H)*0.001*V
        (H,I,J)^(-1)*F(H,I,J)^(-
        1)+(C_d+Cr*tr)*K_G*L(H)*(Dia(H,I,J)+2*D(H,I,J))*(D(H,I,J)^1
        .2)*(V(H,I,J)^0.2)/(1000*A(H)*D_s(H)*F(H,I,J)*(V_Gs(H)^0.2)
        );

      !here, calculate the expected loss;

      E_loss(H,J) = K1(H)*Sigma(H)*(-
      Tol(H)*((1/((2*Pi)^0.5))*@EXP(-

```

```

0.5*(z1^2))))+K1(H)*Sigma(H)^2
      *(0.5-Pz1(H,I,J))+K2(H)*Sigma(H)*(-
Tol(H))*((1/((2*Pi)^0.5))*@EXP(-0.5*(z2^2)))
      +K2(H)*(Sigma(H)^2)*(1-Pz2(H,I,J)-0.5);

K1(H) = (C_UM(1,3,2)+Cost(H,I,J)+C_i(H,I,J))/(Tol(H)^2);
K2(H) = (Cost(H,I,J)+C_i(H,I,J))/(Tol(H)^2);

!Calculate the value of standard normal probability density
function;

      Pz1(H,I,J) = @PSN(z13);
      Pz2(H,I,J) = 1-@PSN(z23);

!Calculate the value of cumulative standard normal
probability distribution;

@FREE(z13);
z13 = (-Tol(H))/Sigma(H);
z23 = Tol(H)/Sigma(H);

Sigma(H) = Tol(H)*(3*Cp(H,I,J))^(-1);    !calculate the
Sigma;

);
);
);

!-----calculate the machining cost of component 2
(cylinder)-----;

!1)calculate the machining cost of drilling process;

@FOR (COMPONENT(H) | H #EQ# 2:
      @FOR ( STAGES(I) | I #EQ# 1:
            @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(H,I,J) #EQ# 1:

                  C_UM(H,I,J) =
(C_u(H)+(Cost(H,I,J)+C_i(H,I,J))*(1+Pz1(H,I,J)))/(1-
Pz2(H,I,J));

                  Cost(H,I,J) = Pi*L(H)*0.001*Dia(H,I,J)*V(H,I,J)^(-
1)*F(H,I,J)^(-1)*(Cm(H,J)+((-66.48+13.467*Dia(H,I,J)-
0.8633*Dia(H,I,J)^2+0.0237*Dia(H,I,J)^3-
0.000206*Dia(H,I,J)^4)+Cr*tr)*(V(H,I,J)^U_T(J)*F(H,I,J)^Z_T
(J)*K_T(J)^(-1)*Dia(H,I,J)^(-X_T(J)))^W_T(J));

```

```

Pz1(H,I,J) = @PSN(z14);
Pz2(H,I,J) = 1-@PSN(z24);

@FREE(z14);
z14 = -3*Cp(H,I,J);
z24 = 3*Cp(H,I,J);

);
);
);

!2)calculate the machining cost of rough, semi-finish and
finish boring process;

@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #EQ# 2:
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:

      C_UM(H,I,J) =
      (C_UM(2,1,1)+(Cost(H,I,J)+C_i(H,I,J))*(1+Pz1(H,I,J)))/(1-
      Pz2(H,I,J));

      Cost(H,I,J) = Pi*L(H)*0.001*Dia(H,I,J)*V(H,I,J)^(-
      1)*F(H,I,J)^(-
      1)*(Cm(H,J)+(C_tB+Cr*tr)*(V(H,I,J)^U_T(J)*D(H,I,J)^Y_T(J)*F
      (H,I,J)^Z_T(J)*K_T(J)^(-1))^W_T(J));

      Pz1(H,I,J) = @PSN(z15);
      Pz2(H,I,J) = 1-@PSN(z25);

      @FREE(z15);
      z15 = -3*Cp(H,I,J);
      z25 = 3*Cp(H,I,J);

    );
  );
);

@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #EQ# 3:
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:

      C_UM(H,I,J) =
      (C_UM(2,2,2)+(Cost(H,I,J)+C_i(H,I,J))*(1+Pz1(H,I,J)))/(1-
      Pz2(H,I,J));

      Cost(H,I,J) = Pi*L(H)*0.001*Dia(H,I,J)*V(H,I,J)^(-

```



```

1)*F(H,I,J)^(-
1)*(Cm(H,J)+(C_tB+Cr*tr)*(V(H,I,J)^U_T(J)*D(H,I,J)^Y_T(J)*F
(H,I,J)^Z_T(J)*K_T(J)^(-1))^W_T(J));

```

```

Pz1(H,I,J) = @PSN(z16);
Pz2(H,I,J) = 1-@PSN(z26);

```

```

@FREE(z16);
z16 = -3*Cp(H,I,J);
z26 = 3*Cp(H,I,J);

```

```

);
);
);

```

!3)calculate the machining cost of semi-finish and finish reaming pass;

```

@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #EQ# 3 :
    @FOR (PROCESSES(J) | J #EQ# 3 #AND# m(H,I,J) #EQ# 1 :

```

```

      C_UM(H,I,J) =
      (C_UM(2,2,2)+(Cost(H,I,J)+C_i(H,I,J))*(1+Pz1(H,I,J)))/(1-
Pz2(H,I,J));

```

```

      Cost(H,I,J) = Pi*L(H)*0.001*Dia(H,I,J)*V(H,I,J)^(-
1)*F(H,I,J)^(-
1)*(Cm(H,J)+(C_tr+Cr*tr)*(V(H,I,J)^U_T(J)*D(H,I,J)^Y_T(J)*F
(H,I,J)^Z_T(J)*K_T(J)^(-1)*Dia(H,I,J)^(-X_T(J))^W_T(J));

```

```

Pz1(H,I,J) = @PSN(z17);
Pz2(H,I,J) = 1-@PSN(z27);

```

```

@FREE(z17);
z17 = -3*Cp(H,I,J);
z27 = 3*Cp(H,I,J);

```

```

);
);
);

```

!4)calculate the machining cost of grinding pass;

```

@FOR (COMPONENT(H) | H #EQ# 2:

```

```

@FOR (STAGES(I) | I #EQ# 4:
  @FOR (PROCESSES(J) | J #EQ# 4 #AND# m(H,I,J) #EQ# 1:

    C_UM(H,I,J) =
(m(2,3,2)*C_UM(2,3,2)+m(2,3,3)*C_UM(2,3,3)+(Cost(H,I,J)+C_i
(H,I,J))*(1+Pz1(H,I,J))+E_loss(H,J))/(1-Pz2(H,I,J));

    Cost(H,I,J)=Cm(H,J)*Pi*(Dia(H,I,J)-
2*D(H,I,J))*L(H)*0.001*V(H,I,J)^(-1)*F(H,I,J)^(-
1)+(C_d+Cr*tr)*K_G*L(H)*(Dia(H,I,J)-
2*D(H,I,J))*(D(H,I,J)^1.2)*(V(H,I,J)^0.2)/(1000*A(H)*D_s(H)
*F(H,I,J)*(V_Gs(H)^0.2));

!here, calculate the expected loss;

E_loss(H,J) = K1(H)*Sigma(H)*(-
Tol(H)*((1/((2*Pi)^0.5))*@EXP(-
0.5*(z1^2)))+K1(H)*Sigma(H)^2*(0.5-
Pz1(H,I,J))+K2(H)*Sigma(H)*(-
Tol(H))*((1/((2*Pi)^0.5))*@EXP(-
0.5*(z2^2)))+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I,J)-0.5);

K1(H) = (Cost(H,I,J)+C_i(H,I,J))/(Tol(H)^2);
K2(H) =
(m(2,3,2)*C_UM(2,3,2)+m(2,3,3)*C_UM(2,3,3)+Cost(H,I,J)+C_i(
H,I,J))/(Tol(H)^2);

!Calculate the value of cumulative standard normal
probability distribution;

Pz1(H,I,J) = @PSN(z18);
Pz2(H,I,J) = 1-@PSN(z28);

@FREE(z18);
z18 = (-Tol(H))/Sigma(H);
z28 = Tol(H)/Sigma(H);

Sigma(H) = Tol(H)*(3*Cp(H,I,J))^(-1);    !calculate the
Sigma;

);

@FOR (PROCESSES(J) | J #EQ# 3 #AND# m(H,I,J) #EQ# 1:

  C_UM(H,I,J) =
(m(2,3,2)*C_UM(2,3,2)+m(2,3,3)*C_UM(2,3,3)+(Cost(H,I,J)+C_i
(H,I,J))*(1+Pz1(H,I,J))+E_loss(H,J))/(1-Pz2(H,I,J));

```

```

Cost(H,I,J) = Pi*L(H)*0.001*Dia(H,I,J)*V(H,I,J)^(-
1)*F(H,I,J)^(-
1)*(Cm(H,J)+(C_tR+Cr*tr)*(V(H,I,J)^U_T(J)*D(H,I,J)^Y_T(J)*F
(H,I,J)^Z_T(J)*K_T(J)^(-1)*Dia(H,I,J)^(-X_T(J)))^W_T(J));

```

!here, calculate the expected loss;

```

E_loss(H,J) = K1(H)*Sigma(H)*(-
Tol(H)*((1/((2*Pi)^0.5))*@EXP(-
0.5*(z1^2))))+K1(H)*Sigma(H)^2*(0.5-
Pz1(H,I,J))+K2(H)*Sigma(H)*(-
Tol(H))*((1/((2*Pi)^0.5))*@EXP(-
0.5*(z2^2)))+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I,J)-0.5);

```

```

K1(H) = (Cost(H,I,J)+C_i(H,I,J))/(Tol(H)^2);
K2(H) =
(m(2,3,2)*C_UM(2,3,2)+m(2,3,3)*C_UM(2,3,3)+Cost(H,I,J)+C_i(
H,I,J))/(Tol(H)^2);

```

!Calculate the value of cumulative standard normal probability distribution;

```

Pz1(H,I,J) = @PSN(z19);
Pz2(H,I,J) = 1-@PSN(z29);

```

```

@FREE(z19);
z19 = (-Tol(H))/Sigma(H);
z29 = Tol(H)/Sigma(H);

```

```

Sigma(H) = Tol(H)*(3*Cp(H,I,J))^(-1);    !calculate the
Sigma;

```

```
);
```

```

@FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:

```

```

C_UM(H,I,J) =
(m(2,3,2)*C_UM(2,3,2)+m(2,3,3)*C_UM(2,3,3)+(Cost(H,I,J)+C_i
(H,I,J))*(1+Pz1(H,I,J))+E_loss(H,J))/(1-Pz2(H,I,J));

```

```

Cost(H,I,J) = Pi*L(H)*0.001*Dia(H,I,J)*V(H,I,J)^(-
1)*F(H,I,J)^(-
1)*(Cm(H,J)+(C_tB+Cr*tr)*(V(H,I,J)^U_T(J)*D(H,I,J)^Y_T(J)*F
(H,I,J)^Z_T(J)*K_T(J)^(-1))^W_T(J));

```

!here, calculate the expected loss;

```

E_loss(H,J) = K1(H)*Sigma(H)*(-
Tol(H)*((1/((2*Pi)^0.5))*@EXP(-
0.5*(z1^2))))+K1(H)*Sigma(H)^2*(0.5-
Pz1(H,I,J))+K2(H)*Sigma(H)*(-
Tol(H))*((1/((2*Pi)^0.5))*@EXP(-
0.5*(z2^2)))+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I,J)-0.5);

K1(H) = (Cost(H,I,J)+C_i(H,I,J))/(Tol(H)^2);
K2(H) =
(m(2,3,2)*C_UM(2,3,2)+m(2,3,3)*C_UM(2,3,3)+Cost(H,I,J)+C_i(
H,I,J))/(Tol(H)^2);

!Calculate the value of cumulative standard normal
probability distribution;

Pz1(H,I,J) = @PSN(z110);
Pz2(H,I,J) = 1-@PSN(z210);

@FREE(z110);
z110 = (-Tol(H))/Sigma(H);
z210 = Tol(H)/Sigma(H);

Sigma(H) = Tol(H)*(3*Cp(H,I,J))^(-1);    !calculate the
Sigma;

);
);
);

@GIN(n);
@FOR( LINKS: @BIN(m));

!-----constraints-----
--;
!constraints for product quality requirement;

@FOR (COMPONENT(H):
    @FOR (STAGES(I) | I #EQ# 4:
        @FOR(PROCESSES(J) | m(H,I,J) #EQ# 1:

            1-Pz1(H,I,J)-Pz2(H,I,J) >= 1-a1(H);

        );
    );
);

```

```

@FOR (COMPONENT(H):
  @FOR (STAGES(I):
    @SUM(PROCESSES(J):m(H,I,J)) = 1;
  );
);
!-----
----;
!@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I):
    @SUM(PROCESSES(J):m(H,I,J)*t(H,I,J)) <= 0.012;
  !);
  !);

t(1,4,2) <= 0.013;
!t(1,4,2) < t(1,3,2);
m(2,4,2)*t(2,4,2)+ m(2,4,3)*t(2,4,3)+ m(2,4,4)*t(2,4,4) <=
0.014;
!-----
---;

@FOR ( LINKS(H,I,J) | m(H,I,J) #EQ# 0:
  V(H,I,J)=0;
  F(H,I,J)=0;
  D(H,I,J)=0;
  t(H,I,J)=0;
);

!let the last process's diameter equal to the working
diameter;

@FOR (COMPONENT(H):
  @FOR (STAGES(I) | I #EQ# 4:
    @FOR (PROCESSES(J) | m(H,I,J) #EQ# 1:

      Dia(H,I,J) = Design(H);
      Tol(H) = t(H,I,J);

    );
  );
);

!@FOR (COMPONENT(H):
  @FOR (STAGES(I) | I #EQ# 4:

    @SUM (PROCESSES(J): m(H,I,J)*t(H,I,J))
      < @SUM (PROCESSES(L): m(H,I-1,L)*tL(H,I-1,L));
  );
);

```

```

!);
!);

!process parameter constraints;

@FOR ( LINKS(H,I,J) | m(H,I,J) #EQ# 1:

@BND(VL(H,I,J),V(H,I,J),VU(H,I,J));           !process speed
restriction;
@BND(FL(H,I,J),F(H,I,J),FU(H,I,J));           !process feed rate
restriction;
@BND(DL(H,I,J),D(H,I,J),DU(H,I,J));           !process depth
restriction;
@BND(tL(H,I,J),t(H,I,J),tU(H,I,J));           !process tolerance
restriction;

);

Dia(1,1,1) = Do - 2*n*D(1,1,1);

!limit the diameters relationship between two processes;

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I) | I #GE# 2:

    @SUM (PROCESSES(J): m(H,I,J)*(Dia(H,I,J)+2*D(H,I,J)))
      = @SUM (PROCESSES(L): m(H,I-1,L)*Dia(H,I-1,L));

  );
);

@FOR (COMPONENT(H) | H #EQ# 2:
@FOR (STAGES(I) | I #GE# 2:

  @SUM (PROCESSES(J): m(H,I,J)*(Dia(H,I,J)-2*D(H,I,J)))
    = @SUM (PROCESSES(L): m(H,I-1,L)*Dia(H,I-1,L));

);
);

!machining allowance constraints;

@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #GE# 2:

    (@SUM (PROCESSES(J): m(H,I,J)*(Dia(H,I,J)-t(H,I,J)-
2*DL(H,I,J)))-

```

```

        @SUM (PROCESSES(L): m(H,I-1,L)*(Dia(H,I-1,L)+t(H,I-
1,L))))/2 >= 0;

        (@SUM (PROCESSES(J): m(H,I,J)*(Dia(H,I,J)+t(H,I,J)-
2*DU(H,I,J)))-
        @SUM (PROCESSES(L): m(H,I-1,L)*(Dia(H,I-1,L)-t(H,I-
1,L))))/2 <= 0;

);
);

@FOR (COMPONENT(H) | H #EQ# 1:
    @FOR (STAGES(I) | I #GE# 2:

        (@SUM (PROCESSES(J): m(H,I-1,J)*(Dia(H,I-1,J)-t(H,I-
1,J)))-
        @SUM (PROCESSES(L):
m(H,I,L)*(Dia(H,I,L)+t(H,I,L)+2*DL(H,I,L))))/2 >= 0;

        (@SUM (PROCESSES(J): m(H,I-1,J)*(Dia(H,I-1,J)+t(H,I-
1,J)))-
        @SUM (PROCESSES(L): m(H,I,L)*(Dia(H,I,L)-
t(H,I,L)+2*DU(H,I,L))))/2 <= 0;

);
);

!Constraints WC;

@SUM( LINKS(H,I,J) | I #EQ# 4: m(H,I,J)*t(H,I,J) ) <= T_A;

!Constraints RSS;

!(@SUM( LINKS(H,I,J) | I #EQ# 4:
m(H,I,J)*(t(H,I,J))^2 ))^0.5 <= T_A;

!process total depth of cut constraints;

@FOR (COMPONENT(H) | H #EQ# 1:

Design(H)+2*@SUM(MLINKS(I,J) | I #GE# 2:
m(H,I,J)*D(H,I,J))+2*n*D(1,1,1) = Do;

);

@FOR (COMPONENT(H) | H #EQ# 2:

```

```

2*@SUM(MLINKS(I,J) | I #GE# 2: m(H,I,J)*D(H,I,J))+Dia(2,1,1)
= Design(H);

);

!constraints of cutting force,power,and surface roughness
for component 1;

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR ( STAGES(I) | I #LE# 2:
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(H,I,J) #EQ# 1:

      Bf*V(H,I,J)^alphaf*F(H,I,J)^betaf*D(H,I,J)^gammaf <=
Fmax(H,J);
      Bp*V(H,I,J)^alphap*F(H,I,J)^betap*D(H,I,J)^gammap <=
Pmax(H,J);

    );
  );
);

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR ( STAGES(I) | I #EQ# 2:
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(H,I,J) #EQ# 1:

      Br*r^eta*BHN^(theta)*V(H,I,J)^alphan*F(H,I,J)^betar <=
RTFmax;

    );
  );
);

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR ( STAGES(I) | I #EQ# 3:
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:

      86.4*E*(V(H,I,J)*F(H,I,J)*D(H,I,J))^0.7*0.8*1.05*V_Gs(H)^(-
1) <= Fmax(H,J);
      1.44*(V(H,I,J)*F(H,I,J)*D(H,I,J))^0.7*0.8*1.05 <= Pmax(H,J);
      0.2*(V(H,I,J)*D(H,I,J)*V_Gs(H)^(-
1))^0.5*D_dr(H)^0.25*S_d^0.5+R_slack1 = RGRmax;

    );
  );
);

@FOR (COMPONENT(H) | H #EQ# 1:

```



```

@FOR ( STAGES(I) | I #EQ# 4:
  @FOR (PROCESSES(J) | J #EQ# 2:

86.4*E*(V(H,I,J)*F(H,I,J)*D(H,I,J))^0.7*0.8*1.05*V_Gs(H)^(-
1) <= Fmax(H,J);
1.44*(V(H,I,J)*F(H,I,J)*D(H,I,J))^0.7*0.8*1.05 <= Pmax(H,J);
0.2*(V(H,I,J)*D(H,I,J)*V_Gs(H)^(-
1))^0.5*D_dr(H)^0.25*S_d^0.5+R_slack2 = RGFmax;

);
);
);

!constraints of cutting force,power,and surface roughness
for component 2;
!calculate the Cutting force,torque and power of drilling
process;

@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #EQ# 1:
    @FOR (PROCESSES(J) | J #EQ# 1 #AND# m(H,I,J) #EQ# 1:

      K_F(J)*Dia(H,I,J)^X_F(J)*F(H,I,J)^Z_F(J) = Force(I,J);
      K_M(J)*Dia(H,I,J)^X_M(J)*F(H,I,J)^Z_M(J) = Torque(I,J);
      Force(I,J) <= Fmax(H,J);
      Power(I,J) =
Force(I,J)*Torque(I,J)/(7018760*1.36*1000);
      Power(I,J) <= Pmax(H,J);

    );
  );
);

!calculate the Cutting force,power of boring processes;

@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #GE# 2:
    @FOR (PROCESSES(J) | J #EQ# 2 #AND# m(H,I,J) #EQ# 1:
      K_F(J)*Dia(H,I,J)^X_F(J)*D(H,I,J)^Y_F(J)*F(H,I,J)^Z_F(J) =
Force(I,J);
      Bp*V(H,I,J)^alphap*F(H,I,J)^betap*D(H,I,J)^gammap =
Power(I,J);
      Force(I,J) <= Fmax(H,J);
      Power(I,J) <= Pmax(H,J);

    );
  );
);

```

!calculate the Cutting force,turque and power of reaming processes;

```
@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #GE# 3:
    @FOR ( PROCESSES(J) | J #EQ# 3 #AND# m(H,I,J) #EQ#
1:
  K_F(J)*Dia(H,I,J)^X_F(J)*D(H,I,J)^Y_F(J)*F(H,I,J)^Z_F(
J) = Force(I,J);
  K_M(J)*Dia(H,I,J)^X_M(J)*D(H,I,J)^Y_M(J)*F(H,I,J)^Z_M(
J) = Torque(I,J);
  Force(I,J) <= Fmax(H,J);
  Power(I,J) = Force(I,J)*Torque(I,J)/(7018760*1.36);
  Power(I,J) <= Pmax(H,J);
);
);
);
```

!calculate the Cutting force,power and surface roughness of grinding processes;

```
@FOR (COMPONENT(H) | H #EQ# 2:
  @FOR (STAGES(I) | I #GE# 4:
    @FOR ( PROCESSES(J) | J #EQ# 4 #AND# m(H,I,J) #EQ# 1:
      58.8*E*(V(H,I,J)*F(H,I,J)*D(H,I,J))^0.7*0.8*1.25*V_Gs(
H)^(-1) = Force(I,J);
      0.98*(V(H,I,J)*F(H,I,J)*D(H,I,J))^0.7*0.8*1.25 =
Power(I,J);
      0.2*(V(H,I,J)*D(H,I,J)*V_Gs(H)^(-
1))^0.5*D_dr(H)^0.25*S_d^0.5 = RG;
      Force(I,J) <= Fmax(H,J);
      Power(I,J) <= Pmax(H,J);
      RG <= RGmax;
);
);
);
```

END

Appendix E

Source code in Lingo language for Example 4

!This cost model 2 for an assembly (example 4) including four quality characteristics (base and top) is presented. The inputs are based on all practical data from handbooks and reliable sources;

MODEL:

SETS:

FUNCTION /1..4/:T_Q;
COMPONENT /1..8/:DESIGN,d_e,E_LOSS,Sigma,K1,K2,N;
STAGES /1..2/:Cp,C_i,DL,DÜ,tL,tU,vL,vU,fL,fU;
LINKS (COMPONENT,STAGES):C_UM,v,f,d,t,pz1,pz2,cost,z1,z2;

ENDSETS

DATA:

aa = 5;

T_Q = 0.58 0.28 0.17 0.23;
DESIGN = 25.4 50.8 76.2 101.6 25.4 25.3492 50.8 76.1492;
d_e = 30 20 20 30 50 20 20 40;
L = 40;
Dc = 16;
Nt = 3;
a1 = 0.053;
pi = 3.1415926;
Cp = 1.2 0.9;
C_i = 1.0 1.5;
DL = 0.8 0.08;
DU = 3 0.8;
tL = 0.08 0.01;
tU = 0.5 0.1;
vL = 50 90;
vU = 90 140;
fL = 0.05 0.01;
fU = 0.09 0.06;
C_u = 30;
C_u1 = 25;

```

Crm =1.0;
trm = 1;
Cmm = 3.0;
C_tm = 18;
E_ = 0.75;
K_T = 53;
X_T = 0.45;
U_T = 1;
Y_T = 0.5;
Z_T = 0.5;
V_T = 0.1;
S_T = 0.1;
W_T = 3.03;

K_F1 = 0.642;
K_F2 = -0.86;
K_F3 = 0;
K_F4 = 0.72;
K_F5 = 0.86;

K_P1 = 0.0107;
K_P2 = -0.86;
K_P3 = 1;
K_P4 = 0.72;
K_P5 = 0.86;

Fmax = 0.8;
Pmax = 1.5;

ENDDATA

INIT:

N = 2 2 2 2 2 2 2 2;

t = 0.15 0.05
    0.2 0.05
    0.2 0.04
    0.3 0
    0.4 0
    0.1 0.03
    0.15 0.025
    0.2 0.05;
f = 0.07 0.04
    0.07 0.04
    0.07 0.04
    0.09 0

```

```

0.09 0
0.07 0.04
0.07 0.04
0.07 0.04;

d = 2.3 0.4
2.3 0.4
2.3 0.4
2.5 0
2.5 0
2.3 0.4
2.3 0.4
2.3 0.4;

v = 60 100
60 100
60 100
70 0
70 0
60 100
60 100
60 100;

ENDINIT

!MIN = Sum_cost;
!Max = Sum_tol;
Max = Dummy;                                !Fuzzy GP;

Sum_tol = @SUM(LINKS(H,I): t(H,I));
Sum_cost =
C_um(1,2)+C_um(2,2)+C_um(3,2)+C_um(4,1)+C_um(5,1)+C_um(6,2)
+C_um(7,2)+C_um(8,2);

!WC constraints;
!DUMMY <= (Sum_tol-1.12)/1.62;                !Fuzzy GP;
!DUMMY <= (273.734-Sum_cost)/3.6444;

!RSS constraints;
DUMMY <= (Sum_tol-1.12)/1.76956;              !Fuzzy GP;
DUMMY <= (277.4795-Sum_cost)/7.3899;

!-----component 1-----
-----;

@FOR (COMPONENT(H) | H #EQ# 1:
      @FOR (STAGES(I) | I #EQ# 1:

```

```

C_UM(H,I) = (C_u+(Cost(H,I)+C_i(I))*(1+Pz2(H,I)))/(1-
Pz1(H,I));

Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = design(H)+5-N(H)*d(H,I);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = -3*Cp(I);
z2(H,I) = 3*Cp(I);

);
);

@FOR (COMPONENT(H) | H #EQ# 1:
  @FOR (STAGES(I) | I #EQ# 2:

C_UM(H,I) =
(C_um(1,1)+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_loss(H))/(1-
Pz1(H,I));

Cost(H,I) = Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = dia(1,1)-d(H,I);

E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*
Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2)))+
K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_UM(1,1)+cost(H,I)+C_i(I))/(t(H,I)^2);
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

```

```

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^( -1);

);
);

!-----composant 2-----
-----;

@FOR (COMPONENT(H) | H #EQ# 2:
      @FOR (STAGES(I) | I #EQ# 1:

C_UM(H,I) = (C_u+(Cost(H,I)+C_i(I))*(1+Pz2(H,I)))/(1-
Pz1(H,I));

Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = design(H)+5-N(H)*d(H,I);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = -3*Cp(I);
z2(H,I) = 3*Cp(I);

);
);

@FOR (COMPONENT(H) | H #EQ# 2:
      @FOR (STAGES(I) | I #EQ# 2:

C_UM(H,I) =
(C_um(2,1)+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_loss(H))/(1-
Pz1(H,I));

Cost(H,I) = Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

```

```

!Dia(H,I) = dia(2,1)-d(H,I);

E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*
Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2))
+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_UM(2,1)+cost(H,I)+C_i(I))/(t(H,I)^2);
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^( -1);

);
);

!-----composant 3-----
-----;

@FOR (COMPONENT(H) | H #EQ# 3:
      @FOR (STAGES(I) | I #EQ# 1:

C_UM(H,I) = (C_u+(Cost(H,I)+C_i(I))*(1+Pz2(H,I)))/(1-
Pz1(H,I));

Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = design(H)+5-N(H)*d(H,I);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = -3*Cp(I);
z2(H,I) = 3*Cp(I);

);

```



```

);

@FOR (COMPONENT(H) | H #EQ# 3:
    @FOR (STAGES(I) | I #EQ# 2:

C_UM(H,I) =
(C_um(3,1)+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_loss(H))/(1-
Pz1(H,I));

Cost(H,I) = Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = dia(3,1)-d(H,I);

E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2))))+K1(H)*
Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2))))
+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_UM(3,1)+cost(H,I)+C_i(I))/(t(H,I)^2);
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^(-1);

);
);

!-----composant 4-----
-----;

@FOR (COMPONENT(H) | H #EQ# 4:
    @FOR (STAGES(I) | I #EQ# 1:

C_UM(H,I) =
(C_u+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_Loss(H))/(1-
Pz1(H,I));

```

```
Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);
```

```
!Dia(H,I) = design(H)+5-N(H)*d(H,I);
```

```
E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*
Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2)))+
K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);
```

```
K1(H) = (C_u+cost(H,I)+C_i(I))/(t(H,I)^2);
```

```
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);
```

```
Pz1(H,I) = @PSN(z1(H,I));
```

```
Pz2(H,I) = 1-@PSN(z2(H,I));
```

```
@FREE(Z1(H,I));
```

```
z1(H,I) = (-t(H,I))/Sigma(H);
```

```
z2(H,I) = t(H,I)/Sigma(H);
```

```
Sigma(H) = t(H,I)*(3*Cp(I))^(-1);
```

```
);
```

```
);
```

```
!-----componnent 5-----
-----;
```

```
@FOR (COMPONENT(H) | H #EQ# 5:
```

```
  @FOR (STAGES(I) | I #EQ# 1:
```

```
    C_UM(H,I) =
```

```
    (C_u+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_Loss(H))/(1-
    Pz1(H,I));
```

```
Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);
```

```
!Dia(H,I) = design(H)+5-N(H)*d(H,I);
```

```
E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*
```

```

Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2)))
+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_u+cost(H,I)+C_i(I))/(t(H,I)^2);
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^( -1);
);
);

!-----composant 6-----
-----;

@FOR (COMPONENT(H) | H #EQ# 6:
      @FOR (STAGES(I) | I #EQ# 1:

C_UM(H,I) = (C_u1+(Cost(H,I)+C_i(I))*(1+Pz2(H,I)))/(1-
Pz1(H,I));

Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = design(H)+5-N(H)*d(H,I);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = -3*Cp(I);
z2(H,I) = 3*Cp(I);

);
);

@FOR (COMPONENT(H) | H #EQ# 6:
      @FOR (STAGES(I) | I #EQ# 2:

```

```

C_UM(H,I) =
(C_um(6,1)+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_loss(H))/(1-
Pz1(H,I));

Cost(H,I) = Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = dia(6,1)-d(H,I);

E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*
Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2)))+
K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_UM(6,1)+cost(H,I)+C_i(I))/(t(H,I)^2);
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^(-1);

);
);

!-----composant 7-----
-----;

@FOR (COMPONENT(H) | H #EQ# 7:
@FOR (STAGES(I) | I #EQ# 1:

C_UM(H,I) = (C_u1+(Cost(H,I)+C_i(I))*(1+Pz2(H,I)))/(1-
Pz1(H,I));

Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = design(H)+5-N(H)*d(H,I);

```

```

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = -3*Cp(I);
z2(H,I) = 3*Cp(I);

);
);

@FOR (COMPONENT(H) | H #EQ# 7:
      @FOR (STAGES(I) | I #EQ# 2:

C_UM(H,I) =
(C_um(7,1)+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_loss(H))/(1-
Pz1(H,I));

Cost(H,I) = Pi*Dc*(L+(d(H,I)*(Dc-
d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+
Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_
T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = dia(7,1)-d(H,I);

E_loss(H) = K1(H)*Sigma(H)*(-
t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*
Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-
t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2)))+
K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_UM(7,1)+cost(H,I)+C_i(I))/(t(H,I)^2);
K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^(-1);

);
);

!-----composant 8-----

```

-----;

@FOR (COMPONENT(H) | H #EQ# 8:

@FOR (STAGES(I) | I #EQ# 1:

C_UM(H,I) = (C_u1+(Cost(H,I)+C_i(I))*(1+Pz2(H,I)))/(1-Pz1(H,I));

Cost(H,I) = N(H)*Pi*Dc*(L+(d(H,I)*(Dc-d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = design(H)+5-N(H)*d(H,I);

Pz1(H,I) = @PSN(z1(H,I));

Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));

z1(H,I) = -3*Cp(I);

z2(H,I) = 3*Cp(I);

);

);

@FOR (COMPONENT(H) | H #EQ# 8:

@FOR (STAGES(I) | I #EQ# 2:

C_UM(H,I) = (C_um(8,1)+(Cost(H,I)+C_i(I))*(1+Pz2(H,I))+E_loss(H))/(1-Pz1(H,I));

Cost(H,I) = Pi*Dc*(L+(d(H,I)*(Dc-d(H,I)))^0.5+1)/(1000*Nt*v(H,I)*f(H,I))*(Cmm+(C_tm+Crm*trm)*(v(H,I)^U_T*d(H,I)^Y_T*f(H,I)^Z_T*d_e(H)^V_T*Nt^S_T/(K_T*Dc^X_T))^W_T);

!Dia(H,I) = dia(8,1)-d(H,I);

E_loss(H) = K1(H)*Sigma(H)*(-t(H,I)*((1/((2*pi)^0.5))*@EXP(-0.5*(z1(H,I)^2)))+K1(H)*Sigma(H)^2*(0.5-Pz1(H,I))+K2(H)*Sigma(H)*(-t(H,I))*((1/((2*pi)^0.5))*@EXP(-0.5*(z2(H,I)^2)))+K2(H)*(Sigma(H)^2)*(1-Pz2(H,I)-0.5);

K1(H) = (C_UM(8,1)+cost(H,I)+C_i(I))/(t(H,I)^2);

K2(H) = (cost(H,I)+C_i(I))/(t(H,I)^2);

```

Pz1(H,I) = @PSN(z1(H,I));
Pz2(H,I) = 1-@PSN(z2(H,I));

@FREE(Z1(H,I));
z1(H,I) = (-t(H,I))/Sigma(H);
z2(H,I) = t(H,I)/Sigma(H);

Sigma(H) = t(H,I)*(3*Cp(I))^( -1);

);
);

!-----Constraints-----
-----;

1-Pz1(1,2)-Pz2(1,2) >= 1-a1;
1-Pz1(2,2)-Pz2(2,2) >= 1-a1;
1-Pz1(3,2)-Pz2(3,2) >= 1-a1;
1-Pz1(4,1)-Pz2(4,1) >= 1-a1;
1-Pz1(5,1)-Pz2(5,1) >= 1-a1;
1-Pz1(6,2)-Pz2(6,2) >= 1-a1;
1-Pz1(7,2)-Pz2(7,2) >= 1-a1;
1-Pz1(8,2)-Pz2(8,2) >= 1-a1;

@FOR ( COMPONENT(H) | H #LE# 3:
      @FOR ( STAGES(I):

@BND(vL(I),v(H,I),vU(I));
@BND(fL(I),f(H,I),fU(I));
@BND(dL(I),d(H,I),dU(I));
@BND(tL(I),t(H,I),tU(I));

);
);

@FOR ( COMPONENT(H) | H #GE# 6:
      @FOR ( STAGES(I):

@BND(vL(I),v(H,I),vU(I));
@BND(fL(I),f(H,I),fU(I));
@BND(dL(I),d(H,I),dU(I));
@BND(tL(I),t(H,I),tU(I));

);

```

```

);

@BND(50,v(4,1),90);
@BND(0.05,f(4,1),0.09);
@BND(0.8,d(4,1),3);
@BND(0.08,t(4,1),0.5);

@BND(50,v(5,1),90);
@BND(0.05,f(5,1),0.09);
@BND(0.8,d(5,1),3);
@BND(0.08,t(5,1),0.5);

d(1,2)-t(1,1)-t(1,2) >= DL(2);
d(1,2)+t(1,1)+t(1,2) <= DU(2);
d(2,2)-t(2,1)-t(2,2) >= DL(2);
d(2,2)+t(2,1)+t(2,2) <= DU(2);
d(3,2)-t(3,1)-t(3,2) >= DL(2);
d(3,2)+t(3,1)+t(3,2) <= DU(2);
d(6,2)-t(6,1)-t(6,2) >= DL(2);
d(6,2)+t(6,1)+t(6,2) <= DU(2);
d(7,2)-t(7,1)-t(7,2) >= DL(2);
d(7,2)+t(7,1)+t(7,2) <= DU(2);
d(8,2)-t(8,1)-t(8,2) >= DL(2);
d(8,2)+t(8,1)+t(8,2) <= DU(2);

t(5,2) = 0;
t(4,2) = 0;

N(1)*d(1,1)+d(1,2) = aa;
N(2)*d(2,1)+d(2,2) = aa;
N(3)*d(3,1)+d(3,2) = aa;
N(4)*d(4,1) = aa;
N(5)*d(5,1) = aa;
N(6)*d(6,1)+d(6,2) = aa;
N(7)*d(7,1)+d(7,2) = aa;
N(8)*d(8,1)+d(8,2) = aa;

@FOR (COMPONENT: @GIN(N));

!RSS constraints;
((t(4,1))^2+(t(5,1))^2)^0.5 <= T_Q(1);
((t(1,2))^2+(t(2,2))^2+(t(7,2))^2+(t(8,2))^2)^0.5 <= T_Q(2);
((t(2,2))^2+(t(3,2))^2+(t(6,2))^2+(t(7,2))^2)^0.5 <= T_Q(3);
((t(3,2))^2+(t(6,2))^2+(t(4,1))^2)^0.5 <= T_Q(4);

!WC constraints;

```



```

!t(4,1)+t(5,1) <= T_Q(1);
!t(1,2)+t(2,2)+t(7,2)+t(8,2) <= T_Q(2);
!t(2,2)+t(3,2)+t(6,2)+t(7,2) <= T_Q(3);
!t(3,2)+t(6,2)+t(4,1) <= T_Q(4);

K_P1*Dc^K_P2*v(4,1)^K_P3*f(4,1)^K_P4*D(4,1)^K_P5*d_e(4)*Nt
<= Pmax;
K_F1*Dc^K_F2*v(4,1)^K_F3*f(4,1)^K_F4*D(4,1)^K_F5*d_e(4)*Nt*
E <= Fmax;
K_P1*Dc^K_P2*v(5,1)^K_P3*f(5,1)^K_P4*D(5,1)^K_P5*d_e(5)*Nt
<= Pmax;
K_F1*Dc^K_F2*v(5,1)^K_F3*f(5,1)^K_F4*D(5,1)^K_F5*d_e(5)*Nt*
E <= Fmax;

@FOR ( COMPONENT(H) | H #LE# 3:
    @FOR ( STAGES(I):

        K_P1*Dc^K_P2*v(H,I)^K_P3*f(H,I)^K_P4*D(H,I)^K_P5*d_e(H)*Nt
        <= Pmax;
        K_F1*Dc^K_F2*v(H,I)^K_F3*f(H,I)^K_F4*D(H,I)^K_F5*d_e(H)*Nt*
        E <= Fmax;

    );
);

@FOR ( COMPONENT(H) | H #GE# 6:
    @FOR ( STAGES(I):

        K_P1*Dc^K_P2*v(H,I)^K_P3*f(H,I)^K_P4*D(H,I)^K_P5*d_e(H)*Nt
        <= Pmax;

        K_F1*Dc^K_F2*v(H,I)^K_F3*f(H,I)^K_F4*D(H,I)^K_F5*d_e(H)*Nt*
        E <= Fmax;

    );
);

END

```